

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q.1,2,4

(3 marks, 3 min.)

[9, 9]

Subjective Questions (no negative marking) Q.3,5,6,7

(4 marks, 5 min.)

[16, 20]

Match the Following (no negative marking) Q.8

(8 marks, 8 min.)

[8, 8]

1. If $f(x) = \frac{x-1}{x+1}$, then $f(f(ax))$ in terms of $f(x)$ is equal to

(A) $\frac{f(x)-1}{a(f(x)-1)}$

(B) $\frac{f(x)+1}{a(f(x)-1)}$

(C) $\frac{f(x)-1}{a(f(x)+1)}$

(D) $\frac{f(x)+1}{a(f(x)+1)}$

2. If $f(x) = ((\operatorname{sgn} x)^{\operatorname{sgn} x})^n$; n is an odd integer. Then

(A) $f(x)$ is an odd function

(B) $f(x)$ is an even function

(C) $f(x) = 0$

(D) none of these

3. Let $f(x) = \frac{2}{4^x + 2}$ for real numbers x . Evaluate : $f\left(\frac{1}{2011}\right) + f\left(\frac{2}{2011}\right) + \dots + f\left(\frac{2010}{2011}\right)$.

4. In which of the following functions, range is singleton set.

(A) $f(x) = [x] + [-x]$

(B) $f(x) = \{x\} + \{-x\}$

(C) $f(x) = |\operatorname{sgn}(x)|$

(D) $f(x) = [\sqrt{x - [x]}]$

where $[x]$, $\{x\}$ and $\operatorname{sgn}(x)$ are greatest integer function, fractional part function and signum function respectively.

5. Evaluate (i) $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$ (ii) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+x \sin x} - \sqrt{\cos 2x}}{\tan^2(x/2)} \right)$

6. Solve the inequality : $\left(2 \log_3^2 x - 3 \log_3 x - 8 \right) \left(2 \log_3^2 x - 3 \log_3 x - 6 \right) \geq 3$.

7. Let $f(x) = \frac{|x^3 - 6x^2 + 11x - 6|}{x^3 - 6x^2 + 11x - 6}$. Find the set of points 'a' where $\lim_{x \rightarrow a} f(x)$ does not exist.

8. **Column - I**

Column - II

(A) $\lim_{x \rightarrow 0} [\sin |x| - |x|] =$

(p) 0

(B) $\lim_{x \rightarrow 0} \left[\frac{x}{[x]} \right] =$

(q) 1

(C) $\lim_{x \rightarrow \frac{1}{2}} \left[x \left[\frac{1}{x} \right] \right] =$

(r) Does not exist

(D) $\lim_{x \rightarrow -1} \left[\frac{[x]}{x} \right] =$

(s) -1

($\because$ where $[.]$ denotes greatest integer function)

Answers Key

1. (C) 2. (A) 3. 1005 4. (D)

5. (i) $\frac{1}{\sqrt{2}}$ (ii) 6

6. $\left(0, \frac{1}{\sqrt{27}}\right] \cup \left[\frac{1}{3}, \sqrt{243}\right] \cup [27, \infty)$

7. $a = 1, 2, 3.$

8. $(A) \rightarrow (s), (B) \rightarrow (r), (C) \rightarrow (p), (D) \rightarrow (q)$