"Balancing"

$\vec{F}_{net} = 0$, $\vec{M}_{net} = 0$

Unbalanced Forces

Revolving masses

mew^2

Const. → mag

change → dir^n

Reciprocating mass

$mrw^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

Const. → dir^n

Change → mag.

Balancing

Static
(single plane)

$(\vec{F}_{net} = 0)$

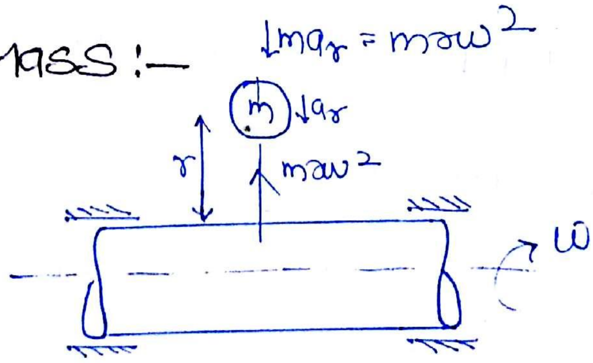
By default
$(\vec{M}_{net} = 0)$

Dynamic
(More than 1 plane)

$\vec{F}_{net} = 0$

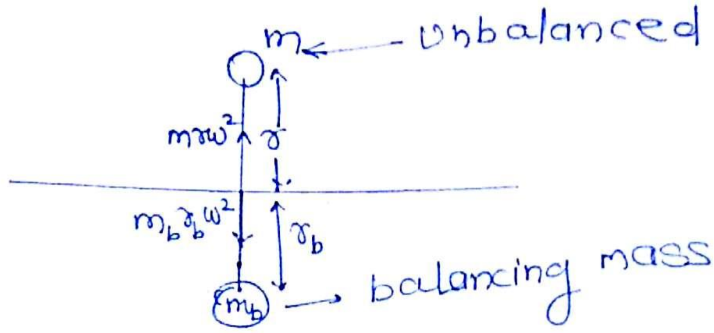
$\vec{M}_{net} = 0$

Revolving Mass :-



① Single Revolving mass :-

(i) static



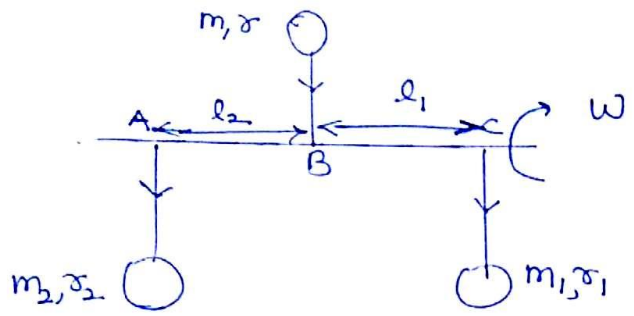
$$\vec{F}_{net} = 0$$

$$mrw^2 - m_b r_b w^2 = 0$$

$$\boxed{mr = m_b r_b}$$

only single mass require

(ii) Dynamic



A, B, C are in a plane

$$\vec{F}_{net} = 0$$

$$mrw^2 = m_1 r_1 w^2 + m_2 r_2 w^2$$

$$\boxed{mr = m_1 r_1 + m_2 r_2}$$

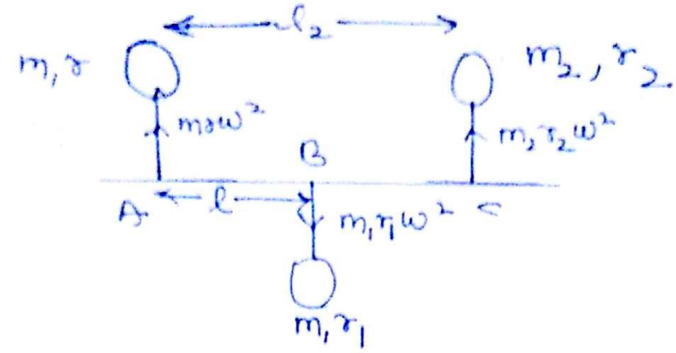
$$\vec{M}_{net} = 0$$

$M_B = 0$ $\quad m_1 r_1 w^2 l_1 = m_2 r_2 w^2 l_2$

$$\boxed{m_1 r_1 l_1 = m_2 r_2 l_2}$$

⟹ More than one mass required to balance dynamically.

*



$\vec{F}_{net} = 0 \qquad mr + m_2 r_2 = m_1 r_1$

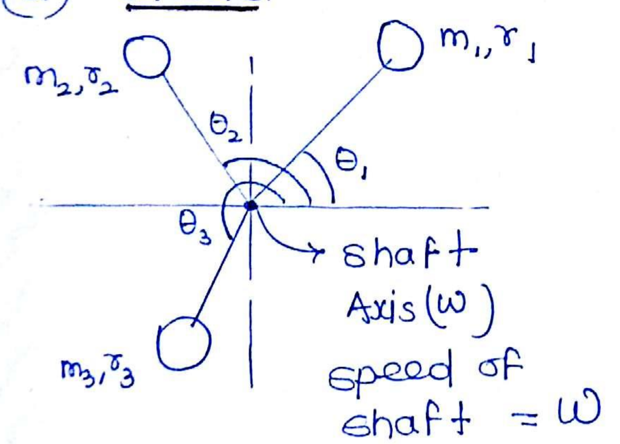
$\vec{M}_{net} = 0 \quad , \vec{M}_\bullet = 0$

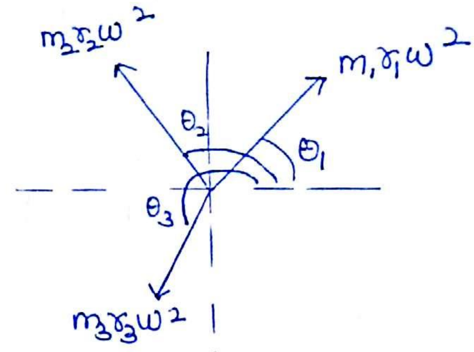
$$m_1 r_1 w^2 l_1 = m_2 r_2 w^2 l_2$$

$$\boxed{m_1 r_1 l_1 = m_2 l_2 r_2}$$

(2) Several Revolving masses :-

(a) Static:-





Analytical $\quad \vec{F}_{net} = 0$

$\vec{F}_x = 0$

$$m_1 r_1 w^2 \cos\theta_1 + m_2 r_2 w^2 \cos\theta_2 + m_3 r_3 w^2 \cos\theta_3 + m_b r_b w^2 \cos\theta_b = 0$$

$$m_b r_b \cos\theta_b = (-) \Sigma mr \cos\theta \quad -(1)$$

$F_y = 0 \qquad m_b r_b \sin\theta_b = (-) \Sigma mr \sin\theta \quad -(2)$

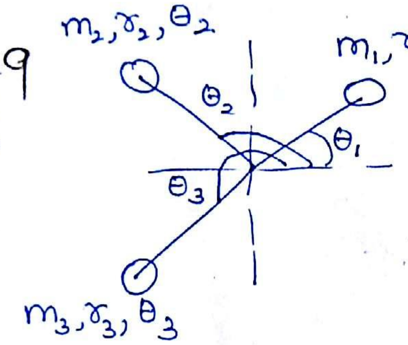
$① ^2 + ② ^2$

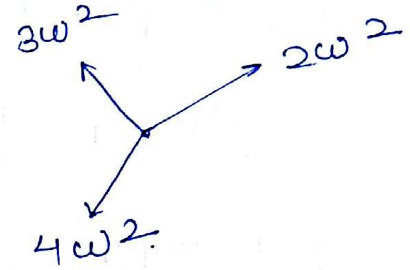
$$m_b r_b = \sqrt{\left(-\Sigma(mr\cos\theta)\right)^2 + \left(-\Sigma(mr\sin\theta)\right)^2}$$

$$\boxed{\tan\theta_b = \frac{-\Sigma\, mr\cos\theta}{-\Sigma\, mr\sin\theta}}$$

take with -ve sign only

Q. 29
Pg-36
W B.

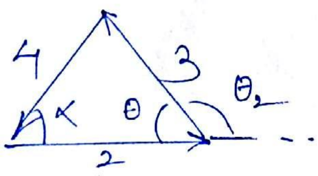




$$2\omega^2\cos\theta_1 + 3\omega^2\cos\theta_2 + 4\omega^2\cos\theta_3 = 0$$

$$2\cos\theta_1 + 3\cos\theta_2 + 4\cos\theta_3 = 0 \quad ①$$

$$2\sin\theta_1 + 3\sin\theta_2 + 4\sin\theta_3 = 0 \quad ②$$



$$\cos\theta = \frac{4+9-16}{2\times2\times3}$$

$\theta = 104.4775 \rightarrow \theta_2 = 75.5224$

$$\cos\alpha = \frac{4+16-9}{2\times2\times4}$$

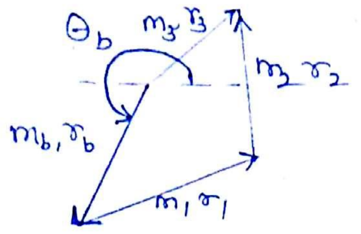
$\alpha = 46.56$

$\theta_3 = 226.56^\circ$

Graphical

from polygon →

$$m_1\vec{r}_1\omega^2 + m_2 r_2\omega^2 + m_3\vec{r}_3\omega^3 + m_b r_b\omega^2 = 0$$

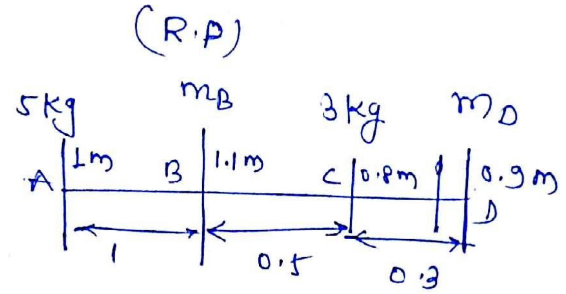


For Angle measure anticlockwise

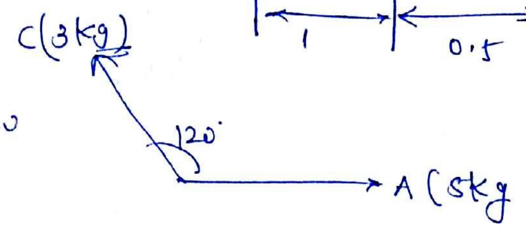
Dynamic balancing

Q 36
Pg. 37
WB

Side view



front view



R.P. reference plane

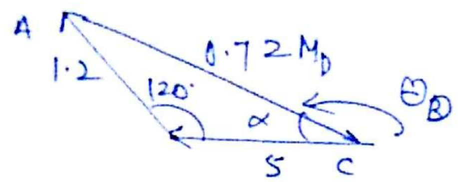
take distance from R.P. – right side +ve
left side –ve

Planes	m	r	mr	l	mrl
A	5	1	5	-1	-5
B	m_B	1.1	$1.1\,m_B$	0	0
C	3	0.8	2.4	0.5	1.2
D	m_D	0.9	$0.9\,M_D$	0.8	$0.72\,M_D$

$$\frac{F}{\omega^2} = mR, \quad \frac{M}{\omega^2} = mrl$$

–ve represent opposite dir^n as given in front view

Moment polygon



$0.72\, m_D = 5.695$

$m_D = 7.91$ kg

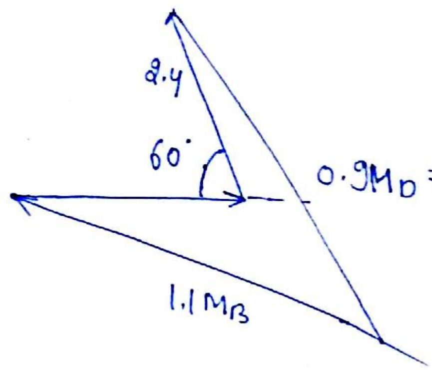
for Direction

$$360° - \alpha$$

$$= 360° - \cos^{-1}\left[\frac{5^2 + (5.69)^2 - (1.2)^2}{2 \times 5 \times 5.69}\right]$$

$$\theta_D = 349.48°$$

force polygon:

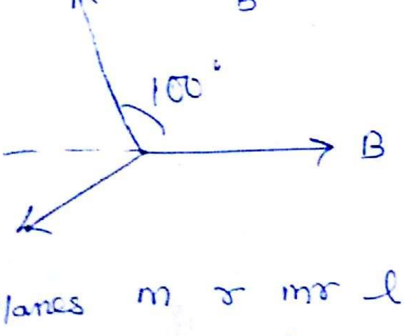


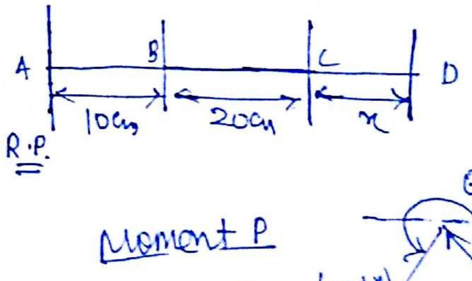
Q.32 $m_B = 18$ kg, $m_C = 12.5$ kg $r_B = r_C = 6$ cm

$r_A = r_D = 8$ cm.

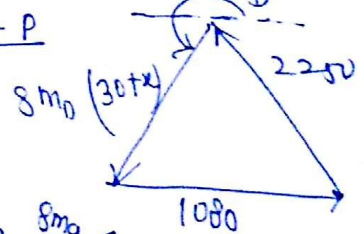
$\theta_C - \theta_B = 100°$

$\theta_B - \theta_A = 190°$



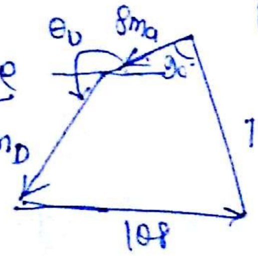


Moment P



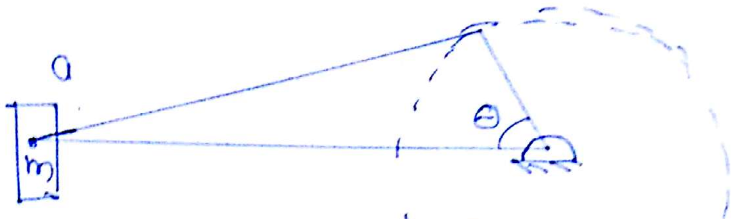
Planes	m	r	mr	l	mrl
A	m_B	8	$8m_a$	0	0
B	18	6	108	10	1080
C	12.5	6	75	30	2250
D	m_D	8	$8m_C$	30+x	$8m_D(30+x)$

Force



Calcu m_D & m_A

Balancing of Reciprocating Mass:-



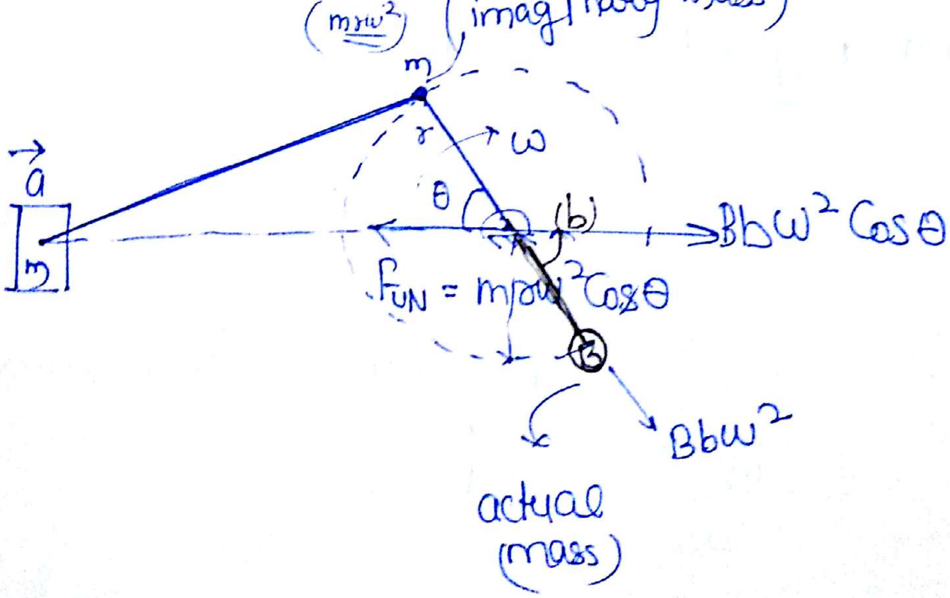
$$f_{un} = ma = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

{force on frame because of reciprocating mass}

$$f_{un} = \underbrace{mr\omega^2 \cos\theta}_{\text{primary}} + \underbrace{mr\omega^2 \frac{\cos 2\theta}{n}}_{\text{secondary}}$$

Primary >> Secondary.

Balancing of Primary force.



$$Bb\omega^2 = mr\omega^2$$

$$Bb = mr$$

$$(F_{net})_{\text{along LOS}} = mr\omega^2\cos\theta - Bb\omega^2\cos\theta$$

$$(F_{net})_{\text{along LOS}} = 0$$

$$(F_{net})_{\text{~~along~~ Ler to LOS}} = Bb\omega^2\sin\theta = mr\omega^2\sin\theta$$

So complate balancing of single reciprocating mass is not possible because one new force in direction Ler to line of stroke is generated. Now we will balance reciprocating mass partially.

Partial balancing of Primary Reciprocating mass.

$$Bb \neq mr$$

$$Bb = cmr \qquad 0 < c < 1$$

c- Fraction of reciprocating mass to be balanced

$$(F_{net})_{\text{along LOS}} = mr\omega^2 \cos\theta - Bb\omega^2 \cos\theta$$

$$(F_{net})_{\text{along LOS}} = mr\omega^2 \cos\theta \left[1-c\right]$$

$$(F_{net})_{\perp \text{ to LOS}} = Bb\omega^2 \sin\theta = cmr\omega^2 \cancel{\cos\theta} \sin\theta$$

$$(F_{net})_{\text{primary}} = mr\omega^2 \sqrt{\cos^2\theta\,(1-c)^2 + c^2\sin^2\theta}$$

$$\tfrac{1}{2} < c < \tfrac{3}{4}$$

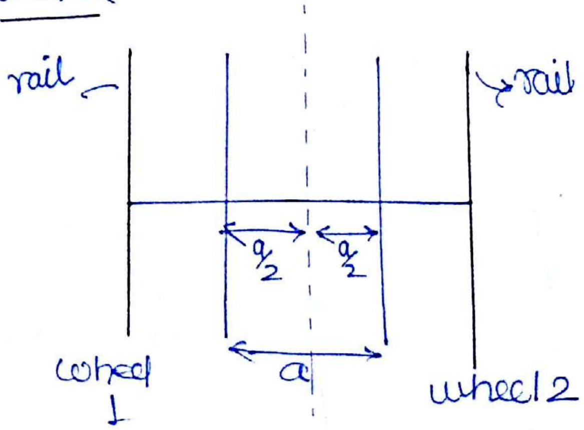
when $c = \tfrac{1}{2}$ $\Rightarrow$ $(F_{net})_{\text{primary}} \rightarrow$ min

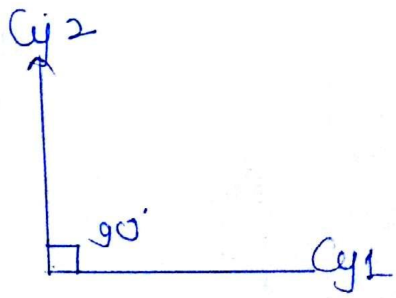
$$\boxed{(F_{net})_{\text{primary min}} = \frac{mr\omega^2}{2}}$$

So we can reduce primary unbalance force upto $\frac{mr\omega^2}{2}$ by partial balancing

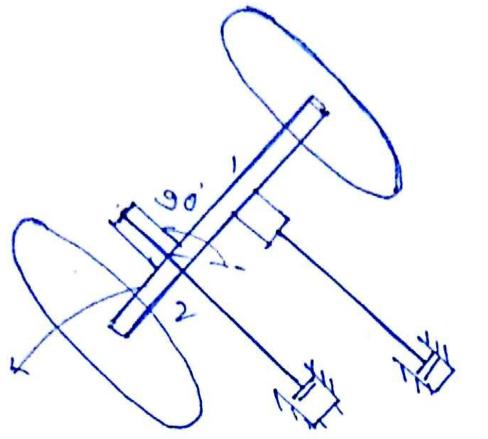
Two Cylinder Locomotive :-
→ inside Cy. locomotive
→ outside Cy. locomotive

Inside Cy. locomotive





Firing order 90°



Effect of partial balancing on two-Cylinder locomotives

(i) Variation in tractive force:-

The resultant of unbalanced primary forces due to two cylinders along the line of stroke is known as tractive force.

$(1-c)mr\omega^2\cos\theta$ — Cy ① θ

$(1-c)mr\omega^2\cos(90+\theta)$ — Cy ② $(90+\theta)$

$$\boxed{\text{Tractive force} = F_t = mr\omega^2(1-c)\left[\cos\theta - \sin\theta\right]}$$

$$\frac{dF_T}{d\theta} = mr\omega^2(1-c)(-\sin\theta - \cos\theta)$$

$$\frac{dF_T}{d\theta} = 0, \quad \tan\theta = -1$$

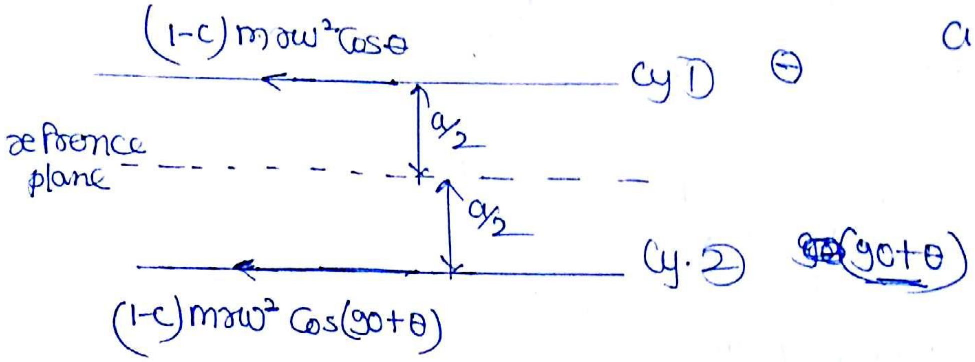
$$\boxed{\theta = 135^\circ,\ 135+180 = 315^\circ}$$

$$(\cos\theta - \sin\theta)_{\theta=135^\circ} = \sqrt{2}$$

$$(\cos\theta - \sin\theta)_{\theta=315^\circ} = -\sqrt{2}$$

Variation in tractive force: $\boxed{F_T = \pm\sqrt{2}\,(1-c)\,mr\omega^2}$

(ii) Variation in swaying couple:-



a = dist. b/w two cy.

$$(M)_{r.p.} = (1-c)mr\omega^2 \cos\theta \left(\frac{a}{2}\right) - (1-c)mr\omega^2 \cos(90+\theta)\left(\frac{a}{2}\right)$$

$$\text{Swaying couple} = (1-c)mr\omega^2 \left(\frac{a}{2}\right)(\cos\theta + \sin\theta)$$

$$\frac{d(\text{swaying couple})}{d\theta} = mr\omega^2(1-c)\frac{a}{2}(-\sin\theta + \cos\theta) = 0$$

$$\boxed{\begin{aligned}\tan\theta &= 1 \\ \theta &= 45, 225^\circ\end{aligned}}$$

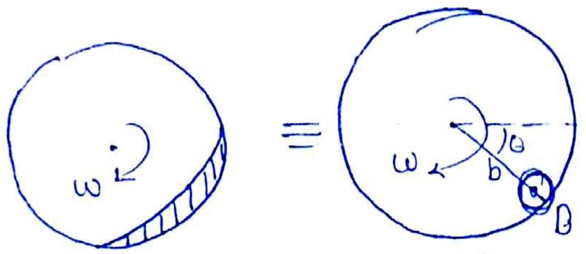
$$(\cos\theta + \sin\theta)_{\substack{\theta=45 \\ \theta=225}} = \sqrt{2}, -\sqrt{2}$$

$$\text{Variation in swaying couple} = \pm\frac{a}{\sqrt{2}}(1-c)(mr\omega^2)$$

Hammar Blow :-

The maximum Valve of the unbalanced force $\perp$er to the line of stroke is called hammar blow.

$$Bb \neq cmr$$

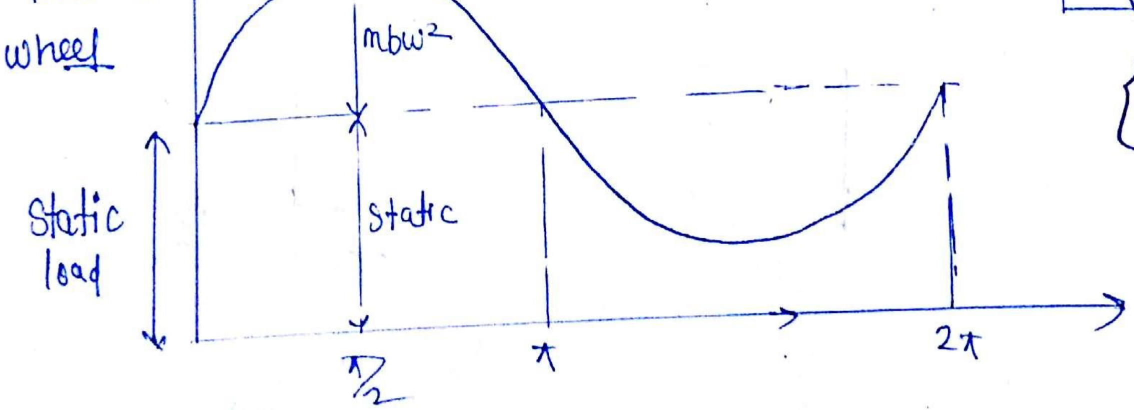


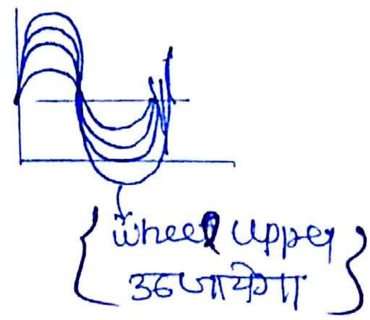
wheel

$$\left(F_t\right)_{\perp er\ to\ LOS} = Bb\omega^2 \sin\theta$$

$$\text{Max. Value} = \underbrace{Bb\omega^2}_{\text{Hammar blow}}$$

B = Balancing mass on wheel required to balance reciprocating mass only





KPBJ

load on wheel = static load + $Bb\omega^2 \sin\theta$

<u>for safe condition</u>

$$Bb\omega^2 < \text{static load} = \frac{\text{weight of locomotive}}{\text{no. of wheel.}}$$

<u>Maximum (ω)</u>

$$\underbrace{Bb\omega^2}_{max} = \text{static load}$$

<u>Que 31</u>
Pg. 37
UoR

Assume Alway
* on crank pin

$a = 70$ cm

$r = 0.3$ m

$c = 2/3$ m

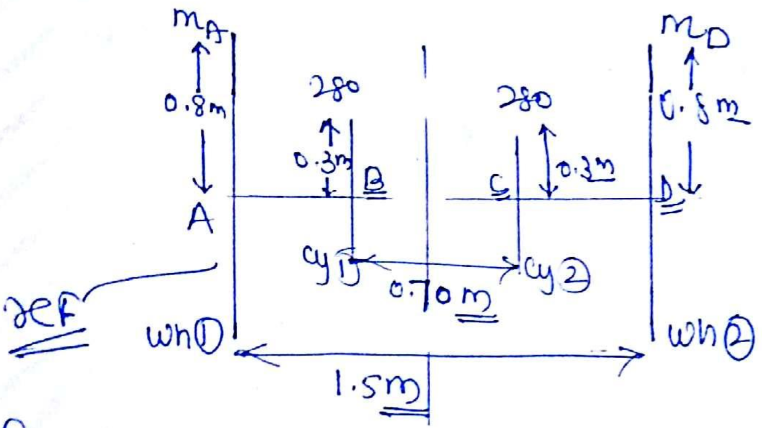
~~rotating~~

<u>revolving mass</u> (B) = 160 kg

reciprocation (m) = 180 kg.

$\omega = 300$ rpm

<u>b = ?</u> <u>B = ?</u>

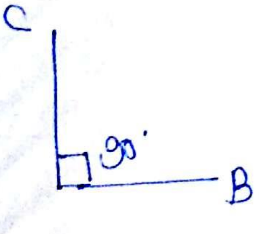


$m_{revol} = 160$ kg

$m_{reci} = 180$ kg

$m_{plane} = 160 + \frac{2}{3} \times 180$

$= 280$ kg



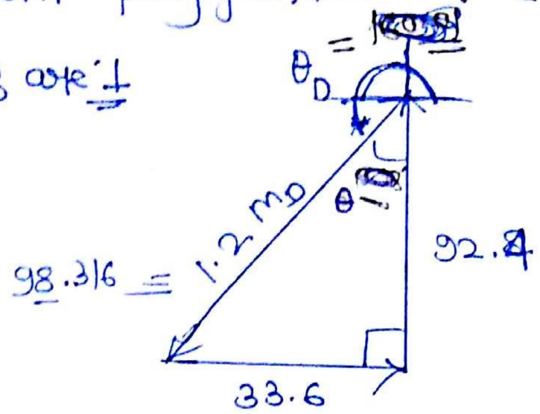
Planes	m	r	mr	l	mrl
A (R.F.)	m_A	0.8	$0.8m_A$	0	0
B	280	0.3	84	0.4	33.6
C	280	0.3	84	1.1	92.4
D	m_D	0.8	$0.8m_D$	1.5	$1.2M_D$.

Moment polygon:—

C & B are ⊥

$\theta_D = 280.02$

$\theta = 19.98$

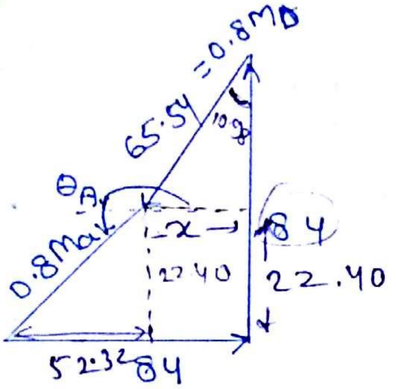


$$1.2\,M_D = \sqrt{(92.4)^2 + (33.6)^2}$$

$$M_D = 81.93 \text{ kg}$$

$$0.8\,M_D = 65.54 \text{ kg}$$

force polygon



$$\cos 19.98 = \frac{(84)^2 + (65.54)^2 - (x)^2}{2 \times 84 \times 65.54}$$

$a = 31.67$

$m_a = 71.14$ kg

✶ Hammer blow $= Bb\omega^2$ → only for reciprocating mass!

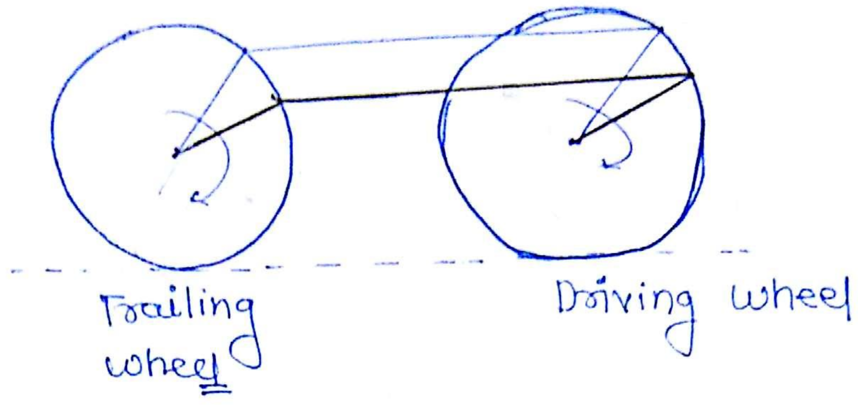
280 kg — 81.93 kg

120 kg — $\frac{81.93}{280} \times 120 = 35.11$ kg.

$$\text{Hammer blow} = 35.11 \times 0.8 \times \left(\frac{2\pi \times 300}{60}\right)^2$$

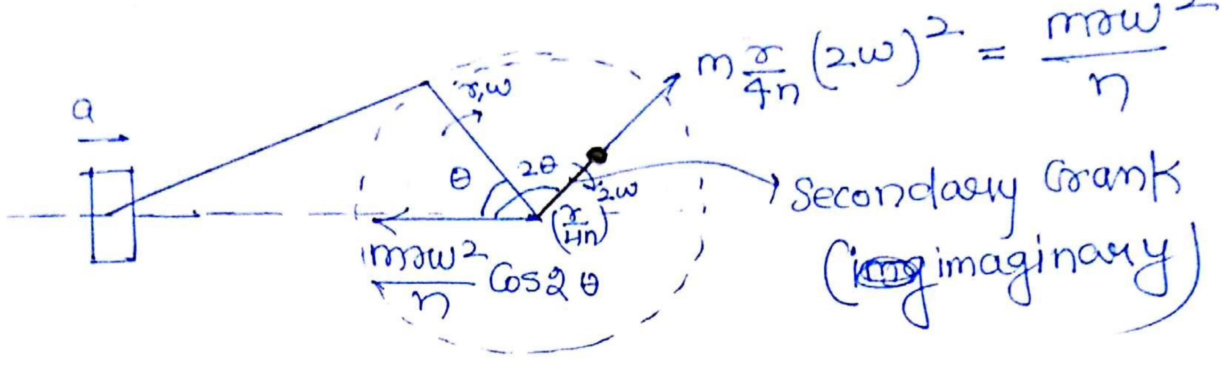
$$= 27.72 \text{ kN}$$

Use of Coupling rod in locomotive :-



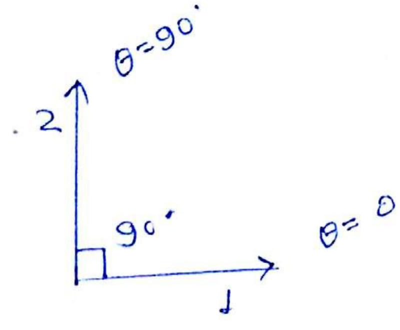
Coupling rod was basically ~~reduce~~ introduce to decrease the amount of Hammer blow by decrease the balacnce mass requirement in order to balance the reciprocating mass of the cylinder through spliting the reciprocating b/w the driving wheel & trailing wheel this was the development from the passenger locomotive to express locomotive in which two - two wheels are coupled to the super fast locomotive in which three-three wheel were coupled.

Balancing of secondary force :-

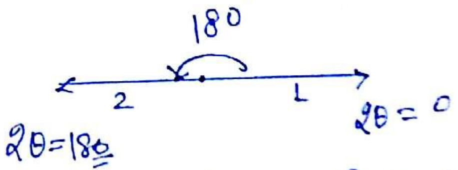


Primary		Secondary
m	$\longrightarrow$	m
r	$\longrightarrow$	$r/4n$
$ω$	$\longrightarrow$	$2ω$
$θ$	$\longrightarrow$	$2θ$

Case-1 Two-Cylinder Inline engine:-

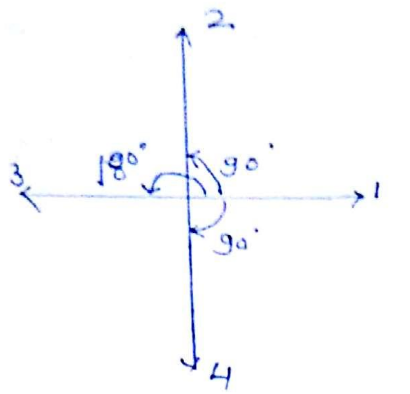


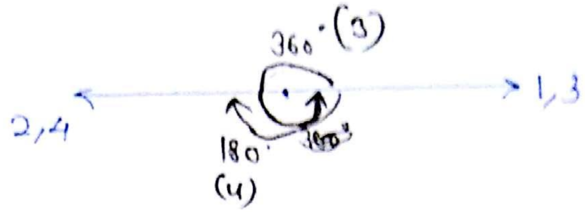
Primary Cranks (actual)



Secondary Cranks (Imaginary)

Case-2 Four cylinder Inline engine:-



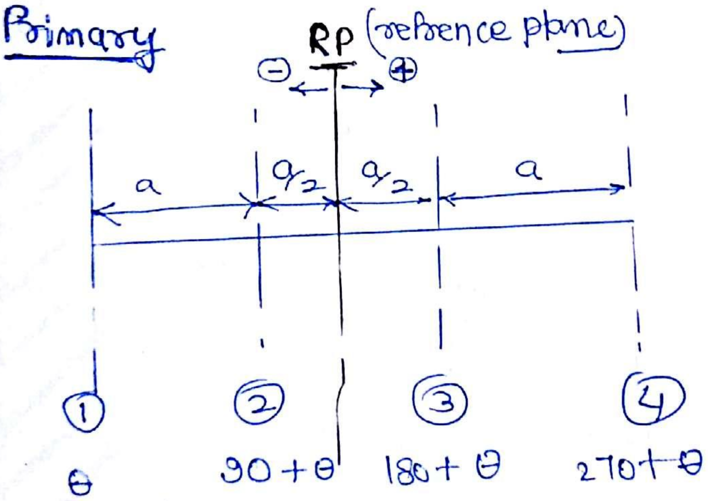


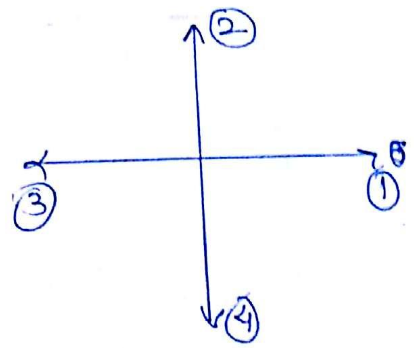
Primary cranks (Actual).

Secondary cranks (Imaginary)

Four Cylinder Inline engine:-

Primary





$$(F_{net})_{primary} = mrw^2\left[\cos\theta + \cos(90+\theta) + \cos(180+\theta) + \cos(270+\theta)\right]$$

$$(F_{net})_{primary} = mrw^2\left[\cos\theta - \sin\theta - \cos\theta + \sin\theta\right] = 0$$

$$(M_{net})_{primary} = \cancel{mrw^2\left[\cos\theta\left(\frac{-3a}{2}\right) + \cos(90+\theta)\left(\frac{-a}{2}\right) + \cos(180)\left(\frac{a}{2}\right)\right.}$$

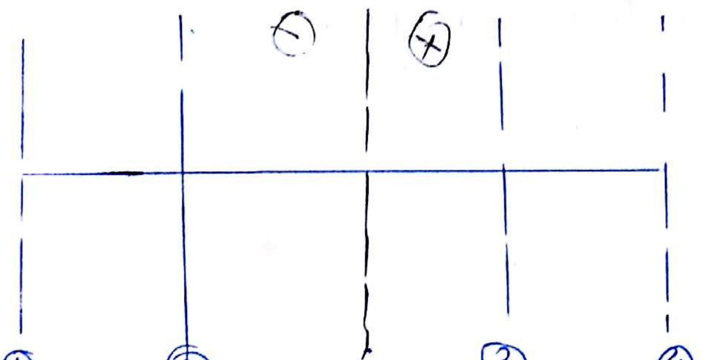
$$(M_{net})_{primary} = mrw^2\left[\cos\theta\left(\frac{-3a}{2}\right) + \cos(90+\theta)\left(\frac{-a}{2}\right) + \cos(180+\theta)\left(\frac{+a}{2}\right) + \cos(270+\theta)\left(\frac{3a}{2}\right)\right]$$

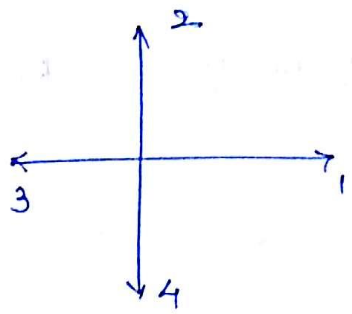
$$= mrw^2 a\left[\frac{-3\cos\theta}{2} + \frac{\sin\theta}{2} - \frac{\cos\theta}{2} + \frac{3\sin\theta}{2}\right]$$

$$= mrw^2 a\left[-2\cos\theta + 2\sin\theta\right]$$

$$\boxed{(M_{net})_{primary} = 2\sqrt{2}\, mrw^2 a}$$

*



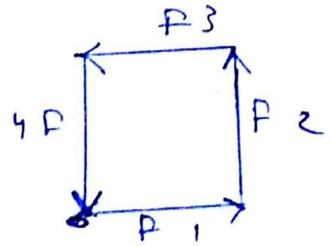


①	②	③	④
$mrw^2 = F$	F	F	F
$mrw^2\left(\frac{-3a}{2}\right)$	$-\frac{M}{2}$	$\frac{M}{2}$	$\frac{3M}{2}$

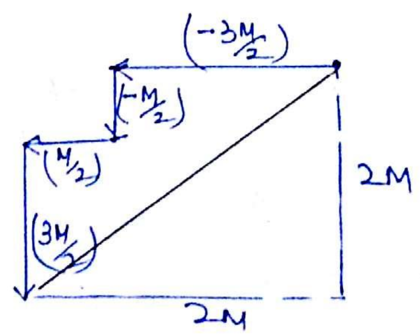
$mrw^2 a\left(\frac{-3}{2}\right)$

$= \frac{-3M}{2}$

$M = mrw^2 a$



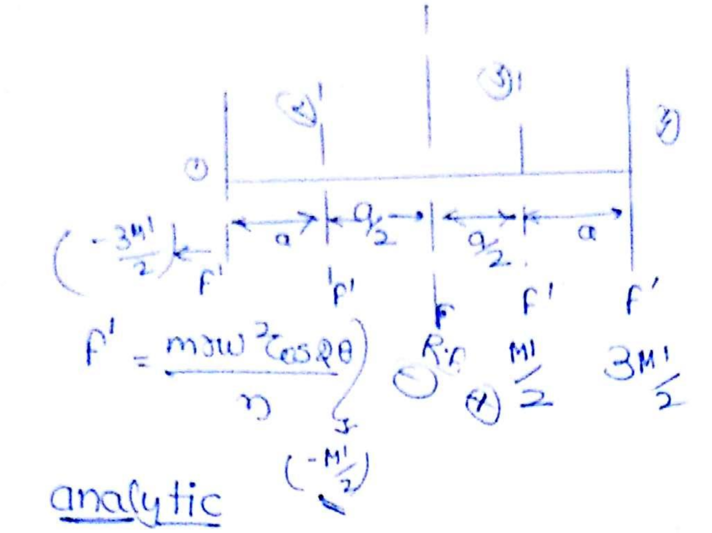
Primary force balancing



$$\left(\left(M_{net}\right)_{primary}\right)_{max} = 2\sqrt{2}M$$

$$= 2\sqrt{2}\, mrw^2 a$$

Secondary force



analytic

secondary force

$$\left(F_{net}\right)_{sec.} = \frac{m r \omega^2}{n}\left[\cos 2\theta + \cos(180+2\theta) + \cos(360+2\theta) + \cos(540+2\theta)\right]$$

$$= \frac{m r \omega^2}{n}\left[\cos 2\theta - \cos 2\theta + \cos 2\theta - \cos 2\theta\right]$$

$$\left(F_{net}\right)_{sec.} = 0$$

Moment

$$\left(M_{net}\right)_{sec.} = \frac{m r \omega^2}{n}\left[\cos 2\theta\left(-\frac{3a}{2}\right) - \cos 2\theta\left(-\frac{a}{2}\right) + \cos 2\theta\left(\frac{a}{2}\right) + \cos 2\theta\left(-\frac{3a}{2}\right)\right]$$

$$= \frac{m r \omega^2 a}{n}\left[-\frac{3\cos 2\theta}{2} + \frac{\cos 2\theta}{2} + \frac{\cos 2\theta}{2} - \frac{3\cos 2\theta}{2}\right]$$

$$\left(M_{net}\right)_{sec.} = -\frac{m r \omega^2 a}{n}\left(2\cos 2\theta\right)$$

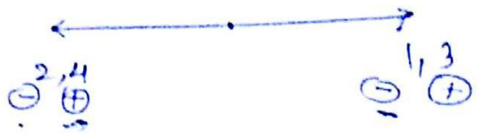
$$M' = \frac{m r \omega^2 a}{n}$$

$$\left(M_{net}\right)_{sec} = -2M'\cos 2\theta$$

$$\left(M_{net}\right)_{sec.} = 2M'$$

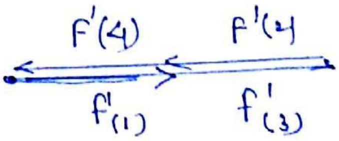
Alternative

using Secondary Crank



Secondary force polygon (SFP)

$$f' = \frac{mr\omega^2}{n}\cos 2\theta$$

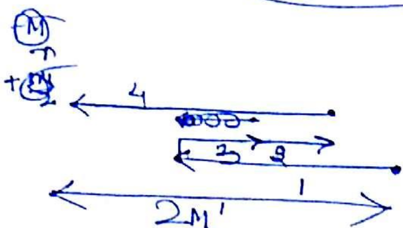


$\boxed{f_{net} = 0}$

1-3-2-4 — firing order

(SMP)

See moment polygon



$$M_1 = \frac{mr\omega^2}{n}\cos 2\theta\left(-\frac{3a}{2}\right)$$

① → $-\frac{3M'}{2}$

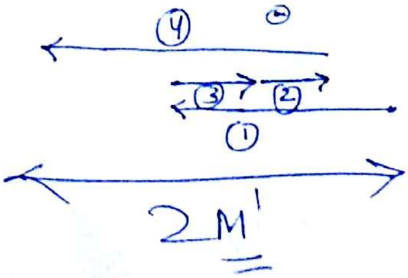
② → $-\frac{M'}{2}$

③ → $+\frac{M'}{2}$

④ → $+\frac{3M'}{2}$

$$M_{ax} = -\frac{3M'}{2}$$

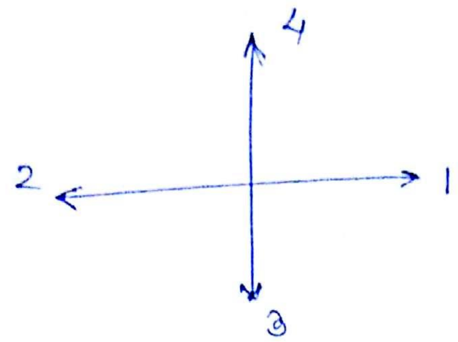
$$M' = \frac{mr\omega^2 a}{n}$$



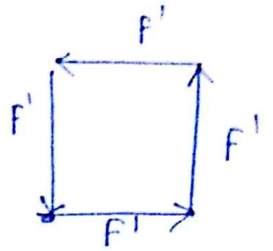
$$M_{net} = 2M'$$

Case-2

① - ④ - ② - ③

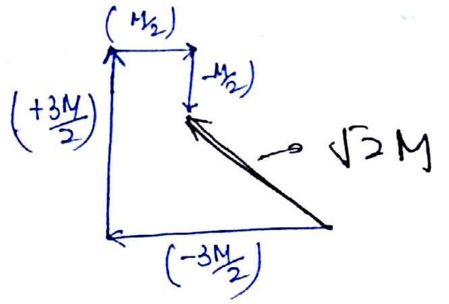


(Primary) force polygon (PFP)



$(F_{net}) = 0$

Primary Moment polygon (PMP)



Net Moment $= \sqrt{2}M$

Secondary

order

1 - 4 - 2 - 3

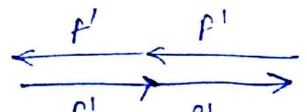
$M_1 = -\frac{3M'}{2}$

$M_2 = -M'/2$

$M_3 = M'/2$

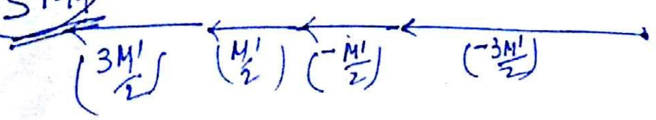
$M_4 = \frac{3M'}{2}$

SFP



$(F_{net}) = 0$

SMP

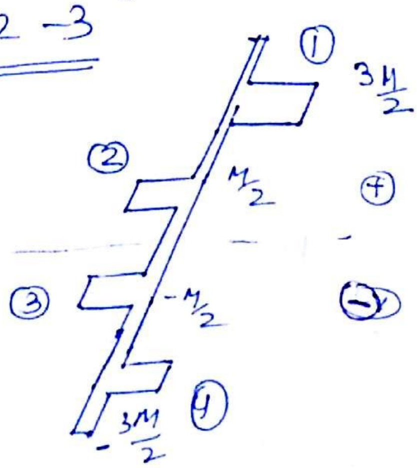


$(M_{net}) = 4M'$

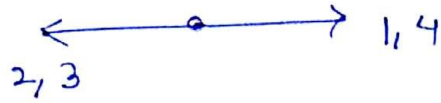
Note: As we can see with change in ~~for~~ firing order net unbalance force and moment changes, so we wish to have such a firing order for which net unbalance force and moment are minimum.

Real firing order of 4-cylinder engine:-

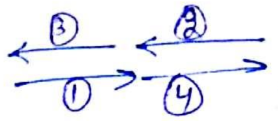
1-4-2-3



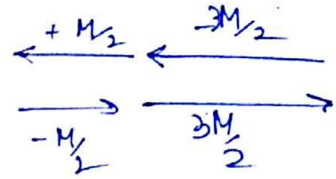
Primary crank



PFP

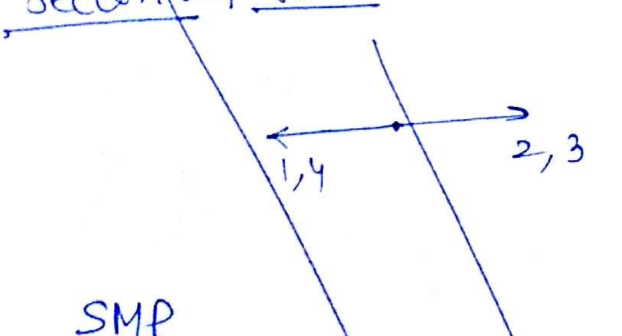


$(F_{net}) = 0$

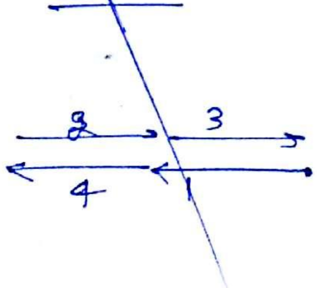


$(M)_{net} = 0$

Secondary cran

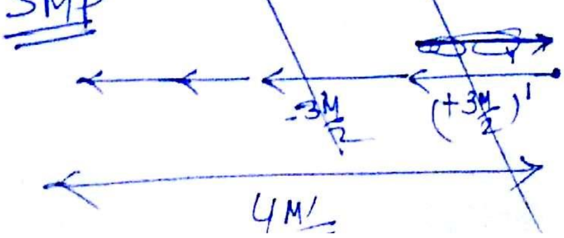


SFP



$(F_{net}) = 0$

SMP



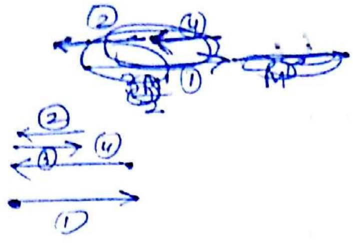
Secondary Crank

SFP

$$(F)_{net} = 4F'$$

$$(F)_{sec} = \frac{4\, m\, r\, \omega^2}{n}$$

SMP

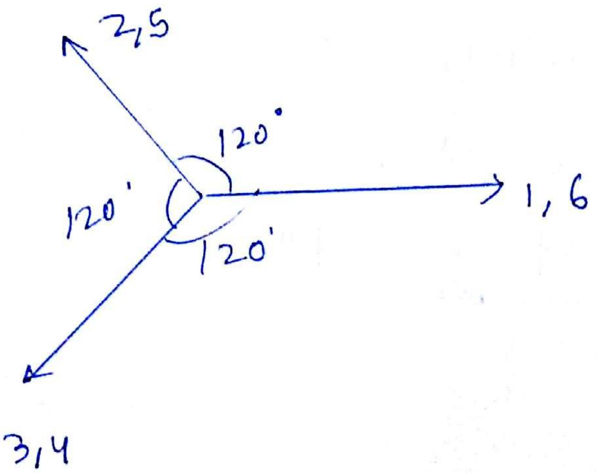


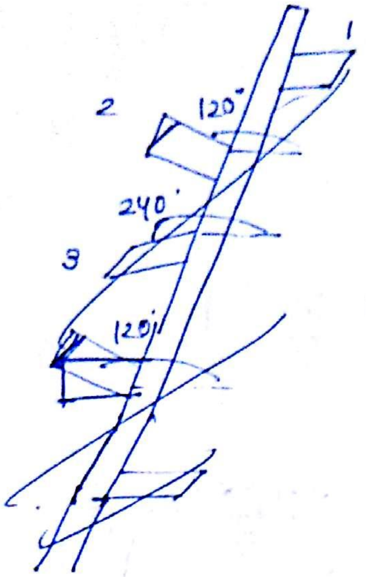
$$(M)_{net} = 0$$

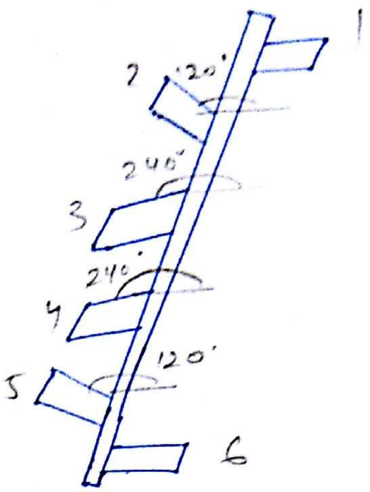
Note Real Four cylinder inline engine is not complately balance (because of net secondary force), to have complate balancing we require mini six cylinder inline engine.

Real firing order for 6-cylinder complatey balance in line engine (1-6-2-5-3-4)

Primary Crank

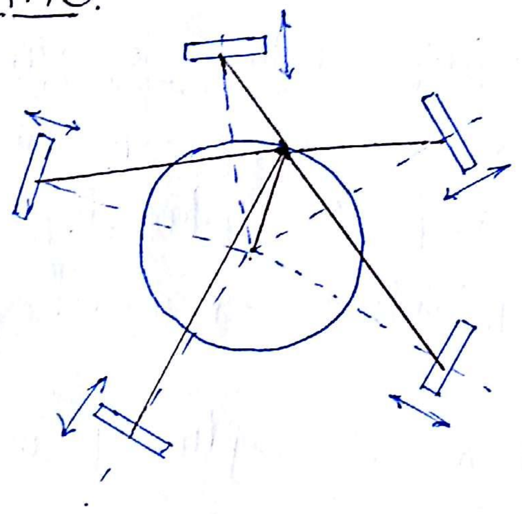






* Cranks are at 120°.

Radial Engine:

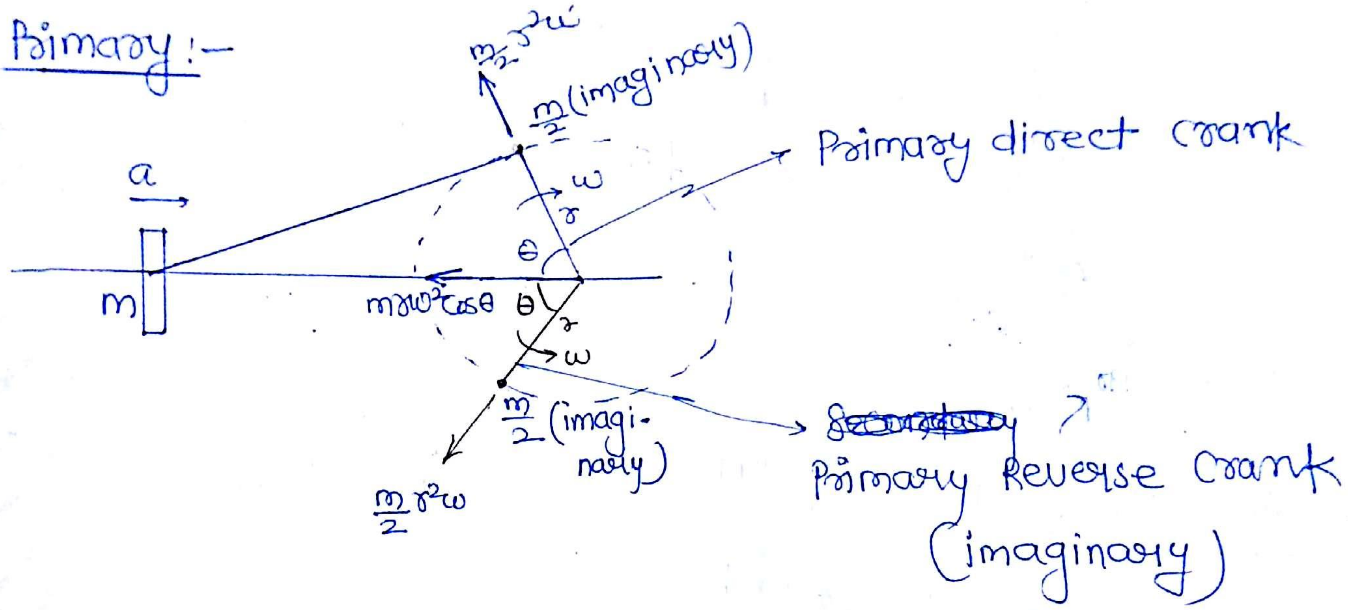


In Radial engines there is only single crank and whose plane of rotation remains same for all the cylinder so there is no unbalanced primary & secondary moment or couple.

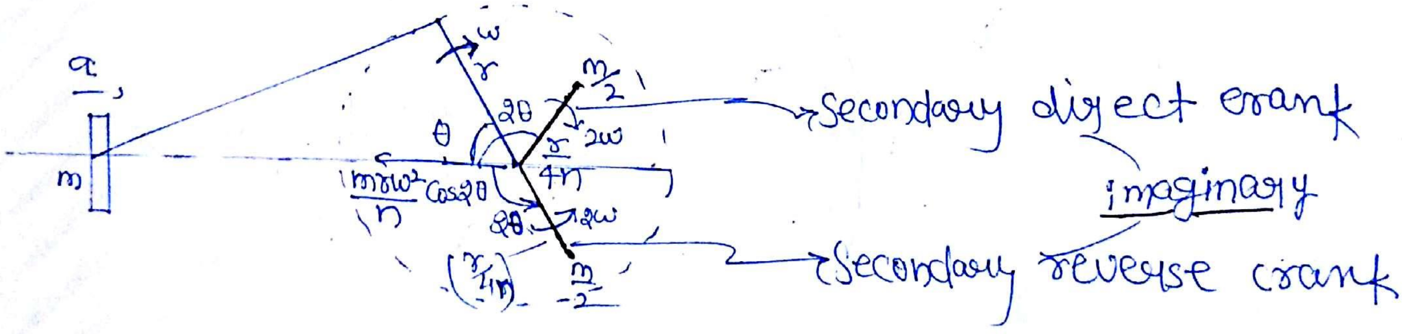
Direct and Reverse crank Method:-

This method is applied in the multicylinder radial engine to get primary & Secondary force information along with their magnitude and direction

Primary:-

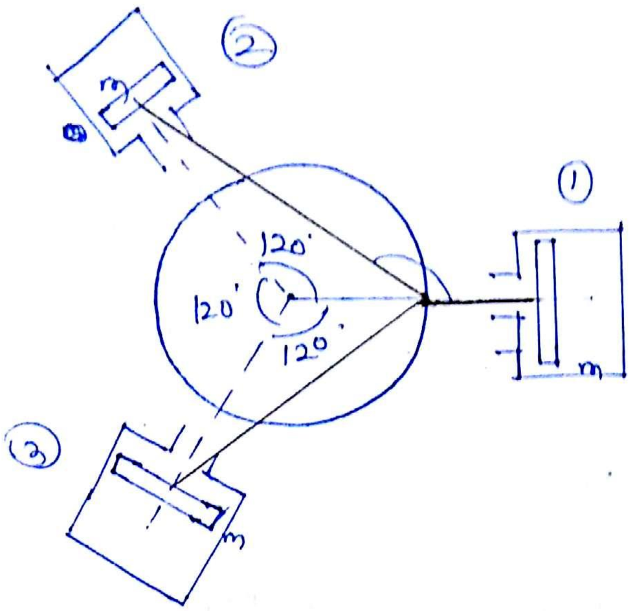


Secondary:-



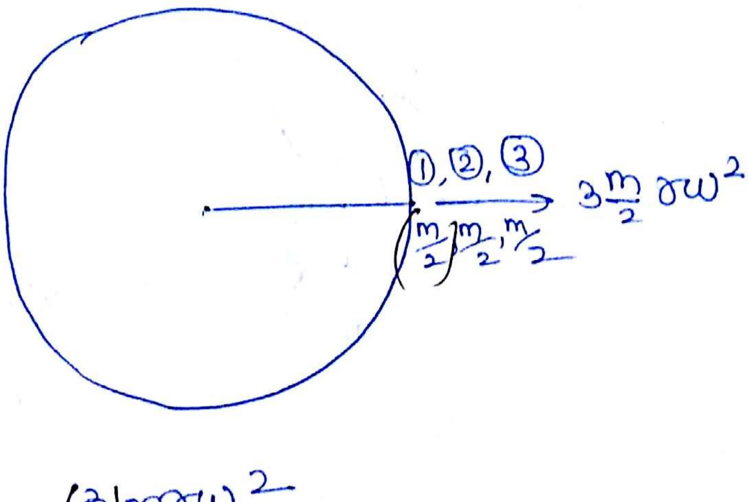
Three Cylinder 120° Radial Engine :-

speed of crank ω



Primary :- speed $= \omega$

Primary Direct crank
$\omega \to \theta$ (same sense)

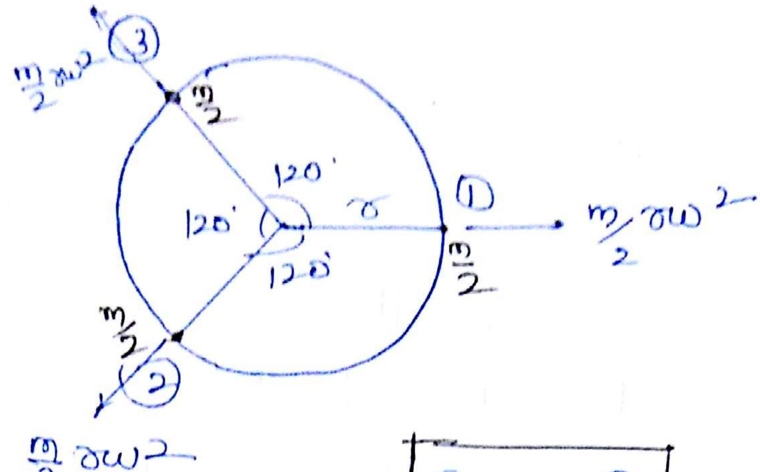


$$F_{net} = \left(\frac{3}{2}\right) m r \omega^2$$

* $\boxed{\left(F_{net}\right)_{primary} = \frac{3}{2} m r \omega^2 \; (\longrightarrow)}$ Net primary force

Primary Reverse crank

$\omega \to \theta$ (opp dir^n)

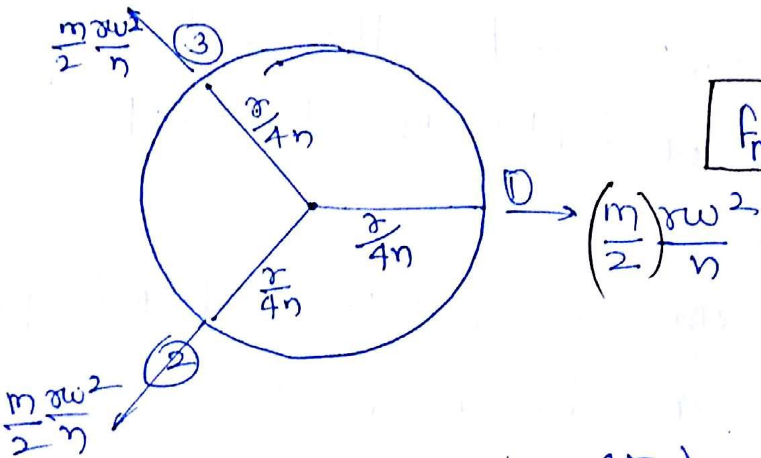


$$\boxed{F_{net} = 0}$$

Secondary

Secondary direct crank $\left(\frac{r}{4n}\right)$

$(2\omega) \to (2\theta)$ same dir^n

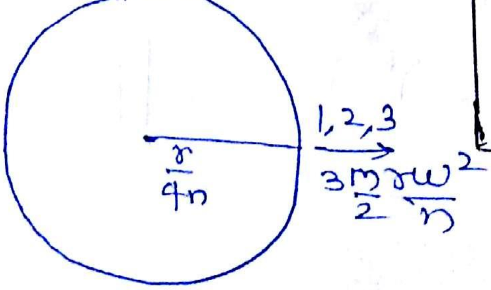


$$\boxed{F_{net} = 0}$$

Secondary reverse crank $\left(\frac{r}{4n}\right)$

$(2\omega) \to (2\theta)$ (opposite dir^n)

$(F_{net}) = \frac{3}{2}\frac{m r\omega^2}{n}$



$$\boxed{(F_{net})_{sec} = \frac{3}{2}\frac{m r\omega^2}{n} \ (\rightarrow)}$$

Net sec. force

Point to remember:-

1. How to draw radial engine

Draw the circle, draw the radial lines according to the number of cylinder (line of strokes of piston) and then fix a crank in circle and then join the respecting connecting rods of all cylinders to their pistons with this single crank point.

2. How to draw primary & secondary crank.

First draw the circle then decide the cylinder (system), then see if Line of stroke and the crank and the angle made by from if L.O.S. and then procedes according the concept (direct/reverse)

Que For 90' V- engine find max. and min primary and secondary unbalance forces.

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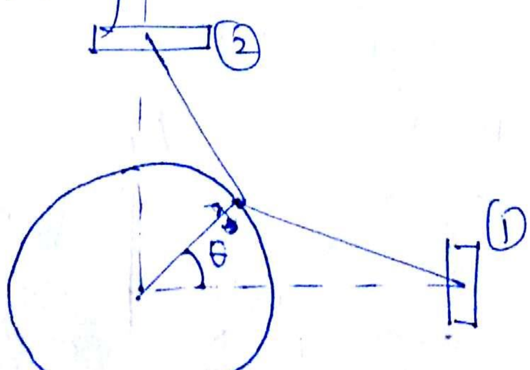
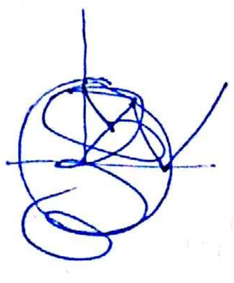
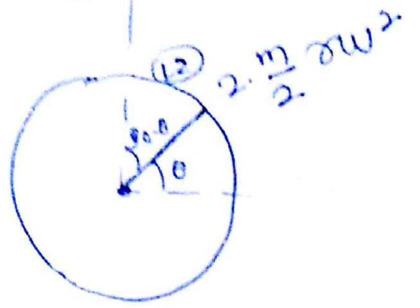


90'-V engine

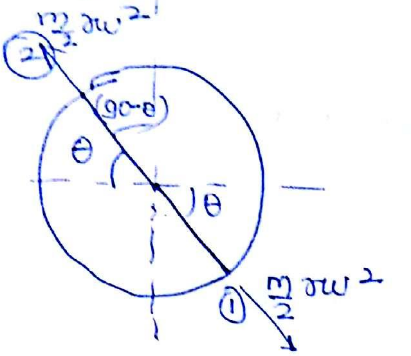


Primary direct crank



$$F_{net} = m r \omega^2$$

primary reverse crank

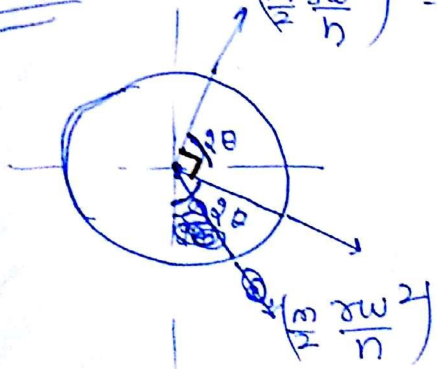


$$(F_{net})_{primary} = m r \omega^2$$

secondary direct

$2\omega - 2\theta$

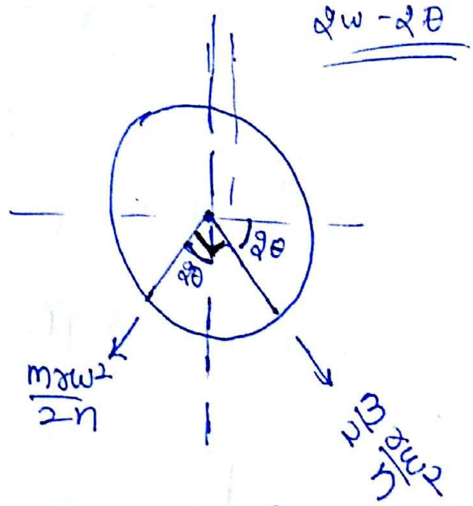
$$\left(\frac{m}{2}\frac{r\omega^2}{n}\right) = \frac{m}{2}\frac{r}{4n}(2\omega)^2$$



$$F_{net} = \cancel{\frac{m r\omega^2}{\sqrt{2}\, n}}$$

secondary reverse

$2\omega - 2\theta$



$$F_{net} = \cancel{\frac{m r\omega^2}{\sqrt{2}\, n}}$$

$$(F_x)_{secon} = \frac{m}{2}\frac{r\omega^2}{n}\left[\cos 2\theta + \sin 2\theta + \cos 2\theta - \sin 2\theta\right]$$

$$(F_x)_{sec.} = \frac{m r \omega^2}{n} \cos 2\theta$$

$$(F_y)_{sec} = \frac{m}{2}\frac{r\omega^2}{n}\left[\sin 2\theta - \cos 2\theta - \sin 2\theta - \cos 2\theta\right]$$

$$(F_y)_{sec.} = -\frac{mr\omega^2}{n}\cos 2\theta$$

$$(F_{net})_{sec.} = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= \frac{mr\omega^2}{n}\sqrt{\cos^2 2\theta + \cos^2 2\theta}$$

$$(F_{net})_{sec.} = \frac{\sqrt{2}\, mr\omega^2}{n}\cos 2\theta$$

max when $\theta = 0$

$$(F_{sec.})_{max} = \frac{\sqrt{2}\, mr\omega^2}{n}$$

min $2\theta = 90, 270^\circ$ $\theta = 45, 135$

$$(F_{sec.})_{min} = 0$$