* Time varying Fields, maxwells eqⁿ:-

* The electric field is produced by time varying magnetic field $\left(\nabla \times E = -\frac{\partial B}{\partial t}\right)$ Produce

michael Faraday (Experimental rerch)

* The magnetic field is produced by time varying electric field

$$(\nabla \times \vec{H}) = \underbrace{\frac{\partial \vec{D}}{\partial t}}_{(maxwell)} \longrightarrow \text{James clerk maxwell (Theoretical research)}$$

Production of Em waves → $\vec{E}$ produces $\vec{H}$; $\vec{H}$ produces $\vec{E}$

* Faraday's Law →

* This law states that the time varying magnetic flux (magnetic field) produces an electric motive force (EMF) which produces a current in a closed path

mathematicaly

$$\boxed{\underset{(\text{induced})}{emf} = -\frac{d\phi}{dt}}$$

minus sign is due to Lenz's law, which states that the emf induced opposes the changing magnetic flux.

$\frac{d\phi}{dt}$ is not 0 in the following cases →

(i) A stationary loop in a time varying magnetic field (Xmer emf)

(ii) A moving loop in a static magnetic field (motional emf)

(iii) Both the cases (i.e. a moving loop in a time varying magnetic field) (Xmer + motional emf)

* Maxwells Equations → (derivation not required)

(1) Faraday's law →

$$emf = \frac{-\partial \phi}{\partial t} \quad \text{———(i)}$$

$$emf = \oint E \cdot dl \quad \text{———(ii)} \qquad \left\{ \begin{array}{l} V(t) = -\oint E \cdot dl \\ emf \text{ is opposite of } V(t) \end{array} \right\}$$

By defination $\phi = \iint \vec{B} \cdot \vec{ds}$ ———(iii)

From (ii), (iii) in (i)

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \iint \vec{B} \cdot \vec{ds} \quad \text{———(iv)}$$

By stoke theorem.

$$\oint E \cdot dl = \iint (\bar{\nabla} \times \bar{E}) \cdot \vec{ds} \quad \text{———(v)}$$

from (v) in (iv)

$$\iint (\bar{\nabla} \times \bar{E}) \, ds = \iint \left(\frac{-\partial B}{\partial t} \right) ds$$

Compare both side

$$\boxed{(\bar{\nabla} \times \bar{E}) = \frac{-\partial B}{\partial t}}$$ differential form of maxwell eqn (∇)

(2) Ampere circuital law →

$$\oint \bar{H} \cdot \bar{dl} = I_{enclosed} = I$$

$$\nabla \times \bar{H} = \bar{J} \quad \text{———(i)}$$

Eqn (i) do not satisfy the continuity eqn, so maxwell introduced $\frac{\partial \bar{D}}{\partial t}$

$$\boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}}$$

where; $\bar{J} = \sigma \bar{E} = \frac{I_c}{area}$ = Conduction current density $= \bar{J_c}$

$$\frac{\partial \bar{D}}{\partial t} = \frac{I_d}{area} = \frac{\frac{d\psi}{dt}}{area} = \bar{J_d} = \text{displacement Current density}$$

So total Current density in any material

$$\bar{J_t} = \bar{J_c} + \bar{J_d}$$

(In a cond^r $J_d \to 0$, because no electric dipole)
(In a dielectric $J_c \to 0$, because $J_c = nqu$
$\because n = 0$ free $e^- = 0$)

Note → Maxwell eqⁿ:–

Law	integral form ($\int$)	differential form (∇)
(1.) Faraday's law	$\oint \vec{E}\cdot d\vec{l} = \iint \left(\frac{-\partial \vec{B}}{\partial t}\right)\cdot d\vec{s}$	$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
(2.) Ampere circuital	$\oint \vec{H}\cdot d\vec{l} = I + \iint \left(\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}$	$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
(3.) Gauss law of magnetic field	$\oiint \vec{D}\cdot d\vec{s} = Q$	$\vec{\nabla}\cdot\vec{D} = \rho_v$
(4.) Gauss law of electric field	$\oiint \vec{B}\cdot d\vec{s} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$