Long Answer Type Questions – I

Q. 1. Prove that 33! Is divisible by 2^{15}, what is the largest integer n such that 33! Is divisible by 2^n?

Sol. $33! = 33 \times 32 \times 31 \times 30 \times 29 \ldots \times 16$

$\times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 33 \times (2)^5 \times 31 \times 30 \times 29 \times \ldots \times (2)^4 \ldots$

$\times (2)^3 \times 7 \times 6 \times 5 \times (2)^2 \times 3 \times (2)^1 \times 1$

Now take all 2's out we get

$33! = (2)^5 \cdot (2)^4 \cdot (2)^3 \cdot (2)^2 \cdot (2)^1$ $[33 \times 31 \times 30 \times 29 \times \ldots \times 1]$

$[33 \times 31 \times 30 \times 29 \times \ldots \times 1]$ contains numbers 6, 10, 12, 14, 18, 20, 22, 24, 26, 28, 30, which have 2 as one of their factors.

So, the product of 2 present in these numbers

$= 2 \times 2 \times 2^2 \times 2 \times 2 \times 2^2 \times 2 \times 2^3 \times 2 \times 2^2 \times 2$

$= 2^{16}$

Therefore, we have $2^{15} \times 2^{16} = 2^{31}$

Thus, 31 is the largest integer such that 33! Is divisible by $2n$.

Q. 2. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

Sol. VCVCVCVC : vowels can occupy V places and consonants can occupy c places.

Possible combinations of arrangements = 4!

= 24

Similar 4 consonants (different) and 4 spaces combinations of arrangements = 4! = 24

Thus, total combinations = $24 \times 24 = 576$

Q. 3. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice. [DDE-2017]

B A
Sol. Number of seats □ □ □ □ □ □ □ □ □ □
A B

Assume two particular person sit by $= 2 \times 10 \times 9$

$= 180$ ways

8 persons sit by $= 8!$ Ways!

Hence total persons sit by $= 180 \times 8!$

$= 180 \times 40320$

$= 7257600$

Q.4. Using the digits 0,1,2,2,3 how many numbers greater than 20000 can be made?

Sol. Total number of digits = 0,1,2,2,3

Total number formed by these digit = $\frac{5!}{2!} = \frac{120}{2} = 60$

Total number formed by starting 0 = $\frac{4!}{2!} = \frac{24}{2} = 12$

Total number formed by starting 1 = $\frac{4!}{2!} = \frac{24}{2} = 12$

Total number formed greater than 2000

$= 60 - 12 - 12 = 36$

Q. 5. A polygon has 35 diagonals. Find the number of its sides.

Sol. Let n be the number of sides of a polygon and D be the number of diagonals of that polygon

We know that, D = n_{c_2} - $- n = \frac{n(n-3)}{2}$

$\therefore 35 = \frac{n^2 - 3n}{2}$

$\Rightarrow n^2 - 3n - 70 = 0$

$\Rightarrow (n - 10)(n + 7) = 0$

$\Rightarrow n = 10, -7$

Since, sides cannot be negative, therefore $n = 10$.

Hence, polygon is a decagon.

Q. 6. Using the letters of the word,' ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both H occur together. [DDE-2017]

Sol. There are 11 letters in the word 'ARRANGEMENT' out of which 2A's, 2E's, 2R's and 2M's.

Considering both A, both E, both R and both M together, 8 letters should be counted as 4.

So, there are total 7 letters (AA EE RR MM G M T)

These 7 letters can be arranged in 7! Ways

Hence, total ways = 7!

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

Q. 7. Find the number of word with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in dictionary, what will be the 50th word? [KVS 2016 Guwahati]

Sol. Number of words made from AGAIN $= \frac{5!}{2!}$

$= 60$

To get the number of words starting with A, we fix the letter A at the extreme left position, we then rearrange the remaining 4 letters taken all at a time. There will be as many arrangements of these 4 letters taken 4 at a time as there are permutations of 4 different things taken 4 at a time.

Hence, the number of words starting with $A = 4!$

$= 24$

Then, starting with G, the number of words $= \frac{4!}{2!} = 12$

As after placing G at the extreme left position, we are left with the letters $A, A\ I$ and N. Similarly, there are 12 words starting with the next letter I.

Total number of words so far obtained $= 24 + 12 + 12+ = 48$

The 49th word is NAAGI. The 50th word is NAAIG.

Q. 8. If $^nC_r : {}^nC_{r+1} = 1:2$ and $^nC_{r+1} : {}^nC_{r+2} = 2:3$, determine the values of n and r.

Sol. Given,

$$\Rightarrow \frac{\left(\frac{n!}{r!\,(n-r)!}\right)}{\left(\frac{n!}{(r+1)!\,(n-r-1)!}\right)} = \frac{1}{2}$$

$$\Rightarrow \frac{(r+1)!\,(n-r-1)!}{r!\,(n-r)!} = \frac{1}{2}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{1}{2}$$

$\Rightarrow n = 3r + 2$ (i)

Similarly,

$$\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{2}{3}$$

$$\Rightarrow \frac{\left(\frac{n!}{(r+1)!\,(n-r-1)!}\right)}{\left(\frac{n!}{(r+2)!\,(n-r-2)!}\right)} = \frac{2}{3}$$

$$\Rightarrow \frac{(r+2)!\,(n-r-2)!}{(r+1)!\,(n-r-1)!} = \frac{2}{3}$$

$$\Rightarrow \frac{r+2}{n-r-1} = \frac{2}{3}$$

$\Rightarrow 3r + 6 = 2n - 2r - 2$

$\Rightarrow 2n = 5r + 8$ (ii)

Solving (i) and (ii), we get

$6r + 4 = 5r + 8$

$\Rightarrow r = 4$

And $n = 14$

Q. 9. Find n if 16. ${}^{n+2}C_8 = 57\ {}^{n-2}P_4$ [DDE-2017]

Sol. Given,

$$\frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$$

$$\Rightarrow \frac{\left(\frac{(n+2)!}{8!\,(n+2-8)!}\right)}{\left(\frac{(n-2)!}{(n-2-4)!}\right)} = \frac{57}{16}$$

$$\frac{\left(\frac{(n+2)!}{8!\,(n-6)!}\right)}{\left(\frac{(n-2)!}{(n-6)!}\right)} = \frac{57}{16}$$

$$\frac{\left(\frac{(n+2)!}{8!}\right)}{\left(\frac{(n-2)!}{1!}\right)} = \frac{57}{16}$$

$$\Rightarrow \frac{(n+2)!}{(n-2)!} = \frac{57}{16} \times 8!$$

$$\Rightarrow \frac{(n+2)(n+1)(n)(n-1)(n-2)}{(n-2)!}$$

$$= \frac{57}{16} \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 57 \times 7 \times 6 \times 5 \times 4 \times 3$$

As L.H.S is the product of four consecutive natural numbers, hence

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 19 \times 3 \times 7 \times 6 \times 5 \times 4 \times 3$$

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 19 \times (3 \times 7) \times (4 \times 5) \times (6 \times 3)$$

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 19 \times 21 \times 20 \times 18$$

On rearranging, we get

$$\Rightarrow (n+2)(n+1)(n)(n-1) = 19 \times 21 \times 20 \times 18$$

$$\Rightarrow n + 2 = 21$$

$$\Rightarrow n = 21 - 2 = 19$$

Q.10. A committee of 7 has to be formed out of 9 boys and 4 girls. In how many ways can be this be done when the committee consist of

(i) exactly 3 girls

(ii) At most 3 girls? [KVS 2016, Mumbai]

Sol. A committee of 7 has to be formed from 9 B and 4 G.

(i) exactly 3 girls

$$= {}^9C_4 \times {}^4C_3$$

$$= \frac{\lfloor 9}{\lfloor 4 \lfloor 5} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 1} = \frac{9 \times 8 \times 7 \times 6 \times \lfloor 5}{3 \times 2 \times \lfloor 5}$$

$$= 72 \times 7 = 504$$

(ii) at most 3 girls

(a) No girl and 7 boys
(b) 1 girl and 6 boys
(c) 2 girls and 5 boys
(d) 3 girls and 4 boys

∴ The committee consists of at most 3 girls

$$= {}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$$

$$= 36 + 336 + 1296 + 504$$

$$= 2172$$

Q.11. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has

(i) no girl
(ii) at least 3 girls
(iii) at least one girl and one boy? [DDE-2017]

Sol. (i) no girl

	Total No.	**No. to be chosen**	**No. of ways to Choose**
Girls	4	0	${}^4C_0$
Boys	7	5	${}^7C_5$

Total number of ways = ${}^4C_0 \times {}^7C_5$

$$= \frac{4!}{0!\,(4-0)!} \times \frac{7!}{5!\,(7-5)!}$$

$$= \frac{4!}{4!} \times \frac{7!}{5!\,2!}$$

$$= 1 \times \frac{7 \times 6}{2}$$

$$= 21$$

(ii) at least 3 girls?

Since, the team has to consist of at least 3 girls, the team can consist of

(a) 3 girls and 2 boys
(b) 4 girls and 1 boy

(a) 3 girls and 2 boys

	Total No.	No. to be chosen	No. of ways to Choose
Girls	4	3	${}^4C_3$
Boys	7	2	${}^7C_2$

Number of ways selecting = ${}^7C_2 \times {}^4C_3$

$= \frac{7!}{2!\,5!} \times \frac{4!}{3!\,1!}$

$= \frac{7 \times 6}{2 \times 1} \times 4$

$= 84$

(b) 4 girls and 1 boy

	Total No.	No. to be chosen	No. of ways to Choose
Girls	4	4	${}^4C_4$
Boys	7	1	${}^7C_1$

Number if ways selecting = ${}^7C_1 \times {}^4C_4$

$= \frac{7!}{1!\,6!} \times \frac{4!}{4!\,0!}$

$= 7 \times 1$

$= 7$

∴ Total number of ways $= 84 + 7 = 91$

(iii) at least one girl and one boy?

A group giving at least one boy and one girl will consist of

(a) 1 boy and 4 girls
(b) 2 boys and 3 girls
(c) 3 boys and 2 girls
(d) 4 boys and 1 girl

Number of ways of selecting 1 boy and 4 girls = ${}^7C_1 \times {}^4C_4 = 7$

Number of ways of selecting 2 boys and 3 girls = ${}^7C_1 \times {}^4C_3 = 84$

Number of ways of selecting 3 boys and 2 girls = ${}^7C_2 \times {}^4C_2 = 210$

Number of ways of selecting 4 boys and 1 girl = ${}^7C_4 \times {}^4C_1 = 140$

Hence, total number of ways = 7 + 84 + 210 + 140 = 441 ways.

Q.5. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together? [DDE-2017]

Sol. Since there is no condition for positive (+) sign, fix them in a row.

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There are 6 places in between each plus and one before and one before and one after these positive (+) sign.

i.e., these are 8 places for negative (-) sign and 5(-) negative signs are there.

∴ These negative (-) signs can be placed in $^8C_5$ ways.

$= {}^8C_5$

$$= \frac{8!}{3!\,5!}$$

$$= \frac{8.7.6}{3.2.1} = 56$$

Q.7. A student has to answer 10 question, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questons?

Sol. Combination from A and from B :: $\frac{4}{6}; \frac{5}{5}; \frac{6}{4}$

Number of way to get $\frac{4}{6}$ pattern = $^6C_4 \times {}^7C_6$

$$= \frac{6!}{2!\,4!} \times \frac{7!}{1!\,6!}$$

$$= \frac{6.5}{2.1} \times \frac{7}{1}$$

$$= 15 \times 7 = 105$$

Number of ways to get $\frac{5}{5}$ pattern = $^6C_5 \times {}^7C_5$

$$= \frac{6!}{1!\,5!} \times \frac{7!}{2!\,5!}$$

$$= \frac{6}{1} \times \frac{7 \times 6}{2 \times 1}$$

$$= 6 \times 21 = 126$$

Number of ways to get the $\frac{6}{4}$ pattern = $^6C_6 \times {}^7C_4$

$= \frac{6!}{0!\,6!} \times \frac{7!}{3!\,4!}$

$= 1 \times \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}$

$= 35$

Hence, total number of ways$= 105 + 126 + 35$

$= 266$

Q.8. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized? [DDE-2017]

Sol. Case 1: Both join

In case both the students decide to join,

${}^{13}C_8 \times {}^{2}C_2$ ways = ${}^{13}C_8 = \frac{13!}{8!5!}$ = 1287

Case 2: In case none of them join, it will be ${}^{13}C_{10}$

Ways ${}^{13}C_{10} = \frac{13!}{3!10!}$ = 286

Total number of cases are = ${}^{13}C_8 + {}^{13}C_{10}$

1287 + 286 = 1573

Q.9. How many different products can be obtained by multiplying two or more of the numbers 2,5,6,7,9?

Sol. The given numbers are 2,5,6,7,9.

The number of different products when 2 or more is taking = the number of ways of taking product of 2 number + number of ways of taking product 3 numbers + number of ways of taking product of 4 number + number of ways taking 5 together

$= {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= \frac{5!}{3!\,2!} + \frac{5!}{2!\,3!} + \frac{5!}{1!\,4!} + \frac{5!}{0!\,5!}$

$= \frac{5 \cdot 4}{2 \cdot 1} + \frac{5 \cdot 4}{2 \cdot 1} + \frac{5}{1} + 1$

$= 10 + 10 + 5 + 1$

$= 26$

Q.10. Determine the number of 5 cards combinations out of pack of 52 cards if at least 3 out of 5 cards are ace cards?

Sol. There are 4 ace cards in a pack of 52 cards, therefore we can choose maximum 4 ace cards.

Case 1: 3 cards of ace and 2 cards out of remaining 4 cards

i.e. 5 cards combinations out of 52 cards = $^4C_3 \times {}^{48}C_2$

$$= \frac{4!}{3!\,1!} \times \frac{48!}{2!\,46!}$$

$$= 4 \times \frac{48 \times 47}{2}$$

$$= 4512$$

Case 2: 4 cards of ace and 1 card out of 52 remaining cards

i.e. 5 cards combinations out of 52 cards

= $^4C_4 \times {}^{48}C_1$

$$= \frac{4!}{4!\,0!} \times \frac{48!}{1!\,7!}$$

$$= 1 \times 48$$

Hence, total number of combinations are

$$= 4512 + 48$$

$$= 4560$$

Q.11. Find the number of all possible arrangement of the letters of the word "MATHEMATICS" taken four from at a time. [DDE-2017]

Sol. The word MATHEMATICS consist of 11 letters:

(M,M), (A,A), (T,T), H,E,I,C,S

Case 1: In this case 2 similar and 2 similar letters are selected, number of arrangements

= $^3C_2 \times \frac{4!}{2!2!}$ = 756

Case 2: In this case all 4 letters selected are different, number of arrangements = $^8C_4 \times 4!$

= 1680

Therefore, total number of arrangement = 18 + 756 + 1680

= 2454

Q.12. Three married couple are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all ladies sit together. [DDE-2017]

Sol. (i) Three couples can be seated in a row in $^3P_3 = 3!$ Ways.

Now, in each couple, the spouses can be arranged in $^2P_2 = 2!$, ways

Thus for three couples, number of arrangements $= 2! \times 2! \times 2!$

$\therefore$ Total number of ways in which spouses are seated next to each other $= 3! \times 2! \times 2! \times 2! = 6 \times 2 \times 2 \times 2 = 48$ ways.

(ii) Now, if the three ladies are to be seated together, there are regard 3 ladies as one block. Therefore, there are now 4 people can be arranged in $^4P_4 = 4! = 24$ ways.

But 3 ladies can interchange their position in $3! = 6$ ways

$\therefore$ Total number of arrangements in which 3 ladies sit together $= 24 \times 6 = 144$.