Algorithms Test 4

Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Suppose 'G' is an undirected graph, Then which of the following is TRUE?

I. If G is a tree, there is a unique path between any 2 vertices in G.

II. If $G = (V, E)$ is connected and $E = V - 1$, then G is a tree.

III. Deleting an edge from a cycle cannot disconnect a graph.

(A) Only II (B) Only III

(C) Only I, II (D) I, II and III

2. Consider an array with 'n' values, to return the maximum sum of 3 values in an array. What is the time complexity of performing this task?

(A) $O(n)$ (B) $O(n^2 \log n)$

(C) $O(n^2)$ (D) $O(\log n)$

3. What is the length of longest increasing subsequence of '6, 4, 5, 2, 7, 11, 8, 12, 13, 9, 10'?

(A) 2 (B) 3

(C) 4 (D) 5

4. Consider 2 sequences, S = gacggattag, and X = gatcggaatag. What is the length of Longest common subsequence?

(A) 7 (B) 8

(C) 9 (D) 11

5. How does the key in a node compared to the keys of its children in

I. Binary search tree

II. A Max HEAP

P. node. key > node.left.key, node.right.key

Q. node.left.key < node.key < node.right.key

Which of the following is correct?

(A) I–P, II–Q (B) I–P, II–P

(C) I–Q, II–P (D) I–Q, II–Q

6. A hash table has 11 slots, uses the hash function $h(k) = k$ mod 11 and collisions are resolved by separate chaining, what is the minimum chain length in the hash table, after inserting these elements:

3, 43, 8, 11, 14, 25

(A) 0 (B) 1

(C) 2 (D) 3

7. Which priority queue implementations, with N elements allow for new entries to be inserted in $O(1)$ time?

(A) Sorted array (B) MAX HEAP

(C) MIN HEAP (D) Unsorted array

8. In order to search for an element in a dynamic set, which of the following techniques is the asymptotically most time efficient in the worst case for the search operation?

(A) Store the element in an unsorted array and apply linear search.

(B) Store the element in a hash table and use hashing.

(C) Store the element in a sorted array and apply binary search.

(D) All the above

9. Which of the following problems have solutions (algorithms) that run in $\theta(n)$ time in the worst case?

(A) Finding the median of 'n' integers

(B) Finding the sum of 'n' integers

(C) Finding the largest of 'n' integers

(D) All the above

10. Consider the following: (Here $h(\ )$ is a hash function).

I. $h(k_1) = h(k_2)$ even for $k_1 \neq k_2$

II. $h(k_1) \neq h(k_2)$ for $k_1 < k_2$ always

III. $h(k_1) = h(k_2)$ for $k_1 > k_2$ always

Which of the following is TRUE?

(A) I only (B) I and II

(C) II and III (D) I, II and III

11. Let 'OPT' be an optimal solution and 'x' the solution we found.

If OPT equals x then we are done otherwise, we can find another optimal solution that 'agrees more with x' which of the following algorithm has in first attempt 'OPT = x'?

(A) Fractional knapsack

(B) Travelling sales person problem

(C) Kruskals algorithm

(D) None of the above

12. Match the following algorithm with its time complexity.

I	Dijkstras algorithm	P.	$O(E + V) \log V$
II	Prims algorithm	Q.	$O(E + V)$
III	Breadth first search		
IV	Depth first search		

(A) I–P, II–P, III–P, IV–Q

(B) I–P, II–P, III–Q, IV–Q

(C) I–Q, II–Q, III–P, IV–P

(D) I–Q, II–P, III–P, IV–Q

13. The distance matrix of a graph with vertices A, B, C, and D is given below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & \infty & \infty \\ \infty & 0 & 2 & 4 \\ 3 & \infty & 0 & 1 \\ 1 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

The shortest path from B to D consists of edge(s) ________.

(A) BC and CD

(B) BD

(C) *AB* and *CD*
(D) There is no shortest path

14. Match the following:

	Sorting algorithm		Programming paradigm
I	Insertion sort	P	Dynamic programming
II	Selection sort	Q	Divide and conquer
III	Merge sort	R	Greedy algorithm
IV	Quick sort	S	Back tracking

(A) I–P, II–R, III–Q, IV–Q
(B) I–P, II–R, III–Q, IV–S
(C) I–R, II–P, III–Q, IV–Q
(D) I–R, II–P, III–Q, IV–S

15. Given a set of objects with (Weight, Profit) pair, and a knapsack of limited weight capacity (M), find a subset of objects for the knapsack to maximize total profit 'p'.
Objects (Weight, Profit)
= {(3, 2), (4, 3), (10, 21), (6, 4)}
$M = 9$ (using 0/1 knapsack)?

(A) 5 (B) 6
(C) 21 (D) 7

16. There are multiple ways to order the multiplication of 4 matrices A, B, C, D: $(A(BC)D)$, $A(B(CD))$, $(AB)(CD)$, $((AB)C)D$, $A((BC)D)$.
Efficiency depends on number of scalar multiplications, in the case of $(A(BC))D$ it is

$$1 \times 4 \times 3 + 5 \times 1 \times 3 + 5 \times 3 \times 6 = 117$$

In the case of $(A(B(CD)))$, it is

$$4 \times 3 \times 6 + 1 \times 4 \times 6 + 5 \times 1 \times 6 = 126$$

What are the dimensions of A, B, C, D respectively?
(A) 5×1, 1×4, 4×6, 6×3
(B) 4×3, 3×5, 5×6, 6×1
(C) 5×1, 1×4, 4×3, 3×6
(D) 1×4, 4×3, 3×5, 5×6

17. Suppose that the symbols a, b, c, d, e occur with frequencies $\frac{1}{36}, \frac{1}{36}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}$ then what is the Huffman Encoding of the alphabets a, b, c respectively?
(A) 1101, 111, 1101 (B) 1100, 1101, 111
(C) 1100, 10, 0 (D) 1101, 1100, 111

18. We are given 'n' positive integers $a_1, \ldots, a_n$. The goal is to select a subset of the numbers with maximal sum such that no three consecutive numbers are selected.
Sequence is '7 5 6 3 8 12 9 13 14 10 11'
What is the maximal sum?
(A) 66 (B) 68
(C) 69 (D) 72

19. Consider the coin change problem with coin values 1, 5, 6, Does the greedy algorithm always find an Optional solution?
(A) No, for even numbers
(B) Yes, for odd numbers
(C) Yes, for even numbers
(D) No

20. If a data structure supports an operation 'foo' such that a sequence of 'n' foo operations take $\theta(n \log n)$ time to perform in the worst case, then the amortized time of a 'foo' operation is θ(_____)
(A) $\theta(n)$ (B) $\theta(\log n)$
(C) $\theta(\log n)$ (D) $\theta(n)$

21. Consider the following statements:
I. Computing the median of 'n' elements takes $\Omega(n \log n)$ time for any algorithm working in the comparison based model.
II. Let 'T' be a minimum spanning tree of G, then for any pair of vertices 'a' and 'b' the shortest path from 'a' to 'b' in G is the path from 'a' to 'b' in T.

Which of the following is CORRECT?
(A) I–TRUE, II–False (B) I–TRUE, II–TRUE
(C) I–False, II–TRUE (D) I–False, II–False

22. Below are the contents of an array of some sorting algorithm sorting it.
Identify the algorithm from given steps:
12 39 2 94 23 77 52 9
12 $\underline{39}$ 2 94 23 77 52 9
$\underline{2}$ 12 39 94 23 77 52 9
2 12 39 $\underline{94}$ 23 77 52 9
2 12 $\underline{23}$ 39 94 77 52 9
2 12 23 39 $\underline{77}$ 94 52 9
2 12 23 39 $\underline{52}$ 77 94 9
2 $\underline{9}$ 12 23 39 52 77 94
(A) Quick sort (B) Selection sort
(C) Insertion sort (D) Bubble sort

23. Below are the contents of an array of some sorting algorithm sorting it.
Identify the algorithm from given steps:
12 39 02 94 23 77 52 09
1$\underline{2}$ 0$\underline{2}$ 5$\underline{2}$ 2$\underline{3}$ 9$\underline{4}$ 7$\underline{7}$ 3$\underline{9}$ 0$\underline{9}$
$\underline{0}$2 $\underline{0}$9 $\underline{1}$2 $\underline{2}$3 $\underline{3}$9 $\underline{5}$2 $\underline{7}$7 $\underline{9}$4
(A) Selection sort (B) Heap sort
(C) Bubble sort (D) Radix sort

24. Suppose that we have possible keys $[0, 1, \ldots, n^2 - 1]$ for a hash table of size 'n', what is the greatest number of distinct keys the table can hold with each of the following separate chaining, Linear probing, Quadratic probing, collision resolution strategies respectively?
(A) n, n, n^2 (B) n^2, n, n^2
(C) n^2, n, n (D) n^2, n^2, n^2

25. Given these 2 states of an array, the second one results from the first pass of which sorting algorithm?

14	5	21	4	16	31	17	8	11

5	14	4	16	21	17	8	11	31

(A) Insertion sort (B) Selection sort
(C) Merge sort (D) Bubble sort

Answer Keys

1. D	**2.** C	**3.** B	**4.** C	**5.** C	**6.** A	**7.** D	**8.** B	**9.** D	**10.** A
11. C	**12.** B	**13.** A	**14.** A	**15.** B	**16.** C	**17.** B	**18.** C	**19.** D	**20.** C
21. D	**22.** C	**23.** D	**24.** C	**25.** D					

Hints and Explanations

1. All the statements are TRUE. Choice (D)

2. First sort the given array with 'n' values. In the worst case sorting takes $O(n^2)$ time.
Last '3' values in the array can be added in constant time. Choice (C)

3. 6 4 5 $\boxed{2\ 7\ 11}$ $\boxed{8\ 12\ 13}$ 9 10
Longest increasing subsequence length is 3
(i) 2 7 11 (ii) 8 12 13. Choice (B)

4.
$$S = \underline{gac}\ \underline{gga}\ t\ \underline{tag}$$
$$X = \underline{ga}\ t\ c\ \underline{gga}\ a\ \underline{tag}$$
$$\text{LCS} = \{g\ a\ c\ g\ g\ a\ t\ a\ g\}$$
Length = 9. Choice (C)

5. **Binary search tree:**
node · left. key < node. key < node × right. key
max heap:
node. key > node. left . key, node. right. key.
Choice (C)

6. Hash function is $h(k) = k \bmod 11$
3 mod 11 = 3
43 mod 11 = 10
8 mod 11 = 8
11 mod 11 = 0
14 mod 11 = 3
25 mod 11 = 3
The minimum chain length is '0', from the hash values it is clear that, the chains with slot numbers 1, 2, 4, 5, 6, 7, 9 are empty.
We do not need to construct a hash table to check the chain lengths. Choice (A)

7. To insert an element in a sorted array we need to check for the correct position of the element. In the worst case it will take 'n' comparisons.
In max-heap and min-heap, we need (log n) time to insert an element.
In unsorted array simply keep the element at the end of the array. Choice (D)

8. Searching an element in a hash table using hashing takes constant time usually, it is the efficient search method, but its implementation depends on hash function, table size etc. Choice (B)

9.
- Median of 'n' integers can be finded by traversing the entire list. It takes θ(n) time.
- Sum of 'n' integers take $(n - 1)$ computation which requires $\theta(n)$ time.
- For finding largest of 'n' number it requires $(n - 1)$ comparisions which requires $\theta(n)$ time.

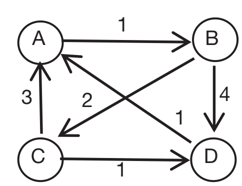
Choice (D)

10. Suppose $h(k) = k \bmod 5$
$k_1 = 10, k_2 = 15$
I. I–TRUE
II. $k_1 = 6, k_2 = 11$
6 mod 5 = 1, 11 mod 5 = 1
II–False
III. $k_1 = 16, k_2 = 14$
16 mod 5 = 1, 14 mod 5 = 4
III–false. Choice (A)

11. Kruskals algorithm is used to construct a minimum spanning tree, in the first attempt itself we will get optimal solution. Choice (C)

12. Dijkstras Algorithm and prims algorithm implementation is similar.
Time complexity is $O(E + V) \log V$
Depth first search and Breadth first search has same time complexity $O(E + V)$. Choice (B)

13. From the given matrix, the graph is drawn



Shortest path from B to $D \Rightarrow = (\overline{BC} - \overline{CD})$
= 2 + 1 =3. Choice (A)

14.
- Insertion sort is a quadratic time sorting algorithm. It is an example of dynamic programming.
- Selection sort is a quadratic time sorting algorithm, It is an example of greedy algorithm.
- Merge sort and Quick sort are examples of Divide and conquer approach. Choice (A)

15. $M = 9$
(i) Weight = 3 + 4 ≤ 9
Profit = 2 + 3 = 5
(ii) Weight = 3 + 6 ≤ 9
Profit = 2 + 4 = 6. Choice (B)

16. $(A\,(B\,(CD))) = 4 \times 3 \times 6 + 1 \times 4 \times 6 + 5 \times 1 \times 6$
First $(CD) = 4 \times 3 \times 6$
$[C]_{4\times3} \times [D]_{3\times6} = [\text{Res } 1]_{4\times6}$
$(B \text{ Res } 1) = 1 \times 4 \times 6$
$[B]_{1\times4}\ [\text{Res } 1]_{4\times6} = [\text{Res } 2]_{1\times6}$
$(A \text{ Res } 2) = 5 \times 1 \times 6$
$[A]_{5\times1}\ [\text{Res } 2]_{1\times6} = [\text{Res } 3]_{5\times6}$
$\therefore \quad A = 5 \times 1,$

$B = 1 \times 4,$
$C = 4 \times 3,$
$D = 3 \times 6.$ Choice (C)

17. $a = \frac{1}{36} = 0.02$

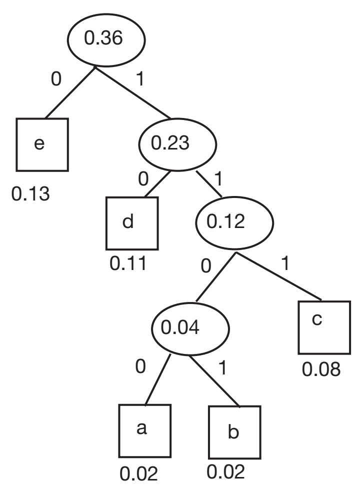
$b = \frac{1}{36} = 0.02$

$c = \frac{1}{12} = 0.08$

$d = \frac{1}{9} = 0.11$

$e = \frac{5}{36} = 0.13$

Huffman tree:



Huffman codes
$a = 1100$, $b = 1101$, $c = 111$, $d = 10$, $e = 0$. Choice (B)

18. $\boxed{7\ 5}$ 6 $\boxed{3\ 8}$ 12 $\boxed{9\ 13}$ 14 $\boxed{10\ 11}$
$12 + 11 + 22 + 21 = 66$
7 $\boxed{5\ 6}$ 3 $\boxed{8\ 12}$ 9 $\boxed{13\ 14}$ 10 $\boxed{11}$
$11 + 20 + 27 + 11 = 69.$ Choice (C)

19. Coin values 1, 5, 6.
Value = 20

(i) $\dfrac{\text{Greedy Algorithm}}{6+6+6+1+1}$

$\Rightarrow$ 5 coins

(ii) $\dfrac{\text{Brute-Force}}{5+5+5+5}$

$\Rightarrow$ 4 coins
Value = 21

(iii) $\dfrac{\text{Greedy Algorithm}}{6+6+6+1+1+1}$

$\Rightarrow$ 6 coins

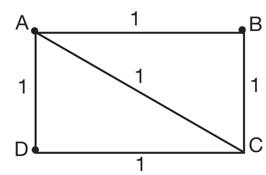
(iv) $\dfrac{\text{Brute Force}}{5+5+5+5+1}$

$\Rightarrow$ 5 coins. Choice (D)

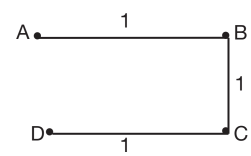
20. Amortized time is θ (log n)
Worst case time is θ (n log n). Choice (C)

21.
- Computing a median of 'n' elements takes $\theta(n)$ time as it has to traverse 'n' elements.
- The shortest path between 2 vertices need not be same in the Minimum spanning tree and in graph.

Consider following graph G



The MST for the above graph 'G' is



The shortest path from A to C is 1 in graph 'G' where as, it is 2 in MST. Choice (D)

22. $\boxed{12\ 39}$ 2 94 23 77 52 9

$\boxed{2\ 12\ 39}$ 94 23 77 52 9

$\boxed{2\ 12\ 39\ 94}$ 23 77 52 9

$\boxed{2\ 12\ 23\ 39\ 94}$ 77 52 9

$\boxed{2\ 12\ 23\ 39\ 77}$ 94 52 9

$\boxed{2\ 12\ 23\ 39\ 77}$ 52 9

2 12 23 39 52 77 94 9

$\therefore$ 2 9 12 23 39 52 77 94

$\therefore$ Insertion sort. Choice (C)

23. Given elements 12 39 02 94 23 77 52 09

First sort elements based on least significant digit

12 02 52 23 94 77 39 09.

Sort the above elements based on second least significant digit.

02 09 12 23 39 52 77 94

$\therefore$ The above sorting algorithm is 'Radix–Sort'.

Choice (D)

24. The available elements are $[0, 1, 2, ..., n^2 - 1]$

There are n^2 elements.

Separate chaining has unlimited space, it can save all the n^2 elements.

Linear probing:
It will have '*n*' slots, so maximum we can store '*n*' values.
Quadratic probing:
Like linear probing, it will also have '*n*' slots.
We can store only '*n*' values.

Choice (C)

25. Bubble sort, compares adjacent elements, if not in order swaps them.

14	5	21	4	16	31	17	8	11

5 14 21 4 16 31 17 8 11

5 14 4 21 16 31 17 8 11

5 14 4 16 21 31 17 8 11

5 14 4 16 21 17 31 8 11

5 14 4 16 21 17 8 31 11

5 14 4 16 21 17 8 11 31.

Choice (D)