Surface Areas and Volumes

Case Study Based Questions

Case Study 1

Avantika join four cubical open boxes of edge 20 cm each to make a pot for planting saplings of pudina in her kitchen garden. The saplings are cylindrical in shape with diameter 14.2 cm and height 11 cm.

Based on the above information, solve the following questions:

Q1. If Avantika wants to paint the outer surface of the pot, then how much area she needs to paint?

a. 6400 cm^2

b. 5600 cm^2

c. 4200 cm^2

d. 2025 cm^2

Q2. What is the volume of the pot formed?

a. 32000 cm^3

b. 20250 cm^3

c. 40000 cm^3

d. 10125 cm^3

Q3. If Avantika decorates the four walls of the pot with coloured square paper of side 10 cm each, then how many pieces of papers would be required?

a. 120

b. 54

c. 160
d. 40

Q4. Find the volume of 1 sapling.
a. 1742.75 cm^3
b. 4548.16 cm^3
c. 1764.08 cm^3
d. None of these

Q5. If Avantika planted 4 saplings in the pot with some soil and compost up to the brim of the pot, then how much soil and compost are there in the pot?
a. 12612 cm^3
b. 25029 cm^3
c. 21975 cm^3
d. None of these

Solutions

1. Given, edge of each cubical box (a) = 20 cm
:- Area to be painted = Area of (24-6-4) ie., 14 square faces
= $14 a^2 = 14 (20)^2 = 5600$ cm^2
So, option (b) is correct.

2. From figure,
Length of the pot (l) = 20 cm
Breadth of the pot (b) = 20 x 4 = 80 cm
and height of the pot (h) = 20 cm
Volume of pot = lbh = 20 x 80 x 20
= 32000 cm^3
So, option (a) is correct.

3. Area of four walls = 2(l + b) x h
=2 (20+80) × 20 = 4000 cm^2
Given, side of coloured square paper = 10 cm
Now, area of a square paper = $(10)^2 = 100$ cm^2

$\therefore$ Number of pieces of paper required $= \frac{4000}{100} = 40$

So, option (d) is correct.

4. Given, height of sapling (H) = 11 cm
and diameter of sapling = 14.2 cm

$\therefore$ Radius of sapling $(R) = \frac{14.2}{2} = 7.1$ cm

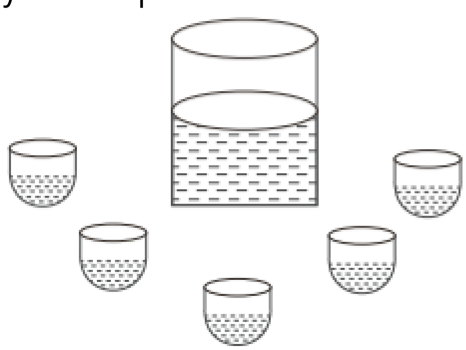
$$\therefore \text{Volume of 1 sapling} = \pi R^2 H = \frac{22}{7} \times (7.1)^2 \times 11$$
$$= 1742.75 \text{ cm}^3$$

So, option (a) is correct.

5. Total volume of pot = 32000 cm^3
Volume of 4 saplings = 1742.75 x 4 = 6971 cm^3
:- Volume of compost and soil
32000 - 6971
=25029 cm^3
So, option (b) is correct.

Case Study 2

In some Muslim countries, eating in public during day-light hours in Ramadan is crime. The sale of alcohol becomes prohibited during Ramadan and alcohol is completely restricted in Ramadan Mela. At a Ramadan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small glasses, each small glass consist of a 6 cm long cylindrical portion attached to a hemisphere of radius 3 cm and sold for Rs15 each.



Based on the above information, solve the following questions:

Q1. The volume of juice in the vessel is:

a. $1200\ \pi\ cm^3$

b. $4200\ \pi\ cm^3$

c. $5200\ \pi\ cm^3$

d. $7200\ \pi\ cm^3$

Q2. The capacity of each small glass is:

a. $72\ \pi\ cm^3$

b. $42\ \pi\ cm^3$

c. $32\ \pi\ cm^3$

d. $64\ \pi\ cm^3$

Q3. Number of glasses of juice that are sold:

a. 10

b. 50

c. 100

d. 90

Q4. How much money does the stall keeper receive by selling the juice completely?

a. 1500

b. 750

c. 1250

d. 1750

Q5. If 1/4 part of juice fall initially by stall keeper and then sold remaining juice for 25 each. How much money does the stall keeper receive by selling the remaining juice completely?

a. 550

b. 1875

c. 650

d. 750

Solutions

1. Given that.

Radius of the cylindrical vessel (R) = 15 cm

and height of the cylindrical vessel (H) = 32 cm

:- The volume of juice in the vessel
=Volume of the cylindrical vessel
=$лR^2H$
= x 15 x 15 x 32 = 7200 cm^3
So, option (d) is correct.

2. Given that,
Height of the small glass (h) = 6 cm
and radius of the small glass (r)
= Radius of cylinder = Radius of hemisphere
= 3 cm
:- The capacity of juice in each glass can hold
=Volume of each small glass
= Volume of small cylinder
+ Volume of small hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(3h + 2r)$$

$$= \frac{\pi(3)^2}{3}(3 \times 6 + 2 \times 3)$$

$$= 3\pi\ (18 + 6) = 72\pi\ cm^3$$

So, option (a) is correct.

3. The number of glasses of juice that are sold

$$= \frac{\text{Volume of the vessel}}{\text{Volume of each glass}}$$

$$= \frac{7200\pi}{72\pi} = 100$$

So, option (c) is correct.

4. Amount received by the stall keeper
= 15 x 100 = 1500
So, option (a) is correct.

5. The volume of juice in the vessel = 7200π cm^3

$\therefore$ Volume of $\frac{1}{4}$ part of juice

$$= \frac{1}{4} \times 7200\pi = 1800\pi \text{ cm}^3$$

$\therefore$ Volume of remaining juice

$$= 7200\pi - 1800\pi = 5400\pi \text{ cm}^3$$

$\therefore$ The number of glasses of juice that are sold

$$= \frac{5400\pi}{72\pi} = 75$$

$\therefore$ Amount received by the stall keeper

$$= ₹\ 25 \times 75 = ₹\ 1875$$

So, option (b) is correct.

Case Study 3

A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions 14 cm x 17cm x 4 cm. On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height 3 cm and radius 2 cm.

Based on the above information, solve the following questions: [CBSE 2023]

Q1. Find the volume of wood carved out to make one cylindrical hollow.

Q2. Find the lateral surface area of the cuboid to paint it with green colour.

Q3. (a) Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows.

Or

(b) Find the surface area of the top surface of the cuboid to be painted yellow.

Solutions

1. Given, height of the hollow cylinder (h) = 3 cm
and radius of the hollow cylinder (r) = 2 cm
So, the volume of wood carved out to make one cylindrical hollow
= volume of a hollow cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times (2)^2 \times 3 = \frac{22}{7} \times 12 = \frac{264}{7} cm^3$$

2. Given, length of the cuboidal wooden block (() = 14 cm,
breadth of the cuboidal wooden block (b) = 17 cm
and height of the cuboidal wooden block (h) = 4 cm
:- Lateral surface area of the cuboid to paint it with green colour
= 2 (l + b) x h
=2 (14+17) × 4
=2x 31 x 4 = 248 cm^2

3. Volume of cuboidal wooden block = lxbxh
= 14 x 17 x 4
= 952 cm^2
and volume of seven cylindrical hollows
= 7 x volume of a hollow cylinder

$$= 7 \times \frac{264}{7} = 264 cm^3$$

:- The volume of wood in the remaining cuboid after carving out seven cylindrical hollows.
= volume of cuboidal block volume of seven cylindrical hollows.
=952-264-688 cm^3

Or

Surface area of the top surface of the cuboid
=lx b=14x17=238 cm^2
and curved surface area of seven circular region

$= 7 \times \pi r^2 = 7 \times \frac{22}{7} \times 4 = 88 cm^2$

;- The surface area of the top surface of the cuboid to be painted yellow surface area of the top surface of the cuboid - C.S.A of seven circular region
= 238-88 = 150 cm^2.

Case Study 4

In a coffee shop, coffee is served in two types of cups. One is cylindrical in shape with diameter 7 cm and height 14 cm and the other is hemispherical with diameter 21 cm.

Based on the above information, solve the following questions: [CBSE 2023]

Q1. Find the area of the base of the cylindrical cup.

Q2. What is the curved surface area of the cylindrical cup?

Q3. What is the capacity of the hemispherical cup?

Or

Find the capacity of the cylindrical cup.

Solutions

1. Let rand h be the radius and height of the cylindrical cup respectively.
Given, diameter of the base = 7 cm

$\therefore$ Its radius $(r) = \frac{7}{2}$ cm

So, base area of the cylindrical cup = π r2

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2}$$
$= 38.5 cm^2$.

2. In cylindrical cup.

Given, radius $(r) = \frac{7}{2}$ cm and height $(h) = 14$ cm

$\therefore$ Curved surface area of the cylindrical cup

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 14 = 308 \text{ cm}^2.$$

3. Let R be the radius of the hemispherical cup.

.. Given, diameter of hemispherical cup = 21 cm

Its radius $(R) = \frac{21}{2}$ cm

So, capacity of the hemispherical cup

$$= \frac{2}{3}\pi R^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3 = \frac{11 \times 21 \times 21}{2}$$

$$= 2425.5 \text{ cm}^3.$$

Or

Given, height of the cylindrical cup $(h) = 14$ cm

and its radius $(r) = \frac{7}{2}$ cm

Capacity of the cylindrical cup

$$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 14$$

$$= 11 \times 7 \times 7 = 539 \text{ cm}^3.$$

Case Study 5

On diwali festival, a big company decided to gift his employees an electric kettle which was in a shape of cylinder and gift wrapped in the cubical box. The dimension of box is 20 cm x 15 cm × 30 cm and the radius and height of electrical kettle are 14 cm and 25 cm.

Based on the above information, solve the following questions:

Q1. Find the volume of the box.

Q2. Find the maximum length of rod that can be kept in the box.

Q3. Find the area of the wrapping sheet that covers the box exactly.

Or

Find the total surface area of an electric kettle.

Solutions

1. Given, dimension of a box is l = 20 cm, b = 15 cm
and h= 30 cm
The volume of the box = lbh
= 20 x 15 x 30 = 9000 cm^3,
Hence, volume of the box is 9000 cm^3.

2.

$\therefore$ The maximum length of rod

$$= \text{length of diagonal of a cuboid}$$
$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(20)^2 + (15)^2 + (30)^2}$$
$$= \sqrt{400 + 225 + 900} = \sqrt{1525}$$
$$= 5\sqrt{61}\ \text{cm}$$

Hence, a maximum length that can be kept in the box is $5\sqrt{61}$ cm.

3. The area of the wrapping sheet that covers the box is equal to the surface area of the box.
:- Surface area of the box = 2 (lb+bh+hl)
= 2 (20 × 15 +15 × 30 + 30 x 20)
= 2 (300+ 450+600)
= 2 (1350) = 2700 cm^2.
Hence, the area of the wrapping sheet that covers the box exactly is 2700 cm^2.

Or

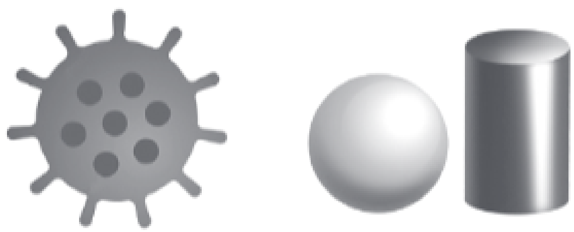
Given, radius and height of an electric kettle are
r=14 cm and h=25 cm.
:- The total surface area of an electric kettle
= total surface area of cylinder
= $2\pi r (h+r)$
= 2 x 3.14 x 14 (25 +14)
= 87.92 x 393428.88 cm^2
Hence, surface area of an electric kettle is 3428.88 cm^2.

Solutions for Questions 6 to 20 are Given Below

Case Study 6

Science Project

Arun a 10^{th} standard student makes a project on corona virus in science for an exhibition in his school. In this project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes, each of volume 1540 cm^3 with length 10 cm.



Based on the above information, answer the following questions.

(i) Diameter of the base of the cylinder is

(a) 7 cm (b) 14 cm (c) 12 cm (d) 16 cm

(ii) Diameter of the sphere is

(a) 40 cm (b) 42 cm (c) 21 cm (d) 20 cm

(iii) Total volume of the shape formed is

(a) 85541 cm^3 (b) 45738 cm^3 (c) 24625 cm^3 (d) 55748 cm^3

(iv) Curved surface area of the one cylindrical shape is

(a) 850 cm^2 (b) 221 cm^2 (c) 440 cm^2 (d) 540 cm^2

(v) Total area covered by cylindrical shapes on the surface of sphere is

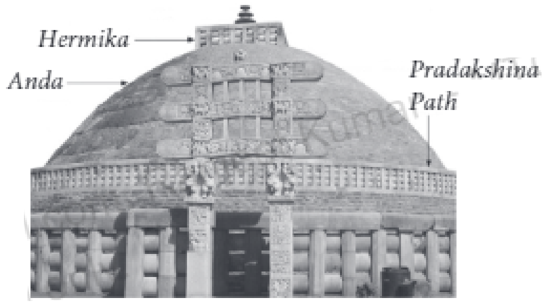
(a) 1694 cm^2 (b) 1580 cm^2 (c) 1896 cm^2 (d) 1470 cm^2

Case Study 7

Visit to Sanchi Stupa

Ajay is a Class X student. His class teacher Mrs Kiran arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in

India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as *Anda*. It also contains a cubical shape part called *Hermika* at the top. Path around *Anda* is known as *Pradakshina Path*.



Based on the above information, answer the following questions.

(i) Find the lateral surface area of the *Hermika*, if the side of cubical part is 8 m.
(a) 128 m^2 (b) 256 m^2 (c) 512 m^2 (d) 1024 m^2

(ii) The diameter and height of the cylindrical base part are respectively 42 m and 12 m. If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.
(a) 1663200 (b) 1580500 (c) 1765000 (d) 1865000

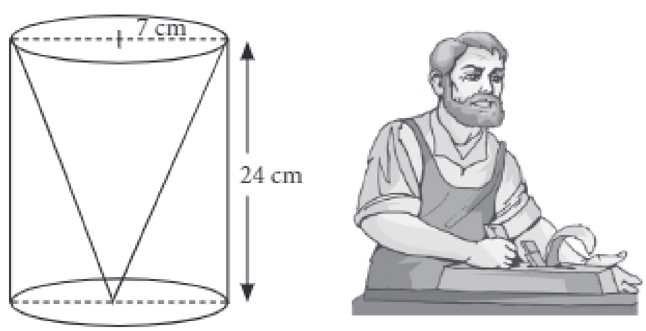
(iii) If the diameter of the *Anda* is 42 m, then the volume of the *Anda* is
(a) 17475 m^3 (b) 18605 m^3 (c) 19404 m^3 (d) 18650 m^3

(iv) The radius of the *Pradakshina path* is 25 m. If Buddhist priest walks 14 rounds on this *path*, then find the distance covered by the priest.
(a) 1860 m (b) 3600 m (c) 2400 m (d) 2200 m

(v) The curved surface area of the *Anda* is
(a) 2856 m^2 (b) 2772 m^2 (c) 2473 m^2 (d) 2652 m^2

Case Study 8

One day Rinku was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Rinku's mind. Help Rinku to find the answer of the following questions.



(i) After carving out cone from the cylinder,
(a) Volume of the cylindrical wood will decrease.
(b) Height of the cylindrical wood will increase.
(c) Volume of cylindrical wood will increase.
(d) Radius of the cylindrical wood will decrease.

(ii) Find the slant height of the conical cavity so formed.

(a) 28 cm (b) 38 cm (c) 35 cm (d) 25 cm

(iii) The curved surface area of the conical cavity so formed is

(a) 250 cm^2 (b) 550 cm^2 (c) 350 cm^2 (d) 450 cm^2

(iv) External curved surface area of the cylinder is

(a) 876 cm^2 (b) 1250 cm^2 (c) 1056 cm^2 (d) 1025 cm^2

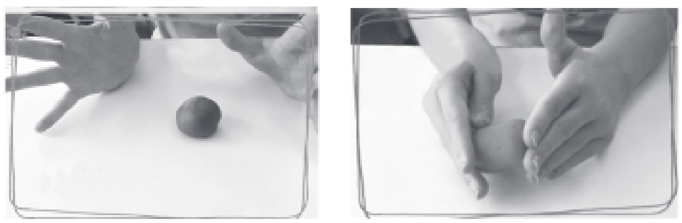
(v) Volume of conical cavity is

(a) 1232 cm^3 (b) 1248 cm^3 (c) 1380 cm^3 (d) 999 cm^3

Case Study 9

Classroom Activity

To make the learning process more interesting, creative and innovative, Amayra's class teacher brings clay in the classroom, to teach the topic - Surface Areas and Volumes. With clay, she forms a cylinder of radius 6 cm and height 8 cm. Then she moulds the cylinder into a sphere and asks some questions to students.



(i) The radius of the sphere so formed is

(a) 4 cm (b) 6 cm (c) 7 cm (d) 8 cm

(ii) The volume of the sphere so formed is

(a) 905.14 cm^3 (b) 903.27 cm^3 (c) 1296.5 cm^3 (d) 1156.63 cm^3

(iii) Find the ratio of the volume of sphere to the volume of cylinder.

(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 3 : 1

(iv) Total surface area of the cylinder is

(a) 528 cm^2 (b) 756 cm^2 (c) 625 cm^2 (d) 636 cm^2

(v) During the conversion of a solid from one shape to another the volume of new shape will

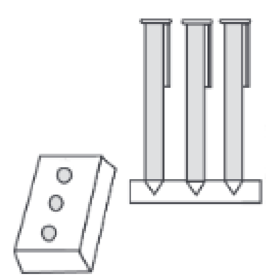
(a) be increase (b) be decrease (c) remain unaltered (d) be double

Case Study 10

Pen Holder

A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, conical. Aarav went to his shop and asked him to make a pen stand as explained below.

Pen stand must be of the cuboidal shape with three conical depressions, which can hold 3 pens. The dimensions of the cuboidal part must be 20 cm × 15 cm × 5 cm and the radius and depth of each conical depression must be 0.6 cm and 2.1 cm respectively.



Based on the above information, answer the following questions.

(i) The volume of the cuboidal part is
   (a) 1250 cm^3 (b) 1500 cm^3 (c) 1625 cm^3 (d) 1500 cm^3

(ii) Total volume of conical depressions is
   (a) 2.508 cm^3 (b) 1.5 cm^3 (c) 2.376 cm^3 (d) 3.6 cm^3

(iii) Volume of the wood used in the entire stand is
   (a) 631.31 cm^3 (b) 3564 cm^3 (c) 1502.376 cm^3 (d) 1497.624 cm^3

(iv) Total surface area of cone of radius r is given by
   (a) $\pi rl + \pi r^2$ (b) $2\pi rl + \pi r^2$ (c) $\pi r^2 l + \pi r^2$ (d) $\pi rl + 2\pi r^3$

(v) If the cost of wood used is ₹ 5 per cm^3, then the total cost of making the pen stand is
   (a) ₹ 8450.50 (b) ₹ 7480 (c) ₹ 9962.14 (d) ₹ 7488.12

Case Study 11

Stack of Coins

Meera and Dhara have 12 and 8 coins respectively each of radius 3.5 cm and thickness 0.5 cm. They place their coins one above the other to form solid cylinders.

Based on the above information, answer the following questions.

(i) Curved surface area of the cylinder made by Meera is
   (a) 144 cm^2 (b) 132 cm^2 (c) 154 cm^2 (d) 142 cm^2

(ii) The ratio of curved surface area of the cylinders made by Meera and Dhara is
   (a) 2 : 5 (b) 3 : 2 (c) 1 : 2 (d) 2 : 7

(iii) The volume of the cylinder made by Dhara is
   (a) 154 cm^3 (b) 144 cm^3 (c) 132 cm^3 (d) 142 cm^3

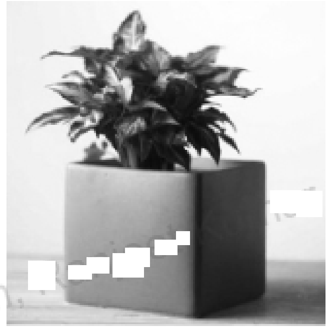
(iv) The ratio of the volume of the cylinders made by Meera and Dhara is
   (a) 1 : 2 (b) 2 : 5 (c) 3 : 2 (d) 4 : 3

(v) When two coins are shifted from Meera's cylinder to Dhara's cylinder, then
   (a) Volume of two cylinder become equal
   (b) Volume of Meera's cylinder > Volume of Dhara's cylinder
   (c) Volume of Dhara's cylinder > Volume of Meera's cylinder
   (d) None of these

Case Study 12

Ceramic Flower Vase

Ankit wants a beautiful ceramic cuboidal flower vase for the decoration of his room. So, he visit to ceramicists and explained him about, what kind of flower vase he wants. According to his requirement, the ceramicists carved out a sphere of maximum size from a cuboidal ceramic block of dimensions 24 cm by 24 cm by 27 cm.



Based on the above information, answer the following questions.

(i) What is the maximum radius of the sphere that can be carved out from the block of ceramic?
(a) 23 cm (b) 17 cm (c) 9 cm (d) 12 cm

(ii) What is the volume of the complete block of ceramic?
(a) 15552 cm^3 (b) 12646 cm^3 (c) 15292 cm^3 (d) 12898 cm^3

(iii) What is the volume of the ceramic carved out?
(a) 1940.4 cm^3 (b) 7241.14 cm^3 (c) 14553.5 cm^3 (d) None of these

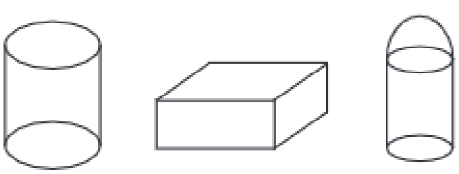
(iv) What is the volume of the cuboidal vase thus formed?
(a) 8853.73 cm^3 (b) 1153.37 cm^3 (c) 8310.86 cm^3 (d) None of these

(v) What is the surface area of the sphere carved out?
(a) 15540 cm^2 (b) 1810.28 cm^2 (c) 2702 cm^2 (d) 1838 cm^2

Case Study 13

Storage Tank for Irrigation

Pankaj's father has to purchase a new water tank to store water for irrigation of their fields. For this purpose, they visit to a shop. The shopkeeper has three types of water tanks as shown below.



Type-I Type-II Type-III

Based on the above information, answer the following questions.

(i) If the radius of type-I tank is 1.5 m and its height is 3.5 m, then find the capacity of tank type-I. (Take $\pi = 3.14$)
(a) 24727.5 litres (b) 10000 litres (c) 13200 litres (d) 90400 litres

(ii) Find the capacity of type-II tank having dimensions 5 m × 4 m × 3.5 m.
(a) 72000 litres (b) 70000 litres (c) 250000 litres (d) 404000 litres

(iii) How much more water type-III tank contains than tank of type-I, if its base radius is 2.5 m and total height is 5.5 m? [Take $\pi = 3.14$]

(a) 12394.5 litres (b) 32200.5 litres (c) 29000.5 litres (d) 66852.5 litres

(iv) If Pankaj's father bought type-II tank and wants to cover it with a cloth costs ₹ 45 per m^2, then find the total cost of cloth used (if cloth is covered on all its faces).

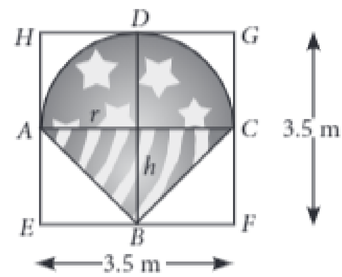
(a) ₹ 4495 (b) ₹ 1500 (c) ₹ 4635 (d) ₹ 1750

(v) Find the ratio of the total surface area of type-I and type-II tanks.

(a) 728 : 275 (b) 275 : 729 (c) 51 : 325 (d) 471 : 1030

Case Study 14

Spinner Toy



Emily purchased a spinner from a shop, which is of the shape as shown in the figure, in which right circular cone and hemisphere lie on opposite sides of a common base of length 3.5 m. Cylindrical box circumscribing them in this position.

Now, answer the following questions.

(i) What will be the volume of the cone?

(a) 6.5 m^3 (b) 2.9 m^3 (c) 40 m^3 (d) 5.614 m^3

(ii) Volume of hemispherical part is

(a) 11.23 m^3 (b) 6.03 m^3 (c) 8 m^3 (d) 9.5 m^3

(iii) Volume of cylinder that circumscribe the cone and hemisphere, is

(a) 31 m^3 (b) 17.19 m^3 (c) 17.5 m^3 (d) 33.69 m^3

(iv) Find the additional space enclosed by the cylinder.

(a) 3.14 m^3 (b) 0.13 m^3 (c) 2.14 m^3 (d) 16.846 m^3

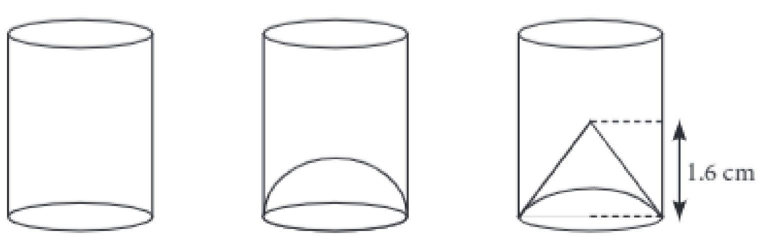
(v) Find the ratio of the curved surface areas of cone and hemisphere.

(a) $1 : \sqrt{2}$ (b) 1 : 5 (c) $1 : \sqrt{5}$ (d) 1 : 3

Case Study 15

Juice Corner

Pinki's class teacher explained the students about the benefits of drinking fruit juice in the morning. So, Pinki went to a juice stall with her friend Bipin. On the stall, they observed that shopkeeper has three types of glasses of inner diameter 4.6 cm to serve customers. The height of each glass is 11 cm. Seeing this, certain questions came into their mind. Help Pinki and Bipin to solve these questions.



(A) (B) (C)

(i) Volume of the type (A) glass is

(a) $275\ cm^3$ (b) $250\ cm^3$ (c) $182.88\ cm^3$ (d) $208\ cm^3$

(ii) Volume of type (B) glass is

(a) $208.6\ cm^3$ (b) $150.5\ cm^3$ (c) $152.4\ cm^3$ (d) $157.39\ cm^3$

(iii) How much more juice can be filled in type (A) glass than glass of type (C)?

(a) 10.48 mL (b) 9.10 mL (c) 98.12 mL (d) 8.6 mL

(iv) Which glass has minimum capacity?

(a) Type (A) (b) Type (B)

(c) Type (C) (d) All glasses have same capacity

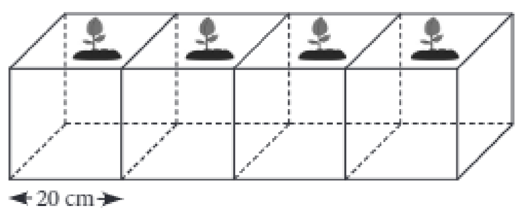
(v) Which mathematical concept has been used in above problem?

(a) Curved surface area (b) Total surface area (c) Volume (d) None of these

Case Study 16

Kitchen Garden

Anjali join four cubical open boxes of edge 20 cm each to make a pot for planting saplings of pudina in her kitchen garden. The saplings are cylindrical in shape with diameter 14.2 cm and height 11 cm.



On the basis of above information, answer the following questions.

(i) If Anjali wants to paint the outer surface of the pot, then how much area she needs to paint?

(a) $6400\ cm^2$ (b) $5600\ cm^2$ (c) $4200\ cm^2$ (d) $2025\ cm^2$

(ii) What is the volume of the pot formed?

(a) $32000\ cm^3$ (b) $20250\ cm^3$ (c) $40000\ cm^3$ (d) $10125\ cm^3$

(iii) If Anjali decorates the four walls of the pot with coloured square paper of side 10 cm each, then how many pieces of papers would be required?

(a) 120 (b) 54 (c) 160 (d) 40

(iv) Find the volume of 1 sapling.

(a) $1742.75\ cm^3$ (b) $4548.16\ cm^3$ (c) $1764.08\ cm^3$ (d) None of these

(v) If Anjali planted 4 saplings in the pot with some soil and compost up to the brim of the pot, then how much soil and compost are there in the pot?

(a) $12612\ cm^3$ (b) $25029\ cm^3$ (c) $21975\ cm^3$ (d) None of these

Case Study 17

Gift Pack

Ritu packed a football as a gift for her brother's birthday in a cuboidal box whose diameter is same as that of length of base of the box having length, breadth and height respectively 23 cm, 23 cm and 28 cm.

(i) The volume of the football is

(a) 3581 cu.cm (b) 6373.19 cu.cm (c) 6451 cu.cm (d) 9807 cu.cm

(ii) Ritu covers the box with a wrapping sheet. The area of the wrapping sheet that covers the box exactly is

(a) 3634 sq.cm (b) 2533 sq.cm (c) 2584 sq.cm (d) 3813 sq.cm

iii) The volume of the box is

(a) 25733 cu.cm (b) 18573 cu.cm (c) 14812 cu.cm (d) 77536 cu.cm

(iv) Half of the remaining volume of the box is filled with thermocol balls. Find the volume of thermocol balls used.

(a) 36150.9 cu.cm (b) 4219.405 cu.cm (c) 2764 cu.cm (d) 4048.05 cu.cm

(v) The surface area of the football is

(a) 691.03 sq.cm (b) 12772 sq.cm (c) 15544 sq.cm (d) 1662.57 sq.cm

Case Study 18

Night Stay in Tent

Alok and his family went for a vacation to Jaipur. There they had a stay in tent for a night. Alok found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42 m and height of the cylinder is 22 m.

Based on the above information, answer the following questions.

(i) How much canvas is needed to make the tent?

(a) 3280 m^2 (b) 4464 m^2 (c) 4818 m^2 (d) None of these

(ii) If each person needs 126 m^2 of floor, then how many persons can be accommodated in the tent?

(a) 17 (b) 11 (c) 19 (d) 15

(iii) Find the cost of the canvas used to make the tent, if the cost of 100 m^2 of canvas is ₹ 425.

(a) ₹ 12944 (b) ₹ 18244 (c) ₹ 24724 (d) ₹ 20476.50

(iv) Find the volume of the tent.

(a) 27248 m^3 (b) 32496 m^3 (c) 39732 m^3 (d) 15874 m^3

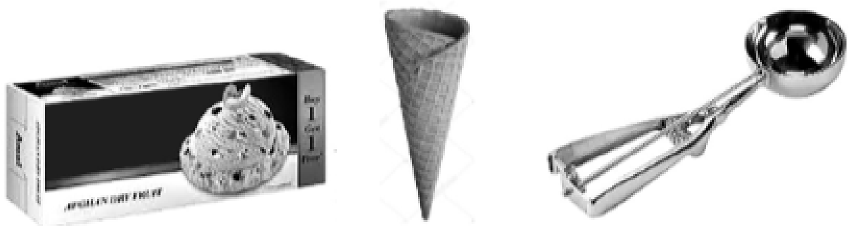
(v) Find the number of persons that can be accommodated in tent, if each person needs 1892 m^3 of space.

(a) 21 (b) 31 (c) 18 (d) 42

Case Study 19

Ice Cream Party

Isha's father brought an ice-cream brick, empty cones and scoop to pour the ice-cream into cones for all the family members. Dimensions of the ice-cream brick are $(30 \times 25 \times 10)$ cm^3 and radius of hemi-spherical scoop is 3.5 cm. Also, the radius and height of cone are 3.5 cm and 15 cm respectively.



Based on the above information, answer the following questions.

(i) The quantity of ice-cream in the brick (in litres) is

(a) 3 (b) 7.5 (c) 2.5 (d) 4.5

(ii) Volume of hemispherical scoop is

(a) 40.6 cm^3 (b) 2509 cm^3 (c) 89.83 cm^3 (d) 20 cm^3

(iii) Volume of a cone is

(a) 148 cm^3 (b) 250.05 cm^3 (c) 145.83 cm^3 (d) 192.5 cm^3

(iv) The minimum number of scoops required to fill one cone upto brim is

(a) 2 (b) 3 (c) 4 (d) 5

(v) The number of cones that can be filled upto brim using the whole brick is

(a) 15 (b) 39 (c) 40 (d) 42

Case Study 20

Rolling Pin

Arpana is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.

(i) Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5 cm and length of each part is 5 cm.

(a) 475 cm^2 (b) 78.57 cm^2 (c) 877 cm^2 (d) 259.19 cm^2

(ii) Find the volume of big cylindrical part.

(a) 190.93 cm^3 (b) 75 cm^3 (c) 77 cm^3 (d) 83.5 cm^3

(iii) Volume of two hemispherical ends having diameter 2.5 cm, is

(a) 4.75 cm^3 (b) 8.18 cm^3 (c) 2.76 cm^3 (d) 75 cm^3

(iv) Curved surface area of two hemispherical ends, is

(a) 17.5 cm^2 (b) 7.9 cm^2 (c) 19.64 cm^2 (d) 15.5 cm^2

(v) Find the difference of volumes of bigger cylindrical part and total volume of the two small hemispherical ends.

(a) 175.50 cm^3 (b) 182.75 cm^3 (c) 76.85 cm^3 (d) 96 cm^3

HINTS & EXPLANATIONS

6. **(i) (b):** We know that, volume of cylinder $= \pi r^2 h$

$\Rightarrow 1540 = \frac{22}{7} \times r^2 \times 10$

$\Rightarrow \frac{154 \times 7}{22} = r^2 \Rightarrow r^2 = 49 \Rightarrow r = 7$ cm

$\therefore$ Diameter of the base of cylinder $= 2r = 2 \times 7 = 14$ cm

(ii) (b): We know that, volume of sphere $= \frac{4}{3}\pi r^3$

$\Rightarrow 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\Rightarrow r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 441 \times 21 = (21)^3 \Rightarrow r = 21$ cm

$\therefore$ Diameter of sphere = 42 cm

(iii) (d): Total volume of shape formed = Volume of cylindrical shapes + Volume of sphere

$= 11 \times 1540 + 38808 = 16940 + 38808 = 55748 \text{ cm}^3$

(iv) (c): Curved surface area of one cylindrical shape $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 10 = 440 \text{ cm}^2$

(v) (a): Area covered by cylindrical shapes on the surface of sphere $= 11 \times \pi r^2 = 11 \times \frac{22}{7} \times 7 \times 7 = 1694 \text{ cm}^2$

7. **(i) (b):** Lateral surface area of *Hermika* which is cubical in shape $= 4a^2 = 4 \times (8)^2 = 256 \text{ m}^2$

(ii) (a): Diameter of cylindrical base = 42 m

$\therefore$ Radius of cylindrical base (r) = 21 m

Height of cylindrical base (h) = 12 m

$\therefore$ Number of bricks used $= \dfrac{\frac{22}{7} \times 21 \times 21 \times 12}{0.01}$

$= 1663200$

(iii) (c): Given, diameter of *Anda* which is hemispherical in shape = 42 m

$\Rightarrow$ Radius of *Anda* (r) = 21 m

$\therefore$ Volume of *Anda* $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 44 \times 21 \times 21 = 19404 \text{ m}^3$

(iv) (d): Given, radius of *Pradakshina Path* (r) = 25 m

$\therefore$ Perimeter of *path* $= 2\pi r$

$= \left(2 \times \frac{22}{7} \times 25\right)$ m

$\therefore$ Distance covered by priest $= 14 \times 2 \times \frac{22}{7} \times 25$

$= 2200$ m

(v) (b): $\because$ Radius of *Anda* (r) = 21 m

$\therefore$ Curved surface area of *Anda* $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 21 \times 21 = 2772 \text{ m}^2$

8. **(i) (a)**

(ii) (d): Slant height of conical cavity, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ cm

(iii) (b): Curved surface area of conical cavity $= \pi rl$

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

(iv) (c): External curved surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056 \text{ cm}^2$

(v) (a): Volume of conical cavity $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 1232 \text{ cm}^3$

9. **(i) (b):** Since, volume of sphere = volume of cylinder

$\Rightarrow \frac{4}{3}\pi R^3 = \pi r^2 h$, where R, r are the radii of sphere and cylinder respectively.

$\Rightarrow R^3 = \frac{6 \times 6 \times 8 \times 3}{4} = (6)^3 \Rightarrow R = 6$ cm

$\therefore$ Radius of sphere = 6 cm

(ii) (a): Volume of sphere $=\frac{4}{3}\pi R^3$

$=\frac{4}{3}\times\frac{22}{7}\times 6\times 6\times 6=905.14\text{ cm}^3$

(iii) (c): $\because$ Volume of sphere = Volume of cylinder

$\therefore$ Required ratio = 1 : 1

(iv) (a): Total surface area of the cylinder $=2\pi r(r+h)$

$=2\times\frac{22}{7}\times 6(6+8)=2\times\frac{22}{7}\times 6\times 14=528\text{ cm}^2$

(v) (c)

10. (i) (b): Volume of cuboidal part $= l\times b\times h$

$=(20\times 15\times 5)\text{ cm}^3=1500\text{ cm}^3$

(ii) (c): Radius of conical depression, $r = 0.6$ cm

Height of conical depression, $h = 2.1$ cm

$\therefore$ Total volume of conical depressions $=3\times\frac{1}{3}\pi r^2 h$

$=\frac{22}{7}\times 0.6\times 0.6\times 2.1=\frac{2376}{1000}=2.376\text{ cm}^3$

(iii) (d): Volume of wood used in the entire stand

= Volume of cuboidal part

– Total volume of conical depressions

$=1500-2.376=1497.624\text{ cm}^3$

(iv) (a)

(v) (d): Cost of wood per cm^3 = ₹ 5

$\therefore$ Total cost of making the pen stand

= ₹ (5 × 1497.624) = ₹ 7488.12

11. We have, radius of each coin = 3.5 cm

$=\frac{35}{10}\text{ cm}=\frac{7}{2}\text{ cm}$

Thickness of each coin = 0.5 cm $=\frac{1}{2}$ cm

So, height of cylinder made by Meera $(h_1)=12\times\frac{1}{2}=6$ cm

and height of cylinder made by Dhara (h_2)

$=8\times\frac{1}{2}=4$ cm

(i) (b): Curved surface area of cylinder made by Meera $=2\times\frac{22}{7}\times\frac{7}{2}\times 6=132\text{ cm}^2$

(ii) (b): Required ratio

$=\frac{\text{Curved surface area of cylinder made by Meera}}{\text{Curved surface area of cylinder made by Dhara}}$

$=\frac{2\pi rh_1}{2\pi rh_2}=\frac{h_1}{h_2}=\frac{6}{4}=\frac{3}{2}$ *i.e.*, 3 : 2

(iii) (a): Volume of cylinder made by Dhara $=\pi r^2 h_2$

$=\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 4=154\text{ cm}^3$

(iv) (c): Required ratio

$=\frac{\text{Volume of cylinder made by Meera}}{\text{Volume of cylinder made by Dhara}}$

$=\frac{\pi r^2 h_1}{\pi r^2 h_2}=\frac{h_1}{h_2}=\frac{6}{4}=\frac{3}{2}$ *i.e.*, 3 : 2

(v) (a): When two coins are shifted from Meera's cylinder to Dhara's cylinder, then length of both cylinders become equal.

So, volume of both cylinders become equal.

12. (i) (d): Let r be the radius of the sphere.

Then, diameter of sphere = 24 cm

$\therefore$ Radius $(r)=\frac{24}{2}=12$ cm

(ii) (a): Volume of ceramic block $= l\times b\times h$

$=24\times 24\times 27=15552\text{ cm}^3$

(iii) (b): Volume of ceramic carved out $=\frac{4}{3}\pi r^3$

$=\frac{4}{3}\times\frac{22}{7}\times(12)^3=7241.14\text{ cm}^3$

(iv) (c): Volume of cuboidal vase = Volume of ceramic block – Volume of sphere

$=15552-7241.14=8310.86\text{ cm}^3$

(v) (b): Surface area of the sphere carved out $=4\pi r^2$

$=4\times\frac{22}{7}\times(12)^2=1810.28\text{ cm}^2$

13. (i) (a): Type-I tank is cylindrical in shape with $r = 1.5$ m and $h = 3.5$ m.

$\therefore$ Required volume $=\pi r^2 h=(3.14\times 1.5^2\times 3.5)\text{ m}^3$

$=24.7275\text{ m}^3$

Now, 1 m^3 = 1000 litres

$\therefore$ Capacity of type-I tank = (24.7275 × 1000) litres

= 24727.5 litres

(ii) (b): Capacity of type-II tank $= l\times b\times h$

$=5\times 4\times 3.5\text{ m}^3=70\text{ m}^3=(70\times 1000)$ litres

= 70000 litres

(iii) (d): Volume of type-III tank

$=\pi r^2 h+\frac{2}{3}\pi r^3=3.14\times(2.5)^2\left[(5.5-2.5)+\frac{2}{3}(2.5)\right]$

$=91.58\text{ m}^3=91.58\times 1000$ litres = 91580 litres

$\therefore$ Required difference = 91580 – 24727.5

= 66852.5 litres

(iv) (c): TSA of type-II tank $=2(lb+bh+hl)$

$=2(5\times 4+4\times 3.5+3.5\times 5)$

$= 2(20 + 14 + 17.5) = 103\ m^2$

$\therefore$ Cost of cloth required = ₹ (45 × 103) = ₹ 4635

(v) (d): Required ratio $= \dfrac{2\pi r(r+h')}{2(lb+bh+hl)}$

$= \dfrac{2\times3.14\times1.5(1.5+3.5)}{103} = \dfrac{471}{1030}$ *i.e.*, 471 : 1030

14. (i) (d): Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\times\dfrac{22}{7}\times\dfrac{3.5}{2}\times\dfrac{3.5}{2}\times\dfrac{3.5}{2}$

$[\because r = \dfrac{3.5}{2}$ and $h = 3.5 - \dfrac{3.5}{2} = \dfrac{3.5}{2}]$

$= 5.614\ m^3$

(ii) (a): Volume of hemisphere $= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3}\times\dfrac{22}{7}\times\dfrac{3.5}{2}\times\dfrac{3.5}{2}\times\dfrac{3.5}{2} = 11.23\ m^3$

(iii) (d): Volume of cylinder that circumscribe the cone and hemisphere $= \dfrac{22}{7}\times\dfrac{3.5}{2}\times\dfrac{3.5}{2}\times3.5$

$= 33.69\ m^3$

(iv) (d): Additional space enclosed by cylinder = Volume of cylinder – (volume of cone + volume of hemisphere)

$= 33.69 - (11.23 + 5.614) = 16.846\ m^3$

(v) (a): Required ratio

$= \dfrac{\text{Curved surface area of cone}}{\text{Curved surface area of hemisphere}} = \dfrac{\pi r\sqrt{r^2+h^2}}{2\pi r^2}$

$= \dfrac{\sqrt{2r^2}}{2r} = \dfrac{\sqrt{2}r}{2r} = \dfrac{1}{\sqrt{2}}$ *i.e.*, $1:\sqrt{2}$

15. Diameter of each glass = 4.6 cm

$\therefore$ Radius of each glass = 2.3 cm

Height of each glass = 11 cm

(i) (c) : Volume of type (A) glass $= \pi r^2 h$

$= \dfrac{22}{7}\times2.3\times2.3\times11 = 182.88\ cm^3$

(ii) (d) : Volume of type (B) glass

= Volume of type (A) glass – Volume of hemisphere

$= 182.88 - \dfrac{2}{3}\pi r^3 = 182.88 - \dfrac{2}{3}\times\dfrac{22}{7}\times2.3\times2.3\times2.3$

$= 182.88 - 25.49 = 157.39\ cm^3$

(iii) (d) : Volume of type (C) glass = Volume of type (A) glass – Volume of cone

$= 182.88 - \dfrac{1}{3}\pi r^2 h = 182.88 - \dfrac{1}{3}\times\dfrac{22}{7}\times2.3\times2.3\times1.6$

$= 182.88 - 8.86 = 174.02\ cm^3$

$\therefore$ Required difference = 182.88 – 174.02

$= 8.86\ cm^3 = 8.86$ mL

(iv) (b) : Glass of type B has minimum capacity.

(v) (c)

16. (i) (b): Area to be painted = Area of 14 square faces $= 14 \times (20)^2 = 5600\ cm^2$

(ii) (a) : Height of pot = 20 cm

Length of pot = 20 × 4 = 80 cm

Breadth of pot = 20 cm

$\therefore$ Volume of pot $= 20 \times 80 \times 20 = 32000\ cm^3$

(iii) (d) : Required area $= 2(l + b) \times h$

$= 2(80 + 20) \times 20 = 4000\ cm^2$

Side of coloured square paper = 10 cm

$\therefore$ Number of pieces of paper required $= \dfrac{4000}{10\times10} = 40$

(iv) (a) : We have,

Radius $(r) = \dfrac{14.2}{2}$ cm = 7.1 cm

Height (h) = 11 cm

$\therefore$ Volume of each sapling $= \pi r^2 h = \dfrac{22}{7}\times(7.1)^2\times11$

$= 1742.75\ cm^3$

(v) (b) : Total volume of pot $= 32000\ cm^3$

Volume of 4 saplings $= 1742.75 \times 4 = 6971\ cm^3$

So, volume of compost and soil = 32000 – 6971

$= 25029\ cm^3$

17. Diameter of football = Length of base of the box = 23 cm

$\therefore$ Radius of football $= \left(\dfrac{23}{2}\right)$ cm

(i) (b) : Volume of the football $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3}\times\dfrac{22}{7}\times\dfrac{23}{2}\times\dfrac{23}{2}\times\dfrac{23}{2} = 6373.19\ cm^3$

(ii) (a) : Area of wrapping sheet = Total surface area of the cuboidal box

$= 2(lb + bh + hl) = 2(23 \times 23 + 23 \times 28 + 28 \times 23)$

$= 2(529 + 644 + 644) = 3634\ cm^2$

(iii) (c) : Volume of the box $= l \times b \times h$

$= 23 \times 23 \times 28 = 14812\ cm^3$

(iv) (b) : Volume of thermocal balls used

$= \dfrac{1}{2}$ (Volume of box – Volume of football)

$= \dfrac{1}{2}(14812 - 6373.19) = \dfrac{1}{2}\times8438.81 = 4219.405\ cm^3$

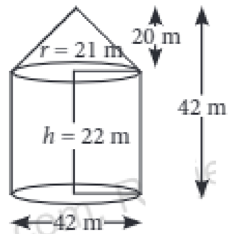
(v) (d) : Surface area of the football $= 4\pi r^2$

$= 4\times\dfrac{22}{7}\times\dfrac{23}{2}\times\dfrac{23}{2} = 1662.57\ cm^2$

18. **(i) (c) :** Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh = \pi r(l + 2h)$

$= \frac{22}{7} \times 21\ (29 + 44) = 4818\ \text{m}^2$



$\left[\because\ l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29\ \text{m}\right]$

(ii) (b) : Area of floor $= \pi r^2$

$= \frac{22}{7} \times 21 \times 21 = 1386\ \text{m}^2$

Number of persons that can be accommodated in the tent $= \frac{1386}{126} = 11$

(iii) (d) : Since, cost of 100 m^2 of canvas = ₹ 425

$\therefore$ Cost of 1 m^2 of canvas = ₹ 4.25

Thus, cost of 4818 m^2 of canvas = ₹ 20476.50

(iv) (c) : Volume of tent = Volume of cone + Volume of cylinder $= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h = \pi r^2\left(\frac{1}{3}h_1 + h\right)$

$= \frac{22}{7} \times (21)^2\left[\frac{20}{3} + 22\right] = \frac{9702}{7} \times \frac{86}{3} = 39732\ \text{m}^3$

(v) (a) : Required number of persons

$= \frac{\text{Volume of tent}}{\text{Space required by one person}} = \frac{39732}{1892} = 21$

19. **(i) (b) :** Quantity of ice-cream in the brick

= volume of the brick $= (30 \times 25 \times 10)\ \text{cm}^3 = 7500\ \text{cm}^3$

$= \frac{7500}{1000}\ l$ $\quad [\because\ 1\ l = 1000\ \text{cm}^3]$

$= 7.5\ l$

(ii) (c) : Volume of hemispherical scoop $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = \frac{1886.5}{21} = 89.83\ \text{cm}^3$

(iii) (d) : Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 15 = \frac{4042.5}{21} = 192.5\ \text{cm}^3$

(iv) (a) : Number of scoops required to fill one cone

$= \frac{\text{Volume of a cone}}{\text{Volume of a scoop}} = \frac{192.5}{89.83} = 2.14 \approx 2$

(v) (b) : Number of cones that can be filled using the whole brick $= \frac{\text{Volume of brick}}{\text{Volume of 1 cone}}$

$= \frac{7500}{192.5} = 38.96 \approx 39$

20. **(i) (b) :** Curved surface area of two identical cylindrical parts $= 2 \times 2\pi rh = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times 5$

$= 78.57\ \text{cm}^2$

(ii) (a) : Volume of big cylindrical part $= \pi r^2 h$

$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12 = 190.93\ \text{cm}^3$

(iii) (b) : Volume of two hemispherical ends $= 2 \times \frac{2}{3}\pi r^3$

$= \frac{2 \times 2}{3} \times \frac{22}{7} \times \left(\frac{2.5}{2}\right)^3 = 8.18\ \text{cm}^3$

(iv) (c) : Curved surface area of two hemispherical ends $= 2 \times 2\pi r^2 = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times \frac{2.5}{2} = 19.64\ \text{cm}^2$

(v) (b) : Difference of volume of bigger cylinder to two small hemispherical ends $= 190.93 - 8.18 = 182.75\ \text{cm}^3$