CBSE Board

Class XII Mathematics

Sample Paper 5 – Solution

Part A

1. Correct option: A

Explanation:-

A is a matrix of order 2 × 2.

For A to be an identity matrix,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\Rightarrow \cos\alpha = 1 \text{ and } \sin\alpha = 0$

$\Rightarrow \cos\alpha = \cos 0° \text{ and } \sin\alpha = \sin 0°$

$\Rightarrow \alpha = 0°$

Thus, for $\alpha = 0°$, A is an identity matrix

2. Correct option: D

Explanation:-

$$I = \int \frac{1}{\sqrt{9-25x^2}}dx$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}dx$$

$$= \frac{1}{5} \times \frac{5}{3}\sin^{-1}\left(\frac{x}{\frac{3}{5}}\right) + c$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{5x}{3}\right) + c$$

3. Correct option: B

Explanation:-

Equation of a line through (−2, 1, 3) and parallel to $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-1}{5} = \frac{z-3}{6}$$

4. **Correct option:** C

Explanation:-

The plane is x + 2y +3z – 6 = 0

$\Leftrightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)=6$

$\Leftrightarrow \hat{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)=6$

Hence, the vector normal to the plane x + 2y +3z – 6 = 0 is $\hat{i}+2\hat{j}+3\hat{k}$.

5. **Correct option:** C

Explanation:-

Given: $\vec{a}\perp\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b}=0$

$\Rightarrow\left(3\hat{i}+\hat{j}-2k\right)\left(\hat{i}+\lambda\hat{j}-3k\right)=0$

$\Rightarrow 3\times1+1\times\lambda+(-2)\times(-3)=0$

$\Rightarrow \lambda+9=0$

$\Rightarrow \lambda=-9$

6. **Correct option:** B

Explanation:-

The equation of the plane ZOX is y = 0.

A plane parallel to it is of the form, y = a.

As y-intercept of the plane is 3, we have

a = 3.

Thus, the equation of the required plane is y = 3.

7. **Correct option:** D

Explanation:-

Let E_1: A is selected

E_2: B is selected

Now, P (E_1) = $\frac{1}{4}$ and P (E_2) = $\frac{1}{6}$

E_1 and E_2 are independent events.

∴ P (both are selected) = P ($E_1 \cap E_2$) = P (E_1) × P (E_2) = $\frac{1}{24}$.

8. **Correct option:** B

Explanation:-

Let $\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta-\sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta-\sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta-\sin\gamma\sin\delta \end{vmatrix}$

Applying $[C_3 \to C_3 + (\sin\delta)C_1 - (\cos\delta)C_2]$

$$\Rightarrow \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}$$

$\Rightarrow \Delta = 0$ ... [Expanded through C_3]

Hence, $\Delta = 0$.

9. Correct option: B

Explanation:-

Range of principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Let $\sin^{-1}\left(\frac{-1}{2}\right) = \theta$

Then, $\sin\theta = \frac{-1}{2} = \sin\left(-\frac{\pi}{6}\right)$, where $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

10. Correct option: C

Explanation:-

Ranges of principal values of $\sin^{-1}$ and $\cos^{-1}$ are $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively.

Let $\cos^{-1}\left(\frac{1}{2}\right) = \theta_1$

$\Rightarrow \theta_1 = \frac{\pi}{3}$ ... (As $\theta_1 \in [0, \pi]$)

Let $\sin^{-1}\left(\frac{1}{2}\right) = \theta_2$

$\Rightarrow \theta_2 = \frac{\pi}{6}$ .... As $\theta_2 \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{2\pi}{3}$

11. Correct option: D

Explanation:-

Given: $R = \{(a, b): a = b^2\}$

(i) Let $a = 2$,

We know that $2 \neq 2^2 \Rightarrow (2, 2) \notin R$

i.e. 2 is not related to 2.

Therefore, R is not reflexive.

(ii) Now for (4, 2), $4 = 2^2 \Rightarrow 4 \text{ R } 2$.
But, $2 \neq 4^2 \Rightarrow 2$ is not related to 4.
Therefore, R is not symmetric.
(iii) For (16, 4), $16 = 4^2 \Rightarrow 16 \text{ R } 4$.
For (4, 2), $4 = 2^2 \Rightarrow 4 \text{ R } 2$
But $16 \neq 2^2 \Rightarrow 16$ is not related to 2.
Therefore, R is not transitive.

12. Correct option: B
Explanation:-

Given: $f(x) = \begin{cases} \dfrac{x^2-25}{x-5} & ,x \neq 5 \\ k & ,x = 5 \end{cases}$

f is continuous at x = 5, then

$$\lim_{x\to5} f(x) = f(5)$$

$$\Rightarrow \lim_{x\to5} \frac{x^2-25}{x-5} = k$$

$$\Rightarrow \lim_{x\to5} \frac{(x-5)(x+5)}{x-5} = k$$

$$\Rightarrow \lim_{x\to5} x+5 = k$$

$$\Rightarrow 10 = k$$

13. Correct option: C
Explanation:-

Let $y = \dfrac{x^2}{(1+x^2)}$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1-y)$$

$$\Rightarrow x = \sqrt{\frac{y}{1-y}}$$

For x to be real, we must have $\dfrac{y}{1-y} \geq 0$ and $1 - y \neq 0$

$\Rightarrow 0 \leq y < 1$
Hence, range (f) = $\{y \in R: 0 \leq y < 1\}$

14. Correct option: D
Explanation:-
Given: $R(x) = 3x^2 + 40x + 10$

$$\Rightarrow MR = \frac{dR}{dx}$$

$$= \frac{d}{dx}\left(3x^2 + 40x + 10\right)$$

$$= 6x + 40$$

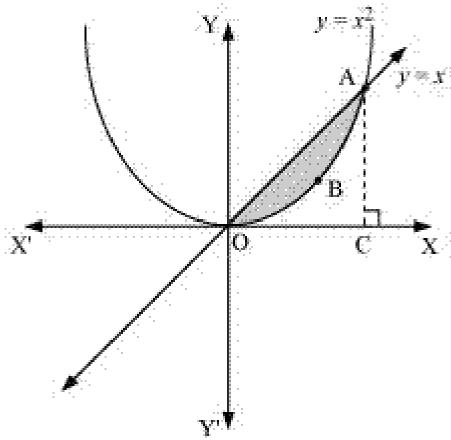
$$\Rightarrow [MR]_{x=5} = 6 \times 5 + 40 = 70.$$

Hence, the marginal revenue is Rs. 70.

15. Correct option: A

Explanation:-

The required area is represented by OBAO below:



Therefore, the area of OBAO = Area (△ OAC) – Area (OCABO).

Required area = $\int_0^1 x\,dx - \int_0^1 x^2\,dx$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6} \text{ units}$$

16. Correct option: B

Explanation:-

Given differential equation is $\frac{dy}{dx} + y = 1$

$$\Rightarrow \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{1-y} = dx$$

$$\Rightarrow \int \frac{dy}{1-y} = \int dx$$

$$\Rightarrow -\log|1-y| = x + C$$

$$\Rightarrow \log|1-y| + x = C$$

17. Correct option: A

Explanation:-

Given differential equation is $\left(x^2+1\right)\frac{dy}{dx} = xy$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{\left(x^2+1\right)}$$

$$\Rightarrow \frac{dy}{y} = \frac{xdx}{\left(x^2+1\right)}$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{x}{\left(x^2+1\right)} dx$$

$$\Rightarrow \int \frac{dy}{y} = \frac{1}{2} \int \frac{2x}{\left(x^2+1\right)} dx$$

$$\Rightarrow \log|y| = \frac{1}{2}\log\left(x^2+1\right) + \log C$$

$$\Rightarrow \log|y| = \log\sqrt{x^2+1} + \log C$$

$$\Rightarrow y = C_1\sqrt{x^2+1}$$

18. Correct option: B

Explanation:-

Let I = $\int \frac{dx}{1+\sin x}$

$$I = \int \frac{dx}{1+\sin x}$$

$$= \int \left(\frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\right) dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx = \int \left(\sec^2 x - \sec x \tan x\right) dx$$

$$= \int \sec^2 x dx - \int \sec x \tan x \cdot dx$$

$$= \tan x - \sec x + c$$

19. Correct option: C

Explanation:-

$$\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3}\left(\frac{d}{dx}x^3\right) = e^{x^3}\left(3x^2\right) = 3x^2 e^{x^3}$$

20. Correct option: B

Explanation:-

Given: $f(x) = 2x^2 - 3x$

$\Rightarrow f'(x) = 4x - 3$

Take $f'(x) = 0 \Rightarrow 4x - 3 = 0$ i.e. $x = \frac{3}{4}$

Now, the point $x = \frac{3}{4}$ divides the number line in two intervals $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$

$f'(x) < 0$ in the interval $\left(-\infty, \frac{3}{4}\right)$ and $f'(x) > 0$ in the interval $\left(\frac{3}{4}, \infty\right)$

Hence, f(x) is increasing in the interval $\left(\frac{3}{4}, \infty\right)$.

Part B

21. $I = \int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\sin^2 x}{\sin 2x}\right)\right)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\sin^2 x}{2\sin x\cos x}\right)\right)dx$

$= \int_0^{\frac{\pi}{2}} \left(\log\left(\frac{\tan x}{2}\right)\right)dx$ ... (i)

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\tan\left(\frac{\pi}{2} - x\right)}{2}\right)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{\cot x}{2}\right)dx$ ... (ii)

Additing (i) & (ii)

$$2I=\int_0^{\frac{\pi}{2}}\left[\log\left(\frac{\tan x}{2}\right)+\log\left(\frac{\cot x}{2}\right)\right]dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log\left[\left(\frac{\tan x}{2}\right)\left(\frac{\cot x}{2}\right)\right]dx$$

$$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\log\left(\frac{1}{4}\right)dx$$

$$\Rightarrow I=\frac{1}{2}\log\left(\frac{1}{4}\right)\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\log\left(\frac{1}{4}\right)^{\frac{1}{2}}\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\log\left(\frac{1}{2}\right)\times\left(\frac{\pi}{2}\right)$$

$$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$$

OR

$$I=\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx \qquad ...(1)$$

Using the proporty $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4 x+\sin^4 x}dx \qquad ...(2)$$

Adding (1) and (2),

$$2I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}.\sin x\cos x\right)}{\sin^4 x+\cos^4 x}\,dx$$

$$\Rightarrow I=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\left[\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\right]dx$$

$$=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\left[\frac{\frac{\sin x\cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x}+1}\right]dx$$

$$=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\tan x\sec^2 x}{\tan^4 x+1}dx$$

Put $\tan^2 x = z$

$\therefore\ 2\tan x\sec^2 x\,dx = dz$

$\Rightarrow\ \tan x\sec^2 x\,dx = \dfrac{dz}{2}$

When $x = 0, z = 0$ and when $x = \dfrac{\pi}{2}, z = \infty$

$$\therefore I=\frac{\pi}{4}\int_0^{\infty}\frac{\frac{dz}{2}}{z^2+1}$$

$$\Rightarrow I=\frac{\pi}{8}\int_0^{\infty}\frac{dz}{1+z^2}$$

$$=\frac{\pi}{8}\left|\tan^{-1}(z)\right|_0^{\infty}$$

$$=\frac{\pi}{8}\ \tan^{-1}\ \infty\ -\tan^{-1}\ 0$$

$$=\frac{\pi}{8}\left(\frac{\pi}{2}-0\right)$$

$$=\frac{\pi^2}{16}$$

22. Given differential equation is $(x + 1)\dfrac{dy}{dx}=2e^{-y}-1$ ... (i)

$$\Rightarrow\frac{dy}{2e^{-y}-1}=\frac{dx}{(x+1)}$$

Integrating both sides, we get

$$\int\frac{dy}{2e^{-y}-1}=\int\frac{dx}{(x+1)}$$

$$\Rightarrow \int \frac{e^y dy}{2-e^y} = \int \frac{dx}{(x+1)}$$

$$\Rightarrow -\int -\frac{e^y dy}{2-e^y} = \int \frac{dx}{(x+1)}$$

$$\Rightarrow -\log\left|2-e^y\right| = \log\left|x+1\right| + c$$

$$\Rightarrow \log\left|(x+1)\left(2-e^y\right)\right| = -c$$

$$\Rightarrow \left|(x+1)\left(2-e^y\right)\right| = e^{-c}$$

$$\Rightarrow (x+1)\left(2-e^y\right) = \pm e^{-c} = A(\text{say})$$

$$\Rightarrow (x+1)\left(2-e^y\right) = A \text{ .... (ii)}$$

Given : $x = 0, y = 0$

$$(0+1)\left(2-e^0\right) = A$$

$$\Rightarrow 1(2-1) = A$$

$$\Rightarrow A = 1$$

Substituting in (ii), we get

$$(x+1)\left(2-e^y\right) = 1$$

23.

i. (a, b) * (c, d) = (ac, ad + b)
(c, d) * (a, b) = (ca, cb + d)
(ac, ad + b) $\neq$ (ca, cb + d)
So, ‘*’ is not commutative.

ii. Let (a, b) (c, d) , (e, f) $\in$ A, Then
((a, b)* (c, d)) * (e, f) = (ac, ad + b) * (e, f) = ((ac)e, (ac)f + (ad + b))
= (ace, acf + ad + b)
(a, b)* ((c, d)* (e, f)) = (a, b) * (ce, cf +d) = (a(ce), a(cf + d) +b) = (ace, acf +ad +b)
((a, b)* (c, d)) * (e, f) = (a, b) * ((c, d)* (e, f))
Hence, ‘*’ is associative.

iii. Let (x, y) $\in$ A. Then (x, y) is an identity element, if and only if
(x, y) * (a, b) = (a, b) = (a, b) * (x, y), for every (a, b) $\in$A
Consider, (x, y) * (a, b) = (xa, xb + y)
(a, b) * (x, y) = (ax, ay +b)
(xa, xb + y) = (a, b) = (ax, ay +b)
$ax = x\,a = a \Rightarrow x = 1$
$xb + y = b = ay + b \Rightarrow b + y = b = ay + b \Rightarrow y = 0 = ay \Rightarrow y = 0$
Therefore, (1, 0) is the identity element.

24. Given: $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

Let $\cos^{-1}x = X, \cos^{-1}y = Y, \cos^{-1}z = Z$

$\Rightarrow x = \cos X, y = \cos Y, z = \cos Z$

Hence, $X + Y + Z = \pi$

Consider $x^2 + y^2 + z^2 + 2xyz$

$= \cos^2 X + \cos^2 Y + \cos^2 Z + 2\cos X \cos Y \cos Z$

$= \frac{1+\cos 2X}{2} + \frac{1+\cos 2Y}{2} + \frac{1+\cos 2Z}{2} + 2\cos X \cos Y \cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + 2\cos X \cos Y \cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [\cos(X+Y) + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [\cos(\pi - Z) + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) + [-\cos Z + \cos(X-Y)]\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z + \cos(X-Y)\cos Z$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z + \cos(X-Y)\cos(\pi - (X+Y))$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z - \cos(X-Y)\cos(X+Y)$

We know that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$\therefore x^2 + y^2 + z^2 + 2xyz$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \cos^2 Z - \cos^2 X + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{1+\cos 2Z}{2} - \frac{1+\cos 2X}{2} + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - 1 - \frac{\cos 2Z + \cos 2X}{2} + \sin^2 Y$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{\cos 2Z + \cos 2X}{2} - \cos^2 Y \quad [\because -1 + \sin^2 Y = -\cos^2 Y]$

$\therefore x^2 + y^2 + z^2 + 2xyz$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{\cos 2Z + \cos 2X}{2} - \frac{1+\cos 2Y}{2}$

$= \frac{3}{2} + \frac{1}{2}(\cos 2X + \cos 2Y + \cos 2Z) - \frac{1}{2} - \frac{\cos 2Z + \cos 2Y + \cos 2X}{2}$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$

25. Given: $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(1+\sin^2\theta)$

$= 2(1+\sin^2\theta)$

The value of $\sin\theta$ lies in the range of -1 and 1.

$\Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow 0+1 \le (1+\sin^2\theta) \le 1+1$

$\Rightarrow 1 \le (1+\sin^2\theta) \le 2$

$\Rightarrow 2(1) \le 2(1+\sin^2\theta) \le 2(2)$

$\Rightarrow 2 \le 2(1+\sin^2\theta) \le 4$

$\Rightarrow 2 \le |A| \le 4$

$\Rightarrow |A| \in [2,\ 4]$

So value of $|A|$ lies in the interval [2, 4]

26. $x = a(\boldsymbol{\theta} - \sin\boldsymbol{\theta}),\ y = a(1 + \cos\boldsymbol{\theta})$

Differentiating both sides w.r.t. $\boldsymbol{\theta}$

$\frac{dx}{d\theta} = a(1-\cos\theta)$ ... (1)

$\frac{dy}{d\theta} = a(-\sin\theta)$ ... (2)

Dividing (2) by (1),

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= -\cot\frac{\theta}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\cot\frac{\theta}{2}$$

Differentiating w.r.t. θ,

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\cot\frac{\theta}{2}\right)\frac{d\theta}{dx}$$

$$= -\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2} \times \frac{1}{a(1-\cos\theta)}$$

$$= -\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2} \times \frac{1}{2a\sin^2\frac{\theta}{2}}$$

$$= -\frac{1}{4a\sin^4\frac{\theta}{2}}$$

As y = a$(1 + \cos\boldsymbol{\theta})$

$$\Rightarrow y = 2a\cos^2\frac{\theta}{2} \Rightarrow \sin^2\frac{\theta}{2} = 1 - \frac{y}{2a} = \frac{2a-y}{2a}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{4a\times\left(\frac{2a-y}{2a}\right)^2} = -\frac{a}{(2a-y)^2}$$

27. The given line is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Given point is (2, 4, −1)

The distance of a point whose position vector is $\overrightarrow{a_2}$

from a line whose vector equation is $\vec{r} = \vec{a_1} + \lambda\vec{v}$ is

$$d = \frac{\left|\vec{v}\times(\vec{a_2}-\vec{a_1})\right|}{\left|\vec{v}\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times\left((2\hat{i}+4\hat{j}-1k)-(-5\hat{i}-3\hat{j}+6k)\right)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times\left((2\hat{i}+4\hat{j}-1k)-(-5\hat{i}-3\hat{j}+6k)\right)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9k\right)\times(7\hat{i}+7\hat{j}-7k)\right|}{\left|\left(\hat{i}+4\hat{j}-9k\right)\right|}$$

$$\left(\hat{i}+4\hat{j}-9k\right)\times(7\hat{i}+7\hat{j}-7k) = \begin{vmatrix}\hat{i} & \hat{j} & k\\ 1 & 4 & -9\\ 7 & 7 & -7\end{vmatrix} = 35\hat{i}-56\hat{j}-21k$$

$$= \frac{7}{\sqrt{98}}\left|\left(5\hat{i}-8\hat{j}-3k\right)\right| = \frac{7}{\sqrt{98}}\sqrt{98} = 7 \text{ units}$$

28. Take $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

$$\text{Let } x = A\left[\frac{d}{dx}(8+x-x^2)\right] + B$$

$x = A(1 - 2x) + B$

$\Rightarrow x = -2Ax + (A+B)$

$\Rightarrow -2A = 1; A+B = 0$

$\Rightarrow A = -\frac{1}{2}; B = \frac{1}{2}$

$$\therefore I = \int \frac{-\frac{1}{2}(1-2x)+\frac{1}{2}}{\sqrt{8+x-x^2}}dx$$

$$\Rightarrow I = -\frac{1}{2}\int \frac{(1-2x)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{8+x-x^2}}dx = I_1 + I_2 (\text{say})$$

$$I_1 = -\frac{1}{2}\int \frac{(1-2x)}{\sqrt{8+x-x^2}}dx;$$

Let $t = 8+x-x^2 \therefore dt = (1-2x)dx$

$$\Rightarrow I_1 = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\left[2\sqrt{t}\right] = -\sqrt{8+x-x^2}$$

$$I_2 = \frac{1}{2}\int \frac{1}{\sqrt{8+x-x^2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\frac{33}{4} - \left(x-\frac{1}{2}\right)^2}} = \frac{1}{2}\sin^{-1}\frac{\left(x-\frac{1}{2}\right)}{\frac{\sqrt{33}}{2}} = \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right)$$

$$\text{So } I = -\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right) + C$$

29. Let E_1, E_2 and E_3 be the events of a driver being a scooter driver, car driver and truck driver respectively.

Let A be the event that the person meets with an accident

There are 2000 insured scooter drivers, 4000 insured car drivers and 6000 insured truck drivers.

Total number of insured vehicle drivers = 2000 + 4000 + 6000 = 12000

$$\therefore P\ E_1 = \frac{2000}{12000} = \frac{1}{6}, P\ E_2 = \frac{4000}{12000} = \frac{1}{3}, P\ E_3 = \frac{6000}{12000} = \frac{1}{2}$$

Also, we have:

$$P(A|E_1) = 0.01 = \frac{1}{100}$$

$P(A|E_2) = 0.03 = \dfrac{3}{100}$

$P(A|E_3) = 0.15 = \dfrac{15}{100}$

Now, the probability that the insured person who meets with an accident is a scooter driver is $P(E_1|A)$.

Using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1) \times P(A|E_1)}{P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2) + P(E_3) \times P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}}$$

$$= \frac{1}{6} \times \frac{6}{52}$$

$$= \frac{1}{52}$$

30. Given: $f(x) = |x - 2|$

Now, $f(2) = |2 - 2| = 0$

$$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)$$

$$= \lim_{h \to 0} |2+h-2|$$

$$= \lim_{h \to 0} |h|$$

$$= \lim_{h \to 0} h$$

$$= 0$$

$$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h)$$

$$= \lim_{h \to 0} |2-h-2|$$

$$= \lim_{h \to 0} |-h|$$

$$= \lim_{h \to 0} |h|$$

$$= 0$$

$$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2) = 0.$$

So, $f(x)$ is continuous at $x = 2$.

Now,

$$\text{R.H.D} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$= \lim_{h\to 0}\frac{|2+h-2|-0}{h}$$
$$= \lim_{h\to 0}\frac{|h|}{h} = \lim_{h\to 0}\frac{h}{h} = 1$$

And,

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$$
$$= \lim_{h\to 0}\frac{|2-h-2|-0}{-h}$$
$$= \lim_{h\to 0}\frac{|-h|}{-h} = \lim_{h\to 0}\frac{h}{-h}$$
$$= -1$$

Thus, L.H.D $\neq$ R.H.D.

Hence, f(x) is not differentiable at x = 2.

OR

$f(x) = \left|1 - x + |x|\right|,\ x \in R$

Let $g(x) = 1 - x + |x|, x \in R$

$h(x) = |x|, x \in R$

$h[g(x)] = h[1 - x + |x|]$

$= \left|1 - x + |x|\right| = f(x)$

$1 - x$, being a polynomial function is continuous

$|x|$, being a modulus function is continuous

If f and g are two continuous functions, then f+g is a continuous function

$\therefore g(x) = 1 - x + |x|, x \in R$ is a continuous function

If is continuous function, then $|f|$ is continuous.

In the given problem,

$g(x) = 1 - x + |x|$

$\therefore h[g(x)] = |g(x)|$

Since g is a continuous function , $h[g(x)] = |g(x)|$, is a continuous function

$h[g(x)] = f(x) = \left|1 - x + |x|\right|$

$\therefore f(x)$ is continuous.

31. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if their scalar product is zero

$\vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -\lambda & 5 \end{vmatrix} = 0$$

$2(10-3\lambda)+1(5+9)+1(-\lambda-6)=0$

$\Rightarrow 20-6\lambda+14-\lambda-6=0$

$\Rightarrow -7\lambda+28=0$

$\Rightarrow -7\lambda=-28$

$\Rightarrow \lambda=4$

OR

Let $\vec{a}=10\hat{i}+3\hat{j}$, $\vec{b}=12\hat{i}-5\hat{j}$ and $\vec{c}=a\hat{i}+11\hat{j}$

are the position vectors of the three points A, B, and C

$\overrightarrow{AB}=\left(12\hat{i}-10\hat{i}\right)+\left(-5\hat{j}-3\hat{j}\right)=2\hat{i}-8\hat{j}$

$\overrightarrow{BC}=\left(a\hat{i}-12\hat{i}\right)+\left(11\hat{j}+5\hat{j}\right)=(a-12)\hat{i}+16\hat{j}$

Since A, B, and C are collinear

So, $\overrightarrow{AB}=m\overrightarrow{BC}$

$\Rightarrow 2\hat{i}-8\hat{j}=m\left[(a-12)\hat{i}+16\hat{j}\right]$

$16m\hat{j}=-8\hat{j} \Rightarrow m=-\frac{1}{2}$

$\Rightarrow -\frac{1}{2}(a-12)=2$

$\Rightarrow a=8$

Part C

32. Let the award money given for honesty, regularity and hard work be Rs. x, Rs. y and Rs. z respectively.

Since total cash award is Rs. 6,000.

$\therefore$ x + y + z = 6,000 ... (1)

Three times the award money for hard work and honesty amounts to Rs. 11,000.

$\therefore$ x + 3z = 11,000

$\Rightarrow$ x + 0×y + 3 z = 11,000 ... (2)

Award money for honesty and hard work is double that given for regularity.

$\therefore$ x + z = 2y

$\Rightarrow$ x – 2y + z = 0 ... (3)

The above system of equations can be written in matrix form AX = B as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

Here,

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}$$

$$|A|=1(0+6)-1(1-3)+1(-2-0)=6\neq 0$$

Thus, A is non-singular. Hence, it is invertible.

$$\text{Adj } A=\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}\text{ adj} A=\frac{1}{6}\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}$$

$$X=A^{-1}B=\frac{1}{6}\begin{bmatrix}6 & -3 & 3\\ 2 & 0 & -2\\ -2 & 3 & -1\end{bmatrix}\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}=\frac{1}{6}\begin{bmatrix}36000-33000+0\\ 12000+0-0\\ -12000+33000-0\end{bmatrix}=\frac{1}{6}\begin{bmatrix}3000\\ 12000\\ 21000\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}500\\ 2000\\ 3500\end{bmatrix}$$

Hence, x = 500, y = 2000, and z = 3500.

Thus, award money given for honesty, regularity and hard work is Rs. 500, Rs. 2000 and Rs. 3500 respectively.

The school can include awards for obedience.

33. Equation of the plane passing through the intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 is:

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$

$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-(1+5\lambda)=0$$

This plane has to be perpendicular to the plane x-y+z=0.

Thus,

$$(1+2\lambda)1+(1+3\lambda)(-1)+(1+4\lambda)1=0$$

$$1+2\lambda-1-3\lambda+1+4\lambda=0$$

$$1+3\lambda=0$$

$$\lambda=-\frac{1}{3}$$

Thus, the equation of the plane is :

$$\left(1+2\left(-\frac{1}{3}\right)\right)x+\left(1+3\left(-\frac{1}{3}\right)\right)y+\left(1+4\left(-\frac{1}{3}\right)\right)z-\left(1+5\left(-\frac{1}{3}\right)\right)=0$$

$$\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1-\frac{4}{3}\right)z-\left(1-\frac{5}{3}\right)=0$$

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$$x-z=-2$$

Thus, the distance of this plane from the origin is :

$$\left|\frac{-(-2)}{\sqrt{1^2+0^2+1^2}}\right|=\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$$

OR

To find the image of the point $2\hat{i}+3\hat{j}-4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$

So point A (2, 3, −4) and the plane is $2x - y + z = 3$.

Let the required image be B (α,β, γ)

Direction ratios of AB are (α − 2, β − 3, γ + 4)

Direction ratios of normal to the plane (2, −1, 1)

AB is parallel to the normal to the plane

⇒ α − 2 = 2λ, β − 3 = −λ, γ + 4 = λ,

⇒ α = 2λ + 2, β = −λ + 3, γ = λ − 4

⇒ (2λ + 2, −λ + 3, λ − 4)

Mid-point of AB is $C\left(\frac{2\lambda+2+2}{2},\frac{-\lambda+3+3}{2},\frac{\lambda-4-4}{2}\right)$

$$C\left(\lambda+2,\frac{-\lambda}{2}+3,\frac{\lambda}{2}-4\right)$$

C lies on plane $2x - y + z = 3$

$$2(\lambda+2)-\left(\frac{-\lambda}{2}+3\right)+\left(\frac{\lambda}{2}-4\right)=3$$

$$3\lambda-3=3$$

$$\lambda=2$$

⇒ Image B is given as

B (2 × 2 + 2, −2 + 3, 2 − 4)

B (6, 1, −2)

34. Let the two tailors work for x days and y days respectively,

The problem is to minimise the objective function, C = 150 x + 200 y, subject to the constraints,

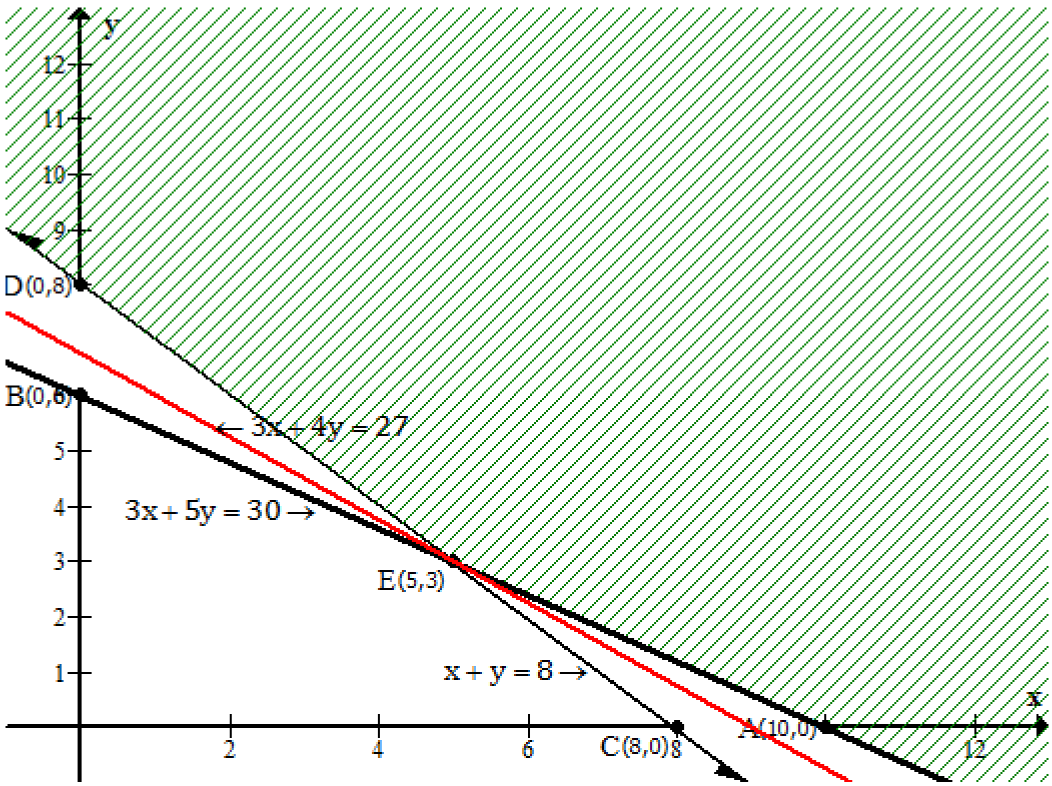
$6x + 10y \geq 60 \Leftrightarrow 3x + 5y \geq 30$

$4x + 4y \geq 32 \Leftrightarrow x + y \geq 8$

And,

$x \geq 0, y \geq 0$

Feasible region is shown shaded.



This region is unbounded.

Corner points	Objective function values C =150x + 200y
A(10, 0)	1500
E(5, 3)	1350
D(0, 8)	1600

The red line in the graph shows the line 150 x + 200y = 1350 or 3x + 4y =27

We see that the region 3x + 4y > 27 has no point in common with the feasible region.

Thus, the function has minimum value at E (5, 3).

Hence, the labour cost is the least when tailor A works for 5 days and Tailor B works for 3 days.

OR

Let x packets be transported to the school at P and y packets to the school at Q from the kitchen at A. Clearly (60 – x – y) packets will be transported to the school at R. Hence, $x \geq 0$, $y \geq 0$, $60 - x - y \geq 0$. The monthly requirements of the school at P is 40 packets and x packets are transported from the kitchen at A, the remaining (40 – x) packets will have to be transported from the kitchen at B. Thus $40 - x \geq 0$.

Similarly (40 – y) and [50 – (60 –x – y)] packets will be transported from the kitchen at B to the schools at Q and R respectively.

So $40 - y \geq 0$

And $50 - (60 - x - y) \geq 0$ i.e $x + y - 10 \geq 0$

Now the objective function or the transportation cost is given by

$C = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$

$= 3x + 4y + 370$

Now our problem is to minimise C subject to the constraints

Minimise $C = 3x + 4y + 370$,

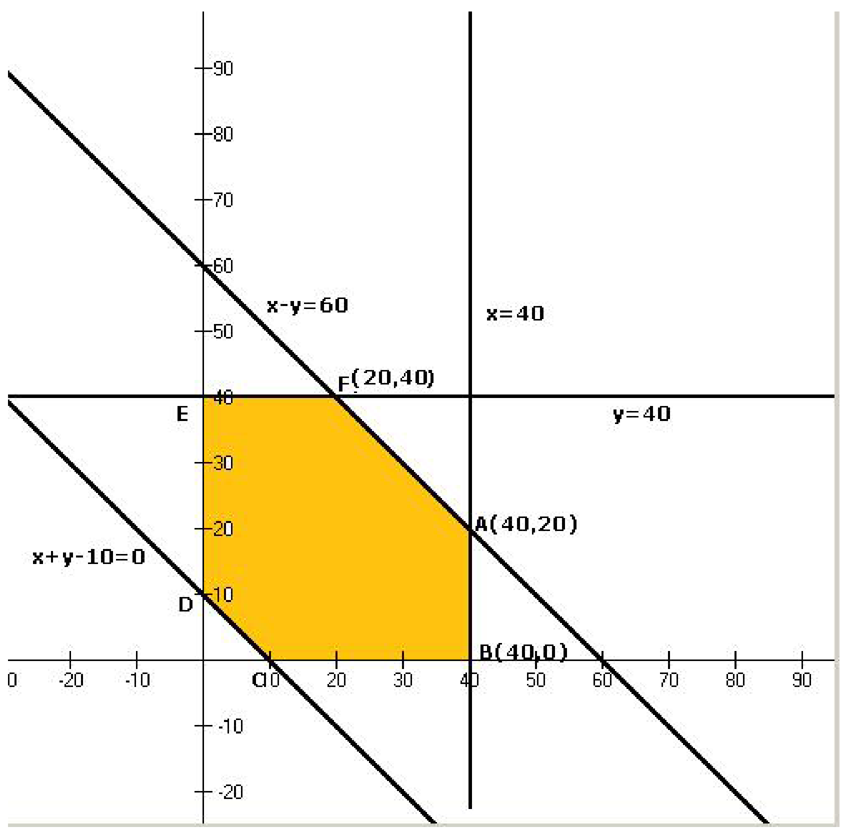
Subject to $60 - x - y \geq 0$

$40 - x \geq 0$

$40 - y \geq 0$

$x + y - 10 \geq 0$

$x \geq 0, y \geq 0$



The shaded region is the feasible region.

The corner points and the corresponding values of transportation cost is in this table.

Corner points	Value of transportation cost
(10, 0)	380
(40, 0)	490
(40, 20)	570
(20, 40)	590
(0 , 40)	530
(0, 10)	410

Thus we find that C is minimum at the point (10, 0)
Therefore, for the cost to be minimum 10, 0 and (60 – 10 - 0) packets will be transported from the factory at A and 30, 40 and 0 packets will be transported from the factory at B to the agencies at P, Q and R respectively.
In the tabular form, the number of packets to be transported is

To/ From	A	B
P	10	30
Q	0	40
R	50	0

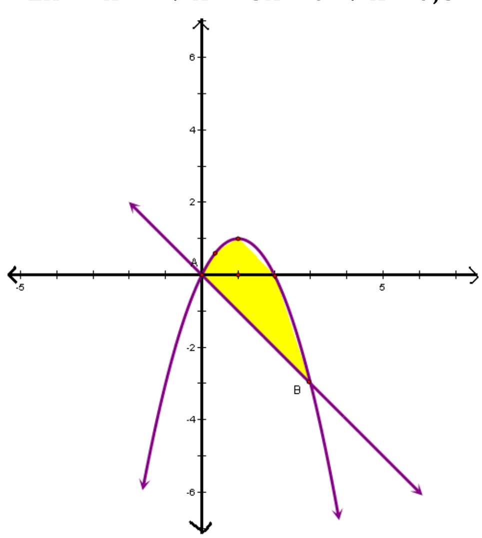
Minimum cost is given by
$C = 3 \times 10 + 4 \times 0 + 370 = \text{Rs. } 400$

35. The curve is $y = 2x - x^2$

$\Rightarrow (x-1)^2 = -(y-1)$

The points of intersection of the parabola and the line $y = -x$ is:

$-x = 2x - x^2 \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0,3$



So points of intersection are $(0,0)$ and $(3,-3)$

Required area$=\left|\int_0^3 \left[\left(2x\text{-}x^2\right)-\left(-x\right)dx\right]\right| = \left|\int_0^3 \left[\left(3x-x^2\right)dx\right]\right|$

$=\left|\frac{3x^2}{2}-\frac{x^3}{3}\right|_0^3 = \left|\frac{27}{2}-\frac{27}{3}\right| = 27\left|\frac{1}{2}-\frac{1}{3}\right|$

$=\frac{9}{2}$ sq.units

OR

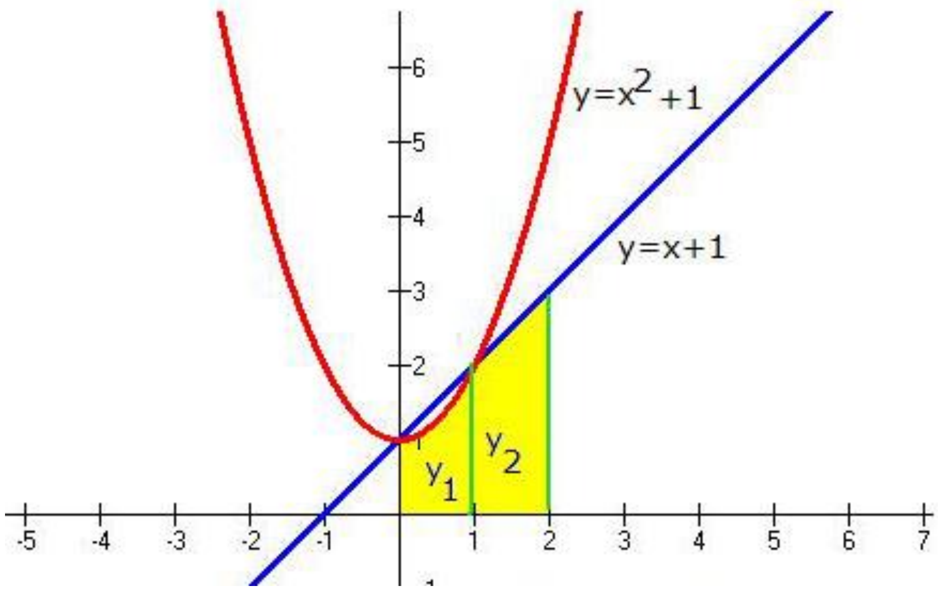
Points of intersection of $y = x^2 + 1$, $y = x + 1$

$x^2 + 1 = x + 1$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0, 1$

So points of intersection are P(0, 1) and Q(1, 2). The graph is represented as



Required area is given by

$A = \int_0^1 y_1\,dx + \int_1^2 y_2\,dx$,

where y_1 and y_2 represent the y co-ordinate of the parabola and straight line respectively.

$\therefore A = \int_0^1 \left(x^2+1\right)dx + \int_1^2 \left(x+1\right)dx$

$=\left(\frac{x^3}{3}+x\right)\Big]_0^1 + \left(\frac{x^2}{2}+x\right)_1^2$

$=\left[\left(\frac{1}{3}+1\right)-0\right]+\left[\left(2+2\right)-\left(\frac{1}{2}+1\right)\right]=\frac{23}{6}$ sq. units

36. Given: $f(x)=\frac{4x^2+1}{x},(x\neq 0)$

$\Rightarrow f(x)=\left(4x+\frac{1}{x}\right),(x\neq 0)$

$\Rightarrow f'(x)=4-\frac{1}{x^2}$

$\Rightarrow f'(x)=\frac{4x^2-1}{x^2}$ ... (i)

a) f(x) is increasing

$\Rightarrow f'(x) \geq 0$

$\Rightarrow \frac{4x^2-1}{x^2}\geq 0$ .... From (i)

$\Rightarrow \; 4x^2-1 \; \geq 0$ ... $[x^2>0]$

$\Rightarrow \; 2x-1 \;\; 2x+1 \; \geq 0$

$\Rightarrow 2\left(x-\frac{1}{2}\right)\cdot 2\left(x+\frac{1}{2}\right)\geq 0$

$\Rightarrow \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)\geq 0$

$\Rightarrow \left[\left(x-\frac{1}{2}\right)\geq 0 \text{ and } \left(x+\frac{1}{2}\right)\geq 0\right]$

or $\left[\left(x-\frac{1}{2}\right)\leq 0 \text{ and } \left(x+\frac{1}{2}\right)\leq 0\right]$

$\Rightarrow \left[x\geq\frac{1}{2} \text{ and } x\geq-\frac{1}{2}\right]$

or $\left[x\leq\frac{1}{2} \text{ and } x\leq-\frac{1}{2}\right]$

$\Rightarrow x\geq\frac{1}{2}$ or $x\leq-\frac{1}{2}$

$\Rightarrow x\in\left[\frac{1}{2},\infty\right]$ or $x\in\left(-\infty,-\frac{1}{2}\right]$

$\Rightarrow x\in\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right]$

$\therefore$ f(x) is increasing on $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right]$.

b) f(x) is decreasing

$\Rightarrow f'(x) \leq 0$

$$\Rightarrow \frac{4x^2-1}{x^2} \leq 0 \text{ .... From (i)}$$

$$\Rightarrow 4x^2-1 \leq 0 \text{ ... } [x^2 > 0]$$

$$\Rightarrow 2x-1 \quad 2x+1 \leq 0$$

$$\Rightarrow 2\left(x-\frac{1}{2}\right)\cdot 2\left(x+\frac{1}{2}\right) \leq 0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right) \leq 0$$

$$\Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$\Rightarrow x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$\therefore$ f(x) is decreasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

37. The events A, E_1, E_2, E_3, and E_4 are given by

A = event when doctor visits patients late

E_1 = doctor comes by train

E_2 = doctor comes by bus

E_3 = doctor comes by scooter

E_4 = doctor comes by other means of transport

So, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$, $P(E_4) = \frac{2}{5}$

P (A/E_1) = Probability that the doctor arrives late, given that he is comes by train.

$= \frac{1}{4}$

Similarly P $(A/E_2) = \frac{1}{3}$, P $(A/E_3) = \frac{1}{12}$, P $(A/E_4) = 0$

Required probability of the doctor arriving late by train by using Baye's theorem,

$$P(E_1/A) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right)+P(E_2)P\left(A/E_2\right)+P(E_3)P\left(A/E_3\right)+P(E_4)P\left(A/E_4\right)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{3}{40}\times\frac{120}{18} = \frac{1}{2}$$

Hence the required probability is $\frac{1}{2}$.

OR

B_1: the bulb is manufactured by machine X

B_2: the bulb is manufactured by machine Y

B_3: the bulb is manufactured by machine Z

P $(B_1) = 1000/(1000 + 2000 + 3000) = 1/6$

P $(B_2) = 2000/(1000 + 2000 + 3000) = 1/3$

P $(B_3) = 3000/(1000 + 2000 + 3000)= 1/2$

P $(E|B_1)$ = Probability that the bulb drawn is defective given that it is manufactured by machine X = 1% = 1/ 100

Similarly, P $(E|B_2)$ = 1.5% = 15/ 100 = 3/ 200

P $(E|B_3)$ = 2% = 2/100

$$P(B_1|E) = \frac{P(B_1)P(E|B_1)}{P(B_1)P(E|B_1)+P(B_2)P(E|B_2)+P(B_3)P(E|B_3)}$$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{200}+\frac{1}{2}\times\frac{2}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{2}+1}$$

$$= \frac{1}{1+3+6}$$

$$= \frac{1}{10}$$