

Long Answer Type Questions

[4 Marks]

Que 1. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area.

Sol. Let the sides of the triangle be $12x$, $17x$ and $25x$

Perimeter of the triangle = 540 cm

$$\therefore 12x + 17x + 25x = 540$$

$$\Rightarrow 54x = 540 \Rightarrow x = 10$$

Let $a = 12x = 12 \times 10 = 120$ cm

$$b = 17x = 17 \times 10 = 170$$
 cm

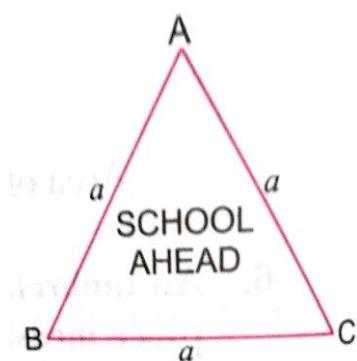
$$c = 25x = 25 \times 10 = 250$$
 cm

$$\therefore s = \frac{a+b+c}{2} = \frac{120+170+250}{2} = \frac{540}{2} = 270 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{270(270-120)(270-170)(270-250)} \\ &= \sqrt{270 \times 150 \times 100 \times 20} = 100\sqrt{27 \times 15 \times 20} \\ &= 100 \times 9 \times 5 \times 2 = 9000 \text{ cm}^2 \end{aligned}$$

Que 2. A traffic signal board indicating 'SCHOOL AHEAD' is an equilateral triangle with side a . Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Sol.



Perimeter of the signal board,

$$2s = a + a + a \Rightarrow s = \frac{3}{2}a$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3a}{2} \left(\frac{3}{2}a - a\right) \left(\frac{3}{2}a - a\right) \left(\frac{3}{2}a - a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4} a^2 \text{ sq. units}$$

Now, if perimeter = 180 cm

$$3a = 180 \Rightarrow a = 60 \text{ cm}$$

$$\therefore \text{Area of signal board } \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3} \text{ cm}^2$$

Que 3. The lengths of the sides of a triangle are 7 cm, 13 cm and 12 cm. Find the length of perpendicular from the opposite vertex to the side whose length is 12 cm.

Sol.

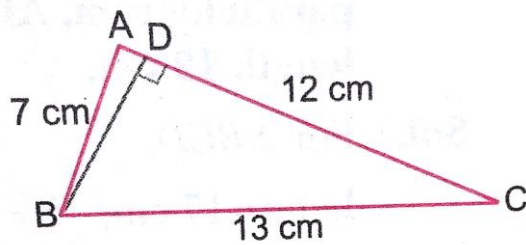


Fig. 12.6

Let, $a = 7 \text{ cm}$, $b = 13 \text{ cm}$, $c = 12 \text{ cm}$

$$\therefore s = \frac{a + b + c}{2} = \frac{7 + 13 + 12}{2} = \frac{32}{2} = 16 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-7)(16-13)(16-12)} \\ &= \sqrt{16 \times 9 \times 3 \times 4} = 24\sqrt{3} \text{ cm}^2 \end{aligned}$$

Also, Area of $\triangle ABC = \frac{1}{2} AC \cdot BD$

$$24\sqrt{3} = \frac{1}{2} \times 12 \times BD \Rightarrow BD = \frac{24\sqrt{3} \times 2}{12} = 4\sqrt{3} \text{ cm}$$

Que 4. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm. Find the cost of polishing the tiles at the rate of 50 p per cm^2 .

Sol. Measures of the sides of the triangular tile are 28 cm, 9 cm and 35 cm.

Let $a = 28 \text{ cm}$, $b = 9 \text{ cm}$, $c = 35 \text{ cm}$

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2} = \frac{28+9+35}{2} = 39 \text{ cm}$$

$$\begin{aligned}
 \therefore \text{Area of one triangular tile} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{36(36-28)(36-9)(36-35)} \\
 &= \sqrt{36 \times 8 \times 27 \times 1} = 36\sqrt{6} \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{So, area of 16 triangular tile} &= 16 \times 36\sqrt{6} \text{ cm}^2 = 576\sqrt{6} \text{ cm}^2 \\
 &= 576 \times 2.45 = 1411.2 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, cost of polishing the tiles at the rate of ₹ } \frac{1}{2} \text{ per cm}^2 &= ₹ \frac{1}{2} \times 1411.2 \\
 &= ₹ 705.60
 \end{aligned}$$

Que 5. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm, and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Sol. Let $a = 26\text{cm}$, $b = 28\text{cm}$, $c = 30\text{cm}$

$$\therefore s = \frac{a+b+c}{2} = \frac{26+28+30}{2} = \frac{84}{2} = 42$$

$$\begin{aligned}
 \therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{42(42-26)(42-28)(42-30)} \\
 &= \sqrt{42 \times 16 \times 14 \times 12} \\
 &= \sqrt{2 \times 3 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 2 \times 2 \times 3} \\
 &= 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 336 \text{ cm}^2
 \end{aligned}$$

Now, Area of parallelogram = Area of triangle

$$\Rightarrow \text{Base} \times \text{height} = 336$$

$$\Rightarrow 28 \times \text{height} = 336$$

$$\Rightarrow \text{height} = \frac{336}{28} = 12 \text{ cm}$$

Que 6. The length of two adjacent sides of a parallelogram are 17 cm and 12 cm. One of its diagonals is 25 cm long. Find the area of the parallelogram. Also find the altitude from vertex on the side of length 12 cm.

Sol.

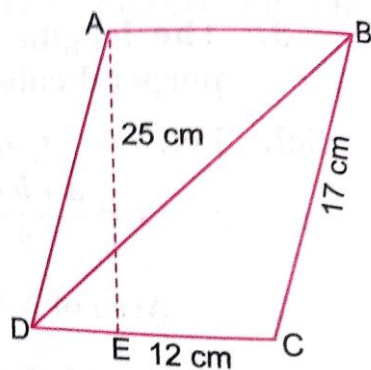


Fig. 12.7

For $\triangle BCD$.

Let $a = 17$ cm, $b = 12$ cm, $c = 25$ cm

So its semi-Perimeter, $s = \frac{a+b+c}{2}$

$$= \frac{17+12+25}{2} = 27 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \triangle BCD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{27(27-17)(27-12)(27-25)} \\ &= \sqrt{27 \times 10 \times 15 \times 2} = 90 \text{ cm}^2 \end{aligned}$$

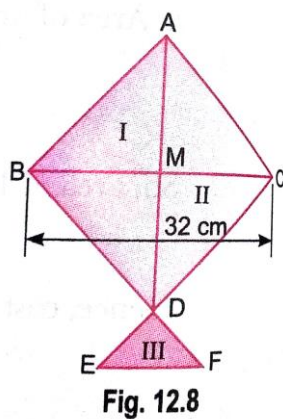
$$\begin{aligned} \text{Now, area of parallelogram ABCD} &= 2 \times \text{Area of } \triangle BCD \\ &= 2 \times 90 = 180 \text{ cm}^2 \end{aligned}$$

Also, area of parallelogram ABCD = DC $\times$ AE

$$\therefore 180 = 12 \times \text{AE}$$

$$\Rightarrow \text{AE} = \frac{180}{12} = 15 \text{ cm}$$

Que 7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and side 6 cm each is to be made of three different shades as shown in Fig. 12.8. How much paper of each shade has been used in it?



Sol. As the diagonals of a square bisect each other at right angle

$$\therefore \text{AM} = \text{DM} = \frac{32}{2} = 16 \text{ cm}$$

Area of shade I = Area of shade II = Area of $\triangle ABD$

$$= \frac{1}{2} \times \text{AD} \times \text{BM} = \frac{1}{2} \times 32 \times 16 = 256 \text{ cm}^2$$

For the area I = Area of shade III

$$\text{Area of isosceles } \triangle DEF = \frac{a}{4} \sqrt{4b^2 - a^2}$$

$$= \frac{8}{4} \sqrt{4}(6)^2 - 8^2$$

$$= 2\sqrt{144 - 64} = 2\sqrt{80} = 8\sqrt{5} \text{ cm}^2$$

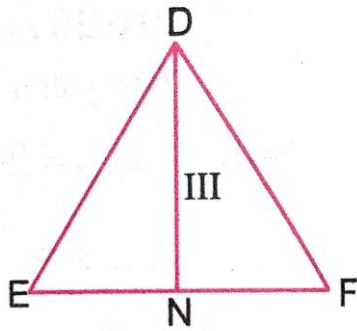


Fig. 12.9

Que 8. The perimeter of triangle is 50cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Sol. Let the length of the smaller side = x

According to the statement, other two sides of the triangle will be x + 4 and 2x - 6

Perimeter of triangle = x + x + 4 + 2x - 6

$$\Rightarrow 50 = 4x - 2 \Rightarrow 4x = 52 \Rightarrow x = 13$$

$\therefore$ Sides of triangle are = 13, (13 + 4), (2 x 13 - 6) = 13 cm, 17 cm, 20 cm

Let, a = 13 cm, b = 17 cm and c = 20 cm

$$\therefore s = \frac{a+b+c}{2} = \frac{13+17+20}{2} = 25 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{25(25-13)(25-17)(25-20)} \\ &= \sqrt{25 \times 12 \times 8 \times 8} = \sqrt{5 \times 5 \times 3 \times 4 \times 4 \times 2 \times 5} \\ &= 20\sqrt{30} = 20 \times 5.48 = 109.6 \text{ cm}^2 \end{aligned}$$

Que 9. The area of a trapezium is 475 cm² and the height is 19 cm. Find the lengths of its parallel sides if one side is 4 cm greater than the other.

Sol.

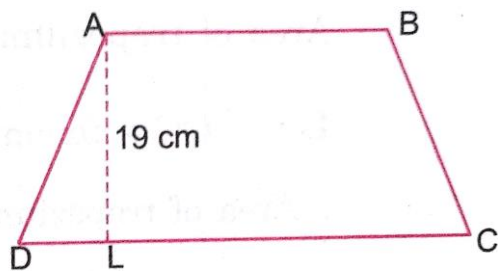


Fig. 12.10

Let $AB = x$ cm $\therefore DC = x + 4$

Area of trapezium $ABCD = \frac{1}{2}(AB + DC)AL$

$$475 = \frac{1}{2}(x + x + 4) \times 19$$

$$950 = (2x + 4)19 \Rightarrow 38x + 76 = 950$$

$$\Rightarrow 38x = 950 - 76 \Rightarrow x = \frac{874}{38} \Rightarrow x = 23$$

$\therefore$ Sides of trapezium are 23, $23 + 4$ i.e., 23, 27 cm

Que 10. The triangle side walls of a flyover is used for advertisement. The sides of walls are 122 m, 22 m and 120 m as shown in figure. The advertisement yields an earning of ₹5000 per m^2 per year. A company fixed one of its wall for 3 months. How much rent did it ray?

Sol. Let the sides of triangle wall be
 $a = 122$ m, $b = 22$ m, $c = 120$ m

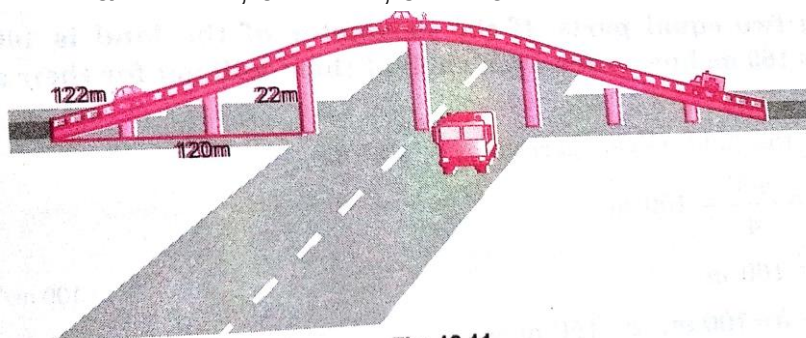


Fig. 12.11

$$\therefore s = \frac{a + b + c}{2} = \frac{122 + 22 + 120}{2} = \frac{264}{2} = 132 \text{ m}$$

$$\begin{aligned} \therefore \text{Area of triangle wall} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{132(132-122)(132-22)(132-120)} \\ &= \sqrt{132 \times 10 \times 110 \times 12} \end{aligned}$$

$$= \sqrt{12 \times 11 \times 10 \times 11 \times 10 \times 12}$$

$$= 10 \times 11 \times 12 = 1320 \text{ m}^2$$

Now, yearly rent = ₹5000 per m²

$$\therefore \text{Monthly rent} = ₹\left(\frac{5000}{12}\right) \text{ Per m}^2$$

$$\therefore \text{Rent paid by company fo the three months} = ₹\frac{5000}{12} \times 3 \times 1320$$

$$= ₹5000 \times 330 = ₹16,50,000$$