*Permutation and Combination *

Permutation ↓ arrangement of objects (Pa)

Combination ↓ selection of objects (Cs)

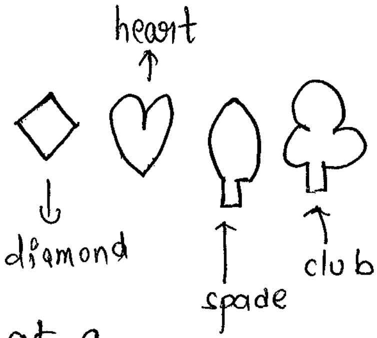
$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

out of n objects we are arranging r objects

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

combinations

$${}^nP_r = {}^nC_r \cdot r!$$



*Fundamental principle of counting

(i) additive rule "OR" [only one thing at a time]

— (10B, 15G)

To make one monitor

$10+15 = 25$ ways

(ii) multiplication rule "AND" [Product rule]

– more than one thing at a time

* Permutation

- Permutation is defined as arrangement of objects in an ordered manner.

For eg:- 3 persons has to be arrange in three chairs, the seating arrangement can be done in 3! ways.

- It is represented by ^nP_r, here ^nP_r is total no. of ways in which r things at a time can be selected and arranged at a time among them.

$$^nP_r = \frac{n!}{(n-r)!}$$

- For ex:- 5 persons has to be arranged in 3 chairs, seating arrangement can be done in $^5P_3$ ways

$$^5P_3 = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2}}{\cancel{2}} = 60 \text{ ways}$$

*Combination

- Combination is defined as <u>selection</u> of objects in which order do not matter

For eg:- 3-player, select 2-player

$$^3C_2 = 3 \text{ ways}$$

selection (combination)

A, B, C

AB ≈ BA
AC ≈ CA
BC ≈ CB

captain	vice-captain
A	B
A	C
B	A
B	C
C	A
C	B

⇒ arrangement (permutation)

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

- It is no. of ways of selecting 'r' things at a time among 'n' things.

<u>NOTE</u>:
- Permutation is the special case of selection & arrangement.

$$^nP_r = {^nC_r} \cdot r!$$

$$\boxed{{}^nC_r = {}^nC_{n-r}}$$

* Binomial Theorem

- Total no. of selection of zero or more things out of n different thing

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots\dots {}^nC_n = 2^n}$$

Eg:- Bhavya has 5 friends, he wants to invite them into a new-year party. He can invite in how many ways?

$${}^5C_0 = 1 + {}^5C_1 = 5 + {}^5C_2 = 10 + {}^5C_3 = 10 + {}^5C_4 = 5 + {}^5C_5 = 1 = \boxed{32 \text{ ways}}$$

OR $2^5 = 32$ ways

* Important Results

- No. of permutation of n different things taken all at a time (when repeatation is not allowed) $= n!$

- No. of permutation of n things out of which p are of same kind then q are alike of second kind and r is alike of third kind and rest of all different

$$= \frac{n!}{p!\,q!\,r!}$$

Q:- How many ~~ways~~ words can be formed by "MISSISSIPPI".

$$= \frac{11!}{4!\,4!\,2!} = \frac{11 \times 10 \times 9 \times \cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{2}}$$

③ No. of selection of r things out of n identical thing = 1

Q:- In how many ways, 3 ballpen can be choosen out of 50 identical ballpen?

= ~~8~~ 1

④ The no. of permutation of n different things taken r at a time when repeatation is allowed $= n^r$

Q:- How many 3 digit can be formed by 1, 3, 5 if repeatation is allowed?

$= 3^3 = 27$ ways

⑤ Selection of r things out of n things if k things are always selected $= {}^{n-k}C_{r-k}$

Q:- Out of 11 football players, 6 are to be invited into party. Such that captain and goalkeeper will always be invited. Then this can be done in ___.

$${}^{11-2}C_{6-2} = {}^{9}C_{4} = \frac{9!}{5!}$$

* CIRCULAR PERMUTATION *

- In case of circular permutation total no. of ways of arrangement is $(n-1)!$

Q:- Seating arrangement of 6 people around a dinning table in 6 chair will be

$(6-1)! = 5!$

NOTE:

- Flowers in garland and beads in necklace can be done in $\frac{(n-1)!}{2!}$

* Probability and Statistics

① Random experiment : An experiment whose outcome is not predictable with certainty

② Althrough the outcome of experiment is known in advance.

Sample space :- The set of all possible outcomes and it is denoted by S.

Event :- Any subset 'E' of sample space is called an events.

For eg:- $S = \{1, 2, 3, 4, 5, 6\}$

$E = \{1\}$; $E \underbrace{\subseteq}_{\text{subset}} S$

Mutually exclusive Events : Two events E and F are mutually exclusive if $E \cap F = \phi$ i.e. $P(E \cap F) = 0$

In other words E occurs, F can't occurs and F occurs, E can't occurs. [Both can't occur together]

Total or Collective Exhaustive Events :- Two events E and F are collectively exhaustive if $E \cup F = S$ i.e. to gather E and F include all possible outcome. Therefore

$P(E \cup F) = P(S) = 1$

Independent Events: Two events E and F are independent Event if $\boxed{P(E \cap F) = P(E) \cdot P(F)}$

$$P\left(\frac{E}{F}\right) = P(E) \quad \text{OR} \quad P\left(\frac{F}{E}\right) = P(F)$$

- Conditional probability is equal to marginal probability i.e. P(E) is not affected by whether F has happened or not, and vice versa.

$$\boxed{P(E) = \frac{\text{no. of fav. cases}}{\text{Total no. of possible cases}}}$$

Axioms of probability

1] $0 \leq P(E) \leq 1$

2] $P(S) = 1$

3] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

if A and B are disjoint sets / mutally exclusive $(A \cap B = \phi)$ $P(A \cap B) = 0$

then $P(A \cup B) = P(A) + P(B)$

Conditional Probability

- Let S be sample space, and A and B are any two event of S, then the conditional probability of A after occurance of B is denoted by $P(A/B)$ and is defined

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}} \; ; P(B) \neq 0$$

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

* Complimentary probability

$$\boxed{P(A) = 1 - P(A^c)}$$

$A^c \cap B^c = (A \cup B)^c$ De Morgan's law

$P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

P (neither A nor B) = 1 − P (either A or B)

Q:- What is the chance that a leap year has 53 sundays.
(for non-leap year)

$52 \times 7 = 364$ days

↳ leap year − 366 days

$366 - 364 = 2$ days

Sat	Sun
Sun	Mon
Mon	Tue
Tue	Wed
Wed	Thurs
Thur	Fri
Fri	Sat

$$\boxed{P(\text{leap}) = \frac{2}{7}}$$

$$\boxed{P(\text{non-leap}) = \frac{1}{7}}$$

Q:- A card is drawn from deck of playing cards, what is the probability that

(A) face card

(B) Heart card

(c) face & heart card

(D) face or heart card

(A) $\frac{12}{52} = \frac{3}{13}$

(c) $\frac{3}{52}$

(B) $\frac{13}{52} = \frac{1}{4}$

(D) $\frac{22}{52} = \frac{11}{26}$

Face − card = 12

Heart $= 13 - 3 = 10$

Q:- From a deck of playing cards, 2-cards are drawn at random. What is probability that both card will be king. (1) First card is not replaced
(2) Replacement is allowed.

(1)

$$P(K) = \frac{\text{No. of favourable cases}}{\text{Total no. of cases}} = \frac{\cancel{4}}{13\ \cancel{52}} \times \frac{\cancel{3}}{\cancel{51}\ 17} = \frac{1}{17 \cdot 13}$$

(2)

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13}$$

Q:- From a card is drawn from a deck of playing cards and a gambler bets that it is a spade or ace. What are the odds (ratio) against his winning the ~~bed~~ bet.

Sol:- $= 13 + 3$

$$P(\text{win}) = \frac{16}{52}$$

$$P(\text{not winning}) = 1 - \frac{16}{52} = \frac{36}{52}$$

$$\text{Ratio} = \frac{\frac{16}{\cancel{52}}}{\frac{36}{\cancel{52}}} = \frac{4}{9}$$

Q:- A husband and wife get appear in interview for 2 vacancy in same post. The probability of husband selection is $1/7$ and wife is $1/5$. What is the probability (1) Both are selected
(2) Only one of them ~~are~~ is selected
(3) None of them is selected.

(1) Both are selected $\qquad P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{1}{7} \cdot \frac{1}{5} = \frac{1}{35}$$

(2) Only one of them is selected $\qquad P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$$= \frac{1}{7} \cdot \frac{4}{5} + \frac{6}{7} \cdot \frac{1}{5} = \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$$

(3) None of them is selected

$$= \frac{6}{7} \cdot \frac{4}{5} = \frac{24}{35} \qquad P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$$

☆ Q:- An integer is selected at random from first 200 digit. What is the probability that integer choosen is divisible by either 6 or 8.

Divisible by 6 $\qquad$ 6, 12, 18, 60, 120, 180, 186, 192, 198

10 $\quad$ 10 $\quad$ 10 $\quad$ 3

$$\frac{200}{6} = 33$$

Divisibly by 8 $\qquad$ 8, 16, 80, 160, 168, ~~176~~, 184, 19~~0~~2, 200

10 $\quad$ 10 $\quad$ 5

$$\frac{200}{8}$$

2 | 6 | 8
2 | 3 | 4
3

24, 48, 72, 96 $\}$ 4+4

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{58}{200} - \frac{8}{200} = \cancel{0.29}$$

$$= \frac{50}{200} = \frac{1}{4} = 0.25$$

Q:- A bag contains 3 red, 6 white and 7 black balls. What is the probability that 2 balls drawn are white & black. (repeatation not allowed)

Sol^n:-

$$\frac{({}^6C_1)({}^7C_1)}{{}^{16}C_2} = \frac{\cancel{3}\cancel{6} \times 7 \times 2}{\cancel{4}\cancel{8}\cancel{16} \times \cancel{15}5} = \frac{7}{20}$$

Q:- A problem in mathematics is given to three students A, B and C whose chances of solving are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. What is the probability that problem is solved.

Sol^n:-

$$\cancel{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = \frac{13}{12}}$$

$P(A) = 1/2$, $P(\bar{A}) = 1/2$

$P(B) = 1/3$, $P(\bar{B}) = 2/3$

$P(C) = 1/4$, $P(\bar{C}) = 3/4$

Problem solved $=$ 1 $-$ Problem not solved $= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= 1 - \tfrac{1}{2} \cdot \tfrac{2}{3} \cdot \tfrac{3}{4} = 1 - \frac{6}{24} = \frac{18}{24} = \frac{3}{4}$$

CS-11

Q:- A deck of five cards (each carrying a distinct number from 1 to 5) is shuffled throughly. Two cards are then removed one at a time. What is the probability that two cards are selected with the number on first card begin one higher than number on second card.

Sol^n:-

~~First~~ ~~Second~~

1, 2, 3, 4, 5

$S = \{ (1,2), (1,3), (1,4), (1,5)$ ×
$(2,1)$, $(2,3)$, $(2,4)$, $(2,5)$
$(3,1)$, $(3,2)$, $(3,4)$, $(3,5)$
$(4,1)$, $(4,2)$, $(4,3)$, $(4,5)$ $\}$
$(5,1)$, $(5,2)$, $(5,3)$, $(5,4)$

$$P(E) = \frac{4}{20} = \frac{1}{5}$$

EE

Q:- P and Q are two random events then

(A) Independence of P & Q $\Leftrightarrow P(P\cap Q) = 0$

(B) Probability of $(P \cup Q) \geq$ Prob(P) + Prob.(Q)

(C) If P and Q are mutually exclusive then they must be independent

✓ (D) $P(P \cap Q) \leq P(P)$

Q:- An exam consist of 2 papers, paper 1 and paper 2. Prob. of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is

(a) 0.5 (b) 0.18 (c) 0.12 (d) 0.06.

Soln:-

$P(1') = 0.3$ fail in paper 1 $= P(A)$

$P(2') = 0.2$ " " paper 2 $= P(B)$

$P(A) = 0.3$ $P(A/B) = 0.6$

$P(B) = 0.2$

$$P(A \cap B) = P(A/B) \cdot P(B) = 0.6 \cdot 0.2 = \underline{\underline{0.12}}$$

Q:- X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^c) = 0.7$. Which one of the following is value of $P(X \cup Y) = ?$ (a) 0.7 (b) 0.5 (c) 0.4 (d) 0.3

Soln:

$P(X \cap Y) = P(X) \cdot P(Y)$ [$\because$ independent events]

$P(X \cup Y^c) = P(X) + P(Y^c) - P(X) P(Y^c)$

$0.7 = 0.4 + P(Y^c)[1 - P(X)]$

$0.3 = P(Y^c)[0.6]$ $P(Y^c) = 0.5$ $\therefore P[Y] = 0.5$

$\therefore P(X \cup Y) = P(X) + P(Y) - P(X)P(Y)$

$= 0.4 + 0.5 - (0.4)(0.5)$

$= 0.9 - 0.2$

$\boxed{P(X \cup Y) = 0.7}$ (a).

Q:- 3 companies x, y, z supply computers to a university. The percentage of computers supplied by them & probability of those being defective are tabulated below:-

Company	% of computer supplied	Probability of being defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

Given that a CPU is defective, the probability that it was supplied by Y is ____.

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

Sol^n:- S → supply by y, D → defective

$$P(S/D) = \frac{P(S \cap D)}{P(D)}$$

$$= \frac{(0.02)(0.3)}{(0.6)(0.01) + (0.3)(0.02) + (0.1)(0.03)}$$

$$= \frac{0.006}{0.006 + 0.006 + 0.003}$$

$$P(S/D) = \frac{0.006}{0.015} = 0.4$$

* Bayes Function

$$P(A_s/B) = \frac{P(B/A_s)\,P(A_s)}{P(B)} = \frac{P(B/A_s)\,P(A_s)}{\sum_{j=1}^{n} P(B/A_j)\,P(A_j)}$$

In above que.

$\lambda_1, \lambda_2, \lambda_3 = P(B/A_s)$

& $P(A_y) = s_1, s_2, \ldots = $ supply

* Random Variable

$S = \{HH, HT, TH, TT\}$

X = no. of heads

↑

discrete random variable

$HH \to 2$

$HT, TH \to 1$

$TT \to 0$

Mean of a random variable

- is denoted by μ or E

$$\boxed{\underbrace{\mu = E}_{\text{expectance}} = \sum x_i \cdot P(x = x_i)}$$

Variance of a random variable σ^2

$$\sigma^2 = \sum (x_i - \mu)^2 \; P(x = x_i)$$

$$= \sum (x_i^2 - 2x_i\mu + \mu^2)\,(P(x = x_i))$$

$$= \sum x_i^2 P(x = x_i) - 2\sum x_i \mu P(x = x_i) + \mu^2 \sum P(x = x_i)$$

$$= \sum x_i^2 P(x = x_1) - 2\mu^2 + \mu^2$$

$$\boxed{\sigma^2 = \sum x_i^2 \, P(x = x_i) - \mu^2}$$

Standard deviation $= \sigma$

* Probability mass function

Let X be discrete R.V. . A probability mass function is defined as,

$$f(x) = P\{x : X(x_i = x)\}$$

* Distribution function

Let X be a discrete R.V. , then distribution function of X is given by

$$F(x) = P(X = x_i) = \sum_{x_i < x} p_i$$

* Probability Density Function (PDF) [continuous]

- Let $P(x)$ be a probability function in the interval $[a,b]$. then P that variable value x to lie in Interval $[a,b]$ is given by,

$$\boxed{P[a \leq x \leq b] = \int_a^b f(x)\,dx}$$

- The $f(x)$ is called PDF if and only if the total probability is unity means

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \& \quad f(x) \geq 0.$$

Q:- Find the value of K for PDF

$$f(x) = \begin{cases} Kx^2 & ; 0 \le x \le 3 \\ 0 & ; \text{otherwise} \end{cases}$$

Sol^n:-

$$P = \int_0^3 f(x)\,dx + \int 0\,dx$$

$$1 = \int_0^3 kx^2\,dx = K\left[\frac{x^3}{3}\right]_0^3 = K \cdot 9$$

$$K = \frac{1}{9}$$

* Binomial Probability distribution [Type: Discrete]

↑

only for n outcomes

p = success

q = failure

$\boxed{p + q = 1}$

$q = 1 - p.$

$$\boxed{P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}}$$

n = repeatation of experiment

frequency = N·p(r)

NOTE: Here N value is small, nearly 10 or 20.

* Mean and S.D. of binomial distribution

1. Mean / First moment (μ_1') about origin

$$\boxed{\text{Mean} = np}$$

2. S.D. / second moment (μ_2') about origin

$$\boxed{\text{S.D.} = \sqrt{npq}}$$

Q:- A coin is tossed 4 times. What is the prob. of getting

(i) 2 head (ii) atleast 2 head

Sol:- $r = 2,\ n = 4 \quad p = \frac{1}{2},\ q = \frac{1}{2}$

$$P(i) = {}^4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2 = \frac{\cancel{4}^{2} \times 3}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$$

$$P(ii) = {}^4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2 + {}^4C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right) + {}^4C_4 \left(\tfrac{1}{2}\right)^4 \cdot \left(\tfrac{1}{2}\right)^0$$

$$P(ii) = \frac{3}{8} + \frac{4}{24} + \frac{1}{24} = \frac{3}{8} + \frac{1}{4} + \frac{1}{16} = \frac{6+4+1}{16} = \frac{11}{16} = 0.6875$$

Q:- The Prob. that bomb dropped from plane will strike the target is $\frac{1}{5}$. If 6 bombs are dropped. Find the prob. that

(i) Exactly two will strike the target.

(ii) Atleast two will strike the target.

Sol:- $n = 6,\ P(r) = \frac{1}{5},\ P(q) = \frac{4}{5}$

$$P(i) = {}^6C_2 \left(\tfrac{1}{5}\right)^2 \left(\tfrac{4}{5}\right)^4 = \frac{6 \times \cancel{8}}{\cancel{2}} \cdot \frac{1}{\cancel{8}} \cdot \frac{1}{5} \cdot \frac{\cancel{4}^{2} \cdot 4 \cdot 4 \cdot 4}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{128 \cdot 6}{125 \cdot 25} = 0.2457$$

$$P(ii) = 1 - {}^6C_1 \left(\tfrac{1}{5}\right)^1 \left(\tfrac{4}{5}\right)^5 - {}^6C_2 \left(\tfrac{1}{5}\right)^2 \left(\tfrac{4}{5}\right)^4 = 1 - 6 \cdot \frac{1}{5} \cdot \frac{4^5}{5^5} - \frac{6\cancel{5}}{2} \cdot \frac{1}{\cancel{5} \cdot 5} \cdot \frac{4^4}{5^4} = 0.60678 - 0.24576$$

$$= \underline{\underline{0.36102}}$$

* Poisson's distribution (Discrete)

p = probability of success

q = " of failure

n is large value

$$P(r = \text{success}) = \frac{e^{-m} \cdot m^r}{r!} \quad ; \quad m = \text{mean} = np$$

n = no. of trials

r = required success

Frequency of r success = n . P(r)

Q:- Find the probability that almost 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective.

Sol:- n = 200

$m = np = 200 \times 0.02 = 4$

$$P = e^{-4} \cdot \left[\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} \right]$$

$$= (2.718)^{-4} \left[\frac{1}{1} + 4 + 8 + \frac{\cancel{64}\ 32}{\cancel{6}\ 3} + \frac{\cancel{64}\ 32}{3 \cdot \cancel{2}} + \frac{4 \cdot \cancel{4} \cdot \cancel{4} \cdot 16\ 2}{5 \cdot \cancel{4} \cdot 3 \times \cancel{2}} \right]$$

$$= (2.718)^{-4} \left[13 + 10.666 + \overset{10.666}{\cancel{64}} + 8.533 \right]$$

$$= (2.718)^{-4} \left[\overset{42.865}{\cancel{08.999}} \right]$$

$$\boxed{P = 0.7851}$$

Q:- A traffic after imposes on an average 5 number of penalties daily on traffic violators. Assume that the no. of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ____.

Sol^n:- $m=5$; $r=0,1,2,3$

$$P = e^{-5}\left[\frac{5^0}{1}+\frac{5^1}{1}+\frac{5^2}{2}+\frac{5^3}{6}\right]$$

$$= (0.00673)(1+5+12.5+20.83)$$

$$\boxed{P = 0.26469}$$

* Normal Distribution (Type I continuous)

$x \rightarrow$ lies in the interval.

$\mu =$ mean & $\sigma =$ standard deviation

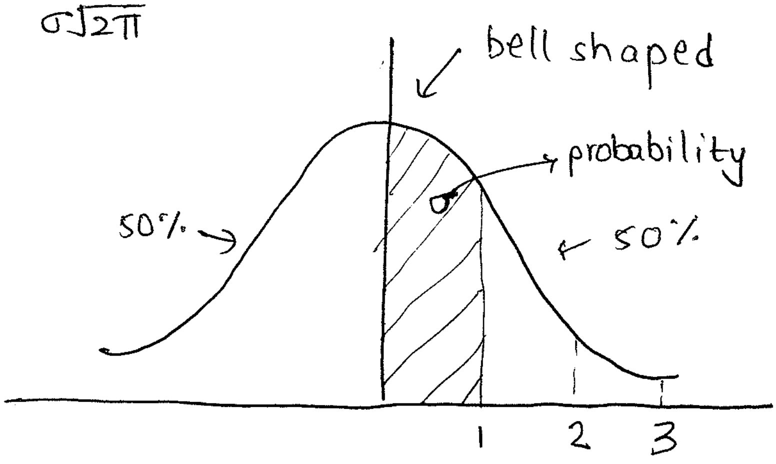
We defined probability density

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$$-\infty < x < \infty$$

If $\mu = 0$

$$\therefore f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$$



shaded area shows P between different interval of z.

Standard normal distribution function
$(\mu > 0, \sigma > 1)$

*Working rule:

Step:1 Convert x into standard normal variable by formula $z = \frac{x-\mu}{\sigma}$

Step:2 Find the limit of z corresponding to limit of x

when $x = a$, then $z = \frac{a-\mu}{\sigma}$

when $x = b$, then $z = \frac{b-\mu}{\sigma}$

Step:3 $P(a \le x \le b) = P\left(\frac{a-\mu}{\sigma} \le z \le \frac{b-\mu}{\sigma}\right)$

Step:4 Use normal table

Remember:

$$\boxed{\begin{array}{l} P(0 \le z \le 1) = 0.3417 \\ P(0 \le z \le 2) = 0.4771 \\ P(0 \le z \le 3) = 0.99 \end{array}}$$

Q:- 1000 students have written an exam with the μ of test is 35 and σ is 5. Assuming distribution to be normal. i) Find how many student got marks between 25 and 40

ii) How many got ~~answe~~ greater than 40 marks

iii) How many got < 20 marks

Sol^n:- (i) $x = (25, 40)$

$$Z = \left(\frac{25-35}{5}, \frac{40-35}{5}\right)$$

$$Z = (-2, 1)$$

$$P(-2 \le z \le 0) + P(0 \le z \le 1)$$

$$0.4771 + 0.3417$$

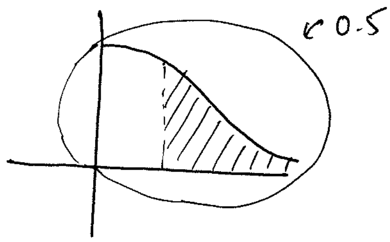
$$= 0.8188$$

No. of student $= 0.8188 \times 1000$
$= 818.8$
$\simeq 819$

(ii) >40 marks

$$z_1 = \frac{40-35}{5}, \quad z_2 = \frac{\infty - 35}{5}$$

$$z_1 = 1 \qquad z_2 = \infty$$



$$Z = (1, \infty)$$

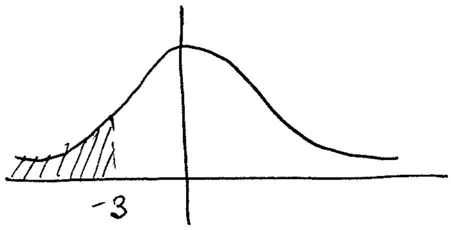
$$P = 0.5 - P(0 \le z \le 1)$$

$$= 0.5 - 0.3417$$

$$P = 0.1583$$

$\therefore$ No. of student $= 0.1583 \times 1000$
$= 158.3 \simeq 158$

(iii) <20 marks



$$z_1 = \frac{-\infty - 35}{5}, \quad z_2 = \frac{20-35}{5}$$

$$z_1 = -\infty \qquad z_2 = -3$$

$$Z = (-\infty, -3)$$

$P = 0.5 - 0.499$

$= 0.001$

$\therefore$ no. of students $= 0.001 \times 1000$

$= 1$

Q:- A class of 1st year B.TECH students is comprises of a batch A, B, C and D. Each consists of 30 students. It is found that sectional mark of student in EG in batch C have a mean of 6.6 and S.D. of 2.3. The mean and S.D. of the marks for entire class are 5.5 and 4.2 respectively. It is decided by the course instructor to normalize the marks of students of all batches to have same mark and S.D. as that of entire class. Due to this, marks of the students in batch C are change from 8.5 to ___.

Sol^n: 120 students are there

$\mu = 5.5$, $\sigma = 4.2$

→ In C batch

$\mu_c = 6.6$, $\sigma_c = 2.3$, $x_c = 8.5$

→ Normal score of class batch C

$$\frac{x_c - \mu_c}{\sigma_c} = \frac{8.5 - 6.6}{2.3} = 0.826$$

Normal score

$$z = \frac{x - \mu}{\sigma} = \frac{x - 5.5}{4.2} = 0.826$$

$x = 8.9$

$\therefore \boxed{x = 9}$

Q:- A nationalized bank has found that daily balance available in its saving account follows a normal distribution with the mean of rupees 500 and S.D. 50. The % of saving account holder who maintains an average daily balance more than rupees 500 is ___.

→ $\mu = 500$, $\sigma = 50$

For >500 $\qquad\qquad$ $500 \le x \le \infty$

$$z_1 = \frac{500-500}{50} \qquad\qquad z_2 = \frac{\infty - 500}{50}$$

$$z_1 = 0 \qquad\qquad z_2 = \infty$$

$$z = (0, \infty)$$

$$P = P(0 \le z < \infty)$$

$$= 0.5$$

$$P = 50\%$$

*Exponential distribution (Type: Continuous R.V.)

- A continuous RV whose PDF is given for some $\lambda > 0$ given by

$$p(r) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \ge 0 \\ 0, & \text{if } x < 0 \end{cases}$$

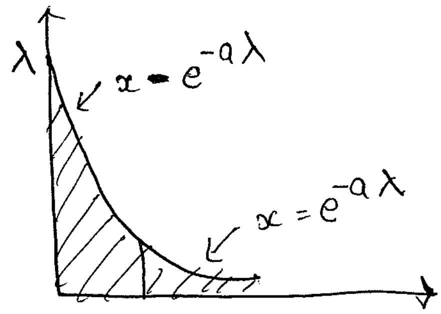
is said to be exponential R.V. with parameter $\lambda > 0$.

*Cumulative Distribution Function (CDF)

$$P[p(x)] = \int_0^{\infty} f(x)\,dx$$

$$= \int_0^{\infty} \lambda e^{-\lambda x} dx = \frac{\lambda e^{-\lambda x}}{-\lambda} = [-e^{-\lambda x}]_0^{\infty}$$

$$= 1$$



$$P[x \le a] = \int_0^a f(x)\,dx$$

$$= \int_0^a \lambda e^{-\lambda x}\,dx$$

$$= [e^{-\lambda x}]_0^a$$

$$= 1 - e^{-a\lambda}$$

* Mean of exponential distribution $= 1/\lambda$

Variance $= \frac{1}{\lambda^2}$

* STATISTICS

* Arithmetic Mean

⇒ Arithmetic mean of row data

$\bar{x} = \frac{\Sigma x}{n}$ x = value of an observation

n = no. of observation

⇒ Arithmetic mean for group data [frequency distribution]

$\bar{x} = \frac{\Sigma f(x)}{\Sigma f}$

* Median [central value]

is the central value of distribution in the sense that the no. of value less than the median is equal to no. of value greater than median.

⇒ Median for row data

- If we have r value of x, they can be arrange in ascending order $x_1, x_2, \ldots, x_n$

Suppose n is odd

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

n is even, we have two middle

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ value} + \left(\frac{n}{2}+1\right)^{th} \text{ value}}{2}$$

Median for group data

1. Identify the median class which contain the middle observation $\left(\frac{N+1}{2}\right)^{th}$ observation. This can be done by observing the 1^{st} ~~order~~ class in which cumulative frequency is equal to or more than $\frac{N+1}{2}$.

   Here $n = \Sigma f$

2. Median can be calculated as follows:

$$\text{Median} = L + \left[\frac{\left(\frac{N+1}{2}\right) - (F+1)}{f_m}\right] \times h$$

L = lower limit of median class

$N = \Sigma f$

F = cumulative frequency of class immediately preceding the median class

f_m = frequency of median class

h = width of median class.

Q:- Four fair six-sided dice are rolled. The probability that the sum of results being 22 is $X/1296$. The value of X is ____.

$6+6+6+6 = 24$ (maximum sum)

(we can interchange position also) $6, 6, 6, 4 = \frac{4!}{3!} = 4$ ways

$6, 6, 5, 5 = \frac{4!}{2! \cdot 2!} = 6$ ways

$P = \frac{6+4}{6 \cdot 6 \cdot 6 \cdot 6} = \frac{10}{1296}$

$$\boxed{X = 10}$$

Mode:

is defined as value of the variable which occurs most frequency

→ Mode for row data:

- In row data, most frequently occuring observation is mode.
- If there is more than one data with highest frequency, then each of them is mode. Thus we have unimodel (single model), bimodel (double model), trimodel (three model).

→ Mode for group data

Mode is the value of x for which frequency is maximum. If the value of x are grouped into classes.

1. Identify the class which has the largest frequency (model class).
2. Calculate mode as follows:-

$$\text{Mode} = L + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2} \right) \cdot h$$

L = low limit of model class

f_0 = largest class (freq. of model class)

f_1 = freq. in class preceeding model class

f_2 = freq. in class next to model class

h = width of model class.

Q:- Data relating to height of 352 students are given. Find the model height

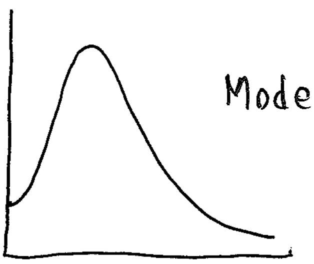
Height (in feet)	no. of student	
3.0 – 3.5	12	
3.5 – 4.0	37	
4.0 – 4.5	79	$= f_1$
L = $\boxed{4.5 - 5.0}$	$\boxed{152}$	$= f_0$
5.0 – 5.5	65	$= f_2$
5.5 – 6.0	7	
	352	

⇒

$$\text{Mode} = 4.5 + \left(\frac{73}{160}\right) 0.5$$

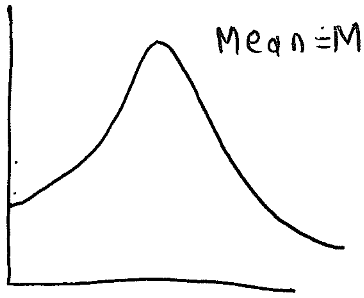
$$= 4.5 + 0.228$$

$$\text{Mode} = 4.728 \text{ feet}$$



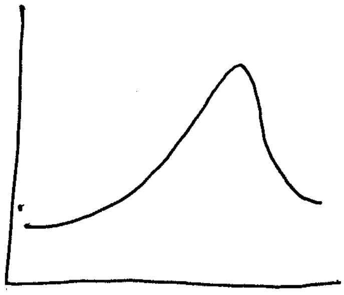
Mode ≤ Median ≤ Mean

(a) positive skewed



Mean = Median = Mode

(b) Symmetric



Mean ≤ Median ≤ Mode

(c) Negative skewed

$$\boxed{MODE = 3MEDIAN - 2MEAN}$$

Q:- The spot speed observed at the road section are 66, 62, 45, 79, 32, 51, 56, 60, 53, 49. Median speed is ___

Ascending order

32, 45, 49, 51, 53, 56, 60, 62, 66, 79

$$\text{Median} = \frac{53+56}{2} = 54.5$$

* Standard deviation

is measure of dispersion (variation) among data.

=) Standard variation of row data

$$\sigma = \sqrt{\frac{\Sigma(x_1 - x)^2}{n}}$$

→ Square of standard deviation is called variance

* Co-efficient of variation

$$\boxed{C.V. = \frac{\sigma}{\mu} \times 100}$$

Q:- If S.D. of spot speed of vehicle in highway 8.8 kmph & mean speed of vehicle is 33 kmph. The co-efficient of variation is

Soln:- $\sigma = 8.8$

$\mu = 33$

$$C.V. = \frac{\sigma}{\mu} = \frac{8.8}{33} = 0.266$$

$$C.V.\% = 26.67\%$$

Q:- Consider the following table giving the marks obtained by student in exam. calculate median.

Mark range	f (no. of students)	c.f.
0-20	2	2
20-40	3	5
40-60	10	15 = F
60-80	15 = f_m	30
80-100	20	50
	50	

Here, $\frac{N+1}{2} = \frac{51}{2} = 25.5$

$$\text{Median} = L + \left(\frac{\frac{N+1}{2} - (F+1)}{f_m}\right) \cdot h$$

$$= 60 + \left(\frac{25.5 - (15+1)}{15}\right) \cdot 20$$

$$= 60 + \frac{9.5}{15} \cdot 20$$

$$= 60 + 12.66$$

$$\text{Median} = 72.66$$