**CBSE Sample Paper -04**
**SUMMATIVE ASSESSMENT –I**
**Class – X Mathematics**

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Time allowed: 3 hours　　　　　　　　　　　　　　　　　　　　Maximum Marks: 90

**General Instructions:**

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

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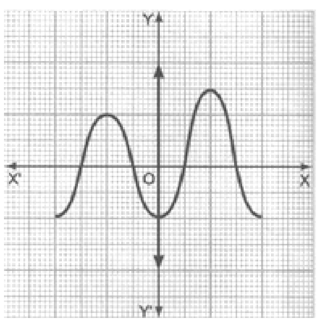
## SECTION – A

1. From the given graph, find the number of zeroes of the corresponding polynomial.




2. Evaluate: $\cos 48^o \cos 42^o - \sin 48^o \sin 42^o$ .

3. Given that $\sin\theta = \frac{a}{b}$, find the value of $\tan\theta$.

4. If the mode of a distribution is 8 and its mean is also 8, then find median.

## SECTION - B

5. A man goes 24 m towards West and then 10 m towards North. How far is the from the starting point?

6. If p(x) = $2x^2 - 3x + 4$, find p(3) and p(–1).

7. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

8. Taking A = 60° and B = 30°, verify that

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

9. In given figure, DE||BC. If AD = x, DB = x-2, AE = x+2 and EC = x-1. Find the value of x.

10. The following data gives the distribution of total household expenditure (in rupees) of manual workers in a city:

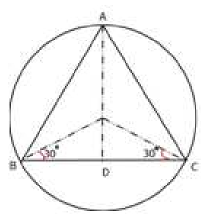
| Expenditure (Rs) | Frequency | Expenditure (Rs) | Frequency |
|---|---|---|---|
| 1000-1500 | 24 | 3000-3500 | 30 |
| 1500-2000 | 40 | 3500-4000 | 22 |
| 2000-2500 | 33 | 4000-4500 | 16 |
| 2500-3000 | 28 | 4500-5000 | 7 |

Find the average expenditure which is being done by the maximum number of manual workers.

**SECTION – C**

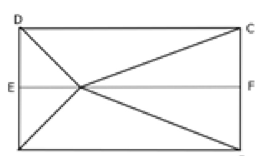
11. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

12. An equilateral triangle is inscribed in a circle of radius 6 cm. Find its side.




13. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of: $\sin A \cos C + \cos A \sin C$

14. A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$




15. If 3 times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also, if 7 times the smaller number is divided by the larger one, we get 5 as quotient and 1 as the remainder. Find the numbers.

16. Draw the graph of the polynomial $f(x) = -4x^2 + 4x - 1$. Also, find the vertex of this parabola.

17. Prove that one of every three consecutive positive integers is divisible by 3.

18. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

19. If the median of the following frequency distribution is 46, find the missing frequencies.

Variable:

| Less than20 | Less than20 | Less than20 | Less than20 | Less than20 | Less than20 | Less than20 | Less than20 | Less than20 |
|---|---|---|---|---|---|---|---|---|
| 0 | 4 | 16 | 30 | 46 | 66 | 82 | 92 | 100 |

20. Prove that if three or more parallel lines are intersected by two transversals, prove that the intercepts made by them on the transversals are proportional.

## SECTION – D

21. If $p$ is a prime number, then prove that $\sqrt{p}$ is irrational.

22. The mean of the following frequency table is 50. But the frequencies $f_1$ and $f_2$ in class 20-40 and 60-80 are missing. Find the missing frequencies.

| Class | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | Total |
|---|---|---|---|---|---|---|
| Frequency | 17 | $f_1$ | 32 | $f_2$ | 19 | 120 |

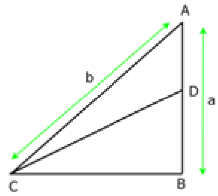
23. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

24. Prove that the areas of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

25. Rama went to a stationary stall and purchased 2 pencils and 3 erasers for Rs 9. Her friend Sonal saw the new variety of pencils and erasers with Rama and she also bought 4 pencils and 6 erasers of the same kind for Rs 18. Represent this situation algebraically and graphically.

26. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

27. In the given figure, AD = DB and ∠B is a right angle. Find $\sin^2\theta + \cos^2\theta$.




28. If $\operatorname{cosec}\theta - \sin\theta = 1 \, and \, \sec\theta - \cos\theta = m$, prove that $l^2m^2(l^2 + m^2 + 3) = 1$.

29. The distribution below gives the marks of 100 students of a class.

| Marks | 0-5 | 2-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 |
|---|---|---|---|---|---|---|---|---|
| No. of Students | 4 | 6 | 10 | 10 | 25 | 22 | 18 | 5 |

Draw a less than type and a more than type ogive from the given data. Hence, obtain the median marks from the graph.

30. Verify that the numbers given alongside the cubic polynomial below are their zeros. Also verify the relationship the between the zero and the coefficients. $x^3 - 4x^2 + 5x - 2;\ 2,1,1$

31. a. After every 6 months, price of petrol increases at the rate of Rs 4 per litre. Taking price of petrol in December 2010 as x and present price of petrol as y, form a linear equation showing the price of petrol in December 2014.

b. Due to continuous rise in the price of petrol, people are more interesting in CNG whose price is increasing at the rate of Rs 3 per litre in a year. Form a linear equation taking price of CNG in December 2010 as a and in December 2014 as b.

c. Which value is depicted by using CNG over petrol?

# CBSE Sample Paper -04
## SUMMATIVE ASSESSMENT –I
## Class – X Mathematics

Time allowed: 3 hours | ANSWERS | Maximum Marks: 90

### SECTION – A

1. The number of zeros is four as the graph intersects the X-axis at four points.

2. $\cos 48^o \cos 42^o - \sin 48^o \sin 42^o = \cos(90^o - 42^o)\cos(90^o - 48^o) - \sin 48^o \sin 42^o$

$= \sin 42^o \sin 48^o - \sin 48^o \sin 42^o \qquad [\because \cos(90-\theta) = \sin\theta]$

$= 0.$

3. $\sin\theta = \dfrac{a}{b}$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a/b}{\sqrt{\dfrac{b^2-a^2}{b}}} = \frac{2}{\sqrt{b^2-a^2}}$$

4. Mode = 8; Mean = 8; Median = ?

Relation among mean, median and mode is

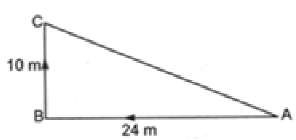
3 median = mode + 2 mean

$3 \times \text{median} = \text{mode} + 2\text{mean}$

$$\text{median} = \frac{8+16}{3} = \frac{24}{3} = 8$$

### SECTION – B

5. By Pythagoras Theorem




$AC^2 = AB^2 + BC^2 = (24)^2 + (10)^2$

$AC^2 = 676$

$AC = 26m$

$\therefore$ The man is 26 m away from the starting point.

6. We have,

$p(x) = 2x^2 - 3x + 4$

$\Rightarrow \quad p(3) = 2 \times 3^2 - 3 \times 3 + 4$

$= 2 \times 9 - 9 + 4$

$= 18 - 9 + 4 = 22 - 9 = 13$

$p(-1) = 2 \times (-1)^2 - 3 \times (-1) + 4$

$= 2 + 3 + 4$

$= 9$

7. Let $x = 0.3\overline{178}$

Then x = 0.3178178178… ..(i)

10 x= 3.178178178… ..(ii)

10000 x = 3178.178178… ..(iii)

On subtracting (ii) from (iii), we get

$$9990x = 3175 \Rightarrow x = \frac{3175}{9990} = \frac{635}{1998}$$

$$\therefore \ 0.3\overline{178} = \frac{635}{1998}$$

8. A = 60° and B = 30°

$\Rightarrow \quad A - B = 60° - 30° = 30°$

$\therefore \quad \sin(A - B) = \sin 30° = \frac{1}{2}$

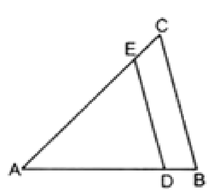
$\sin A\cos B - \cos A\sin B = \sin 60°\cos 30° - \cos 60°\sin 30°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore \quad \sin(A - B) = \sin A\cos B - \cos A\sin B$

9. In $\Delta ABC$, we have,




DE||BC,

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \dfrac{x}{x-2} = \dfrac{x+2}{x-1} \quad \Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$

10. We observe that the class 1500-2000 has the maximum frequency 40. So, it is the modal class such that

$l = 1500$, $h = 500$, $f = 40$, $f_1 = 24$ and $f_2 = 33$

$\therefore$ Mode = $l + \dfrac{f - f_1}{2f - f_1 - f_2} \times h = 1500 + \dfrac{40-24}{80-24-33} \times 500$

$$= 1500 + \frac{16}{23} \times 500 = 1847.826$$

**SECTION – C**

11. Let the required numbers be $x$ and $y$.

Then, $x + y = 16$

And, $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{3} \quad \Rightarrow \quad \dfrac{x+y}{xy} = \dfrac{1}{3} \quad \Rightarrow \quad \dfrac{16}{xy} = \dfrac{1}{3} \quad \Rightarrow \quad xy = 48$

We can write

$x - y = \sqrt{(x+y)^2 - 4xy}$

$= \sqrt{(16)^2 - 4 \times 48}$

$= \sqrt{256 - 192} = \sqrt{64} = \pm 8$

$\therefore \quad x + y = 16 \quad$ ...(i)

$x - y = 8 \quad$ ...(ii)

Or, $\quad x + y = 16 \quad$ ...(iii)

$x - y = -8 \quad$ ...(iv)

On solving (i) and (ii), we get $x = 12$ and $y = 4$

On solving (iii) and (iv), we get $x = 4$ and $y = 12$

Thus, the required numbers are 12 and 4.

12. Let ABC be an equilateral triangle inscribed in a circle of radius 6 cm. Let O be the centre of the circle. Then,

OA = OB = OC = 6 cm

Let OD be perpendicular from O on side BC. Then, D is the mid-point of BC and OB and OC are bisectors of $\angle$B and $\angle$C respectively.

$\therefore \quad \angle OBD = 30°$

In $\Delta OBD$, right angled at D, we have

$\angle OBD = 30°$ and OB = 6 cm

$\therefore \quad \cos\angle OBD = \frac{BD}{OB}$

$\Rightarrow \quad \cos 30° = \frac{BD}{6}$

$\Rightarrow \quad BD = 6\cos 30°$

$\Rightarrow \quad BD = 6\times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$\Rightarrow \quad BC = 2BD = 2 \times 3\sqrt{3} = 6\sqrt{3}$

Thus, the side of the equilateral triangle is $6\sqrt{3}$.

13. We have a right-angled $\Delta ABC$ in which $\angle B = 90^o$

And, $\tan A = \frac{1}{\sqrt{3}}$

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $BC = K$ *and* $AB = \sqrt{3}k$

$\therefore$ By Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (\sqrt{3})^2 + (k)^2 = 3k^2 + k^2$

$\Rightarrow AC^2 = 4k^2 \qquad \therefore AC = 2k$

Now, $\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{k}{2k} = \frac{1}{2}; \qquad \cos A = \frac{Base}{Hypotenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{Perpendicular}{Hypotenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}; \; \cos C = \frac{Base}{Hypotenuse} = \frac{k}{2k} = \frac{1}{2}$

$\sin A.\cos C + \cos A.\sin C = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$

14. Let ABCD be the given rectangle and let O be a point within it. Join OA, OB, OC and OD.

Through O, draw EOF || AB. Then, ABFE is a rectangle.

In right triangles OEA and OFC, we have

$OA^2 = OE^2 + AE^2$amd$OC^2 = OF^2 + CF^2$

$\Rightarrow \quad OA^2 + OC^2 = (OE^2 + AE^2) + (OF^2 + CF^2)$

$\Rightarrow \quad OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ ...(i)

Now, in right triangles OFB and ODE, we have

$OB^2 = OF^2 + FB^2$ and $OD^2 = OE^2 + DE^2$

$\Rightarrow \quad OB^2 + OD^2 = (OF^2 + FB^2) + (OE^2 + DE^2)$

$\Rightarrow \quad OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$

$= OE^2 + OF^2 + CF^2 + AE^2$ [$\because$ DE = CF and AE = BF] ...(ii)

From (i) and (ii), we get

$OA^2 + OC^2 = OB^2 + OD^2$

15. Let the larger number be $x$ and smaller one be $y$. We know that

Dividend = (Divisor × Quotient) + Remainder ...(i)

When $3x$ is divided by $y$, we get 4 as quotient and 3 as remainder. Therefore, by using (i), we get

$3x = 4y + 3 \quad \Rightarrow \quad 3x - 4y - 3 = 0$ ...(ii)

When $7y$ is divided by $x$, we get 5 as quotient and 1 as remainder. Therefore, by using (i), we get

$7y = 5x + 1 \quad \Rightarrow \quad 5x - 7y + 1 = 0$ ...(iii)

Solving equations (ii) and (iii), by cross-multiplication, we get

$$\frac{x}{-4-21} = \frac{-y}{3+15} = \frac{1}{-21+20}$$

$\Rightarrow \quad x = 25$ and $y = 18$

Thus, the required numbers are 25 and 18.

16. Let $y = f(x)$ or $y = -4x^2 + 4x - 1$

The following table gives the values of y for various values of x.

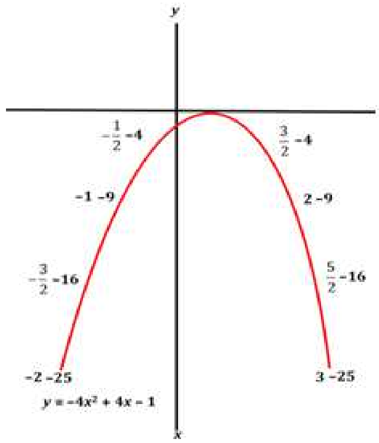
| $x$ | $-2$ | $-\frac{3}{2}$ | $-1$ | $-\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $1$ | $\frac{3}{2}$ | $2$ | $\frac{5}{2}$ | $3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y = -4x^2 + 4x - 1$ | $-25$ | $-16$ | $-9$ | $-4$ | $-1$ | $0$ | $-1$ | $-4$ | $-9$ | $-16$ | $-25$ |

Thus, the following points lie on the graph of $y = -4x^2 + 4x - 1$:

$(-2, -25)$, $\left(-\frac{3}{2}, -16\right)$, $(-1, -9)$, $\left(-\frac{1}{2}, -4\right)$, $(0, -1)$, $\left(\frac{1}{2}, 0\right)$, $(1, -1)$, $\left(\frac{3}{2}, -4\right)$, $(2, -9)$, $\left(\frac{5}{2}, -16\right)$ and

$(3, -25)$.

Plot these points on a graph paper and draw a free hand smooth curve passing through these points.




The shape of the curve is shown in the figure. It is a parabola opening downward having its vertex at $\left(\frac{1}{2}, 0\right)$.

17. Let $n$, $n + 1$ and $n + 2$ be three consecutive positive integers.

We know that $n$ is of the form $3q$, $3q + 1$ or $3q + 2$.

So, we have the following cases:

Case I: When $n = 3q$

In this case, $n$ is divisible by 3 but $n + 1$ and $n + 2$ are not divisible by 3.

Case II: When $n = 3q + 1$

In this case, $n + 2 = 3q + 1 + 2 = 3(q + 1)$,

which is divisible by 3 but $n$ and $n + 1$ are not divisible by 3.

Case III: When $n = 3q + 2$

In this case, $n + 1 = 3q + 1 + 2 = 3(q + 1)$,

which is divisible by 3 but $n$ and $n + 2$ are not divisible by 3.

Thus, one of $n$, $n + 1$ and $n + 2$ is divisible by 3.

18. We have,

LHS $= \left(m^2 + n^2\right)\cos^2\beta$

$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ $\quad \left[\because m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta}\right]$

$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = n^2$ = RHS

19. We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

| Class intervals | Frequency (*f*) | Cumulative frequency (*cf*) |
|---|---|---|
| 20-30 | 4 | 4 |
| 30-40 | 12 | 16 |
| 40-50 | 14 | 30 |
| 50-60 | 16 | 46 |
| 60-70 | 20 | 66 |
| 70-80 | 16 | 82 |
| 80-90 | 10 | 92 |
| 90-100 | 8 | 100 |
| $N = \sum f_i = 100$ | | |

Here, $N = \sum f_i = 100 \Rightarrow \frac{N}{2} = 50$

We observe that the cumulative frequency just greater than $\frac{N}{2} = 50$ is 66 and the corresponding class is 60-70.

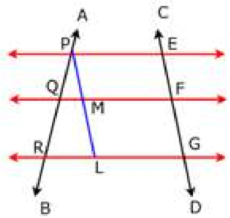
So, 60-70 is the median class.

∴ $l = 60, f = 20$, F = 46 and $h = 10$

Now, median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$= 60 + \frac{50-46}{20} \times 10 = 62$

20. Given: Three parallel lines l, m and n which are cut by the transversals AB and CD in P, Q, R and E, F, G, respectively.




To prove: $\frac{PQ}{QR} = \frac{EF}{FG}$

Construction: Draw PL || CD meeting the lines m and n in M and L, respectively.

Proof: Since PE || MF and PM || EF,

$\therefore$ PMFE is a parallelogram.

$\Rightarrow$ PM = EF ...(i)

Also, MF || LG and ML || FG,

$\therefore$ MLFG is a parallelogram.

$\Rightarrow$ ML = FG ...(ii)

Now, in ΔPRL, we have

QM || RL

$\Rightarrow \frac{PQ}{QR} = \frac{PM}{ML}$ [By Thale's Theorem]

$\Rightarrow \frac{PQ}{QR} = \frac{EF}{FG}$ [Using (i) and (ii)]

## SECTION – D

21. Let *p* be a prime number and if possible, let $\sqrt{p}$ be rational.

Let its simplest form be $\sqrt{p} = \frac{m}{n}$, where m and n are integers having no common factor other than 1, and $n \neq 0$.

Then $\sqrt{p} = \frac{m}{n}$

$\Rightarrow p = \frac{m^2}{n^2}$ [On squaring both sides]

$\Rightarrow pn^2 = m^2$ ...(i)

$\Rightarrow$ *p* divides $m^2$ [$\because$ *p* divides $pn^2$]

$\Rightarrow$ *p* divides *m* [$\because$ *p* is prime and *p* divides $m^2 \Rightarrow p$ divides *m*]

Let $m = pq$ for some integer $q$.

Putting $m = pq$ in (i), we get

$pn^2 = p^2q^2$

$\Rightarrow$ $n^2 = pq^2$

$\Rightarrow$ $p$ divides $n^2$ [$\because p$ divides $pq^2$]

$\Rightarrow$ $p$ divides $n$ [$\because p$ is prime and $p$ divides $n^2 \Rightarrow p$ divides $n$]

Thus, $p$ is a common factor of $m$ and $n$.

But this contradicts the fact that $m$ and $n$ have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{p}$ is rational.

Thus, $\sqrt{p}$ is irrational.

22. Let the assumed mean be A = 50 and $h = 20$.

Calculation of mean

| Class | Frequency $f_i$ | Mid-values | $u_i = \frac{x_i - A}{h}$ | $f_iu_i$ |
|---|---|---|---|---|
| 0-20 | 17 | 10 | –2 | –34 |
| 20-40 | $f_1$ | 30 | –1 | $-f_1$ |
| 40-60 | 32 | 50 | 0 | 0 |
| 60-80 | $f_2$ | 70 | 1 | $f_2$ |
| 80-100 | 19 | 90 | 2 | 38 |
| $N = \sum f_i = 68 + f_1 + f_2 \sum f_iu_i = 4 - f_1 + f_2$ | | | | |

We have,

$N = \sum f_i = 120$ [Given]

$\Rightarrow$ $68 + f_1 + f_2 = 120$

$\Rightarrow$ $f_1 + f_2 = 52$ ...(i)

Now,

Mean = 50

$$\Rightarrow \quad A + h\left\{\frac{1}{N}\sum f_iu_i\right\} = 50$$

$$\Rightarrow \quad 50 + 20 \times \left\{\frac{4 - f_1 + f_2}{120}\right\} = 50$$

$\Rightarrow \quad 50+\frac{4-f_1+f_2}{6}=50$

$\Rightarrow \quad \frac{4-f_1+f_2}{6}=0$

$\Rightarrow \quad 4-f_1+f_2=0$

$\Rightarrow \quad f_1-f_2=4$ ...(ii)

Solving equations (i) and (ii), we get

$f_1 = 28$ and $f_2 = 24$

23. Let the speed of the boat in still water be $x$ km/hr and the speed of the stream be $y$ km/hr.

Then,

Speed upstream = $(x - y)$ km/hr

Speed downstream = $(x + y)$ km/hr

Now,

Time taken to cover 32 km upstream = $\frac{32}{x-y}$ hrs

Time taken to cover 36 km downstream = $\frac{36}{x+y}$ hrs

But, total time of journey is 7 hours

$\therefore \quad \frac{32}{x-y}+\frac{36}{x+y}=7$ ...(i)

Time taken to cover 40 km upstream = $\frac{40}{x-y}$

Time taken to cover 48 km downstream = $\frac{48}{x+y}$

In this case, total time of journey is given to be 9 hours.

$\therefore \quad \frac{40}{x-y}+\frac{48}{x+y}=9$ ...(ii)

Putting $\frac{1}{x-y}$ = $u$ and $\frac{1}{x+y}$ = $v$ in equations (i) and (ii), we get

$32u + 36v = 7 \quad \Rightarrow \quad 32u + 36v - 7 = 0$ ...(iii)

$40u + 48v = 9 \quad \Rightarrow \quad 40u + 48v - 9 = 0$ ...(iv)

Solving these equations by cross-multiplication, we get

$$\frac{u}{36\times-9-48\times-7}=\frac{-v}{32\times-9-40\times-7}=\frac{1}{32\times48-40\times36}$$

$\Rightarrow \quad \frac{u}{-324+336}=\frac{-v}{-288+280}=\frac{1}{1536-1440}$

$\Rightarrow \quad \frac{u}{12}=\frac{v}{8}=\frac{1}{96}$

$\Rightarrow \quad u=\frac{12}{96}=\frac{1}{8}$ and $v=\frac{8}{96}=\frac{1}{8}$

Now, $u = \frac{1}{8} \quad \Rightarrow \quad \frac{1}{x-y}=\frac{1}{8} \quad \Rightarrow \quad x - y = 8$ ...(v)

and, $v = \frac{1}{12} \quad \Rightarrow \quad \frac{1}{x+y}=\frac{1}{12} \quad \Rightarrow \quad x + y = 12$ ...(vi)

Solving equations (v) and (vi), we get $x = 10$ and $y = 2$

Thus, speed of the boat in still water = 10 km/hr

Speed of the stream = 2 km/hr

24. Given: ΔABC~ΔDEF and AX and DY are bisector of ∠A and ∠D respectively.

To prove: $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)}=\frac{AX^2}{DY^2}$

Proof: Since the ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

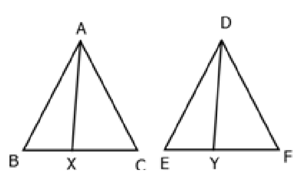
$\therefore \quad \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)}=\frac{AB^2}{DE^2}$ ...(i)

Now, ΔABC~ΔDEF

$\Rightarrow \quad \angle A = \angle D$

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle D$

$\Rightarrow \quad \angle BAX = \angle EDY$




Thus, in triangles ABX and DEY, we have

$\angle BAX = \angle EDY$ and $\angle B = \angle E$ [∵ ΔABC~ΔDEF]

So, by AA-similarity criterion, we have

$\Delta ABX \sim \Delta DEY$

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AX}{DY}$

$\Rightarrow \quad \dfrac{AB^2}{DE^2} = \dfrac{AX^2}{DY^2}$ ...(ii)

From (i) and (ii), we get

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AX^2}{DY^2}$$

25. Let the cost of 1 pencil be Rs$x$ and that of one eraser be Rs$y$.

It is given that Rama purchased 2 pencils and 3 erasers for Rs 9.

$\therefore \quad 2x + 3y = 9$

It is also given that Sonal purchased 4 pencils and 6 erasers for Rs 18.

$\therefore \quad 4x + 6y = 18$

Algebraic representation: The algebraic representation of the given situation is

$2x + 3y = 9$

$4x + 6y = 18$

Graphical representation: In order to obtain the graphical representation of the above pair of linear equations, we find two points on the line representing each equation. That is, we find two solutions of each equation. Let us find these solutions. We will try to find solutions having integral values.

We have,

$2x + 3y = 9$

Putting $x = -3$, we get

$-6 + 3y = 9 \quad \Rightarrow \quad 3y = 15 \quad \Rightarrow \quad y = 5$

Putting $x = 0$, we get

$0 + 3y = 9 \quad \Rightarrow \quad y = 3$

Thus, two solutions of $2x + 3y = 9$ are:

| | | |
|---|---|---|
| $x$ | –3 | 0 |
| $y$ | 5 | 3 |

We have,

$4x + 6y = 18$

Putting $x = 3$, we get

$12 + 6y = 18 \Rightarrow \quad 6y = 6 \quad \Rightarrow \quad y = 1$

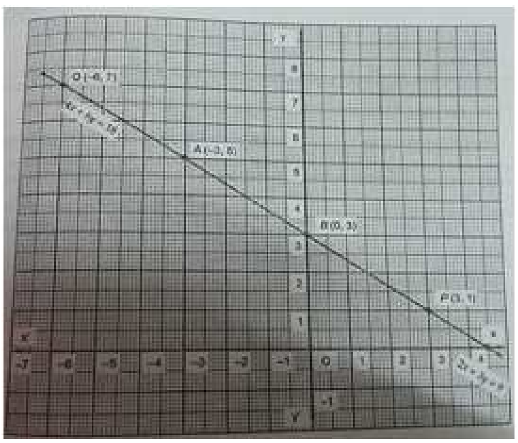
Putting $x = -6$, we get

$-24 + 6y = 18 \Rightarrow \quad 6y = 42 \quad \Rightarrow \quad y = 7$

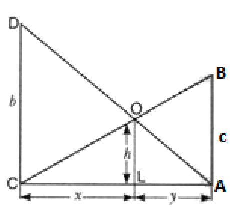
Thus, two solutions of $4x + 6y = 18$ are:

| | | |
|---|---|---|
| $x$ | 3 | −6 |
| $y$ | 1 | 7 |

Now, we plot the points A(−3, 5) and B(0, 3) and draw the line passing through these points to obtain the graph of the line $2x + 3y = 9$. Points P(3, 1) and Q(−6, 7) are plotted on the graph paper and we join them to obtain the graph of the line $4x + 6y = 18$. We find that both the lines AB and PQ coincide.



26. Let AB and CD be two poles of height a and b metres respectively such that the poles are p metres apart i.e., AC = p metres. Suppose the lines AD and BC meet at O such that OL = h metres.




Let CL = x and LA = y. Then, x + y = p.

In $\Delta ABC\ and\ \Delta LOC$, we have

$\angle CAB = \angle CLO$ [Each equal to $90^o$]

$\angle C = \angle C$ [Common]

$\therefore\ \Delta ABC \sim \Delta LOC$ [By AA criterion of similarity]

$\Rightarrow \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{P}{x} = \frac{a}{h}$

$\Rightarrow \; = \dfrac{ph}{a}$ ...(i)

In $\Delta ALO \; and \; \Delta ACD$, we have

$\angle ALO = \angle ACD,$ we have

$\angle ALO = \angle ACD$ $[Each\, equal\, to\, 90^o]$

$\angle A = \angle A$ $[Common]$

$\therefore \; \Delta ALO \sim \Delta ACD$ [By AA criterion of similarity]

$\Rightarrow \dfrac{AL}{AC} = \dfrac{OC}{DC} \quad \Rightarrow \dfrac{y}{p} = \dfrac{h}{b}$

$\Rightarrow y = \dfrac{ph}{b}$ ...(ii)

From (i) and (ii), we have

$x + y = \dfrac{ph}{a} + \dfrac{ph}{b}$

$\Rightarrow \; p = ph\left(\dfrac{1}{a} + \dfrac{1}{a}\right) \; [\because \; x + y = p]$

$\Rightarrow 1 + h\left(\dfrac{a+b}{ab}\right) \Rightarrow h = \dfrac{ab}{a+b} metres.$

Hence, the height of the intersection of the lines joining the top of each pole of the foot of the opposite pole is $\dfrac{ab}{a+b}$ metres.

27. We have,

AB = a

$\Rightarrow$ AD + DB = a

$\Rightarrow$ AD + AD = a

$\Rightarrow$ 2AD = a $\Rightarrow$ AD = $\dfrac{a}{2}$

Thus, AD = DB = $\dfrac{a}{2}$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $b^2 = a^2 + BC^2$

$\Rightarrow$ $BC^2 = b^2 - a^2$

$\Rightarrow$ BC² = $\sqrt{b^2 - a^2}$

Thus, in ΔBCD, we have

Base = BC = $\sqrt{b^2 - a^2}$ and perpendicular = BD = $\frac{a}{2}$

Applying Pythagoras theorem in ΔBCD, we have

$BC^2 + BD^2 = CD^2$

$\Rightarrow \left(\sqrt{b^2 - a^2}\right)^2 + \left(\frac{a}{2}\right)^2 = CD^2$

$\Rightarrow CD^2 = b^2 - a^2 + \frac{a^2}{4}$

$\Rightarrow CD^2 = \frac{4b^2 - 4a^2 + a^2}{4}$

$\Rightarrow CD^2 = \frac{4b^2 - 3a^2}{4}$

$\Rightarrow CD = \frac{\sqrt{4b^2 - 3a^2}}{2}$

Now, $\sin\theta = \frac{BD}{CD} = \frac{\frac{a}{2}}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{a}{\sqrt{4b^2 - 3a^2}}$

And, $\cos\theta = \frac{BC}{CD} = \frac{\sqrt{b^2 - a^2}}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}$

Thus, $\sin^2\theta + \cos^2\theta = \left(\frac{a}{\sqrt{4b^2 - 3a^2}}\right)^2 + \left(\frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}\right)^2$

$= \frac{a^2}{4b^2 - 3a^2} + \frac{4\left(b^2 - a^2\right)}{4b^2 - 3a^2}$

$= \frac{a^2 + 4b^2 - 4a^2}{4b^2 - 3a^2}$

$= \frac{4b^2 - 3a^2}{4b^2 - 3a^2} = 1$

28. L.H.S = $l^2 m^2 \left(l^2 + m^2 + 3\right)$

$= \left(\cos ec\theta - \sin\theta\right)\left(\sec\theta - \cos\theta\right)^2 \left\{\left(\cos ec\theta - \sin\theta\right)^2 + \left(\sec\theta - \cos\theta\right)^2 + 3\right\}$

$$=\left(\frac{1}{\sin\theta}-\sin\theta\right)^2\left(\frac{1}{\cos\theta}-\cos\theta\right)^2\left\{\left(\frac{1}{\sin\theta}-\sin\theta\right)^2+\left(\frac{1}{\cos\theta}-\cos\theta\right)^2+3\right\}$$

$$=\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2+\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2+3\right\}$$

$$=\left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2+\left(\frac{\sin^2\theta}{\cos\theta}\right)+3\right\}$$

$$=\frac{\cos^2\theta}{\sin^2\theta}\frac{\sin^4\theta}{\cos^2\theta}\left\{\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3\right\}$$

$$=\cos^2\theta\sin^2\theta\left\{\frac{\cos^6\theta+\sin^6\theta+3\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}\right\}$$

$$=\cos^6\theta+\sin^2\theta+3\cos^2\theta\sin^2\theta$$

$$=\left[\left(\cos^2\theta\right)^3+\left(\sin^2\theta\right)\right]+3\cos^2\theta\sin^2\theta$$

$$=\left[\left(\cos^2\theta+\sin^2\theta\right)^3-3\cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)\right]+3\cos^2\theta\sin^2\theta$$

$$\left[\because a^3+b^3=\left(a+b\right)^3-3ab\left(a+b\right)\right]$$

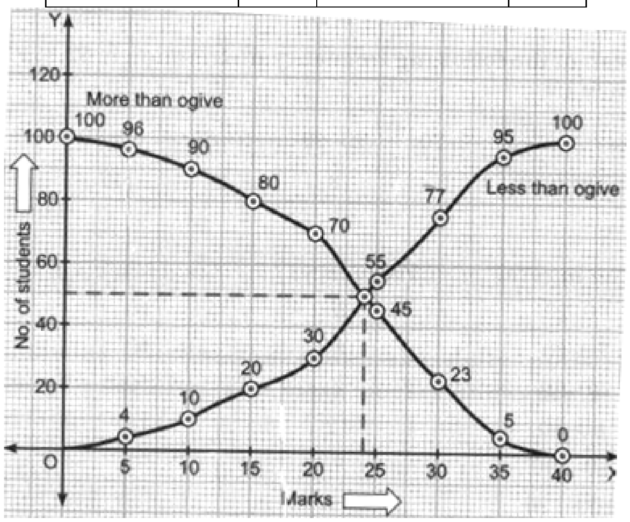
$= 1-3\cos^2\theta\sin^2\theta+3\cos^2\theta\sin^2\theta$ $\quad\left[\because \cos^2\theta+\sin^2\theta=1\right]$

= 1 = R.H.S

29.

| Marks | Cf | Marks | Cf |
|---|---|---|---|
| Less than 5 | 4 | More than 0 | 100 |
| Less than10 | 10 | More than 5 | 96 |
| Less than15 | 20 | More than 10 | 90 |
| Less than 20 | 30 | More than 15 | 80 |
| Less than 25 | 55 | More than 20 | 70 |
| Less than 30 | 77 | More than | 45 |

| | | | |
|---|---|---|---|
| | | 25 | |
| Less than 35 | 95 | More than 30 | 23 |
| Less than 40 | 100 | More than 35 | 5 |




30. Let p(x) = $x^3 - 4x^2 + 5x - 2$

On comparing with general polynomial p(x) = $ax^3 + bx^2 + cx + d$, we get
a = 1, b = -4, c =5 and d = -2
Given zeroes 2, 1, 1
Therefore, p(2) = $(2)^2 - 4(2)^2 + 5(2) - 2$
= 8 – 16 + 10 – 2 = 0
and p(1) = $(1)^3 - 4(1)^2 + 5(1) - 2$
= 1 – 4 + 5 - 2 = 0
Hence, 2, 1 and 1 are the zeroes of the given cubical polynomial.
Again consider, $\alpha = 1$, $\beta = 1$, $\gamma = 1$
Therefore, $\alpha + \beta + \gamma = 2 + 1 + 1 = 4$

and $\alpha + \beta + \gamma = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha$ = (2)(1) + (1)(1) + (1)(2)
= 2 + 1 + 2 = 5

and $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a} = \frac{5}{1} = 5$

$\alpha\beta\gamma$ = (2)(1)(1) = 2

and $\alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a} = \frac{-(-2)}{1} = 2$

31.

a. Price of petrol in December 2010 = x

   Price of petrol in December 2014 = y

   Price of petrol increased in 1 year = 4 × 2 = Rs 8

   Price of petrol increased in 4 years (December 2010- December 2014) = 8 × 4 = Rs 32

   Equation representing the price of petrol in December 2014 = y = x + 32

b. Price of CNG in December 2010 = a

   Price of CNG in December 2014 =b

   Price of CNG increased in 1 year = Rs 3

   Price of CNG increased in 4 years (December 2010- December 2014) = 3 × 4 = Rs 12

   Equation representing the price of CNG in December 2014 = b = a + 12

c. The value depicted by using CNG over petrol is environmental protection.