Determinants

S.No → 1

Art - 1

Determinants :- We know that a matrix is an arrangement of number and not a numbers with every square matrix $A = [c_{ij}]$ be associated a number called the determinants of A and it denoted by det A and $|A|$.

Art - 2

Determinants of 2nd order :- The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ in which any four number a_1, b_1, a_2, b_2 are arranged in square array consisting of two horizontal lines and bounded by two vertical bars is called a determinant of 2nd order.

Art - 3

Expansion of Determinant of 2nd Order :-

Let $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$\therefore \Delta = a_1 b_2 - a_2 b_1$

Note :→ Let $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ be a matrix of order 2×2, then the determinant of A is defined as:

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\Delta = a_{11}a_{22} - a_{21}a_{12}$$

S.No. → 2

Q.1 Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$\therefore \Delta = \cos 15^\circ \cos 75^\circ - \sin 75^\circ \sin 15^\circ$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\therefore \Delta = \cos(15^\circ + 75^\circ)$

$\therefore \Delta = \cos 90^\circ$

$\therefore \boxed{\Delta = 0}$

Q.2 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then show that $|2A| = 4|A|$

Sol. We are given

$A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$\therefore 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$\therefore |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$\therefore |2A| = 8 - 32 = -24$

$\therefore |2A| = -24$ —— (1)

$\therefore |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$

$\therefore |A| = 2 - 8 = -6$

$\therefore 4|A| = 4 \times (-6) = -24$ —— (2)

$\therefore$ from (1) & (2)

$|2A| = 4|A|$

Q.3 If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then write the value of x.

Sol. We are given

$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\therefore 2x^2 - 40 = 18 + 14$

$\therefore 2x^2 = 32 + 40$

$\therefore 2x^2 = 72$

$\therefore x^2 = \cancel{72}^{36} / \cancel{2}_1$

$\therefore x = \sqrt{36}$

$\therefore \underline{x = \pm 6}$

S.No→3

Art–4

Determinants of 3rd Order:– The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ in which nine numbers $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ are arranged in two vertical bars, is called a determinant of third order.

eg:– Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\therefore |A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Q.4 Find the value of Δ, where $\Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$.

Sol. Let $\Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$

$$\therefore \Delta = (-1)^{1+1} 6 \begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} + (-1)^{1+2}(-3) \begin{vmatrix} 2 & 2 \\ -10 & 2 \end{vmatrix} + (-1)^{1+3}(2) \begin{vmatrix} 2 & -1 \\ -10 & 5 \end{vmatrix}$$

$\therefore \Delta = (6)(-2-10) + 3(4+20) + 2(10-10)$

$\therefore \Delta = (6)(-12) + 3(24) + 0$

$\therefore \Delta = -72 + 72 + 0$

$\therefore \underline{\Delta = 0}$

Q.5 Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

S.No → 4

Sol. Let $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

$\therefore \Delta = (1)\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} + 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$

$\therefore \Delta = 1(0-0) - 2(0-0) + 4(-1-12)$

$\therefore \Delta = 0 - 0 + 4(-13)$

$\therefore \Delta = -52$

Q.6 Evaluate the determinant $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ Also prove that $2 \leq \Delta \leq 4$.

Sol. Let $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$\therefore \Delta = 1\begin{vmatrix} 1 & \sin\theta \\ -\sin\theta & 1 \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & \sin\theta \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} -\sin\theta & 1 \\ -1 & -\sin\theta \end{vmatrix}$

$\therefore \Delta = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$\therefore \Delta = 1 + \sin^2\theta + (-0) + \sin^2\theta + 1$

$\therefore \Delta = 2 + 2\sin^2\theta$

We know that

$-1 \leq \sin\theta \leq 1$, for all θ

$\therefore 0 \leq \sin^2\theta \leq 1$, $\forall \theta$

$\therefore 0 \leq 2\sin^2\theta \leq 2$, $\forall \theta$

$\therefore 0+2 \leq 2+2\sin^2\theta \leq 2+2$, $\forall \theta$

$\therefore 2 \leq 2+2\sin^2\theta \leq 4$, $\forall \theta$

$\therefore 2 \leq \Delta \leq 4$, $\forall \theta$ hence proved

Q.7 If $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$ then show that $|2A| = 8|A|$

Sol. Let $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$

... then show that $|3A| = 27|A|$

S.No→5

$$\therefore |2A| = \begin{vmatrix} 8 & 2 & 2 \\ 2 & 8 & 2 \\ 2 & 2 & 8 \end{vmatrix}$$

$$\therefore |2A| = 8\begin{vmatrix} 8 & 2 \\ 2 & 8 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & 8 \end{vmatrix} + 2\begin{vmatrix} 2 & 8 \\ 2 & 2 \end{vmatrix}$$

$$\therefore |2A| = 8(64-4) - 2(16-4) + 2(4-16)$$

$$\therefore |2A| = 8(60) - 2(12) + 2(-12)$$

$$\therefore |2A| = 480 - 24 - 24 = 432$$

$$\therefore |2A| = 432 \text{ ——— } (1)$$

$$\therefore 8|A| = 8\begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix}$$

$$\therefore \quad = 8\left[4\begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}\right]$$

$$\therefore \quad = 8\left[4(16-1) - 1(4-1) + 1(1-4)\right]$$

$$\therefore \quad = 8\left[4(15) - 3 - 3\right]$$

$$\therefore \quad = 8\left[60-6\right]$$

$$\therefore \quad = 8(54) = 432$$

$$\therefore 8|A| = 432 \text{ ——— } (2)$$

from (1) and (2)

$$|2A| = 8|A|$$

Q.8 If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ find the value of $3|A|$.

Sol The given matrix

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

$$\therefore |A| = (6-4)$$

$$\therefore |A| = 2$$

$$\therefore 3|A| = 2 \times 3$$

$$\underline{\underline{3|A| = 6}}$$

Q.9 If $A = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$ than show that $|3A| = 27|A|$

B.Notes.

Sol. We are given

$$A = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$3A = \begin{vmatrix} 9 & 3 & 3 \\ 3 & 9 & 3 \\ 3 & 3 & 9 \end{vmatrix}$$

$$\therefore |3A| = \begin{vmatrix} 9 & 3 & 3 \\ 3 & 9 & 3 \\ 3 & 3 & 9 \end{vmatrix}$$

$$\therefore |3A| = 9\begin{vmatrix} 9 & 3 \\ 3 & 9 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 \\ 3 & 9 \end{vmatrix} + 3\begin{vmatrix} 3 & 9 \\ 3 & 3 \end{vmatrix}$$

$$\therefore \quad = 9(81-9) - 3(27-9) + 3(9-27)$$

$$\therefore \quad = 9(72) - 3(18) + 3(-18)$$

$$\therefore \quad = 648 - 54 - 54$$

$$\therefore |3A| = 540 \quad \text{—— (1)}$$

$$\therefore 27|A| = 27\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$\therefore \quad = 27\left[3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}\right]$$

$$\therefore \quad = 27\left[3(9-1) - 1(3-1) + 1(1-3)\right]$$

$$\therefore \quad = 27\left[3(8) - 1(2) + 1(-2)\right]$$

$$\therefore \quad = 27\left[24 - 2 - 2\right]$$

$$\therefore \quad = 27(20) = 540$$

$$27|A| = 540 \quad \text{—— (2)}$$

from (1) & (2)

$$|3A| = 27|A|$$

Q.10. Find the value of x such that $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

Sol. We are given

S.No→7

$$\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

$\therefore x^2 - x = 6 - 4$

$\therefore x^2 - x = 2$

$\therefore x^2 - x - 2 = 0$

$\therefore x = \dfrac{-(-1) \pm \sqrt{1+8}}{2}$

$\therefore x = \dfrac{1 \pm \sqrt{9}}{2}$

$\therefore x = \dfrac{1 \pm 3}{2}$

$\therefore x = 4/2, \dfrac{-2}{2}$

$\therefore \boxed{x = 2, -1}$

Q.11. Find the value of Δ, $\Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$

Sol. The given determinant is,

$$\Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

$\therefore \Delta = 6\begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ -10 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ -10 & 5 \end{vmatrix}$

$\therefore \Delta = 6(-2-10) + 3(4+20) + 2(10-10)$

$\therefore \Delta = 6(-12) + 3(24) + 0$

$\therefore \Delta = -72 + 72$

$\therefore \underline{\underline{\Delta = 0}}$

Q.12. Prove that the determine $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Sol. The given determined is $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$$\therefore \Delta = x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$

$$\therefore \Delta = x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$\therefore \Delta = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$$

S.No.→8

$$\therefore \Delta = -x^3 - x + x(\sin^2\theta + \cos^2\theta)$$

$$\therefore \Delta = -x^3 - x + x(1)$$

$$\therefore \Delta = -x^3$$

Q.13 Find the value of x such that $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

Sol We are given

$$\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

$$(x+1)(x+2) - (x-3)(x-1) = 12+1$$

$$\therefore \cancel{x^2}+2x+x+2-[\cancel{x^2}-x-3x+3] = 13$$

$$\therefore 2x+x+2+x+3x-3 = 13$$

$$\therefore 7x = 14$$

$$\therefore \boxed{x=2}$$

Q.14 Find the value of x such that $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$.

Sol. We are given

$$\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$$

$$\therefore 2x(x+1) - 2(x+1)(x+3) = 3-15$$

$$\therefore 2x^2+2x-(2x+2)(x+3) = -12$$

$$\therefore \cancel{2x^2}+\cancel{2x}-(\cancel{2x^2}+6x+\cancel{2x}+6) = -12$$

$$\therefore -6x-6 = -12$$

$$\therefore -6x = -6$$

$$\therefore \boxed{x=1}$$

Q.15 Find the value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

Sol We are given that

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

S.No → 9

$\therefore 3 - x^2 = 3 - 8$

$\therefore -x^2 = -5 - 3$

$\therefore -x^2 = -8$

$\therefore x^2 = 8$

$\therefore x = \sqrt{8} = \pm 2\sqrt{2}$

$\therefore \boxed{x = \pm 2\sqrt{2}}$

Q.16 Find the value of x, for which $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

Sol. We are given that

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

$\therefore 2 - 20 = 2x^2 - 24$

$\therefore 2x^2 = +24 - 18$

$\therefore 2x^2 = 6$

$\therefore x^2 = 6/2$

$\therefore x^2 = 3$

$\therefore \boxed{x = \pm\sqrt{3}}$

Q.17 Find the value of x, $\begin{vmatrix} 2x & 3 \\ -5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ -5 & 8 \end{vmatrix}$

Sol we are given that

$$\begin{vmatrix} 2x & 3 \\ -5 & x \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ -5 & 8 \end{vmatrix}$$

$\therefore 2x^2 + 15 = 32 + 15$

$\therefore 2x^2 = 32$

$\therefore x^2 = 16$

$\therefore \boxed{x = 4}$

Q.18. If $A = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$ then show that $|2A| = 8|A|$

Sol. We are given

$$A = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

S.No → 9

$$|2A| = \begin{vmatrix} 6 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{vmatrix}$$

$$\therefore |2A| = 6\begin{vmatrix} 6 & 2 \\ 2 & 6 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & 6 \end{vmatrix} + 2\begin{vmatrix} 2 & 6 \\ 2 & 2 \end{vmatrix}$$

$$\therefore |2A| = 6(36-4) - 2(12-4) + 2(4-12)$$

$$\therefore |2A| = 6(32) - 2(8) + 2(-8)$$

$$\therefore |2A| = 192 - 32$$

$$\therefore |2A| = 160 \quad \text{—— (1)}$$

$$8|A| = 8\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 8\left[3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}\right]$$

$$\therefore = 8[3(9-1) - 1(3-1) + 1(1-3)]$$

$$\therefore = 8[3(8) - 1(2) + 1(-2)]$$

$$\therefore = 8[24-4]$$

$$\therefore = 8(20)$$

$$\therefore 8|A| = 160 \quad \text{—— (2)}$$

from (1) & (2)

$$|2A| = 8|A|$$

Q.19 Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

Sol. We are given. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

$\therefore (b+c)\left(((c+a)(a+b)-cb) - a\left(b(a+b)-cb\right) + a\left(bc - c(c+a)\right)\right] = 4abc$

$\therefore (b+c)(ac+\cancel{bc}+a^2+ab-\cancel{cb}) - a(ab+b^2-cb) + a(bc-c^2-ac) = 4abc$

$\therefore (b+c)(a^2+ac+ab) - a(ab+b^2-cb) + a(bc-c^2-ac) = 4abc$

$\therefore \cancel{a^2b}+abc+\cancel{ab^2}+\cancel{a^2c}+\cancel{ac^2}+abc-\cancel{a^2b}-\cancel{ab^2}+abc+abc-\cancel{ac^2}-\cancel{a^2c} = 4abc$

$\therefore abc+abc+abc+abc = 4abc$

$$\boxed{4abc = 4abc}$$

$\boxed{\text{S.No} \rightarrow 10}$

Q.20 Evaluate
$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

Sol. We are given det.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

$$\therefore \Delta = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

$\therefore \Delta = 0 - 1(0-6) + 2(-3+0)$

$\therefore \Delta = 6 - 6 = 0$

$\therefore \boxed{\Delta = 0}$

Q.21 Evaluate
$$\begin{vmatrix} 212 & 117 & 345 \\ 19 & 9 & 34 \\ 7 & 3 & 5 \end{vmatrix}$$

Sol. The given determinants

$$\begin{vmatrix} 212 & 117 & 345 \\ 19 & 9 & 34 \\ 7 & 3 & 5 \end{vmatrix}$$

$\therefore \Delta = 212(45-102) - 117(95-238) + 345(57-63)$

$\therefore \quad = 212(-57) - 117(-143) + 345(-6)$

$\therefore \quad = -12084 + 16731 - 2070$

$\therefore \quad = -14154 + 16731$

$\therefore \boxed{\Delta = 2577}$

Art :→ 5

Minors and Cofactors

S.No → 11

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

$\therefore |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Minor of $a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$\therefore$ Minor of $a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$\therefore$ Minor of $a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Minor of $a_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$\therefore$ Minor of $a_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

$\therefore$ Minor of $a_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$\therefore$ Minor of $a_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

$\therefore$ Minor of $a_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

S.No → 12

Cofactors

Art -- 5

Let $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{13} = (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$\therefore$ Cofactors of $a_{21} = (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{22} = (-1)^{2+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{23} = (-1)^{2+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

$\therefore$ Cofactors of $a_{31} = (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$\therefore$ Cofactors of $a_{32} = (-1)^{3+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{33} \end{vmatrix}$

$\therefore$ Cofactors of $a_{33} = (-1)^{3+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Note ⇒

∴ Cofactor of $a_{11} = A_{11}$

∴ Minor of $a_{11} = M_{11}$

∴ Cofactors of $a_{ij} = A_{ij}$

$$A_{12} = (-1)^{1+2} M_{12}$$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Art :→ 6

Let A be a square matrix of order n.

$$AB = I_n = BA$$

$$\therefore \boxed{B = A^{-1}}$$

Q.22. Find A the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$\therefore |A| = 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 5\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}$$

$$= (3-1) - 2(2+1) + 5(2+3)$$

$$= 2 - 6 + 25 = 21 \neq 0$$

$|A| \neq 0$

$\therefore A^{-1}$ exists

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$\therefore A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 3 - 1 = 2$$

Q.No→14

$\therefore A_{12} = -\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = -(2+1) = -3$

$\therefore A_{13} = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = (2+3) = 5$

$\therefore A_{21} = -\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -(2-5) = 3$

$\therefore A_{22} = \begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix} = (1+5) = 6$

$\therefore A_{23} = -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -(1+2) = -3$

$\therefore A_{31} = \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = (2-15) = -13$

$\therefore A_{32} = -\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = -(1-10) = 9$

$\therefore A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3-4) = -1$

$\therefore \text{adj} A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}'$

$\therefore \text{adj} A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj} A)$

$\therefore A^{-1} = \frac{1}{21}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$

Q.13 Using matrix method solve the following system of eqⁿ

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

Sol. The given equations

$$x+2y-3z=-4 \quad \text{------ (1)}$$
$$2x+3y+2z=2 \quad \text{------ (2)}$$
$$3x-3y-4z=11 \quad \text{------ (3)}$$

S. No → 15

In matrix form

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\therefore AX=B$$

where $A=\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$$\therefore |A|=\begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$$\therefore |A|=1\begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix}-2\begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix}-3\begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix}$$

$$\therefore |A|=(-12+6)-2(-8-6)-3(-6-9)$$

$$\therefore |A|=(-6)-2(-14)-3(-15)$$

$$\therefore |A|=-6+28+45$$

$$\therefore |A|=67,\ |A|\neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\therefore A_{11}=\begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix}=(-12+6)=-6$$

$$\therefore A_{12}=-\begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix}=-(-8-6)=14$$

$$\therefore A_{13}=\begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix}=(-6-9)=-15$$

$$\therefore A_{21}=-\begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix}=-(-8-9)=17$$

S.No → 16

$$\therefore A_{22} = \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = (-4+9) = 5$$

$$\therefore A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = -(-3-6) = 9$$

$$\therefore A_{31} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (4+9) = 13$$

$$\therefore A_{32} = -\begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -(2+6) = -8$$

$$\therefore A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3-4) = -1$$

$$\therefore \text{adj} A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}'$$

$$\therefore \text{adj} A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj} A)$$

$$A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\therefore AX = B$$

$$\boxed{X = A^{-1}B}$$

$$X = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\therefore X = \frac{1}{67} \begin{bmatrix} 24+34+143 \\ -56+10-88 \\ 60+18-11 \end{bmatrix}$$

$$\therefore X = \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 201/67 \\ \frac{-134}{67} \\ 67/67 \end{bmatrix}$$

S.No → 17

$$\therefore X = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore \boxed{x=3}$, $\boxed{y=-2}$, $\boxed{z=1}$

Q.24 Use Matrix method to solve the following system of eq^n

$x-y+2z=7$

$3x+4y-5z=-5$

$2x-y+3z=12$

Sol. We are given eq^n

$$x-y+2z=7 \quad \text{——(1)}$$
$$3x+4y-5z=-5 \quad \text{——(2)}$$
$$2x-y+3z=12 \quad \text{——(3)}$$

In matrix form

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$AX=B$$

where $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

S.No→18

$\therefore |A| = 1\begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$

$\therefore \quad = 1(12-5) + 1(9+10) + 2(-3-8)$

$\therefore \quad = 7 + 19 - 22$

$\therefore |A| = 4$

$\therefore |A| \neq 0$

$\therefore A^{-1}$ exists

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

$\therefore A_{11} = \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = (12-5) = 7$

$\therefore A_{12} = -\begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -(9+10) = -19$

$\therefore A_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (-3-8) = -11$

$\therefore A_{21} = -\begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -(-3+2) = +1$

$\therefore A_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3-4) = -1$

$\therefore A_{23} = -\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = (-1+2) = -1$

$\therefore A_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = (5-8) = -3$

$\therefore A_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -(-5-6) = 11$

$\therefore A_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = (4+3) = 7$

S.No → 19

$$\therefore \text{adj}\,A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}'$$

$$\therefore \text{adj}\,A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj}\,A)$$

$$\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore AX = B$$

$$\therefore \boxed{X = A^{-1}B}$$

$$\therefore X = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\therefore X = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$\therefore X = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\therefore \underline{\underline{x=2}},\ \underline{\underline{y=1}},\ \underline{\underline{z=3}}$$

Q.15 Use matrix method to solve the following system

$$5x - y + z = 4$$
$$3x + 2y - 5z = 2$$
$$x + 3y - 2z = 5$$

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S. No → 20

Sol. The given eqn

$5x - y + z = 4$ ——(1)

$3x + 2y - 5z = 2$ ——(2)

$x + 3y - 2z = 5$ ——(3)

In form of matrix

$$\begin{bmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$\therefore AX = B$

Where $A = \begin{bmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{vmatrix}$$

$$\therefore |A| = 5\begin{vmatrix} 2 & -5 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$$

$\therefore = 5(-4+15) + 1(-6+5) + 1(9-2)$

$\therefore = 5(11) + (-1) + 7$

$\therefore = 55 + 6$

$\therefore |A| = 61$

$|A| \neq 0$

A^{-1} exists

$$\therefore |A| = \begin{vmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{vmatrix}$$

$$\therefore A_{11} = \begin{vmatrix} 2 & -5 \\ 3 & -2 \end{vmatrix} = (-4+15) = 11$$

S No → 21

$\therefore A_{12} = -\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} = -(-6+5) = 1$

$\therefore A_{13} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = (9-2) = 7$

$\therefore A_{21} = -\begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} = -(2-3) = 1$

$\therefore A_{22} = \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = (-10-1) = -11$

$\therefore A_{23} = -\begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = -(15+1) = -16$

$\therefore A_{31} = \begin{vmatrix} -1 & 1 \\ 2 & -5 \end{vmatrix} = (5-2) = 3$

$\therefore A_{32} = -\begin{vmatrix} 5 & 1 \\ 3 & -5 \end{vmatrix} = -(25-3) = 28$

$\therefore A_{33} = \begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} = 10+3 = 13$

$$\therefore \text{adj } A = \begin{bmatrix} -11 & 1 & 7 \\ 1 & -11 & -16 \\ 3 & 28 & 13 \end{bmatrix}^{t}$$

$$\therefore \text{adj } A = \begin{bmatrix} +11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$A^{-1} = \frac{1}{61}\begin{bmatrix} +11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{61}\begin{bmatrix} +11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

S.No → 22

$$\therefore X = \frac{1}{61}\begin{bmatrix} 4 + 44 + 2 + 15 \\ 4 - 22 + 140 \\ 28 - 32 + 65 \end{bmatrix}$$

$$\therefore X = \frac{1}{61}\begin{bmatrix} 61 \\ 122 \\ 61 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{61}{61} \\ 122/61 \\ 61/61 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \boxed{x = 1}, \quad \boxed{y = 2}, \quad \boxed{z = 1}$$

Q.26 Use matrix method to solve the following system

$4x + 2y + 3z = 2$
$x + y + z = 1$
$3x + y - 2z = 5$

Sol. The given equations are

$4x + 2y + 3z = 2$ —— (1)
$x + y + z = 1$ —— (2)
$3x + y - 2z = 5$ —— (3)

In form of matrix,

$$\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$AX = B$

Where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$ S.No.→23

$$\therefore |A| = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$$

$$\therefore = 4\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$$

$$\therefore = 4(-2-1) - 2(-2-3) + 3(1-3)$$

$$\therefore = 4(-3) - 2(-5) + 3(-2)$$

$$\therefore = -12 + 10 - 6$$

$$\therefore |A| = -8$$

$$\therefore |A| \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\therefore |A| = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$$

$$\therefore A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = (-2-1) = -3$$

$$\therefore A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2-3) = 5$$

$$\therefore A_{13} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = (1-3) = -2$$

$$\therefore A_{21} = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -(-4-3) = 7$$

$$\therefore A_{22} = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} = (-8-9) = -17$$

S.No→24

$\therefore A_{23} = -\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = -(4-6) = 2$

$\therefore A_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = (2-3) = -1$

$\therefore A_{32} = -\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} = -(4-3) = -1$

$\therefore A_{33} = \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = (4-2) = 2$

$\therefore adj.A = \begin{bmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{bmatrix}'$

$\therefore adj\ A = \begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(adj\ A)$

$\therefore A^{-1} = \frac{1}{-8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$

$AX = B$

$\therefore \underline{\underline{X = A^{-1}B}}$

$\therefore X = \frac{1}{-8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$

$\therefore X = \frac{1}{-8}\begin{bmatrix} -6+7-5 \\ 10-17-5 \\ -4+2+10 \end{bmatrix}$

$$\therefore X = -\frac{1}{8}\begin{bmatrix} -4 \\ -12 \\ 8 \end{bmatrix}$$

S.No → 25

$$\therefore X = \begin{bmatrix} 1/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{x = 1/2}, \quad \boxed{y = 3/2}, \quad \boxed{z = -1}$$

Q.27 Use matrix method to solve the following system of equations.

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

Sol. We are given eqn

$$2x + 3y + 3z = 5 \quad \text{—— (1)}$$
$$x - 2y + z = -4 \quad \text{—— (2)}$$
$$3x - y - 2z = 3 \quad \text{—— (3)}$$

In form of matrix,

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$AX = B$$

Where $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$\therefore |A| = 2\begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}$$

S.No → 26

$$\therefore \quad = 2(4+1) - 3(-2-3) + 3(-1+6)$$

$$\therefore \quad = 2(5) - 3(-5) + 3(5)$$

$$\therefore \quad = 10 + 15 + 15$$

$$\therefore |A| = 40$$

$|A| \neq 0$

A^{-1} exists

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$\therefore A_{11} = \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} = (4+1) = 5$$

$$\therefore A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2-3) = 5$$

$$\therefore A_{13} = \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = (-1+6) = 5$$

$$\therefore A_{21} = -\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} = -(-6+3) = 3$$

$$\therefore A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = (-4-9) = -13$$

$$\therefore A_{23} = -\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = -(-2-9) = 11$$

$$\therefore A_{31} = \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = (3+6) = 9$$

$$\therefore A_{32} = -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$$

$$\therefore A_{33} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = (-4-3) = -7$$

$$\therefore \operatorname{Cof} A = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\operatorname{adj} A)$$

$$\therefore A^{-1} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$AX = B$$

$$\therefore \underline{X = A^{-1}B}$$

$$\therefore X = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\therefore X = \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$\therefore X = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{x = 1}, \boxed{y = 2}, \boxed{z = -1}$$

Q.28 Use the matrix method to solve the following system of equations.

$$2x - 3y + 3z = 1$$
$$2x + 2y + 3z = 2$$
$$3x - 2y + 2z = 3$$

Sol. We are given eq^n

$$2x - 3y + 3z = 1 \quad \text{———(1)}$$
$$2x + 2y + 3z = 2 \quad \text{———(2)}$$
$$3x - 2y + 2z = 3 \quad \text{———(4)}$$

S.No → 27

In form of matrix,

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

where $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\therefore \boxed{AX = B}$$

$$\therefore |A| = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\therefore |A| = 2\begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix}$$

$$\therefore |A| = 2(4+6) + 3(4-9) + 3(-4-6)$$

$$\therefore |A| = 2(10) + 3(-5) + 3(-10)$$

$$\therefore |A| = 20 - 15 - 30$$

$$\therefore |A| = -25$$

$$\therefore |A| \neq 0$$

$\therefore$ A^{-1} exist

$$|A| = \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix}$$

$\therefore A_{11} = \begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} = 4+6 = 10$

$\therefore A_{12} = -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -(4-9) = 5$

S.No → 28

$\therefore A_{13} = \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix} = (-4-6) = -10$

$\therefore A_{21} = -\begin{vmatrix} -3 & 3 \\ -2 & 2 \end{vmatrix} = -(-6+6) = 0$

$\therefore A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = (4-9) = -5$

$\therefore A_{23} = -\begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = -(-4+9) = -5$

$\therefore A_{31} = \begin{vmatrix} -3 & 3 \\ 2 & 3 \end{vmatrix} = (-9-6) = -15$

$\therefore A_{32} = -\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = -(6-6) = 0$

$\therefore A_{33} = \begin{vmatrix} 2 & -3 \\ 2 & 2 \end{vmatrix} = (4+6) = 10$

$$\therefore \text{adj } A = \begin{bmatrix} 10 & 5 & -10 \\ 0 & -5 & -5 \\ -15 & 0 & 10 \end{bmatrix}'$$

$$\therefore \text{adj } A = \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$AX = B$

$\therefore X = A^{-1}B$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

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$$\therefore A^{-1} = \frac{1}{-25}\begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

S.No.→29

$$\boxed{X = A^{-1}B}$$

$$\therefore X = \frac{1}{-25}\begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore X = \frac{1}{-25}\begin{bmatrix} 10+0-45 \\ 5-10+0 \\ -10-10+30 \end{bmatrix}$$

$$\therefore X = \frac{1}{-25}\begin{bmatrix} -35 \\ -5 \\ 10 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7/5 \\ 1/5 \\ -2/5 \end{bmatrix}$$

$$\therefore \boxed{x = 7/5}, \boxed{y = 1/5}, \boxed{z = -2/5}$$

Q.29. Solve the following equations by matrix method.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Sol. The given eq^{n}s are,

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \quad \text{——— (1)}$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \quad \text{——— (2)}$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \quad \text{——— (3)}$$

Putting $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$ and we get eqns

$\boxed{S.No \rightarrow 30}$

$\therefore 2a + 3b + 10c = 4$

$4a - 6b + 5c = 1$

$6a + 9b - 20c = 2$

In form of matrix,

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$\underline{\underline{AX = B}}$

Where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$\therefore |A| = 2\begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} - 3\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} + 10\begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix}$$

$\therefore |A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$\therefore |A| = 2(75) - 3(-110) + 10(72)$

$\therefore |A| = 150 + 330 + 720 = 1200$

$\therefore |A| = 1200$

$\therefore |A| \neq 0$

$\therefore \underline{\underline{A^{-1} \text{ exist}}}$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$\therefore A_{11} = \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} = (120 - 45) = 75$

$\therefore A_{12} = -\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = -(-80-30) = 110$

S.No → 31

$\therefore A_{13} = \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} = 36+36 = 72$

$\therefore A_{21} = -\begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = -(-60-90) = 150$

$\therefore A_{22} = \begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} = (-40-60) = -100$

$\therefore A_{23} = -\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = -(18-18) = 0$

$\therefore A_{31} = \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = (15+60) = 75$

$\therefore A_{32} = -\begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = -(10-40) = 30$

$\therefore A_{33} = \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = (-12-12) = -24$

$$\therefore \text{adj}A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$\therefore \text{adj}A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$\therefore A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$\therefore AX = B$

$\therefore \boxed{X = A^{-1}B}$

S.No → 32

$$\therefore X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore X = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\therefore X = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\therefore \quad a = 1/2, \quad b = 1/3, \quad c = 1/5$

$\therefore \boxed{x = 2}, \boxed{y = 3}, \boxed{z = 5}$

Q.30 Solve the following eqⁿ by matrix method

$$\frac{5}{x} - \frac{1}{y} + \frac{1}{z} = 4$$

$$\frac{3}{x} + \frac{2}{y} - \frac{5}{z} = 2$$

$$\frac{1}{x} + \frac{3}{y} - \frac{2}{z} = 5$$

Sol. The given eqⁿs are

$$\frac{5}{x} - \frac{1}{y} + \frac{1}{z} = 4 \text{ —— (1)}$$

$$\frac{3}{x} + \frac{2}{y} - \frac{5}{z} = 2 \text{ —— (2)}$$

$$\frac{1}{x} + \frac{3}{y} - \frac{2}{z} = 5 \text{ —— (3)}$$

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Putting $a = \frac{1}{x}$, $b = \frac{1}{y}$, $c = \frac{1}{z}$

S.No → 33

hence the given eq^{n}s are

$5a - b + c = 4$

$3a + 2b - 5c = 2$

$a + 3b - 2c = 5$

In form of matrix

$$\begin{bmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$AX = B$$

Where $A = \begin{bmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{vmatrix}$$

$$\therefore |A| = 5 \begin{vmatrix} 2 & -5 \\ 3 & -2 \end{vmatrix} + 1 \begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$$

$\therefore \quad = 5(-4+15) + 1(-6+5) + 1(9-2)$

$\therefore \quad = 5(11) + 1(-1) + 1(7)$

$\therefore \quad = 55 - 1 + 7 = 61$

$\therefore |A| = 61$

$\therefore |A| \neq 0$

$\therefore |A| = 61$, A^{-1} exists

$$|A| = \begin{vmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{vmatrix}$$

$\therefore A_{11} = \begin{vmatrix} 2 & -5 \\ 3 & -2 \end{vmatrix} = (-4+15) = 11$

S.No → 34

$$\therefore A_{12} = -\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} = -(-6+5) = 1$$

$$\therefore A_{13} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = (9-2) = 7$$

$$\therefore A_{21} = -\begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} = -(2-3) = 1$$

$$\therefore A_{22} = \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = (-10-1) = -11$$

$$\therefore A_{23} = -\begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = -(15+1) = -16$$

$$\therefore A_{31} = \begin{vmatrix} -1 & 1 \\ 2 & -5 \end{vmatrix} = (5-2) = 3$$

$$\therefore A_{32} = -\begin{vmatrix} 5 & 1 \\ 3 & -5 \end{vmatrix} = -(-25-3) = 28$$

$$\therefore A_{33} = \begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} = (10+3) = 13$$

$$\therefore \text{adj } A = \begin{bmatrix} 11 & 1 & 7 \\ 1 & -11 & -16 \\ 3 & 28 & 13 \end{bmatrix}'$$

$$\therefore \text{adj } A = \begin{bmatrix} 11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$A^{-1} = \frac{1}{61}\begin{bmatrix} 11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}$$

$$AX = B$$

$$\therefore X = A^{-1}B$$

$$\therefore X = \frac{1}{61}\begin{bmatrix} 11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

S.No → 35

$$\therefore X = \frac{1}{61}\begin{bmatrix} 44+2+15 \\ 4-22+140 \\ 28-32+65 \end{bmatrix}$$

$$\therefore X = \frac{1}{61}\begin{bmatrix} 61 \\ 122 \\ 61 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\frac{1}{x} = a$, $\frac{1}{y} = b$, $c = 1/z$

$\therefore \boxed{x = 1}$, $\boxed{y = 1/2}$, $\boxed{z = 1}$

Q.31 If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence the solve equations.

$3x - 4y + 2z = -1$, $2x + 3y + 5z = 7$, $x + z = 2$.

Sol. We are given

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\therefore |A| = 3\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} + 4\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

S.No → 36

$\therefore |A| = 3(3-0) + 4(2-5) + 2(0-3)$

$\therefore |A| = 9 + (-12) - 6 = -9$

$\therefore |A| = -9$

$|A| \neq 0$

A^{-1} exist

$\therefore A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = (3-0) = 3$

$\therefore A_{12} = -\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -(2-5) = 3$

$\therefore A_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = (0-3) = -3$

$\therefore A_{21} = -\begin{vmatrix} -4 & 2 \\ 0 & 1 \end{vmatrix} = -(-4-0) = 4$

$\therefore A_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = (3-2) = 1$

$\therefore A_{23} = -\begin{vmatrix} 3 & -4 \\ 1 & 0 \end{vmatrix} = -(0+4) = -4$

$\therefore A_{31} = \begin{vmatrix} -4 & 2 \\ 3 & 5 \end{vmatrix} = (-20-6) = -26$

$\therefore A_{32} = -\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = -(15-4) = -11$

$\therefore A_{33} = \begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} = (9+8) = 17$

$$\text{adj} A = \begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -11 & 17 \end{bmatrix}'$$

$$\text{adj} A = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$\therefore A^{-1} = \frac{1}{|A|} (adj\, A)$

$\boxed{S.No \rightarrow 37}$

$$\therefore A^{-1} = \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

The given eqn are

$$3x - 4y + 2z = -1 \quad \text{——— (1)}$$
$$2x + 3y + 5z = 7 \quad \text{——— (2)}$$
$$x + z = 2 \quad \text{——— (3)}$$

In form of matrix

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$\boxed{AX = B}$$

$$\boxed{X = A^{-1} B}$$

$$\therefore X = \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$\therefore X = \frac{1}{-9} \begin{bmatrix} -3 + 28 - 52 \\ -3 + 7 - 22 \\ 3 - 28 + 34 \end{bmatrix}$$

$$\therefore X = \frac{1}{-9} \begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$\boxed{x=3}$, $\boxed{y=2}$, $\boxed{z=-1}$

$\boxed{S.No \rightarrow 38}$

Q. 39 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ Find A^{-1} and Solve the equations

$x+y+z=6$, $x+2z=7$, $3x+y+z=12$.

Sol. We are given

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\therefore |A| = 1\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$$\therefore = (0-2) - 1(1-6) + 1(1-0)$$

$$\therefore = -2+5+1$$

$$\therefore |A| = 4$$

$$\therefore |A| \neq 0$$

$\therefore$ A^{-1} exists

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\therefore A_{11} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = (0-2) = -2$$

$$\therefore A_{12} = -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -(1-6) = 5$$

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$\therefore A_{13} = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = (1-0) = 1$

S. No→39

$\therefore A_{21} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -(1-1) = 0$

$\therefore A_{22} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = (1-3) = -2$

$\therefore A_{23} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -(1-3) = 2$

$\therefore A_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1-1) = 0$

$\therefore A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -(2-1) = -1$

$\therefore A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = (0-1) = -1$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & -1 \end{bmatrix}'$$

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 0 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

The given equations are

$$x+y+z = 6$$
$$x+0y+2z = 7$$
$$3x+y+z = 12$$

In form of matrix

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$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$\boxed{\text{S. No} \rightarrow 40}$

$$\underline{AX = B}$$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -2 & 0 & 0 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\boxed{X = A^{-1}B}$$

$$\therefore X = \frac{1}{4} \begin{bmatrix} -2 & 0 & 0 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\therefore X = \frac{1}{4} \begin{bmatrix} -12+0+0 \\ 30-14-12 \\ 6+14-12 \end{bmatrix}$$

$$\therefore X = \frac{1}{4} \begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore \boxed{x = -3}, \boxed{y = 1}, \boxed{z = 2}$$

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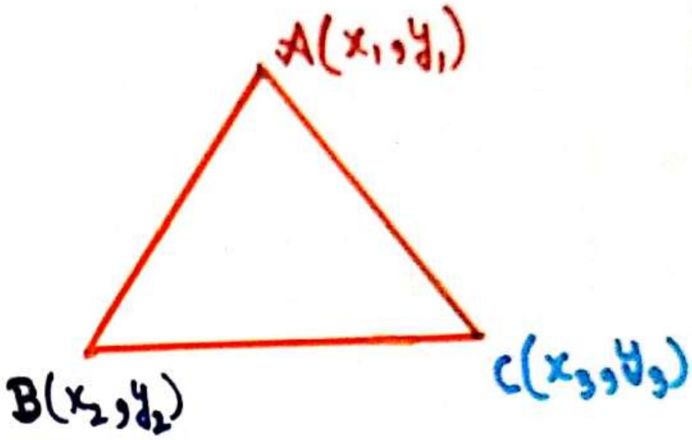
Applications Of Determinants

Art → 7

Area of a triangle:- Let a triangle having vertices $A(x_1, y_1)$ $B(x_2, y_1)$ and $C(x_3, y_3)$.



Then area of Triangle ABC is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ Sq. units.}$$

Q.33 Find the area of the triangle with vector (2,7), (1,1), (10,8)?

Sol. Let Δ be the area of the triangle whose vertices are (2,7), (1,1) and (10,8).

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[2(1-8) - 7(1-10) + 1(8-10)\right]$$

$\therefore = \frac{1}{2}\left[2(-7) - 7(-9) + (-2)\right]$

S. No→42

$\therefore = \frac{1}{2}\left[-14 + 63 - 2\right]$

$\therefore = \frac{1}{2}[47]$

$\therefore \Delta = 23.5$ Square units.

Q. 34 Using determinants, show that the points (11,7), (5,5) and (−1,3) are Collinear.

Sol. Let Δ be the area of the triangle whose vertices are A(11,7), B(5,5) and C(−1,3).

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ -1 & 3 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[11\begin{vmatrix} 5 & 1 \\ 3 & 1 \end{vmatrix} - 7\begin{vmatrix} 5 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 5 \\ -1 & 3 \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[11(5-3) - 7(5+1) + 1(15+5)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[11(2) - 7(6) + 1(20)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[22 - 42 + 20\right]$$

$$\therefore \Delta = \left(\frac{42-42}{2}\right) = 0$$

$$\boxed{\therefore \Delta = 0}$$

Since the area of triangle ABC = 0

$\therefore$ The given points (11,7), (5,5), and (−1,3) are Collinear.

Q.35 Show that the points $(a+5, a-4)$, $(a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.

Sol. Let the given points are

$A(a+5, a-4)$, $B(a-2, a+3)$ and $C(a, a)$ **S.No→43**

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[(a+5)\begin{vmatrix} a+3 & 1 \\ a & 1 \end{vmatrix} - (a-4)\begin{vmatrix} a-2 & 1 \\ a & 1 \end{vmatrix} + 1\begin{vmatrix} a-2 & a+3 \\ a & a \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[(a+5)(\cancel{a}+3-\cancel{a}) - (a-4)(\cancel{a}-2-\cancel{a}) + 1((a-2)a - a(a+3))\right]$$

$$\therefore \Delta = \frac{1}{2}\left[\cancel{3a}+15+\cancel{2a}-8+\cancel{a^2}-\cancel{2a}-\cancel{a^2}-\cancel{3a}\right]$$

$$\therefore \Delta = \frac{1}{2}[15-8]$$

$$\therefore \Delta = \frac{7}{2} \text{ Square.}$$

$$\therefore \Delta \neq 0$$

$\therefore$ given points $(a+5, a-4)$, $(a-2, a+3)$ and (a, a) are not Collinear for any value of a.

Q.36 Find the eq^n of the line joining $A(1,3)$ and $B(0,0)$ by determinantes and for K if $D(K,0)$ a point line that are of the triangle ABD is 3 square units.

Sol. Let the given Points are

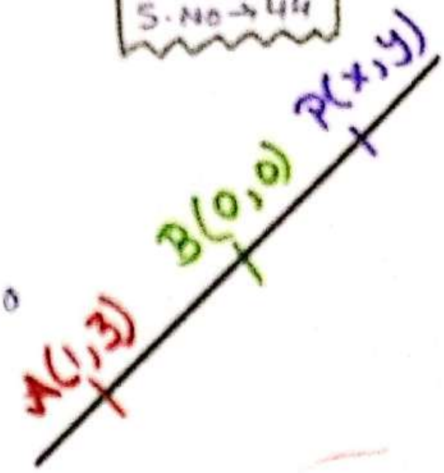
$A(1,3)$ and $B(0,0)$

Let $P(x,y)$ be any point on the line joining $A(1,3)$ and $B(0,0)$.

$\therefore$ area of $\Delta ABP = 0$

S. No → 44

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$



$$\therefore \frac{1}{2}\left[1\begin{vmatrix} 0 & 1 \\ y & 1 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ x & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ x & y \end{vmatrix}\right] = 0$$

$$\therefore \frac{1}{2}\left[1(0-y) - 3(0-x) + 1(0-0)\right] = 0$$

$$\therefore (-y + 3x) = 0$$

$\therefore$ $y = 3x$ is the equation of line.

Let the point D is $(K, 0)$

$\therefore$ from given condition,

Area of $\Delta ABD = 3$

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 3$$

[Note this step]

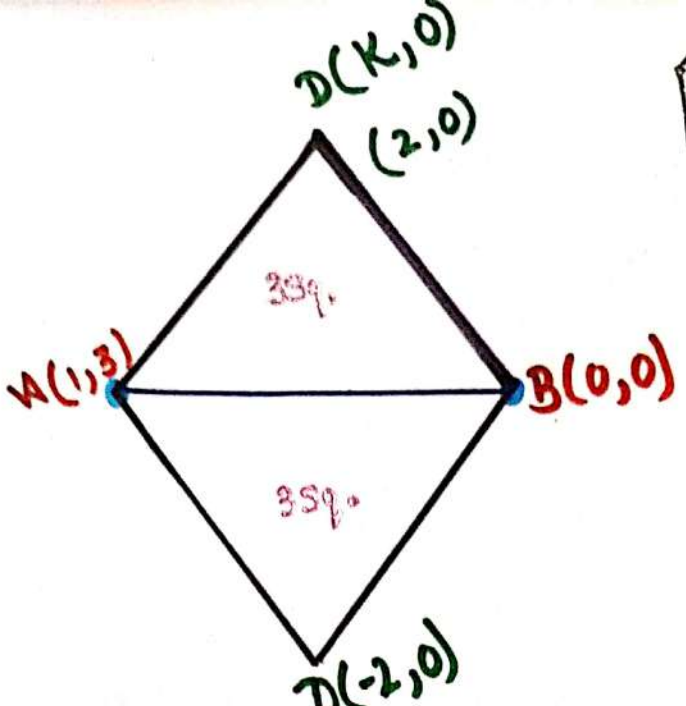
$$\therefore \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 6$$

$$\therefore 1\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ K & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ K & 0 \end{vmatrix} = \pm 6$$

$$\therefore \left[(0-0) - 3(0-K) + 1(0-0)\right] = \pm 6$$

$$\therefore 3K = \pm 6$$

$$\therefore \boxed{K = \pm 2}$$

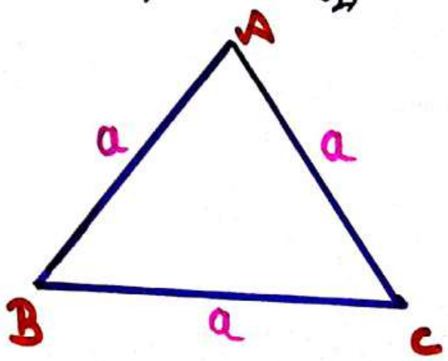


S.No→45

Q. 37. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an equitorial triagle whose each line is equal to a, then prove that

$$\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$$

Sol. Let the given points are $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$



$$\Delta = \frac{\sqrt{3}}{4} a^2 \text{ Square units.}$$

$\therefore$ area of ΔABC is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore 4\Delta = 2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

[Note this step]

Q. No -> 46

$$\therefore 4\Delta = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}$$

[N.T.S]

$$\therefore 16\Delta^2 = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$$

$$\therefore \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 16\left(\frac{\sqrt{3}}{4}a^2\right)^2$$

[$\because$ area of equitorial triangle by det a is $= \frac{\sqrt{3}}{4}a^2$ sq. units]

$$\therefore \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 16\left[\left(\frac{3}{16}\right)a^4\right]$$

$$\therefore \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$$

Q.38 Find the area of the triangle whose vertices are $(3,1)$, $(4,3)$ and $(-5,4)$.

Sol. Let Δ be the area of the triangle whose vertices are $(3,1)$, $(4,3)$ and $(-5,4)$.

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 4 & 3 & 1 \\ -5 & 4 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[3\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ -5 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 3 \\ -5 & 4 \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[3(3-4) - 1(4+5) + 1(16+15)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[3(-1) - 1(9) + 1(31)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[-3-9+31\right]$$

$$\therefore \Delta = \frac{1}{2}\left[-12+31\right] = \frac{19}{2}$$

$$\therefore \Delta = 9.5 \text{ square units}$$

Q. 39 Find the area of the triangle whose vertices $(3,8)$, $(-4,2)$ and $(5,1)$.

Sol. Let Δ be the area of the triangle whose vertices are $(3,8)$, $(-4,2)$ and $(5,1)$.

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & 1 \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[3(2-1) - 8(-4-5) + 1(-4-10)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[3(1) - 8(-9) + 1(-14)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[3+72-14\right]$$

$$\therefore \Delta = \frac{61}{2}, \quad \boxed{\therefore \Delta = 30.5 \text{ Sq. units}}$$

Show that the points $A(a, b+c)$, $B(b, c+a)$ and $C(c, a+b)$ are Collinear

Sol. Let Δ be the area of the triangle whose points are $A(a, b+c)$, $B(b, c+a)$, $C(c, a+b)$

S.No→48

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[a\begin{vmatrix} c+a & 1 \\ a+b & 1 \end{vmatrix} - (b+c)\begin{vmatrix} b & 1 \\ c & 1 \end{vmatrix} + 1\begin{vmatrix} b & c+a \\ c & a+b \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[a(c+a-(a+b)) - (b+c)(b-c) + 1(b(a+b) - c(c+a))\right]$$

$$\therefore \Delta = \frac{1}{2}\left[a(c+\cancel{a}-\cancel{a}-b) - (b^2+\cancel{bc}-\cancel{bc}-c^2) + ab+b^2-c^2-ac\right]$$

$$\therefore \Delta = \frac{1}{2}\left[\cancel{ac} - \cancel{ab} - \cancel{b^2} + \cancel{c^2} + \cancel{ab} + \cancel{b^2} - \cancel{c^2} - \cancel{ac}\right]$$

$$\therefore \Delta = \frac{1}{2}(0)$$

$$\boxed{\therefore \Delta = 0}$$

Since the area of $\Delta = 0$

$\therefore$ The points are $A(a, b+c)$, $B(b, c+a)$ and $C(c, a+b)$ are Collinear.

Q.41. Determine x so that the points $(3, -2)$, $(x, 2)$ and $(8, 8)$ lie on a line.

Sol Let Δ be the Area of triangle and whose points are $A(3, -2)$, $B(x, 2)$ and $C(8, 8)$ lie on a line.

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ 8 & 1 \end{vmatrix} + 2\begin{vmatrix} x & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} x & 2 \\ 8 & 8 \end{vmatrix}\right]$$

$\therefore \Delta = \frac{1}{2}\left[3(2-8)+2(x-8)+1(8x-16)\right]$

$\boxed{S.No \to 49}$

$\therefore \Delta = \frac{1}{2}\left[3(-6)+2(x-8)+(8x-16)\right]$

$\therefore \Delta = \frac{1}{2}\left[-18+2x-16+8x-16\right]$

$\therefore \Delta = \frac{1}{2}\left[10x-50\right]$

$0 = 10x - 50$ $\qquad \boxed{\because \Delta = 0}$

$\therefore 10x = 50$

$\therefore \boxed{x = 5}$

Q. 42 Find the value of x if the area of triangle is 35 Square cm. with vertices $(x,4)$, $(2,-6)$ and $(5,4)$.

Sol. Let the given vertices are $A(x,4)$, $B(2,-6)$ and $C(5,4)$.

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[x\begin{vmatrix} -6 & 1 \\ 4 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -6 \\ 5 & 4 \end{vmatrix}\right]$$

$\therefore \Delta = \frac{1}{2}\left[x(-6-4)-4(2-5)+1(8+30)\right]$

$\therefore 35 = \frac{1}{2}\left[-10x-4(-3)+1(38)\right]$

$\therefore 35 = \frac{1}{2}\left[-10x+12+38\right]$

$\therefore 70 = -10x+50$

$\therefore -10x = 70-50$

$\therefore -10x = 20$

$\therefore$

$\therefore x = -2$

S.No → 50

Q.43 Find values of K if area of triangle of 4 sq. units and vertices are $(K,0)$, $(4,0)$, $(0,2)$.

Sol. Let the given points are $A(4,0)$ and $B(0,2)$

Let $P(x,y)$ be any point on the line joining $A(4,0)$ and $B(0,2)$.

$\therefore$ area of $\Delta ABP = 0$

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \\ x & y & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[4\begin{vmatrix} 2 & 1 \\ y & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ x & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ x & y \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[4(2-y) - 0(0-x) + 1(0-2x)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[8 - 4y - 2x\right]$$

$$\therefore 2x + 4y = 8 \quad \text{—— (1)}$$

$2x + 4y = 8$ is the eq^n of line

Let the Point D is $(K,0)$

$\therefore$ from given Condition

$\therefore$ area of $\Delta ABD = 4$

$$\therefore \frac{1}{2}\begin{vmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 4$$

$$\therefore \frac{1}{2}\left[4\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ K & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ K & 0 \end{vmatrix}\right] = \pm 4$$

S. No→51

$\therefore \frac{1}{2}\left[4(2-0)-0+1(0-2k)\right] = \pm 4$

$\therefore \frac{1}{2}\left[8-2k\right] = \pm 4$

$\therefore 8-2k = \pm 8$

$\therefore -2k = \pm 8 - 8$

$\therefore -2k = -16 \quad , k = 0$

$\therefore \underline{\underline{k = 8, 0}}$

Q. 44 Find the area of triangle whose points are $(-2,-3)$, $(3,2)$, $(-1,-8)$.

Sol. Let Δ be the area of triangle and points are $A(-2,-3)$, $B(3,2)$ and $C(-1,-8)$.

$$\therefore \Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$\therefore \Delta = \frac{1}{2}\left[-2\begin{vmatrix} 2 & 1 \\ -8 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ -1 & -8 \end{vmatrix}\right]$$

$$\therefore \Delta = \frac{1}{2}\left[-2(2+8)+3(3+1)-1(-24+2)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[-2(10)+3(4)-1(-22)\right]$$

$$\therefore \Delta = \frac{1}{2}\left[-20+12+22\right]$$

$$\therefore \Delta = \frac{(-20+34)}{2} = \frac{14}{2}$$

$\therefore \Delta = 7$ Sq. units.

Properties of Determinants

S.No-752

★ **Property I** :- If each element of a row of determinant is zero, then the value of the determinant is zero.

Proof : Let $\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$$\therefore = 0\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - 0\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + 0\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$\therefore = 0 - 0 + 0 = 0$ (Expanding by First Row)

eg ⇒ Let $\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{vmatrix}$

$$\therefore \Delta = 0\begin{vmatrix} 3 & 4 \\ 6 & 8 \end{vmatrix} - 0\begin{vmatrix} 2 & 4 \\ 5 & 8 \end{vmatrix} + 0\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$$

$\therefore \Delta = 0 - 0 + 0$

$\therefore \boxed{\Delta = 0}$

★ **Property II** :- Value of a det is not changed by changing the rows into Columns and Columns into rows.

S. No → 53

$$\therefore \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\therefore \Delta' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\therefore \boxed{\Delta = \Delta'}$$

Note ⇒ $|A| = |A'|$

★ **Property III** :- If two adjacent rows of a determinant are interchanged, then the sign of the determinant is changed but its numerical value is unchanged

eg :- $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 7 \\ 3 & 2 & 5 \end{vmatrix}$

$$\therefore \Delta = 1\begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 7 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}$$

$$\therefore \Delta = 1(15-14) - 2(10-21) + 3(4-9)$$

$$\therefore \Delta = (1) - 2(-11) + 3(-5)$$

$$\therefore \Delta = 1 + 22 - 15$$

$$\therefore \Delta = 8$$

$$\therefore \Delta' = \begin{vmatrix} 2 & 3 & 7 \\ 1 & 2 & 3 \\ 3 & 2 & 5 \end{vmatrix}$$

S. No → 54

$$\therefore \Delta = 2\begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} + 7\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$

$\therefore \Delta = 2(10-6) - 3(5-9) + 7(2-6)$

$\therefore \Delta = 2(4) - 3(-4) + 7(-4)$

$\therefore \Delta = 8 + 12 - 28$

$\therefore \Delta = -8$

$\Delta = 8$, $\Delta = -8$

Cor.: If any row of a determinant Δ is passed over n rows then the resulting determinant Δ' is given by

$$\Delta' = (-1)^n \Delta$$

eg.: Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$\therefore \Delta = (-1)^2 \begin{vmatrix} a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

★ **Property 4:** If two rows of the determinant are identical, then the value of the determinant is zero.

Let $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 7 \\ 1 & 2 & 4 \end{vmatrix} = 0$

$\therefore \underline{\Delta = 0}$

*Property 5 :→ If every element of a row is multiplied by the same constant K, then the value of determinant is multiplied by the constant K.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\therefore \Delta' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\therefore \boxed{\Delta' = K\Delta}$

Note :→ (1) $|2A| = 4|A|$, $A_{2\times2}$

(2) $|2A| = 8|A|$, If $A_{3\times3}$

Note ⇒ If two rows of a determinant are interchanged, then the sign of the determinant is changed.

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix}$

$|A| = (8-6) = 2$

$|A| = 2$

S.No → 56

$\therefore 2A = 2\begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$

$\therefore 2A = \begin{bmatrix} 2 & 6 \\ 4 & 16 \end{bmatrix}$

$\therefore |2A| = \begin{vmatrix} 2 & 6 \\ 4 & 16 \end{vmatrix}$

$\therefore |2A| = 32 - 24 = 8$

$\therefore |2A| = 8$

$\therefore \underline{|2A| = 2 \times |A|}$

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$

$\therefore 2A = \begin{bmatrix} 2 & 6 \\ 4 & 16 \end{bmatrix}$

$\therefore |2A| = \begin{vmatrix} 2 & 6 \\ 4 & 16 \end{vmatrix}$

$\therefore \quad = 2 \times 2 \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix}$

$\therefore \quad = 4|A|$

$\boxed{\therefore |2A| = 4|A|}$

Note :⇒ Let $A = \begin{bmatrix} 4 & 12 \\ 8 & 24 \end{bmatrix}$

$\therefore A = 4\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 4 & 12 \\ 8 & 24 \end{vmatrix}$

$= 4 \times 8 \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}$

$\therefore \; = 32 \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}$ S.No → 57

$\therefore |A| = 32 \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}$

Note :⇒ If A is any square matrix of order n, then

$$|KA| = K^n |A|$$

Let $A = \begin{bmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \cdots & \cdots \\ a_{n1} & a_{n2} \quad a_{nn} \end{bmatrix}$

$\therefore KA = \begin{bmatrix} Ka_{11} & Ka_{12} \cdots Ka_{1n} \\ \cdots & \cdots \\ \cdots & \cdots \\ Ka_{n1} & Ka_{n2} \cdots Ka_{nn} \end{bmatrix}$

$$\therefore \boxed{|KA| = K^n |A|}$$

Cor. :⇒ If two rows of a determinant are proportional then its value is zero.

$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ **Identical**

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ **Proportional**

S.No → 58

Let $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 2 & 4 & 6 \end{vmatrix}$

$\therefore \Delta = 0$

Property 6 :⇒ If each element in any row consist of two terms then the determinant can be expressed as the sum of the two determinant of the same order.

Let $\Delta = \begin{vmatrix} a_1+d_1 & b_1+d_2 & c_1+d_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\therefore \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Q. 45 Prove that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} b & c & a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$

S.No → 59

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} b & c & a \\ b & c & a \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} a & b & c \\ b & c & a \\ a & b & c \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & a & b \\ c & a & b \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & a & b \\ a & b & c \end{vmatrix} + \begin{vmatrix} b & c & a \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$+ \begin{vmatrix} b & c & a \\ b & c & a \\ a & b & c \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ c & a & b \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ c & a & b \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + 0 + 0 + 0 + 0 + 0 + 0 + 0 + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + (-1)^2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Hence proved

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★ **Property 7** :→ The value of a determinant remains unchanged if to each element of a row be added or subtracted equimultiple of the corresponding element of the one or more rows of the determinant.

eg:→ Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Operating $R_1 \leftrightarrow R_1 + lR_2$

$$\therefore \Delta = \begin{vmatrix} a_1 + la_2 & b_1 + lb_2 & c_1 + lc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Operating $R_2 \leftrightarrow R_2 + lR_1$

$$\therefore \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 + la_1 & b_2 + lb_1 & c_2 + lc_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Operating $R_1 \longleftrightarrow R_1 + lR_2 + mR_3$

$$\therefore \Delta = \begin{vmatrix} a_1 + la_2 + ma_3 & b_1 + lb_2 + mb_3 & c_1 + lc_2 + mc_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Note:⇒ (1) $1 + \omega + \omega^2 = 0$

(2) $\omega^3 = 1$

(3) $\omega \cdot \omega^2 = 1$ $\quad \left[\therefore \omega = \frac{1}{\omega^2}, \omega^2 = \frac{1}{\omega} \right]$

Q.46 Without expanding prove that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$

Where ω, ω^2 are imaginary. Cube roots of unity.

Sol. Let $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Operating $R_1 \longleftrightarrow R_1 + R_2 + R_3$

$$\therefore \Delta = \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0 \qquad [\because 1+\omega+\omega^2 = 0]$$

$\therefore \boxed{\Delta = 0}$

Q.47 Write the value of the determinants. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

$$\therefore \Delta = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$$\Delta = (3x) \times 0 = 0$$

$\therefore \boxed{\Delta = 0}$

S.No → 62

Q.48 Without Expansion show that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0$

Sol. Let $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$

Taking (−1) common factor each of three rows.

$$\therefore \Delta = (-1)^3 \begin{vmatrix} 0 & -a & b \\ a & 0 & -c \\ -b & c & 0 \end{vmatrix}$$

$$\therefore \Delta = - \begin{vmatrix} 0 & -a & b \\ a & 0 & -c \\ -b & c & 0 \end{vmatrix}$$

Interchanging Rows and columns

$$\therefore \Delta = - \begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ +b & -c & 0 \end{vmatrix}$$

$$\therefore \Delta = -\Delta$$

$$\therefore 2\Delta = 0$$

$$\therefore \boxed{\Delta = 0}$$

Q.49 Using properties of determinant prove that

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$

Sol Let $\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Operating $C_1 \leftrightarrow C_1 + C_2 + C_3$

$$\therefore \Delta = \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

$$\therefore \Delta = (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

Operating $R_2 \leftrightarrow R_2 - R_1$

$\therefore$ Operating $R_3 \leftrightarrow R_3 - R_1$

$$\therefore \Delta = (a+x+y+z) \begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix}$$

$$\therefore \Delta = (a+x+y+z)\,[1 \cdot a \cdot a]$$

$$\therefore \Delta = a^2(a+x+y+z)$$

Hence Proved

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V.V. Imp.

Q. 50 Prove that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Sol. Let $\Delta = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & -bc \\ 1 & b & -ca \\ 1 & c & -ab \end{vmatrix}$$

S.No → 64

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \left(\frac{\cancel{abc}}{\cancel{abc}}\right)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - (-1)^2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\therefore \Delta = 0$$

V. Imp

Q.51 Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Or $\cdot\ ab+bc+ca+abc$.

Sol. Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking a, b, c Common from R_1, R_2, R_3

$$\therefore \Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Operating $R_1 \leftrightarrow R_1 + R_2 + R_3$,

$$\therefore \Delta = abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix}$$

$$\therefore \Delta = (abc)\left(1+\frac{1}{a}+\frac{1}{b}+1/c\right) \begin{vmatrix} 1 & 1 & 1 \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix}$$

Operating $C_2 \longleftrightarrow C_2 - C_1$
Operating $C_3 \longleftrightarrow C_3 - C_1$

$$\therefore \Delta = (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 0 \\ 1/c & 0 & 1 \end{vmatrix}$$

$$\therefore \Delta = (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)[1\cdot1\cdot1]$$ [∵ product of diagonal elements]

$$\therefore \Delta = (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Q.52 Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.

Sol. The given number a, b, c are in AP

$$\therefore 2b = a + c \quad \text{— (1)}$$

$$\therefore L.H.S = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Operating $R_2 \longleftrightarrow 2R_2$

$$\therefore L.H.S = \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Operating $R_2 \leftrightarrow R_2 - (R_1 + R_3)$

$$\therefore L.H.S = \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 2b-(a+c) \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\therefore L.H.S = \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$\therefore$ **L.H.S = R.H.S** $\quad [\because 2b-(a+c)=0]$

Hence proved

Q.53. Using properties of determinant solve for x

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Sol. The given eq^n is

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Operating $C_1 \leftrightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$

$$\therefore (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

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Operating $R_2 \longleftrightarrow R_2 - R_1$,
operating $R_3 \longleftrightarrow R_3 - R_1$,

$$\therefore (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$$\therefore (3a-x)(4x^2) = 0$$ [∵ of product of diagonal]

$$\therefore x^2(3a-x) = 0$$

$$\therefore x = 0,\ x = 3a$$

$$\therefore \underline{\underline{x = 0, 3a}}$$

Q.54 Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

Sol. Let $\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

∴ multiplying R_1, R_2, R_3 by a, b, c respectively.

$$\therefore \Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix}$$

Taking a,b,c as common

$$\therefore \Delta = \frac{1}{abc}(abc)\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

Operating $R_1 \leftrightarrow R_1 + R_2 + R_3$

$$\therefore \Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$\therefore \Delta = (1+a^2+b^2+c^2) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

Operating $C_2 \leftrightarrow C_2 - C_1$
operating $C_3 \leftrightarrow C_3 - C_1$

$$\therefore \Delta = (1+a^2+b^2+c^2) \begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\therefore \Delta = (1+a^2+b^2+c^2)\{1.1.1\}$$ [∵ of product of diagonal]

$$\therefore \Delta = (1+a^2+b^2+c^2)$$

Q.55 Factories the determinant $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Operating $C_2 \leftrightarrow C_2 - C_1$
operating $C_3 \leftrightarrow C_3 - C_1$

$$\therefore \Delta = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

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$$\therefore \Delta = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

$$\therefore \Delta = (c-a)(b-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

$\therefore$ Expanding by R_1

$$\therefore \Delta = (b-a)(c-a)\left[1\begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix} + 0 + 0\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[(c+\cancel{a}-b-\cancel{a})\right]$$

$$\therefore \Delta = (b-a)(c-a)(c-b)$$

$$\therefore \Delta = (a-b)(b-c)(c-a)$$

Q.56 Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Sol Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Operating $C_2 \leftrightarrow C_2 - C_1$,

Operating $C_3 \leftrightarrow C_3 - C_1$,

$$\therefore \Delta = \begin{vmatrix} 1 & 0 & 0 \\ a^2 & b^2-a^2 & c^2-a^2 \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

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Note :=> (1) $a^2 - b^2 = (a-b)(a+b)$

(2) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$$\therefore \Delta = \begin{vmatrix} 1 & 0 & 0 \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \\ a^3 & (b-a)(b^2+ba+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

$$\therefore \Delta = (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a^2 & (b+a) & (c+a) \\ a^3 & b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix}$$

Expanding by R_1,

$$\therefore \Delta = (b-a)(c-a)\left[1\begin{vmatrix} b+a & c+a \\ b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix} + 0 + 0\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[(b+a)(c^2+ca+a^2) - (c+a)(b^2+ba+a^2)\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[bc^2+abc+a^2b+ac^2+ca^2+a^3-(cb^2+abc+a^2c+ab^2+a^2b+a^3)\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[bc^2+\cancel{abc}+\cancel{a^2b}+ac^2+\cancel{ca^2}+\cancel{a^3}-cb^2-\cancel{abc}-\cancel{a^2c}-ab^2-\cancel{a^2b}-\cancel{a^3}\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[bc^2+ac^2-cb^2-ab^2\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[(bc^2-cb^2)+(ac^2-ab^2)\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[bc(c-b)+a(c^2-b^2)\right]$$

$$\therefore \Delta = (b-a)(c-a)\left[bc(c-b)+a(c-b)(c+b)\right]$$

$$\therefore \Delta = (b-a)(c-a)[c-b]\left[bc+a(c+b)\right]$$

$\therefore \Delta = (b-a)(c-a)(c-b)\left[bc+ac+ab\right]$

$$\boxed{\therefore \Delta = (a-b)(b-c)(c-a)(ab+bc+ac)}$$

Q. 57 If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$,

then prove that $\Delta + \Delta_1 = 0$

Sol. We are given

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad \text{——— (1)}$$

and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$

Operating $C_1 \leftrightarrow xC_1$, $C_2 \leftrightarrow yC_2$, $C_3 \leftrightarrow zC_3$

$\therefore \Delta_1 = \dfrac{1}{xyz}\begin{vmatrix} x & y & z \\ xyz & xyz & xyz \\ x^2 & y^2 & z^2 \end{vmatrix}$

$\therefore \Delta_1 = \dfrac{1}{xyz}(xyz)\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix}$

Operating $R_1 \leftrightarrow R_2$

$\therefore \Delta_1 = -\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

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Interchanging Rows and Columns.

$$\therefore \Delta_1 = -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$\therefore \boxed{\Delta_1 = -\Delta} \qquad [\because \text{ of } (1)]$$

$$\boxed{\therefore \Delta_1 + \Delta = 0}$$

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