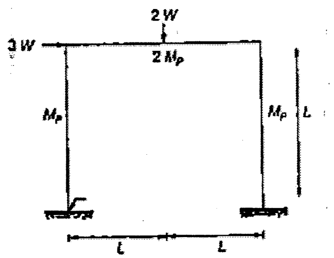
12
CHAPTER

Plastic Analysis and Design

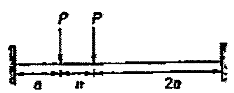
Q.1 What is the collapse load for the portal frame shown in the figure?



(a) $\frac{3M_p}{L}$ (b) $\frac{M_p}{L}$

(c) $\frac{0.5M_p}{L}$ (d) $\frac{1.4M_p}{L}$

Q.2 What is the collapse load for the beam shown in the figure?



(a) $\frac{16M_p}{9a}$ (b) $\frac{16M_p}{3a}$

(c) $\frac{4M_p}{3a}$ (d) $\frac{4M_p}{9a}$

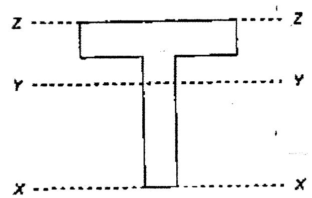
Q.3 The assumption of bilinearly idealised stress-strain relationship for plastic analysis and design is used for

(a) structural steel (mild steel)
(b) high tensile steel
(c) RCC structures
(d) aluminium structures

Q.4 The bending moment at a plastic hinge is

(a) equal to zero
(b) equal to yield moment of section
(c) equal to plastic moment of section
(d) greater than plastic moment of section

Q.5 If Y-Y is the centroidal axis of a T-beam section, subjected to plastic moment M_p, the neutral axis lies



(a) above zz (b) between yy and zz
(c) between xx and yy (d) below xx

Q.6 Which one of the following conditions regarding both elastic and plastic methods of analysis of indeterminate structures has be satisfied?

(a) Yield condition
(b) Mechanism condition
(c) Equilibrium condition
(d) Compatibility condition

Q.7 A beam of square cross-section of side x is composed of material whose yield stress in compression is 1.5 times the yield stress in tension. What is the distance of neutral axis from the centre for the fully plastic condition?

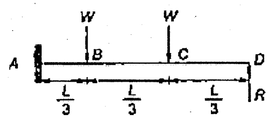
(a) $0.1x$ (b) $0.15x$
(c) $0.2x$ (d) $0.25x$

Q.8 A rectangular steel section of width b and depth h has been stressed upto yield point f_y upto a depth of $h/4$ from both the top and bottom face under the action of a moment M is

(a) $\frac{10}{24}bh^2 \cdot f_y$ (b) $\frac{1}{4}bh^2 \cdot fy$

(c) $\frac{11}{48}bh^2 \cdot fy$ (d) $\frac{13}{36}bh^2 \cdot fy$

Q.9 The plastic collapse load W_p for the propped cantilever supporting two point loads as shown in figure in terms of plastic moment, M_p is given by

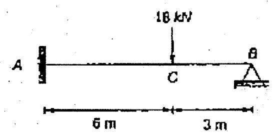


(a) $\frac{3M_p}{L}$ (b) $\frac{4M_p}{L}$

(c) $\frac{5M_p}{L}$ (d) $\frac{6M_p}{L}$

Common data for question 10 and 11

A propped cantilever beam is subjected to collapse load of 18 kN as shown in the figure.



Q.10 The plastic moment capacity of the beam A C B is

(a) 36 kN-m (b) 27 kN-m
(c) 18 kN-m (d) 12 kN-m

Q.11 If the yield stress (f_y = 250 MPa), the plastic section modulus is

(a) 192×10^3 mm³ (b) 108×10^3 mm³
(c) 96×10^3 mm³ (d) 108 mm³

Q.12 Match List-I with List-II and select the correct answer using the codes given below the lists :

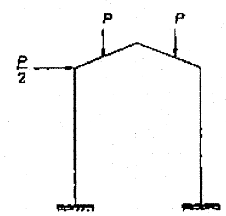
List-I
A. Exact plastic analysis
B. Mechanism method of plastic analysis
C. Equilibrium method of plastic analysis

List-II
1. Equilibrium, sufficient plastic hinges, and non-violation of plastic moment capacity
2. Equilibrium and non-violation of plastic moment capacity
3. Equilibrium and sufficient plastic hinges.

Codes :

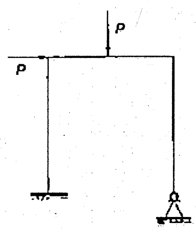
	A	B	C
(a)	3	1	2
(b)	3	2	1
(c)	1	2	3
(d)	1	3	2

Q.13 The number of independent mechanisms the gable frame will have when loaded as shown is



(a) 2 (b) 3
(c) 4 (d) 5

Q.14 The number of possible independent mechanisms (n) for the portal frame show in the given figure is



(a) 2 (b) 4
(c) 1 (d) 3

Q.15 As per IS : 800, in the plastic design, which of the following pairs are correctly matched?

Working Loads	Load factor
1. Dead load	1.7
2. Dead Load + imposed load	1.7
3. Dead load + load due to wind or seismic forces	1.3
4. Dead load + imposed load + load due to wind or seismic forces	1.7

(a) 1 and 2 (b) 1, 2 and 3
(c) 2 and 3 (d) only 1

Q.16 In plastic analysis for flexure, which of the following pairs of shape of section and shape factor are correctly matched?

1. *I*-section 1.4
2. Square 1.5
3. Rectangle 1.5
4. Circle 1.7

Select the correct answer using the codes given below:

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 3 and 4 (d) 1 and 2

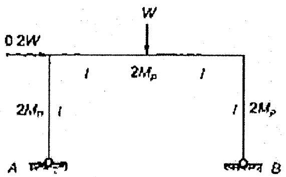
Q.17 At the location of plastic hinge

(a) radius of curvature is infinity
(b) curvature is infinite
(c) moment is infinite
(d) flexural stress is infinite

Q.18 A simply supported beam of span '*L*' supports a concentrated load '*W*' at its midspan. If the cross-section of the beam is an *I*-section, then the length of elastic-plastic zone of the plastic hinge will be

(a) *L*/6 (b) *L*/4
(c) *L*/2 (d) 3*L*/4

Q.19 Collapse moment for the frame shown below has been worked out as $M_p = Wl/5$.



What is the horizontal reaction at *A* at collapse condition?

(a) 0 (b) 0.1W
(c) 0.2W (d) 0.4W

Q.20 The plastic design method is an advantageous replacement over elastic design method for the structures stressed primarily in bending in case of

1. statically loaded structures
2. dynamically loaded structures
3. determinate structures
4. indeterminate structures

Which of these statements are correct?

(a) 1 and 3 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

Q.21 Shape factor depends on

1. yield stress of the material
2. ultimate stress of the material
3. geometry of the section

Which of these statements is/are correct?

(a) Both 1 and 3
(b) Both 1 and 3
(c) Only 3
(d) 1, 2 and 3

Q.22 Consider the following statements:

1. The term load factor is used in the plastic design.
2. Load factor is the ratio of collapse load and working load.
3. Load factor does not depend on support conditions.

Which of these statements are correct?

(a) Both 1 and 3
(b) Both 2 and 3
(c) Both 1 and 2
(d) 1, 2 and 3

Q.23 The load factor in plastic analysis depends on

1. the nature of loading
2. the support conditions
3. geometrical shape of the member

Which of these statements is/are correct?

(a) Both 1 and 2 (b) Both 2 and 3
(c) Only 1 (d) 1, 2 and 3

Q.24 Consider the following statements:

1. In the partial collapse of a structure, the number of plastic hinges is less than (*r* + 1) where '*r*' is the degree of redundancy.
2. The over complete collapse occurs in symmetrically loaded and symmetric structures and in which the plastic hinges do not lie on the axis of the symmetry.
3. Complete collapse occurs when the number of plastic hinges formed is equal to *r* + 1.

Which of these statement/s are correct?

(a) Both 1 and 3 (b) Both 1 and 2
(c) Both 2 and 3 (d) 1, 2 and 3

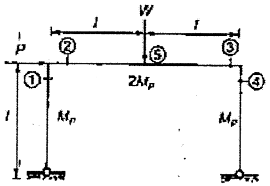
Q.25 Consider the following statements about shape factor:

1. It indicates the increase of strength of a section due to plastic action over elastic strength.
2. It is a ratio of plastic moment of resistance to yield point moment of resistance.
3. Beam sections which have bulk of area near neutral axis will have a low shape factor.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1 and 3
(c) 1 and 2 (d) 2 and 3

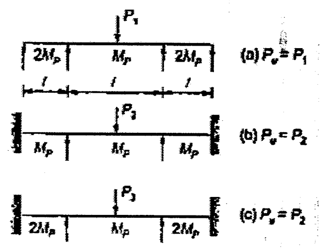
Q.26 A portal frame subjected to central concentrated load and horizontal load is shown in figure:



Likely positions where plastic hinges can form in combined mechanism would include

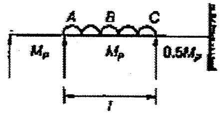
(a) 2, 3 and 5 (b) 1, 4 and 5
(c) 4 and 5 (d) 3 and 5

Q.27 Three types of continuous beams supporting a concentrated load at the central span are shown in figure. Collapse loads (P_u) corresponding to figure (a), (b) and (c) respectively are P_1, P_2, and P_3. Which one of the following conclusion is correct?



(a) $P_3 > P_2 > P_1$
(b) $P_3 = P_2$ and $P_2 > P_1$
(c) $P_1 = P_2 = P_3$
(d) $P_1 = P_3$ and $P_1 > P_2$

Q.28 A continuous beam with plastic moment capacities is shown in figure.



The correct sequence in which the plastic hinges will form in the beam is

(a) *C*, *A*, *B*
(b) *A* and *C* simultaneously, followed by *B*
(c) *B*, *C*, *A*
(d) *B* first, then *A* and *C* simultaneously

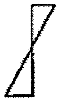
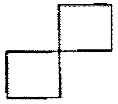
Q.29 A thin, hollow box section of 400 mm x 600 mm deep (outer dimensions) with uniform plate thickness of 10 mm all round is used for a beam. The plastic modulus of section (Z_p) and its shape factor will be

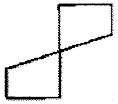
	Z_p (x 10^5 mm^3)	Shape factor
(a)	36	7/6
(b)	36	6/7
(c)	42	6/7
(d)	42	7/6

Q.30 The ratio of collapse load of a propped cantilever of span 'l' carrying a udl throughout the span to that of a simply supported beam carrying the same load is

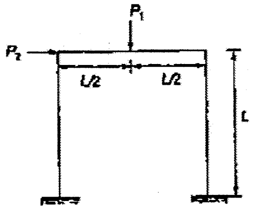
(a) 1.457 (b) 1.500
(c) 2.000 (d) 3.000

Q.31 In a plastic hinge, the actual distribution of strain across the section will be as in

(a)  (b) 

(c)  (d) 

Q.32 Figure shows a portal frame with loads. All members have the same plastic moment of resistance M_p. The ratio P_1 to P_2 for beam and sway mechanism is



(a) 1 (b) 2
(c) 3 (d) 4

Q.33 **Assertion (A) :** The shape factor of a circular section is less than that of a rectangular section.
Reason (R) : Compared to rectangular section, a circular section has more area near the neutral axis than at the extreme fibre.

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.34 For an I-beam the shape factor is 1.12. The factor of safety in bending is 1.5. If the allowable stress is increased by 20% for wind and earthquake loads, then the load factor is

(a) 1.10 (b) 1.25
(c) 1.35 (d) 1.40

Q.35 The plastic modulus of a section is 5×10^{-4} m³. Its shape factor is 1.2 and the plastic moment capacity is 120 kNm. What is the value of the yield stress of the material?

(a) 100 N/mm² (b) 200 N/mm²
(c) 240 N/mm² (d) 288 N/mm²

Q.36 A fixed beam of length 'l' has been loaded with central concentrated load. The beam has been strengthened at the supports with cover plates so that the flexural resisting yield moment capacity at the ends is thrice of that at the centre. If this capacity is to be fully effective resulting in higher collapse load, to what length from the ends should the cover plate extend?

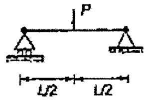
(a) $l/3$ (b) $l/4$
(c) $l/6$ (d) $l/8$

Q.37 In the plastic analysis of structures, the segment between any two successive plastic hinges is assumed to deform as

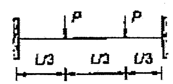
(a) a plastic material
(b) a rigid material
(c) an elastic material
(d) an inelastic material

Q.38 Match List-I (Loaded prismatic beam of uniform M_p) with List-II (Plastic Load) and select the correct answer using the code given below the lists:

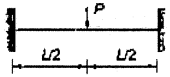
List-I — List-II

A. 

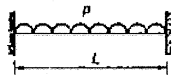
1. $\frac{4M_p}{L}$

B. 

2. $\frac{16M_p}{L^2}$

C. 

3. $\frac{6M_p}{L}$

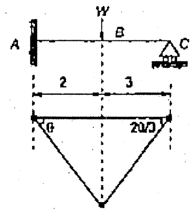
D. 

4. $\frac{8M_p}{L}$

Codes:

	A	B	C	D
(a)	4	3	1	2
(b)	1	2	4	3
(c)	4	2	1	3
(d)	1	3	4	2

Q.39 If a uniform beam as shown in the figure below has the plastic moment capacity M_p for span AB and 0.9 M_p for span BC, then what is the correct virtual work equation?



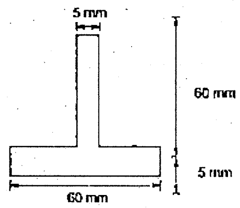
(a) $M_p\theta + M_p\left(\theta + \frac{2\theta}{3}\right) = W \cdot 2\theta$

(b) $M_p\theta + M_p\theta + 0.9M_p\frac{2\theta}{3} = W \cdot 2\theta$

(c) $M_p\theta + 0.9M_p\left(\theta + \frac{2\theta}{3}\right) = W \cdot 2\theta$

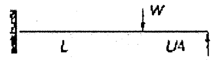
(d) $M_p\theta + 0.9M_p\left(\theta + \frac{2\theta}{3} + \frac{2\theta}{3}\right) = W \cdot 2\theta$

Q.40 The cross-sectional area and plastic section modulus of the given section are respectively



(a) 600 mm², 10,000 mm³
(b) 700 mm², 8650 mm³
(c) 600 mm², 9750 mm³
(d) 700 mm², 10,500 mm³

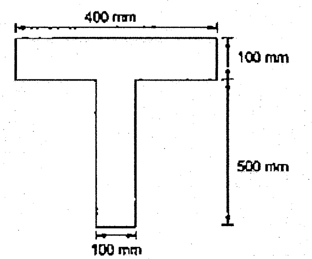
Q.41 A propped cantilever beam of uniform moment capacity M_0 is shown in figure below.



What is the collapse load W?

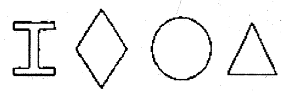
(a) $\frac{12}{L}M_0$ (b) $\frac{8}{L}M_0$
(c) $\frac{6}{L}M_0$ (d) $\frac{3}{L}M_0$

Q.42 In a T-section shown in figure below, what is the distance of plastic neutral axis as measured down from top?



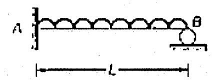
(a) 100 mm (b) 150 mm
(c) 200 mm (d) 300 mm

Q.43 Arrange the following cross-sections in the increasing order of their respective shape factors



(a) 2 < 3 < 4 < 1 (b) 3 < 2 < 1 < 4
(c) 1 < 3 < 2 < 4 (d) 2 < 4 < 1 < 3

Q.44 Position of plastic hinge in the beam shown in figure subjected to collapse load will be

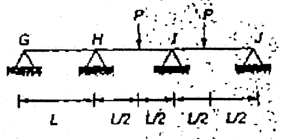


(a) at support A and at mid span
(b) at mid span and at support B
(c) at support A and within $L/2$ from B
(d) at mid span and within $L/2$ from B

Q.45 Which of the following is matched correctly?

(a) Purlin	1. I.S.A.
(b) Girder	2. I.S.H.B.
(c) Joint	3. I.S.F.
(d) Castellated girder	4. I.S.M.C.

Q.46 A continuous beam is loaded as shown in the figure below. Assuming a plastic moment capacity equal to M_P, the minimum load at which the beam would collapse is



(a) $\frac{4M_P}{L}$
(b) $\frac{6M_P}{L}$
(c) $\frac{8M_P}{L}$
(d) $\frac{10M_P}{L}$

Q.47 Plastic deformations in steel structures are
(a) partly recoverable
(b) irrecoverable
(c) recoverable
(d) recoverable depending on the nature of the load

Q.48 The plastic hinge at a section is caused
(a) when most of the material in the section has reached the plastic stress
(b) when all the fibres of the section are under yield stress
(c) when all the material in the section is beyond the elastic limit
(d) none of these

Q.49 The moment-curvature relation at the plastic hinge is
(a) linear
(b) cubic
(c) constant moment for all curvatures
(d) constant curvature for all moments

Q.50 Plastic analysis is applicable to structures made of
(a) any structural materials
(b) ductile and brittle structural materials
(c) ductile materials
(d) brittle materials

Q.51 The method in the plastic analysis of structures based on the upper-bound theorem results in
(a) upper-bound solution of the plastic moment capacity for a given set of forces
(b) upper-bound solution of the loads for a given set of plastic moment capacities
(c) upper bound of the stresses
(d) none of these

Q.52 The design of a structure based on the mechanism method of plastic analysis is on the
(a) safer side
(b) upper bound side
(c) unsafe side
(d) none of these

Q.53 The lower bound theorem in the plastic analysis of structures states that if a state of equilibrium is found such that the stress resultants are in balance with the loads and
(a) the moment anywhere is less than the plastic moment capacity, the structure will fail
(b) the moment anywhere is greater than the plastic moment capacity, the structure will fail
(c) the moment anywhere is less than the plastic moment capacity, the structure will not fail
(d) the mechanism is formed, the structure will fail

Q.54 A beam mechanism in the plastic analysis of structures is possible
(a) in beams only
(b) in beams and slabs only
(c) even in some columns with eccentric load
(d) in all elements

Q.55 Plastic analysis of structures is used in
(a) ultimate strength design
(b) working stress design
(c) limit state design
(d) none of these

Q.56 The effect of axial force and shear force on the plastic moment capacity of a section are
(a) to decrease and to increase the plastic moment respectively
(b) to increase and to decrease the plastic moment respectively
(c) to increase the plastic moment capacity in both cases
(d) to decrease the plastic moment capacity in both cases.

Q.57 Load factor is defined as
(a) $\frac{\text{Ultimate load}}{\text{Yield load}}$
(b) $\frac{\text{Yield load}}{\text{Working load}}$
(c) $\frac{\text{Ultimate load}}{\text{Working load}}$
(d) None of the above

Q.58 Load factor is
(a) always equal to factor of safety
(b) always less than factor of safety
(c) always greater than factor of safety
(d) sometimes greater than factor of safety

Q.59 Other conditions being same, the load factor in indeterminate structures is
(a) equal to load factor in determinate structure
(b) more than the load factor in determinate structure
(c) less than load factor in determinate structure
(d) unpredictable

Q.60 The shape factor for a solid circular section of diameter D is equal to
(a) $\frac{D}{2\pi}$
(b) $\frac{15}{2\pi}$
(c) $\frac{16}{3\pi}$
(d) $\frac{\pi D}{8}$

Q.61 Which one of the following is the correct maximum shear capacity of a prismatic beam under plastic design of steel structures?
(a) $0.5 A_w F_y$
(b) $0.55 A_w F_y$
(c) $0.75 A_w F_y$
(d) $A_w F_y$

Q.62 A prismatic beam (shape factor, S) fixed at both ends carries UDL throughout the span. What is the ratio of collapse load to yield load ?
(a) $\frac{4}{3}S$
(b) $\frac{3}{4}S$
(c) $\frac{5}{3}S$
(d) $\frac{3}{5}S$

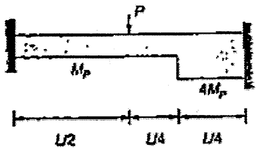
Q.63 A simply supported beam of uniform cross-section has span L and is loaded by a point load P at its mid-span. What is the length of the elastoplastic zone of the plastic hinge ?
(a) $\frac{L}{3}$
(b) $\frac{2L}{3}$
(c) $\frac{L}{2}$
(d) $\frac{3L}{4}$

Q.64 The plastic hinge formed in a collapse mechanism are 4 and the indeterminacy is 3. The collapse is
(a) Partial
(b) Complete
(c) Over complete
(d) Under complete

Q.65 Consider the following assumptions made in the plastic theory
1. Distribution of strain across the section is linear
2. There is an axis of symmetry in the cross-section
3. The influence of normal and shearing forces are neglected
4. Strain energy stored due to elastic bending is ignored

Which of these assumptions are correct?
(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 1, 3 and 4
(d) 1, 2, 3 and 4

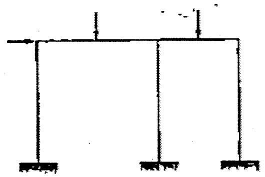
Q.66 For the beam shown in the given figure, the collapse load P is given by



(a) $\frac{16M_p}{L}$ (b) $\frac{14M_p}{L}$
(c) $\frac{12M_p}{L}$ (d) $\frac{10M_p}{L}$

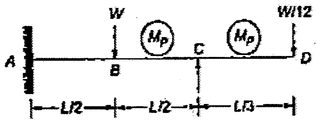
Q.67 Mechanism method and the statical method give
(a) lower and upper bounds respectively on the strength of structure.
(b) Upper and lower bound respectively on the strength of structure.
(c) Lower bound on strength of structure.
(d) Upper bound on strength of structure.

Q.68 Consider a two-bay portal frame subjected to loads in various directions. The total number of independent mechanism are:



(a) 5 (b) 6
(c) 7 (d) 8

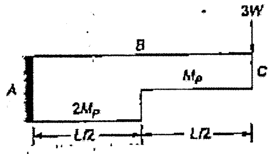
Q.69 A propped cantilever $ABCD$ is loaded as shown below:



The collapse load for beam of uniform cross-section;
(a) $5.63\ M_p/L$ (b) $3.85\ M_p/L$
(c) $6.35\ M_p/L$ (d) $7.25\ M_p/L$

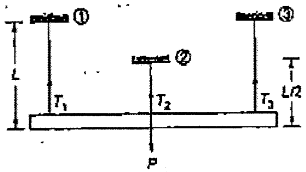
Q.70 Given a fixed ended beam of span L and having I-section loaded uniformly over the entire span. The ratio between ultimate load and load at the first yield is:
(a) 1.123 (b) 1.493
(c) 1.341 (d) 1.262

Q.71 The collapse load for the cantilever beam shown below:



(a) $0.76\ M_p/L$ (b) $0.83\frac{M_p}{L}$
(c) $0.67\ M_p/L$ (d) $0.38\frac{M_p}{L}$

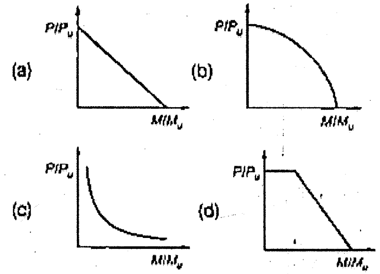
Q.72 Consider a rigid horizontal beam supported by three equally spaced tension bars of same cross-sectional area A as shown below. The length of middle bar is half of the outer bars. The ultimate load P_u on horizontal rigid beam of load is applied at the centre is



(a) $\sigma_y A$ (b) $2\sigma_y A$
(c) $3\sigma_y A$ (d) $4\sigma_y A$

Q.73 When a section is subjected to axial force in additional to bending stress, the plastic moment capacity;
(a) Increases
(b) First increases and then decreases
(c) First decreases and then increases
(d) Decreases

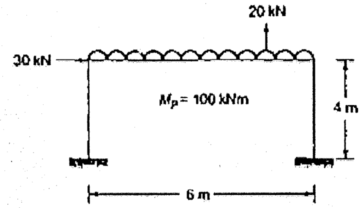
Q.74 Which of following graph represent equation of interaction of axial force on a section having subjected to bending moment?



Q.75 A fixed beam of span L carries a uniformly distributed load $W_{(total)}$ over whole length. The value of collapse load is given as $W_u = \frac{KM_p}{L}$ using plastic theory. The value of K is:
(a) 5.66 (b) 9.66
(c) 11.66 (d) 13.66

Q.76 The shape factor depends upon which of the following factors:
(*i*) Yield stress of the material
(*ii*) Hinge length
(*iii*) Geometry of the section
(*iv*) Redistribution of moments.
(a) (*i*), (*ii*) and (*iii*) only
(b) (*iii*) only
(c) (*ii*), (*iii*) and (*iv*) only
(d) All of the above

Q.77 The minimum collapse load factor the portal frame shown below if plastic moment capacity of all members is 100 kNm considering combined sway mechanism.



(a) 1.699 (b) 1.996
(c) 1.766 (d) 1.688

Q.78 The shape factor for a section of square of side a with its diagonal parallel to x-x-axis is:
(a) 1.5 (b) 2.0
(c) 2.5 (d) 1.8

Q.79 Any correct plastic analysis result, mechanism condition, yield condition ($M \le M_p$) and equilibrium condition are to be satisfied. Which of the above conditions does the statical method of analysis considers
(a) Yield and mechanism condition
(b) Equilibrium and yield condition
(c) Equilibrium condition alone
(d) Equilibrium and mechanism condition

■■■■

Answers Plastic Analysis and Design

1. (b)	2. (c)	3. (b)	4. (c)	5. (b)	6. (c)	7. (a)	8. (c)	9. (b)	10. (b)
11. (b)	12. (d)	13. (c)	14. (a)	15. (a)	16. (b)	17. (b)	18. (a)	19. (a)	20. (b)
21. (c)	22. (c)	23. (d)	24. (d)	25. (c)	26. (b)	27. (c)	28. (b)	29. (d)	30. (a)
31. (d)	32. (b)	33. (d)	34. (d)	35. (c)	36. (b)	37. (b)	38. (d)	39. (c)	40. (c)
41. (c)	42. (b)	43. (c)	44. (c)	45. (b)	46. (b)	47. (b)	48. (b)	49. (c)	50. (c)
51. (b)	52. (c)	53. (c)	54. (c)	55. (c)	56. (d)	57. (c)	58. (c)	59. (b)	60. (c)
61. (b)	62. (a)	63. (a)	64. (b)	65. (d)	66. (b)	67. (b)	68. (b)	69. (c)	70. (b)
71. (c)	72. (c)	73. (d)	74. (b)	75. (c)	76. (b)	77. (b)	78. (b)	79. (b)	

1. (b)

Number of independent mechanisms.

$i = n - r$

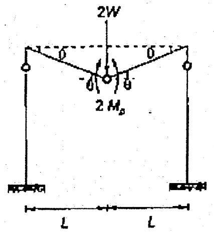
where, n = number of plastic hinges possible = 4,

and r = number of redundants = 2

$\therefore$ $i = 4 - 2 = 2$

Now,

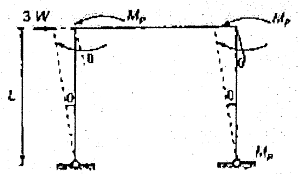
(i) Beam mechanism



$\therefore \quad 2W(L\theta) = M_p(\theta + 2\times 2\theta + \theta)$

$\Rightarrow \quad W = \frac{3M_P}{L}$

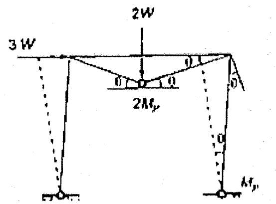
(ii) Panel mechanism



$3W(L\theta) = M_p(\theta + \theta + \theta)$

$\Rightarrow \quad W = \frac{M_P}{L}$

(iii) Combined mechanism



$5WL\theta = 7M_p\theta$

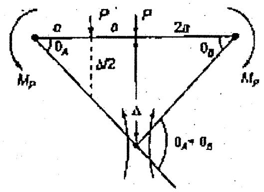
$\therefore \quad W = 1.4\frac{M_P}{L}$

$\therefore$ True collapse load,

$W_u = \frac{M_P}{L}$ (smallest of all)

2. (c)

Of the three hinges needed for collapse, two of them will form at the two ends. The third one can form at any one of the loads. Let it forms at the second load.



$\therefore \quad W_e = P_u\frac{\Delta}{2} + P_u\Delta = \frac{3}{2}P_u\Delta$

$W_i = M_P\theta_B + M_P(\theta_A + \theta_B) + M_P\theta_D$

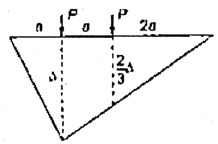
$= 2M_P(\theta_A + \theta_B)$

$= 2M_P\left[\frac{\Delta}{2a} + \frac{\Delta}{2a}\right] = 2M_P\cdot\frac{\Delta}{a}$

$\therefore \quad \frac{3}{2}P_u\Delta = 2M_P\frac{\Delta}{a}$

$\Rightarrow \quad P_u = \frac{4M_P}{3a} = \frac{1.33M_P}{a}$

Let third hinge forms under first load.



$W_e = P_u\Delta + \frac{2}{3}P_u\Delta$

$\Rightarrow \quad \frac{5}{3}P_u\Delta = M_P(\theta_A + \theta_B + \theta_A + \theta_B)$

$\Rightarrow \quad \frac{5}{3}P_u\Delta = 2M_P(\theta_A + \theta_B)$

$\Rightarrow \quad \frac{5}{3}P_u\Delta = 2M_P\left[\frac{\Delta}{a} + \frac{\Delta}{2a}\right]$

$\therefore \quad P_u = \frac{9M_P}{5a} = 1.8\frac{M_P}{a}$

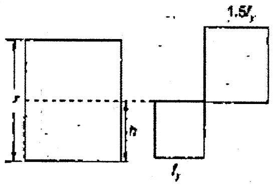
$\therefore$ Collapse load $= \frac{4M_P}{3a}$

6. (c)

In elastic method of analysis, equilibrium equation is used, while in plastic method, equilibrium, yield and mechanism conditions are used.

7. (a)

In fully plastic condition, the stress diagram will be



Area in compression = Area in tension

$x(x - h)1.5\,f_y = xhf_y$

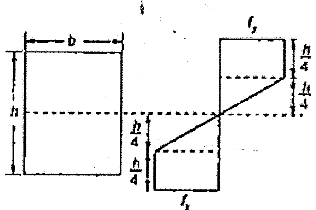
or, $1.5x = 2.5h$

$h = 0.6x$

Distance of N.A. from centre

$= 0.6x - 0.5x = 0.1x$

8. (c)



Cross-section Stress distribution

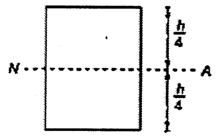
Moment M = Elastic moment + Plastic moment

↓ ↓

MR of the elastic zone MR of the yielded zone

Now, MR of the elastic zone $= f_y \cdot Z_e$

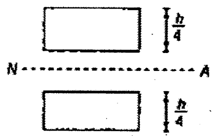
$$= f_y\left[\frac{\frac{b\left(\frac{h}{2}\right)^3}{12}}{\frac{h}{4}}\right] \quad \left\{\because Z_e = \frac{I}{y}\right\}$$



Elastic area

$= \frac{1}{24}f_y \cdot bh^2$

MR of the elastic zone $= f_y \cdot Z_p$



Plastic area

$= f_y \cdot \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

$= f_y \cdot \frac{bh}{2}\left(\frac{3h}{8} + \frac{3h}{8}\right)$

$= \frac{3}{16}f_y \cdot bh^2$

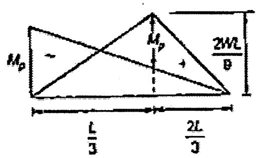
Therefore, $M = \frac{1}{24}bh^2 f_y + \frac{3}{16}f_y bh^2$

$= \frac{11}{48}bh^2 f_y$

9. (b)

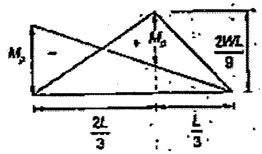
As the degree of static in indeterminacy is 1, hence minimum number of plastic hinges required is 2 for collapse.

Case I : One plastic hinge at fixed support and another at *B*



$$M_P + \frac{2M_P}{3} = \frac{2WL}{9}$$

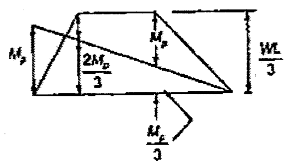
or $$W = \frac{15M_P}{2L}$$



Case II One plastic hinge at *A* and other at *C*

$$M_P + \frac{M_P}{3} = \frac{2WL}{9}$$

$$W = \frac{6M_P}{L}$$



Case III One at *B* and other at *C*

$$M_P + \frac{M_P}{3} = \frac{WL}{3}$$

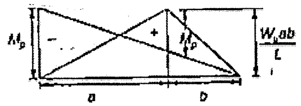
or $$W = \frac{4M_P}{L}$$

Hence collapse load will be minimum of above all cases.

i.e. $$W_u = \frac{4M_P}{L}$$

10. (b)

Degree of static interminancy = 1



Hence, number of plastic hinges required for collapse = 1 + 1 = 2

$$\frac{W_u ab}{L} = M_P + \frac{M_P \cdot b}{L}$$

where, W_u = 18 kN, a = 6 m, b = 3 m, L = 9 m

$$\therefore \quad \frac{18 \times 6 \times 3}{9} = M_P\left(\frac{9+3}{9}\right)$$

or. $$M_P = 27 \text{ kN-m}$$

11. (b)

Plastic section modulus,

$$Z_P = \frac{M_P}{f_y}$$

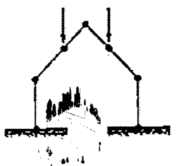
$$= \frac{27 \times 10^6 (\text{Nmm})}{250 (\text{N/mm}^2)}$$

$$= 108 \times 10^3 \text{ mm}^3$$

13. (c)

Number of possible plastic hinges $N = 7$

Number of redundancies $r = 3$



∴ Number of independent mechanism

$$n = N - r$$

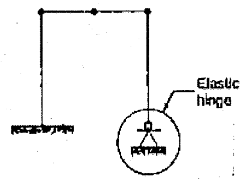
$$= 7 - 3 = 4$$

14. (a)

Number of possible plastic hinges, $N = 4$

Number of redundancies,

r = Total unknown - equilibrium equation



$$r = 5 - 3 = 2$$

$$\therefore \quad n = N - r = 4 - 2 = 2$$

15. (a)

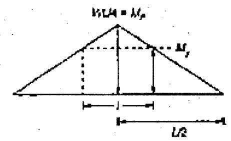
1. For combination of dead load or *DL* + imposed load or *DL* + wind or seismic load,
   Load factor = 1.7
2. For *DL* + imposed load + wind or seismic load,
   Load factor = 1.3

17. (b)

Curvature at plastic hinge is infinite and moment is equal to plastic moment capacity. It means that infinite rotation can occur at fully plastic section.

18. (a)

For I-section, shape factor $\frac{M_p}{M_y}$ = 1.1 to 1.2



From the bending moment diagram

$$\frac{W}{2} \times \frac{l}{2} = M_p - M_y$$

Taking $$M_p = \frac{WL}{4} \text{ and } M_y = \frac{M_p}{1.1}$$

$$l = 0.09L = \frac{L}{11}$$

taking $$M_y = \frac{M_p}{1.2}$$

$$l = \frac{L}{6}$$

So l varies between

$$\frac{L}{6} \text{ to } \frac{L}{11}$$

Take $$l = \frac{L}{6}$$

19. (a)

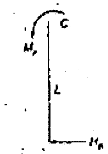
For the complete collapse of the portal frame, two plastic hinges are required

∴ Number of plastic hinges

$$= r + 1 = (4 - 3) + 1 = 2$$

Let 2 plastic hinges be formed under load *W* and at right hand rigid joint above *B*.

Let horizontal reaction at *A* and *B* be $H_A(\leftarrow)$ and $H_B(\leftarrow)$ respectively. Considering equilibrium of right hand column at collapse



$$\Sigma M_c = 0$$

$$\Rightarrow \quad H_B \times L = M_p$$

$$\Rightarrow \quad H_B = \frac{M_p}{L}$$

$$\Rightarrow \quad H_B = \frac{WL}{5L} \qquad \left[\because M_p = \frac{WL}{5}\right]$$

$$\Rightarrow \quad H_B = 0.2\, W$$

Now,

$$H_A + H_B = 0.2\, W$$

$$\therefore \quad H_A = 0$$

21. (c)

The shape factor relates the plastic moment capacity and elastic moment capacity of a section and it depends only on the shape of the section.

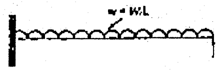
25. (c)

$$\text{Shape factor} = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$

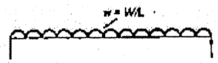
Beams sections which have bulk of area near neutral axis will have a high shape factor.

28. (b)
First the hinge will form at A and C simultaneously because the carry over factor for support A is zero while support C transfers half of moment to fixed support on right side. The hinge at C will form at 0.5 M_p while at A, the moment M_p is required.
Now due to redistribution of moments the hinge at will form.

30. (a)



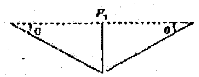
$$w_u = \frac{11.656 M_p}{L}$$



$$w_u = \frac{8M_p}{L}$$

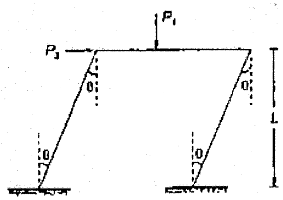
$\therefore$ Ratio of collapse load = $\frac{11.656}{8} = 1.457$

32. (b)



Beam mechanism $P_1 = \frac{8M_p}{L}$

Sway mechanism



$$P_2 = \frac{4M_p}{L}$$

$$\therefore \quad \frac{P_1}{P_2} = 2$$

34. (d)

$$FOS = \frac{\text{Yield stress}}{\text{Allowable stress}}$$

With increase in allowable stress, the FOS will reduce to $\frac{1.5}{1.2} = 1.25$

$\therefore$ Load factor = FS × shape factor
= 1.25 × 1.12 = 1.40

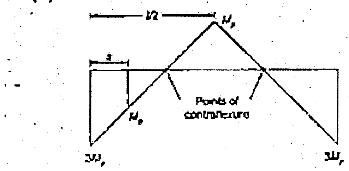
35. (c)

$$M_p = Z_p \times f_y$$

$$\therefore \quad f_y = \frac{120 \times 10^3}{5 \times 10^{-4}}$$

$$= 24 \times 10^7 \text{ N/m}^2 = 240 \text{ N/mm}^2$$

36. (b)



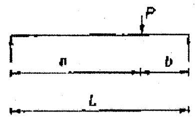
To utilize the capacity effectively, the cover plates should extend from ends to the point where moment is M_p

$$\therefore 3M_p - \frac{4M_p}{l/2} x = M_p$$

$$\therefore \quad x = \frac{l}{4}$$

37. (b)
The initial slopes and deflections of the beam do not affect the virtual work equations. Thus the displacement diagram can be simplified to consists of straight and undefeated lines i.e. rigid deformation.

38. (d)

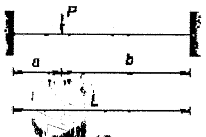


Simply Supported Beam

$$P = \frac{M_pL}{ab}$$

for $a = b = L/2$

$$P = \frac{4M_p}{L}$$

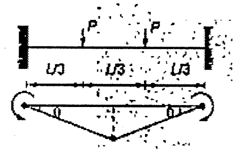


Fixed Beam

$$P = \frac{2M_pL}{ab}$$

for $a = b = L/2$

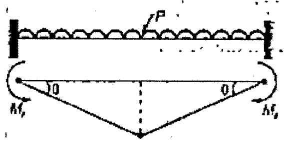
$$P = \frac{8M_p}{L}$$



Assuming hinges at fixed end and under the load. Using virtual work method,

$$M_p(2\theta + 3\theta + \theta) = P \times \frac{L}{3} \times 2\theta + P \times \frac{L}{3} \theta = PL\theta$$

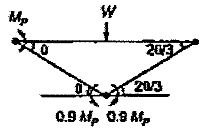
$$P = \frac{6M_p}{L}$$



$$P \times \frac{1}{2} \times l \times \frac{l}{2} \times \theta = M_p(\theta) + M_p(\theta + \theta) + M_p(\theta)$$

$$\Rightarrow \quad P = \frac{16M_p}{l^2}$$

39. (c)
Total number of plastic hinges needed for complete collapse of the beam may be given as
Number of plastic hinges
$= r + 1 = (3 - 2) + 1 = 2$

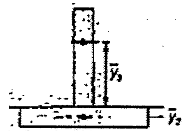


By principle of virtual work, we get

$$-M_p\theta - 0.9M_p\theta - 0.9M_p \times \frac{2\theta}{3} + W \times \theta \times 2 = 0$$

$$\Rightarrow M_p\theta + 0.9M_p\left(\theta + \frac{2\theta}{3}\right) = W \times 2\theta$$

40. (c)
Total area $A = 5 \times 60 + 5 \times 60 = 600 \text{ mm}^2$



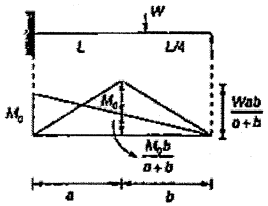
The centroids of half areas on either side of axis are at $\bar{y}_1 = 30$ mm, $\bar{y}_2 = 2.5$ mm
Therefore plastic section modulus

$$= \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$= 300 \times (30 + 2.5)$$

$$= 300 \times 32.5 = 9750 \text{ mm}^3$$

41. (c)
Number of plastic hinges required for complete collapse $= r + 1 = (3 - 2) + 1 = 2$
The plastic hinges will be formed under the load and at the fixed end of the beam respectively.



But $a = L$ and $b = L/4$

$\therefore \quad \dfrac{Wab}{a+b} = M_0 + \dfrac{M_0 b}{a+b}$

$\Rightarrow \quad \dfrac{W \times L \times L/4}{L + L/4} = M_0 + \dfrac{M_0 \times L/4}{L + L/4}$

$\Rightarrow \quad \dfrac{WL^2}{4} = \dfrac{5M_0 L}{4} + \dfrac{M_0 L}{4}$

$\Rightarrow \quad W = \dfrac{6M_0}{L}$

42. (b)

Total area,

$A = 400 \times 100 + 100 \times 500 = 90000 \text{ mm}^2$

$\therefore \quad \dfrac{A}{2} = \dfrac{90000}{2} = 45000 \text{mm}^2$

Thus the plastic neutral axis lies outside the flange because the flange area is only 40000mm^2.

Let the plastic neutral axis lies y mm below the junction of flange and web.

$\therefore \quad y \times 100 = 45000 - 40000$

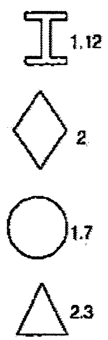
$\Rightarrow \quad y = 50 \text{ mm}$

$\therefore$ Plastic neutral axis as measured from top of flange $= 100 + y$

$= 100 + 50 = 150 \text{ mm}$

43. (c)

Shape Factor



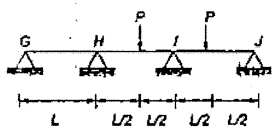
44. (c)

Because maximum bending moment develops at end A and between mid span and end B.

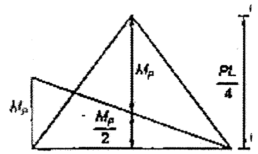
45. (b)

Channel sections are best suited for purlines. An I-section is preferred for Girders

46. (b)



For collapse in IJ

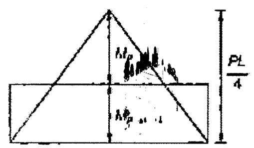


$M_P + \dfrac{M_P}{2} = \dfrac{PL}{4}$

$\Rightarrow \quad \dfrac{3M_P}{2} = \dfrac{PL}{4}$

$\Rightarrow \quad P = \dfrac{6M_P}{L}$

For collapse in HI



$M_P + M_P = \dfrac{PL}{4}$

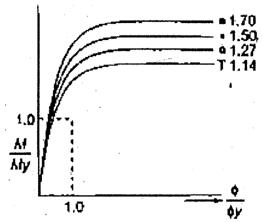
$\Rightarrow \quad P = \dfrac{8M_P}{L}$

$\therefore$ Minimum load for collapse $= \dfrac{6M_P}{L}$

47. (b)

In steel structure deformation are recoverable in elastic zone beyond this zone deformations are not recoverable.

49. (c)



Within the elastic limit the relation between moment and curvature is linear. For all sections beyond the elastic limit it is non linear. As the curvature increases indefinitely M/M_y approaches to shape factor. At the plastic hinge the moment is constant for all the curvature.

51. (b)

Total number of plastic hinges required to make the structures statically determinate is equal to static indeterminacy and total number of plastic hinges required at collapse mechanism is equal to static indeterminacy plus one.

52. (c)

Upper bound theorem

(i) Equilibrium and mechanism conditions must be satisfied.

(ii) Load computed on the basis of an assumed mechanism will always be greater than or be equal to the true ultimate load. Thus design leads to unsafe side.

Lower bound theorem

(i) Equilibrium and yield conditions must be satisfied

(ii) Load computed on the basis of an assumed equilibrium moment diagram in which the moment $M \leq M_p$ is less than or at best equal to true ultimate load thus design leads to safe side.

53. (c)

Upper bound theorem

(i) Equilibrium and mechanism conditions must be satisfied.

(ii) Load computed on the basis of an assumed mechanism will always be greater than or be equal to the true ultimate load. Thus design leads to unsafe side.

Lower bound theorem

(i) Equilibrium and yield conditions must be satisfied

(ii) Load computed on the basis of an assumed equilibrium moment diagram in which the moment $M \leq M_p$ is less than or at best equal to true ultimate load thus design leads to safe side.

54. (c)

Beam mechanism occurs in transversely loaded members only.

59. (b)

This is due to favourable redistribution of moments from zones of high moments to zones of low moments.

61. (b)

The maximum shear capacity is $0.55 A_w f_y$

62. (a)

The collapse load for a prismatic beam fixed at both ends and caries UDL throughout the span is given by

$$W_u = \frac{16M_p}{L^2} \quad \text{...(i)}$$

where M_p is its plastic moment

But $\quad S = \dfrac{\text{Plastic moment}}{\text{Yield moment}}$

$$\Rightarrow \quad S = \frac{M_p}{M_y} \quad \text{...(ii)}$$

$$\text{Also} \quad M_y = \frac{W_y L^2}{12} \quad \text{...(iii)}$$

Substituting value of S from (ii) in (iii), we get

$$\frac{M_p}{S} = \frac{W_y L^2}{12}$$

Substituting value of M_p from (i), we get

$$\frac{W_u L^2}{16S} = \frac{W_y L^2}{12}$$

$$\Rightarrow \quad \frac{W_u}{W_y} = \frac{4}{3}S$$

64. (b)

If the number of plastic hinges in the collapse mechanism are less than $(r+1)$ the collapse is called partial collapse. In such a case, part of the structure may fail making it useless as a whole. If the number of plastic hinges in the collapse mechanism are $(r+1)$ the collapse is called complete collapse. Such a mechanism has only one degree of freedom.

If the number of plastic hinges developed are more than $(r+1)$, the collapse is called over complete collapse. In such a case there are two or more mechanisms for which the corresponding value of the load is the same, this load value being the actual collapse load.

If the degree of indeterminacy is r, and the number of plastic hinges developed is N then

$N < (r+1)$ Partial collapse

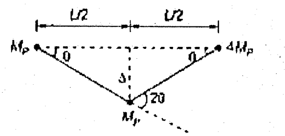
$N = r+1$ Complete collapse

$N > r+1$ Overcomplete collapse

66. (b)

The degree of indeterminacy, $r = 2$

Number of plastic hinges needed for collapse $= r + 1 = 3$

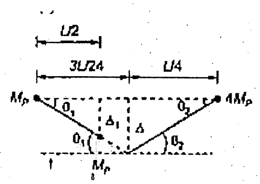


Using virtual work method

$$M_p\theta + M_p(2\theta) + 4M_p\theta = P \times \frac{L}{2}\theta$$

$$\Rightarrow \quad P = \frac{14M_p}{L}$$

The other possible mechanism is



$$\Delta_1 = \frac{2}{3}\Delta;\ \theta_1 = \frac{4\Delta}{3L};\ \theta_2 = \frac{4\Delta}{L}$$

Using virtual work method

$$M_p(\theta_1) + M_p(\theta_1 + \theta_2) + 4M_p\theta_2 = P\Delta_1$$

$$\frac{8M_p}{3L} + \frac{20M_p}{L} = \frac{2}{3}P$$

$$P = \frac{34M_p}{L}$$

Collapse load will the minimum of

$$\frac{14M_p}{L} \text{ and } \frac{34M_p}{L} \text{ i.e., } \frac{14M_p}{L}$$

$$\therefore \quad P = \frac{14M_p}{L}$$

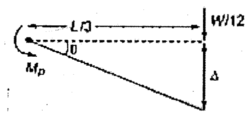
68. (b)

12 plastic hinge locations are possible

Number of mechanism = plastic hinges − redundants

$= 12 - 6 = 6$

69. (c)



For span CD; vertical work method

$$\frac{W}{12} \times \Delta = M_p\theta = \frac{WL}{36}\theta$$

$$\Rightarrow \quad W_u = \frac{36M_p}{L}$$

Span AC:

External work done = work done by load W + negative work done by load $W/12$

$$= \frac{WL}{2}\theta + \left[-\frac{W}{12} \times \frac{L}{3}\theta\right] = \frac{17W_uL}{36}\theta$$

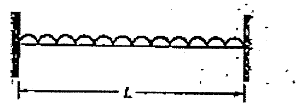
Internal work done

$$= M_p\theta + M_p(\theta + \theta) = 3M_p\theta = \frac{17W_uL}{36}\theta$$

$$W_u = \frac{108 M_p}{17 L} = 6.35\, M_p/L$$

$$\Rightarrow \quad W_u = 6.35\, M_p/L$$

70. (b)



By equilibrium,

$$\frac{P_u L}{8} = 2M_p$$

At elastic limit, the centre moment is one-half the end moments

$$\frac{P_y L}{8} = M_y + \frac{M_y}{2}$$

or, $$\frac{P_y L}{8} = \frac{3}{2}M_y$$

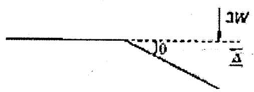
or, $$\frac{P_u}{P_y} = \frac{16M_p}{12M_y}$$

Assuming shape factor $f = 1.12$ for I-section

$$\frac{P_u}{P_y} = \frac{4}{3} \times 1.12 = 1.493$$

71. (c)

Case I: Plastic hinge at point, where cross-section changes.



External work done

$$= 3W_u \times \theta = \frac{3W_uL\theta}{2}$$

Internal work done = moment × rotation

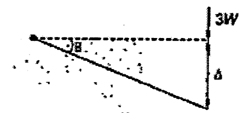
$= M_p\theta$

By principle of virtual work;

$$\frac{3W_uL\theta}{2} = M_p\theta$$

$$\Rightarrow \quad W_u = \frac{2M_p}{3L} = \frac{0.67M_p}{L}$$

Case II: Plastic hinge at the support.



$$3W_u \times \Delta = 2M_p\theta$$

$$\Rightarrow 3W_u \times L\theta = 2M_p\theta$$

$$\Rightarrow \quad W_u = \frac{2M_p}{3L} = \frac{0.67M_p}{L}$$

72. (c)

At plastic stage all three bars have yield stress and the corresponding load is called the ultimate load P_u.

Let T_1 and T_2 = tension in outer and inner bars, respectively.

$$P_u = 2T_1 + T_2$$
$$= 2\sigma_y A + \sigma_y A$$

or, $P_u = 3\sigma_y A$

73. (d)

The plastic moment capacity reduces on application of axial force and reduction depends on shape of section, length of beam column and intensity of compressive force.

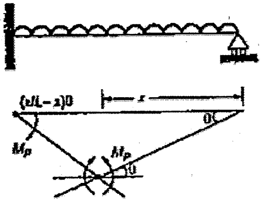
74. (b)

The interaction equation between moment and axial force capacity is given as;

$$\frac{M}{M_P} + \left(\frac{P}{P_y}\right)^2 = 1$$

Here represented by option (b) graph.

75. (c)



Beam mechanism

$$\frac{W_u x\theta}{2} = M_P\left[\frac{x\theta}{L-x}+\theta+\frac{x\theta}{L-x}\right]$$

or $W_u = \frac{2M_P}{L}\times\left(\frac{L+x}{x(L+x)}\right)$

For maximum

$W_u = x_u = 0.414L$

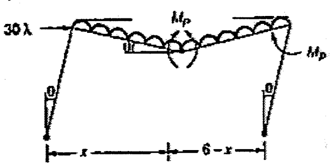
Thus,

$\Rightarrow \quad W_u = \frac{11.66M_P}{L}$

76. (b)

The shape factor is plastic method of analysis depends only on the geometry of the section.

77. (b)



Virtual work equation,

$$30\lambda\times 40 + 120\lambda\times\left(x\frac{0}{2}\right) = 100\left[0+2\left[1+\frac{x}{6-x}\right]0+0\right]$$

$$\lambda = \frac{10(12-x)}{3(2+x)(6-x)} \qquad (i)$$

For minimum value of load factor λ,

$$\frac{d\lambda}{dx} = \frac{10}{3}\frac{\left[-(2+x)(6-x)-(12-x)(4-x)\right]}{(2+x)^2(6-x)^2}$$

$= 0$

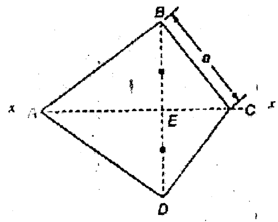
or, $\quad x^2 - 24x + 60 = 0$

or, $\quad x = 2.835$ m

Substituting x = in eq (*i*), we get

$\Rightarrow \quad \lambda = 1.966$

78. (b)



$$AC = \sqrt{a^2+a^2} = \sqrt{2}a$$

$$BE = \frac{BD}{2} = \frac{AC}{2} = \frac{AC}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

Moment of inertia about x-x-axis

$$I_x = \frac{2\sqrt{2}}{12}a\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{12}$$

Elastic section modulus

$$Z_e = \frac{\frac{a^4}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}}$$

Plastic section modulus

$$Z_p = \frac{A}{2}(\bar{y}_1+\bar{y}_2)$$

$$= \frac{a^2}{2}\left(\frac{a}{3\sqrt{2}}+\frac{a}{3\sqrt{2}}\right) = \frac{a^3}{3\sqrt{2}}$$

$\Rightarrow$ Shape factor

$$= \frac{Z_p}{Z_e} = \frac{\frac{a^3}{3\sqrt{2}}}{\frac{a^3}{6\sqrt{2}}} = 2$$

79. (b)

Statical method of analysis doesn't consider mechanism method of analysis. It is considered by kinematic method of analysis.

■■■■