Chapter-3

Quadrilaterals

U2Q5W9

You already know about triangle. A triangle has 3 sides and 3 angles. In this chapter, we shall discuss about simple and closed figures made up of more than 3 line segments.

Observe the following figures.

3.1 Quadrilaterals :

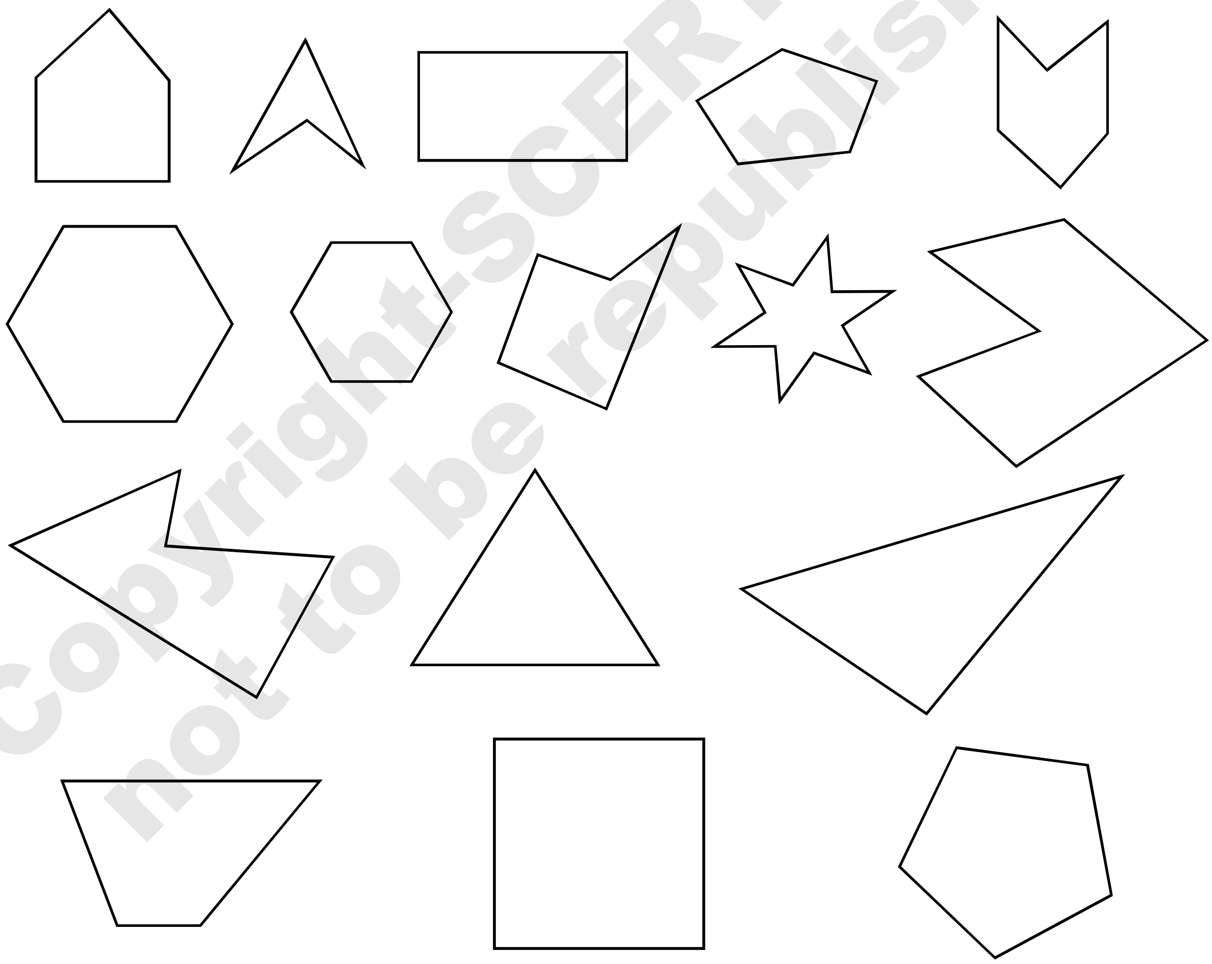


figure - 3.1

What have you seen from the figures?

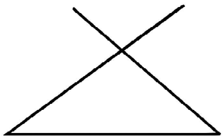
— These are plane figures.

— Figures are made of finite number of line segments.

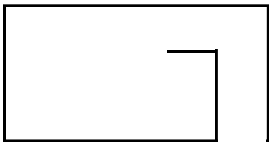
— The figures are simple and closed.

A simple and closed figure in plane made up of finite number of line segments is called a ***polygon****.*

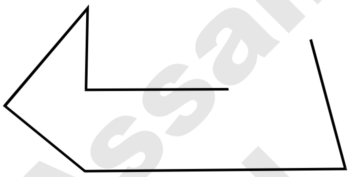
The following figures are **not** polygon. Because, figure (i) is not simple, figures (ii) and (iii) are not closed.



(i)



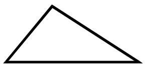
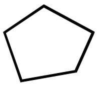
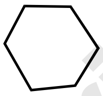
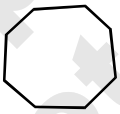
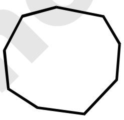
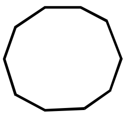
(ii)



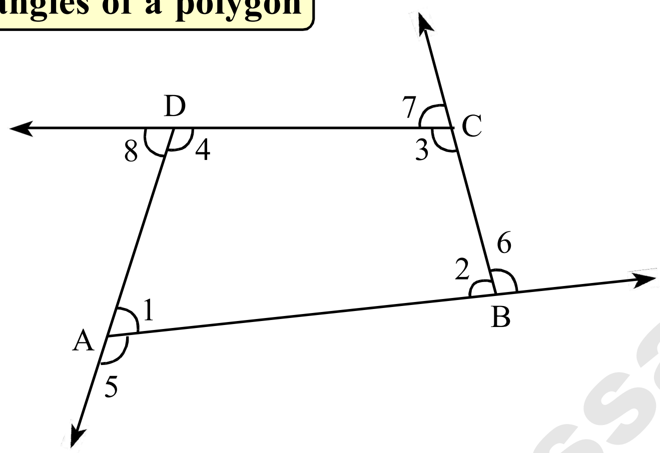
(iii)

3.2 Classification of polygons

Now we classify the polygons according to the number of sides or vertices they have:

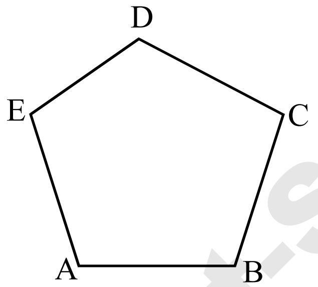
figure	number of sides/vertices	Name
	3	Triangle
	4	Quadrilateral
	5	Pentagon
	6	Hexagon
	7	Heptagon
	8	Octagon
	9	Nonagon
	10	Decagon

3.3 Interior/exterior angles of a polygon

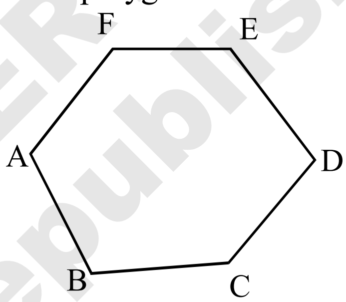


In the quadrilateral ABCD ∠1, ∠2, ∠3 and ∠4 are **interior angles** and ∠5, ∠6, ∠7 and ∠8 are **exterior angles**.

Activity Identify the interior and exterior angles of the following pentagon and hexagon. If necessary extend the sides of the polygons.

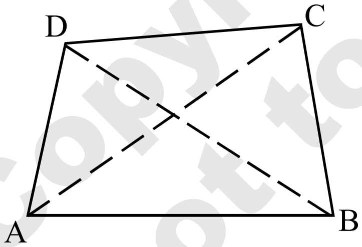


Pentagon

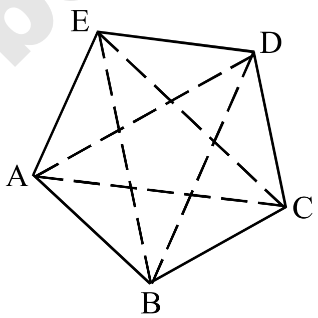


Hexagon

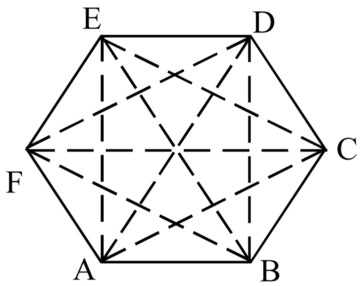
3.4 Diagonals of polygons



The line segment AC connecting the vertices A and C and the line segment BD connecting the vertices B and D are called diagonals of ABCD.



The line segments AD, AC BD, BE, EC are diagonals of the pentagon ABCDE.

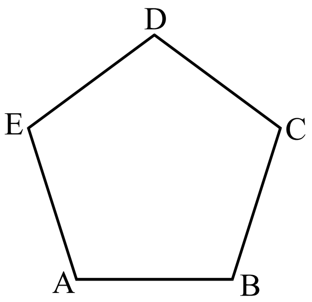


The line segments AC, AD, AE, BF, BE, BD, CF, CE, DF are the diagonals of the hexagon ABCDEF.

Observe the given figures. We can say that the diagonals of a polygon are line segments connecting two non consecutive vertices.

3.5 Adjacent sides, adjacent angles of a ploygon

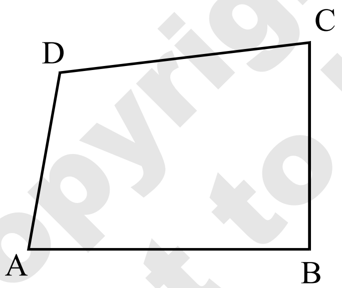
From your earlier class, you know that two angles are said to be adjacent if they have a common side and a common vertex. In polygons, any two interior angles sharing a common side are called adjacent angles. Similarly, two sides of a polygon sharing a common vertex of the polygon are called adjacent sides.

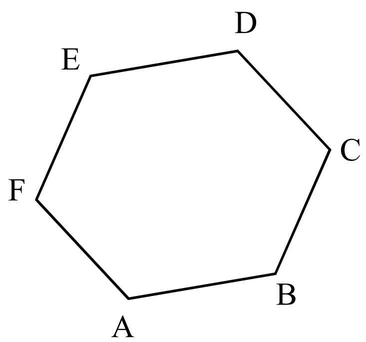


In the adjoining figure AB and BC are adjacent sides. Similarly, BC and CD, CD and DE, DE and EA are also adjacent sides. The angles at the end of a side are adjacent angles. In the adjoining figure, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, $\angle D$ and $\angle E$, $\angle E$ and $\angle A$ are adjacent angles respectively.

Try yourself

Identify the pairs of adjacent sides and adjacent angles of the following quadrilateral and hexagon.





3.6 Convex and concave polygons

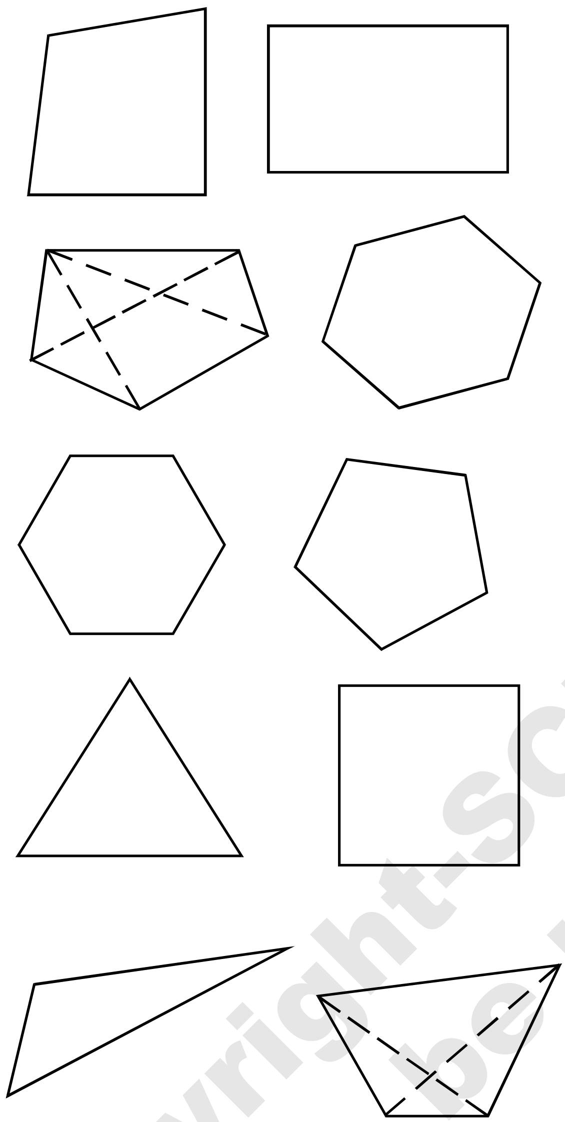


Figure 3(*a*)
Convex polygons

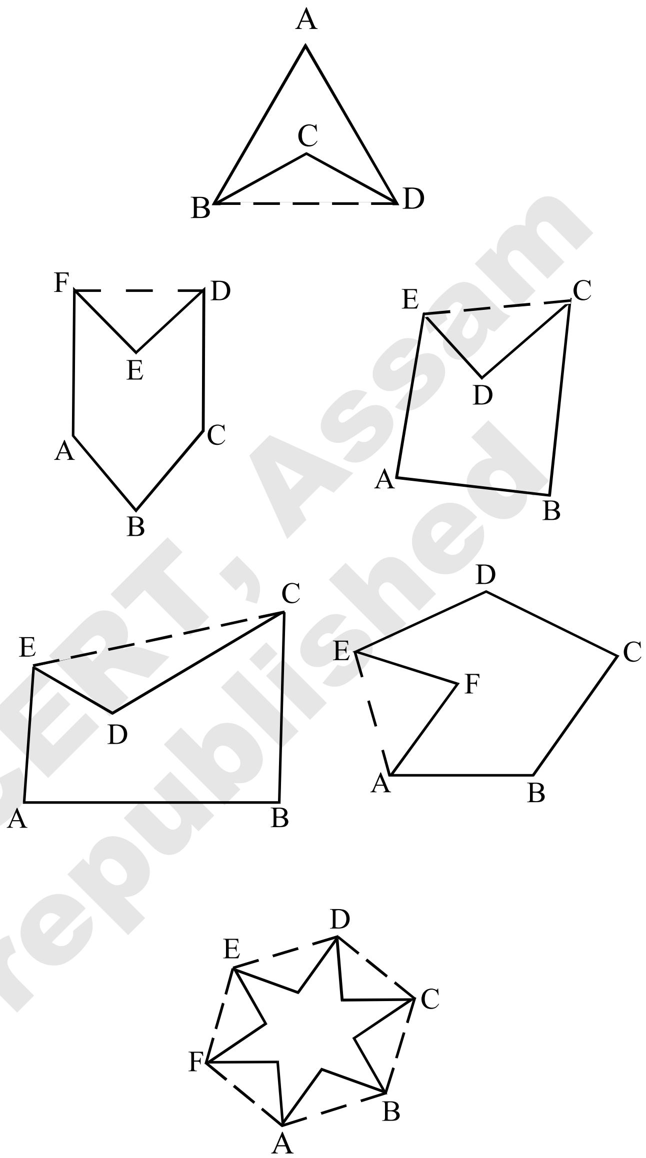


Figure 3(*b*)
Concave polygons

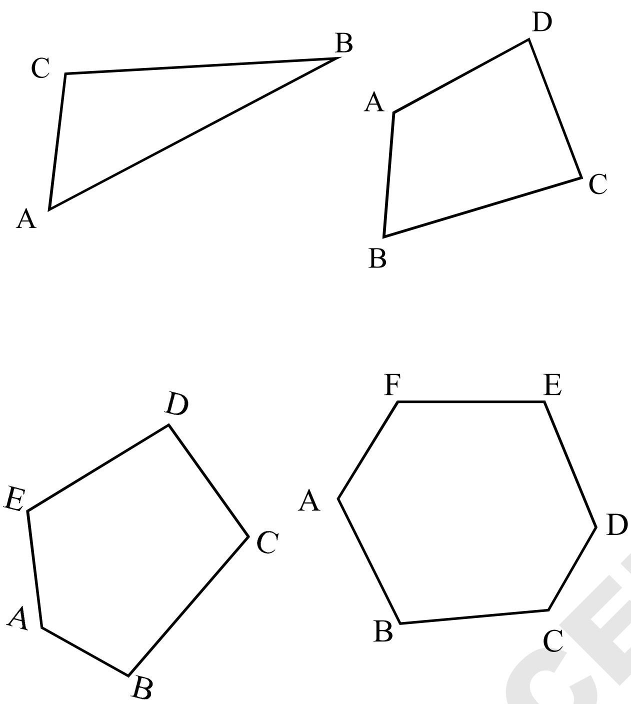
Convex polygon

- ☆ Measures of interior angles are less than 180°
- ☆ Diagonals are in the interior of the polygon.

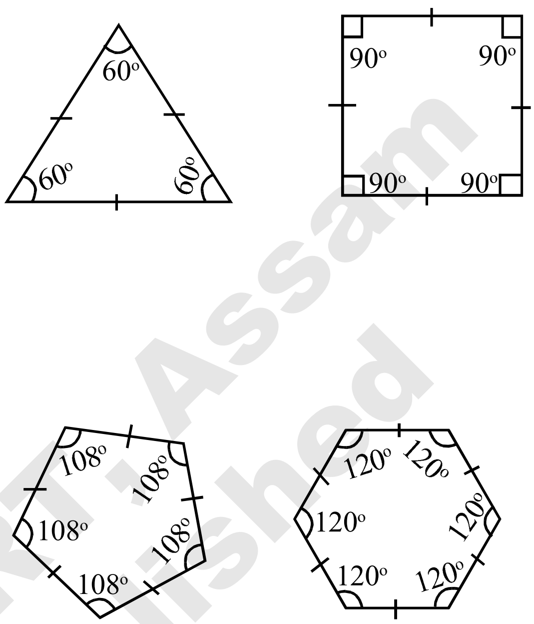
Concave polygon

- ☆ At least one of the interior angles is greater than 180°.
- ☆ At least one of the diagonals is outside the polygon.

Again in fig 3 (a) we name the convex polygons according to their special character.



In these convex polygons length of the sides are not equal and measure of the interior angles are not equal. These polygons are called **irregular polygon**.



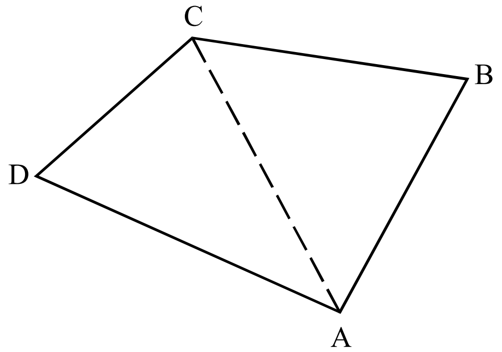
In these convex polygons length of the sides are equal and angles are of equal measure. These convex polygons are called **regular polygon**.

In this chapter we will be dealing with convex polygons only.

3.7 Sum of the measures of angles of a polygon.

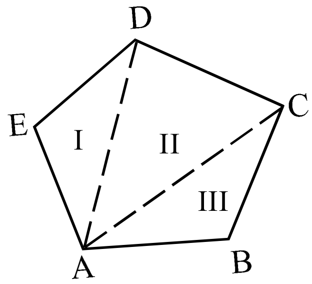
The sum of the measures of the three angles of a triangle is180°. Now, using this property we find the sum of the measures of the interior angles of a polygon.

1. Sum of the measures of interior angles of a quadrilateral.



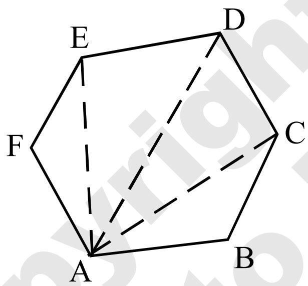
In the adjoining figure, diagonal AC divides the quadrilateral ABCD into two triangles ADC and ABC. Therefore sum of the measures of interior angles of the quadrilateral ABCD = sum of the measures of the interior angles of ABC + sum of the measures of the interior angles of ADC = $180^o + 180^o = 360^o$

2. Sum of the measures of interior angles of a pentagon.



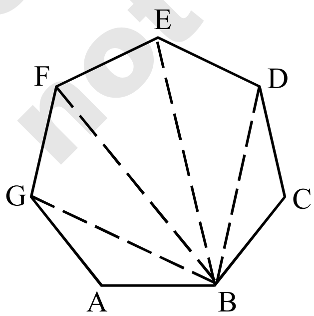
The diagonals AC and AD drawn from the point A divide the pentagon ABCDE into three triangles AED, ADC and ABC. Therefore, sum of the measures of the interior angles of the pentagon ABCDE = sum of the measures of the interior angles of three triangles = $180^o + 180^o + 180^o = 3 \times 180^o = 540^o$.

3. Sum of the measures of interior angles of a hexagon.



The diagonals AE, AD and AC drawn from the point A divide the hexagon ABCDEF into four triangles AFE, AED, ADC and ABC. Therefore, sum of the measures of the interior angles of the hexagon ABCDEF = sum of the measures of the interior angles of four triangles = $180^o + 180^o + 180^o + 180^o = 4 \times 180^o = 720^o$

4. Sum of the measures of interior angles of a heptagon.



The diagonals BD, BE, BF and BG drawn from the point B divide the heptagon ABCDEFG into five triangles. Therefore, sum of the measures of the interior angles of the heptagon = $5 \times 180^o = 900^o$ (why?)

Activities Find the sum of the measures of the interior angles of octagon, nonagon and decagon.

Now we shall try to find the pattern of the sum of the measures of the interior angles of a polygon and develop a formula for sum of interior angles of a polygon.

polygon	number of sides	sum of interior angles	pattern
Triangle	3	180^o	$(3-2)\times 180^o$
Qudrilateral	4	360^o	$(4-2)\times 180^o$
Pentagon	5	540^o	$(5-2)\times 180^o$
Hexagon	6	720^o	$(6-2)\times 180^o$
Heptagon	7	900^o	$(7-2)\times 180^o$
Octagon	8	,,	,,
Nonagon	9	,,	,,
Decagon	10	,,	,,
n sides	n	.....	$(n-2)\times 180^o$ $=(n-2)\times 2\times 90^o$ $=(2n-4)\times 90^o$

Therefore, **the sum of the measures of the interior angles of a polygon with n sides is $(2n-4)\times 90^o$.**

If the polygon is regular then each interior angles

measures $\left(\frac{2n-4}{n}\right)\times 90^o$

[$\because$ sum of the measures of the interior angles of a polygon with n sides is $(2n-4)\times 90^o$ and for a regular polygon angles are of equal measure, so, measure of each interior angle is $\left(\frac{2n-4}{n}\right)\times 90^o$]

Example 1 : Find the measure of the sum of the interior angles of a regular polygon of 12 sides.

Solution : $n = 12$. Therefore, sum of the interior angles is $(2\times 12-4)\times 90^o = 1800^o$

Example 2 : Find the measure of each interior angle of a regular polygon of 15 sides whose sum of the measures of the interior angles is 2340^o.

Solution : Given, sum of the measures of interior angles $= 2340^o$

Number of sides $= 15$

Therefore, measure of each interior angle $= \frac{2340^o}{15}$

$= 156^o$

Example 3 : Find the number of sides of a regular polygon whose each interior angle measures of 160^o

Solution : Let the number of sides of the polygon $= n$

$\therefore$ sum of interior angles $= (2n - 4) \times 90^o$

$\therefore$ measure of each interior angle $= \left(\frac{2n-4}{n}\right) \times 90^o$

According to the question, $\left(\frac{2n-4}{n}\right) \times 90^o = 160^o$

or $\frac{2n - 4}{n} = \frac{160}{90}$

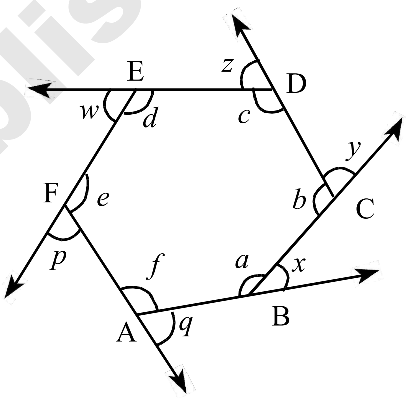
or $18n - 36 = 16n$

or $2n = 36$

or $n = 18$

$\therefore$ Number of sides = 18

3.8 Sum of the measures of the exterior angles



Now we shall try to find the sum of the measures of the exterior angles of a special polygon say hexagon. Let ABCDEF be a hexagon. Total measure of the exterior angles of the hexagon i.e. $\angle x + \angle y + \angle z + \angle w + \angle p + \angle q = ?$

While drawing polygon, moving along the side $\overrightarrow{AB}$ we turn through $\angle x$ at the point B. Again moving along $\overrightarrow{BC}$ we turn through $\angle y$ at the point C and reach the point D. We continue to move in this manner. At the point D we turn through $\angle z$, at E we turn through $\angle w$, at F we turn through $\angle p$ to arrive at the point A. Then we return to the side $\overrightarrow{AB}$ after turning through $\angle q$ at A. Thus, have we not made a complete turn? i.e we have turned through an angle 360^o.

The exterior angle x is formed by extended $\overrightarrow{AB}$ and $\overline{BC}$. Similarly exterior angles y, z, w, p and q can be found. Thus, $\angle x + \angle y + \angle z + \angle w + \angle p + \angle q = 360^o$.

Alternatively,

$\angle x + \angle y + \angle z + \angle w + \angle p + \angle q$

$= 180^0 - \angle a + 180^0 - \angle b + 180^0 - \angle c + 180^0 - \angle d + 180^0 - \angle e + 180^0 - \angle f$

$= 1080^0 - (\angle a + \angle b + \angle c + \angle d + \angle e + \angle f)$

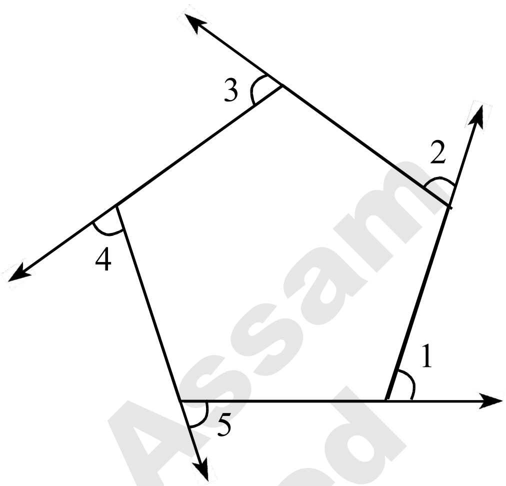
$= 1080^0 - (6 - 2) \times 180^0$

$= 1080^0 - 720^0 = 360^0$

Thus $\angle x + \angle y + \angle z + \angle w + \angle p + \angle q = 360^{o}$. The sum of the measures of the external angles of any polygon is 360^{o}, whatever be the number of sides of the polygon.

Activity Examine whether each exterior angles of the regular polygon are equal or not from the following activities.

1. In the adjoining figure of regular pentagon what is the measure of each interior angle?
2. Is $\angle 1 = \angle 2 = \angle 3 = \angle 4 = \angle 5$? Why?
3. What is the measure of each exterior angle?



Example 4 : Find the number of sides of a regular polygon whose each exterior angle has a measure of 36^{o}.

Solution : Total measure of all exterior angles $= 360^{o}$

Measure of each exterior angle $= 36^{o}$

$\therefore$ Number of exterior angles $= \dfrac{360}{36} = 10$

$\therefore$ Number of sides of the polygon $= 10$

Example 5 : Is it possible to have regular polygon with measure of each exterior angles :

(i) 24^{o} (ii) 22^{o} (iii) 40^{o}

Solution : (i) Total measure of all exterior angles $= 360^{o}$

Each exterior angle $= 24^{o}$

Numbers of exterior angles $= \dfrac{360}{24} = 15$

Thus a regular polygon can be found and number of sides = 15

(ii) Total measure of all exterior angles $= 360^{o}$

Each exterior angle $= 22^{o}$

$\therefore$ Numbers of exterior angles $= \dfrac{360}{22} = 16.36$

Number of exterior angles **cannot be** a decimal number.

Thus it is not possible to have a regular polygon for which measure of each exterior angle is 22^{0}.

(iii) Can be done in the same way.

Example 6 : Is it possible to have a regular polygon with measure of each interior angle as
(i) 24° (ii) 70° (iii) 135°

Solution : (i) The measure of each interior angle of a regular polygon = $\left(\frac{2n-4}{n}\right) \times 90^\circ$

n = number of sides

$$\therefore \left(\frac{2n-4}{n}\right) \times 90^\circ = 24$$

Or, $\frac{2n-4}{n} = \frac{24}{90}$

Or, $180n - 360 = 24n$

Or, $156n = 360$

Or, $n = \frac{360}{156} = 2\frac{48}{156}$

But number of sides cannot be a decimal number or fraction.

Therefore, it is not possible to have a regular polygon whose measure of each angle is 24°

Each interior angle = 24°

$\therefore$ Exterior angle = 156° which is not a divisor of 360°.

Therefore, regular polygon cannot be found.

(ii), (iii) Can be done in the same way.

Example 7 : (i) What is the smallest possible interior angle for a regular polygon and why?
(ii) What is the largest possible exterior angle for a regular polygon and why?

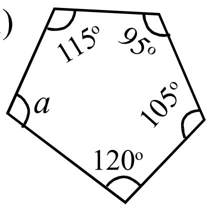
Solution : (i) Since the polygon having smallest number of sides is a triangle and a regular polygonal with three sides is an equilateral triangle, therefore, its interior angles are all equal i.e 60°. Thus, the smallest possible interior angle of a polygon is 60°.

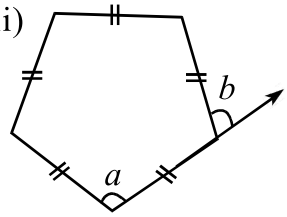
(ii) Following similar arguments, the largest possible exterior angle of a regular polygon is .120°.

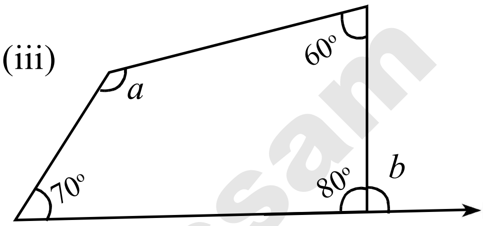
Exercise 3.1

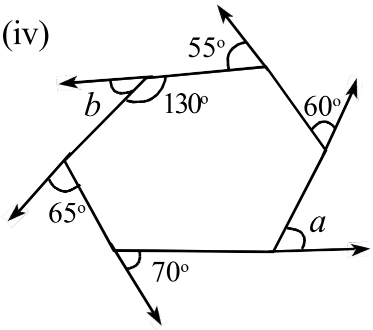
1. Draw the following polygons :
   (i) convex hexagon (ii) concave heptagon (iii) concave pentagon
2. Draw the following convex polygons. Indentify the diagonals in each case and find the total number of diagonals.
   (i) regular hexagon (ii) octagon (iii) nonagon (iv) decagon

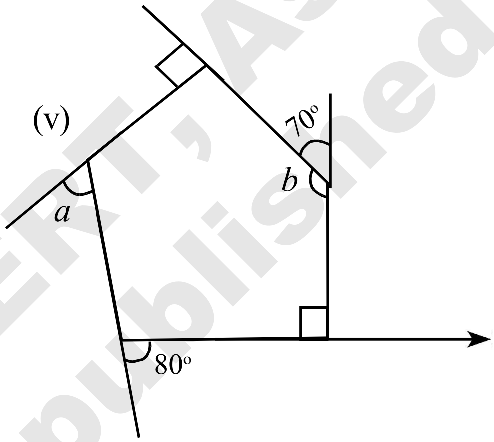
3. Draw the following regular polygons. Find the sum of the angles of each of them. Find the measure of each angle of the polygon.
   (i) regular hexagon (ii) regular nonagon (iii) regular polygon of 12 sides.
4. Find the measures of the angles *a, b* in the following figures.

(i)


(ii)


(iii)


(iv)


(v)


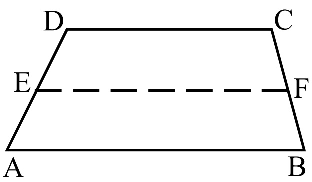
5. Find the number of sides of a regular polygon if one exterior angle has a measure 30°
6. Find the measure of each exterior angle of a regular polygon of 20 sides.
7. The measure of interior angle of a regular polygon is given in the following. Find the number of sides of the polygons.
   (i) 120° (ii) 144° (iii) 156° (iv) 135° (v) 165°
8. The number of sides of polygons are given below. Find the measure of sum of each interior angle of the polygons.
   (i) 12 (ii) 14 (iii) 20 (iv) 24 (v) 25
9. Write with reasons whether the following statements are true or false.
   (i) Each exterior angle of a regular polygon cannot be 25°.
   (ii) Each interior angle of a regular polygon can be 1°.
   (iii) The maximum exterior angle for a regular polygon is 90°.
   (iv) The maximum interior angle for a regular polygon is 180°.
   (v) The minimum interior angle for a regular polygon is 60°.

3.9 Kinds of quadrilateral

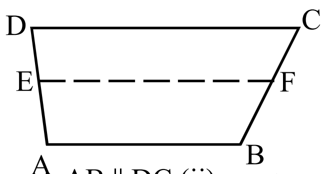
Based on the nature of the sides and angles of a quadrilateral, we get some special quadrilaterals.

1. Trapezium

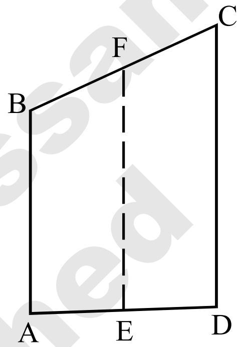
Study the following figures of quadrilateral. In these quadrilaterals any pair of opposite sides are paralle.



AB || DC (i)



AB || DC (ii)



BA || CD

Trapezium *is a quadrilateral with a pair of parallel sides.* The parallel sides are called base sides.

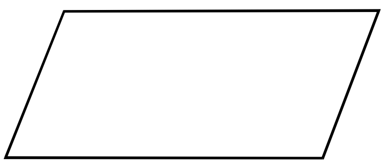
Activity : Draw a trepezium ABCD.

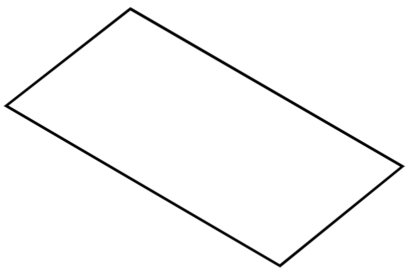
(i) Measure its angles. Is there any relation between the angles?

(ii) Take the measurement of sides. Is $EF = \frac{1}{2}(AB + DC)$?

2. Parallelogram

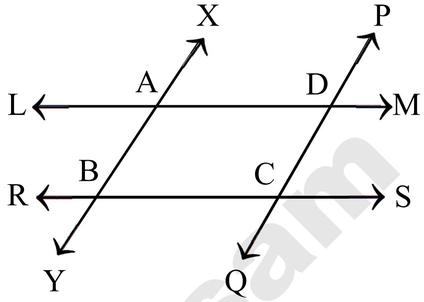
A parallelogram is a quadrilateral whose two pairs of opposite sides are parallel.





In the figure LM and RS are a pair of parallel lines. Another pair of parallel lines XY and PQ intersects LM and RS at A, B, C and D. Does ABCD form a parallelogram?

Now,



(i) Measure the length of the sides AB, BC, CD and DA with the help of a scale and note down. Can you find any relation between these lengths?

(ii) With the help of a Protractor find $\angle A$, $\angle B$, $\angle C$ and $\angle D$ and note down in your copy.

(iii) Is there any relation between the angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

(iv) Identify the adjacent angles of the parallelogram ABCD and try to find the relation between these angles.

3.9.2.1 Properties of parallelogram

Property 1. Opposite sides of a parallelogram are equal.

Verification : Consider the parallelogram ABCD. Diagonal BD is drawn by joining the points B and D.

AB || DC so, $\angle 1 = \angle 2$ and AD || BC therefore $\angle 3 = \angle 4$ [Alternate angle]

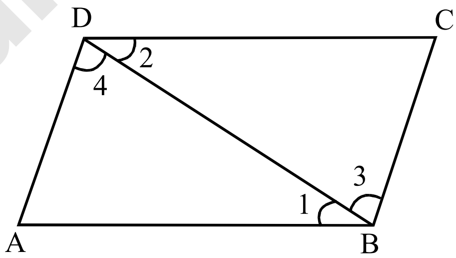
In the ΔABD and ΔBDC,

$\angle 1 = \angle 2$, BD is common, $\angle 3 = \angle 4$

$\therefore$ By ASA congruency criterion

$\Delta BAD \cong \Delta DCB$

This gives AB = DC and AD = BC.



Property 2. The opposite angles of a parallelogram are of equal measure.

Verification : Consider the parallelogram ABCD. The diagonals AC and BD are drawn.

In the ΔABC and ΔADC,

$\angle 1 = \angle 3$ [Alternate angle]

AC is common side

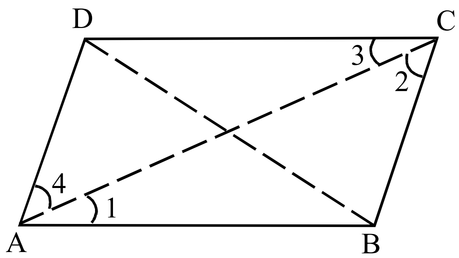
$\angle 2 = \angle 4$ [Alternate angle]

$\therefore$ by ASA congruency condition

$\Delta ABC \cong \Delta CDA$

$\therefore$ $\angle B = \angle D$ [**C**orresponding **P**arts of **C**ongruent **T**riangle]

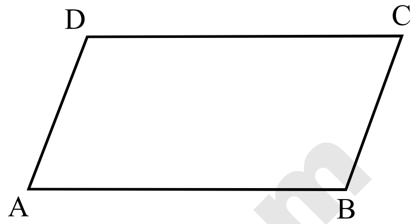
Similarly we can prove that $\angle A = \angle C$



Observe : In your previous activity was $\angle A = \angle C$ and $\angle B = \angle D$?

Property 3. The adjacent angles in a parallelogram are supplementary.

Verification : In the parallelogram ABCD, AB || DC, AD cuts AB and DC



$\therefore$ $\angle A + \angle D = 180^\circ$

[$\because$ sum of the interior angles made by parallel lines and a transversal is 180°]

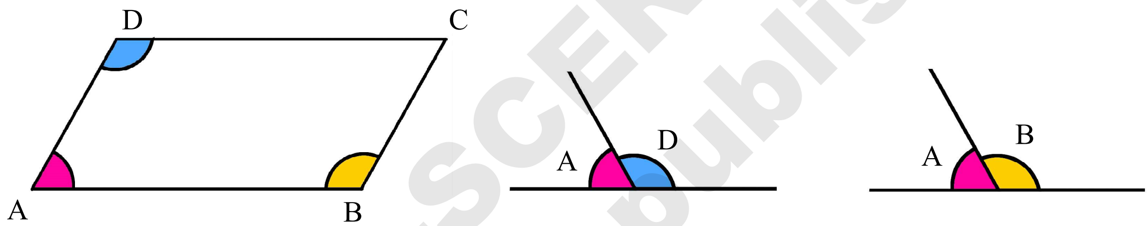
$\therefore$ $\angle A$ and $\angle D$ are supplementary.

Activity

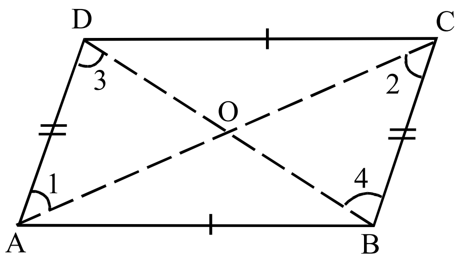
1. Take a cut out of a parallelogram ABCD from a piece of paper.
2. From the parallelogram cut out $\angle A$, $\angle B$ and $\angle D$. Now place first $\angle A$ and $\angle B$ and then $\angle A$ and $\angle D$ in one straight line and match them. What have you seen?

   $\angle A + \angle D = 180^\circ$

   $\angle A + \angle B = 180^\circ$. Isn't it?



Property 4. The diagonals of a parallelogram bisect each other at the point of intersection.



Verification : The diagonals AC and BD of the parallelogram ABCD intersect at O.

To show that AO = CO and BO = DO.

In the ΔAOD and ΔBOC,

$\angle 1 = \angle 2$ [Alternate angle]

AD = BC [Opposite sides of parallelogram]

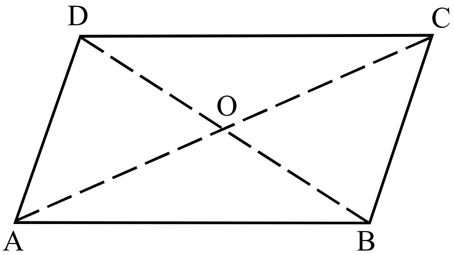
$\angle 3 = \angle 4$ [Alternate angle]

$\therefore$ By applying ASA criterion

$\Delta\, AOD \cong \Delta\, BOC$

$\therefore$ AO = CO and BO = DO

Activity Take a cutout of a parallelogram ABCD from a piece of paper. Fold along both the diagonals AC and BD. You will see that the two folds. meet at a point. Now measure the lengths of diagonal segment between the point and the angular points. Discuss what you have found.



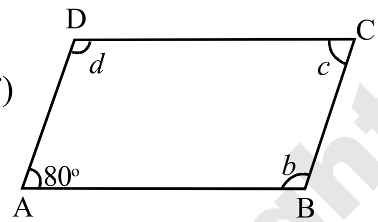
Properties of a parallelogram :

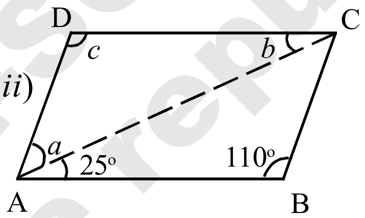
(i) opposite sides are parallel and equal.
(ii) opposite angles are of equal measure.
(iii) adjacent angles are supplementary.
(iv) the two diagonals bisect each other at the point of intersection.

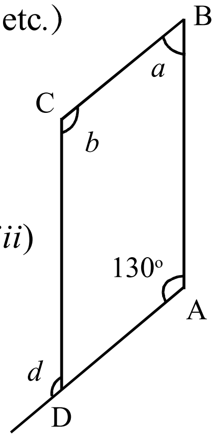
Discuss in group

☆ Can a trapezium be a parallelogram?
☆ Can a parallelogram be a trapezium ?

Example 8 : In the following parallelogram find the unknown (*a, b, c*... etc.)

(*i*)


(*ii*)


(*iii*)


Solution : (*i*) $80^\circ + b = 180^\circ$ (alternative angle)
or $b = 100^\circ$
$b + c = 180^\circ$
or $c = 80^\circ$
Similarly, $d = b = 100^\circ$

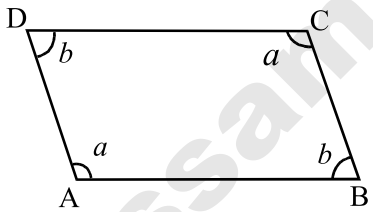
Solution : (*ii*) $a + 25^\circ + 110^\circ = 180^\circ$
or $a = 45^\circ$
$\therefore 25^\circ + 45^\circ + c = 180^\circ$
or $c = 110^\circ$
$b = 25^\circ$

Solution : (*iii*) $\angle a + 130^\circ = 180^\circ$
or $\angle a = 50^\circ$
$\angle b = 130^\circ$
$\angle d = \angle DAB$ [corresponding angle]
$\angle d = 130^\circ$

Example 9 : The measure of two adjacent angles of a parallelogram are in the ratio 5 : 4. Find the measure of each angle of the parallelogram.

Solution : According to the condition $\frac{a}{b} = \frac{5}{4}$ $\therefore a = \frac{5}{4}b$

Now, $a + b = 180$



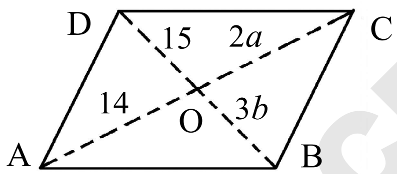
or $\frac{5}{4}b + b = 180$

or $\frac{9}{4}b = 180$

or $b = \frac{180 \times 4}{9} = 80$

Therefore angles of the parallelogram are 100°, 80°, 100°, 80°

Example 10 : Find the unknown quantilies *a, b* in the following figure of parallelogram.



Solution : $OB = 3b = 15 = OD$

$\therefore b = 5$

$OC = 2a = 14 = OA$

$\therefore a = 7$

Example 11 : Is it possible to draw a parallelogram with the given conditions?

(i) AB = CD = 6 cm and BC = AD = 4 cm

(ii) $\angle B = 80°$ and $\angle C = 90°$ (iii) $\angle A = 60°$ and $\angle C = 70°$

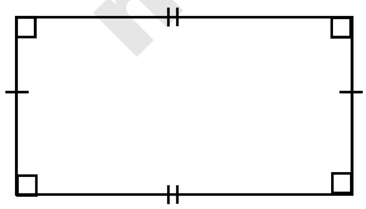
Answer : (i) Possible, as opposite sides are equal.

(ii) Not possible, as the property of adjacent sides of the parallelogram is not satisfied.

(iii) Not possible, as the property of opposite angles of the parallelogram is not satisfied.

3.9.3 Rectangle

Rectangle is a parallelogram with equal angles. Then what could be the measure of each angle? If the measure of each angle is x, then



$x + x + x + x = 360°$

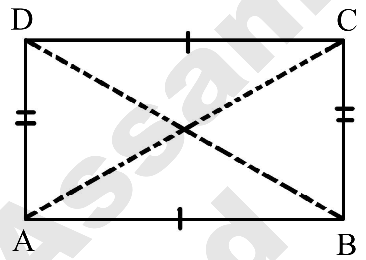
or $4x = 360°$

or $x = 90°$

i.e each angle of a rectangle is 90° or a right angle. Thus a rectangle is a quadrilateral with opposite sides equal, parallel and measure of each angle is a right angle.

Discuss in group (i) Are all parallelograms rectangles?
(ii) Are all rectangles parallelograms ?

3.9.3.1 Property : The diagonals of a rectangle are equal in length.



Verification : The diagonals AC and BD are drawn in the rectangle ABCD.

In ΔABC and ΔABD,

$BC = AD$ (opposite sides)

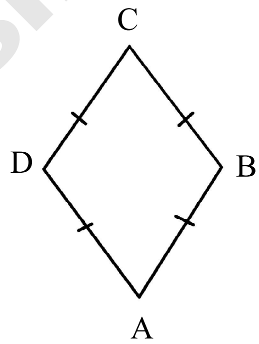
AB is common side

and $\angle A = \angle B = 90^\circ$

$\therefore$ $\Delta ABC \cong \Delta BAD$ (SAS cryterion)

$\therefore$ $AC = BD$

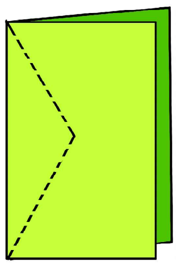
3.9.4 Rhombus



You know that opposite sides of a parallelogram are equal. Rhombus is a parallelogram with all sides of same length. Note that a rhombus is a quadrilateral with sides of equal length.

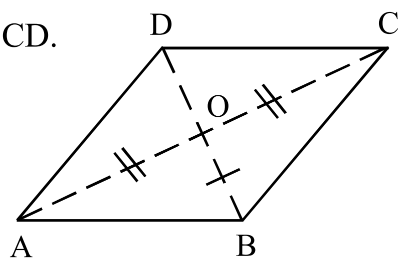
$$AB = BC = CD = DA$$

Activity Fold a piece of paper as shown in the figure. On one side of the fold draw an isosceles triangle. Now along the dotted lines make cut out from both sides of the fold. Then unfold the paper. What will you find? Isn't it a rhombus?



Discuss in group (i) Can a parallelogram be a rhombus?
(ii) Can a rhombus be a parallelogram?

3.9.4.1 Property : The diagonals of a rhombus are perpendicular bisectors of each other.



Verification : AC and BD are diagonals of the rhombus ABCD.

O is their point of intersection.

To show that, $AO = CO$ and $BO = DO$

and $\angle AOB = \angle COB = 90^\circ$

We know that rhombus is a parallelogram and in parallelogram the two diagonals bisect each other.

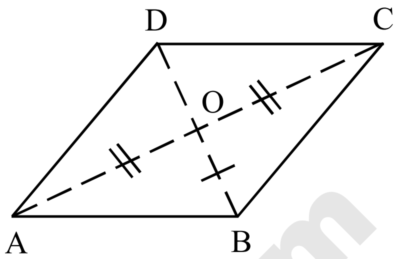
$\therefore$ AO = CO and BO = DO

Now in the ΔAOB and ΔBOC

AO = CO, AB = BC and BO is common side

$\therefore$ $\Delta AOB \cong \Delta COB$ [by SSS cryterion]

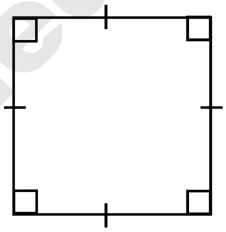
$\therefore$ $\angle AOB = \angle COB = 90^o$ ($\because$ $\angle AOC = 180^o$)



3.9.5 Square

A square is a rhombus with equal angles. Notice that square is a rectangle with equal sides. Similarly a square is a parallelogram with equal sides and equal angles. Again a regular quadrilateral is also a square.

Like the rectangle, the square has equal length of diagonals.



Discuss in group (i) Is square a parallelogram?

(ii) Is square a rectangle?

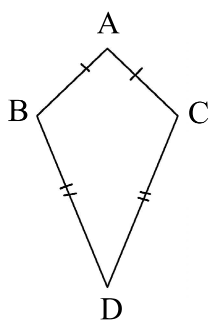
3.9.5.1 Property : Diagonals of a square are perpendicular bisector of each other (Prove yourself)

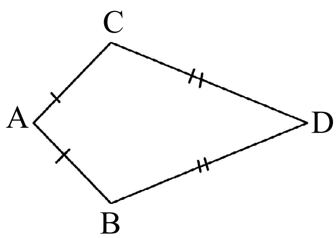
3.9.6 Kite

Observe the following quadrilaterals. The sides with similar marking in each figure are equal. These type of quadrilaterals are called kite.

AB = AC

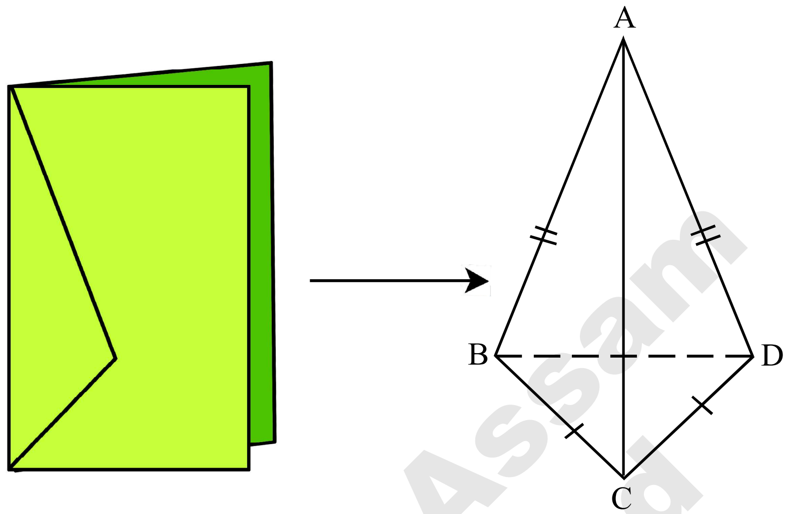
BD = DC





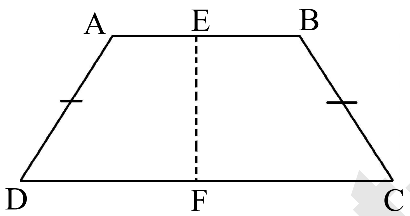
Discuss in group Is rhombus a kite? Is kite a rhombus?

Activity : Take a cut out of a kite from a piece of paper as shown in the figure



Find the lengths of the diagonals AC and BD. Are they equal? Fold AC and BD and use the set square to check if they cut at right angles. Does the diagonal AC divide the kite into used congruent triangles?

3.9.1.1 Isosceles Trapezium

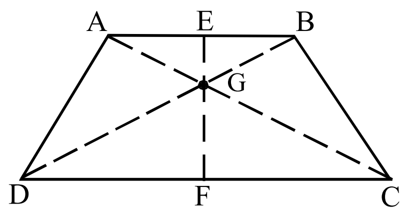


Now we shall discuss a special type of trapezium. Study the given figure . Here the two non parallel sides (AD and BC) are of equal length. These trapezium is called isosceles trapezium.

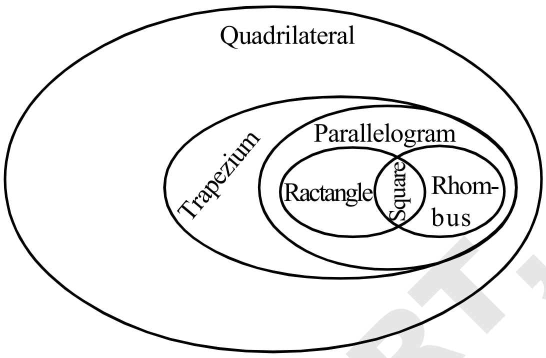
Do yourself

1. Take a cut out of isosceles trapezium ABCD (AB and BC are of equal length) from a piece of paper as shown in the figure.
2. Now fold it placing the point B on A to find middle point of the BA (say E)
3. Does the point C lie on D?
4. Join the points E and F.
5. What does it indicate when the points B lies on A and C lies on D?
6. What does it indicate when the line segment EF divides the trapezium into two equal parts?
7. Mesure the length of AC and BD. Do you get any relation between AC and BD?
8. If the diagonals AC and BD intersect at G, then measure the length of DG, BG, AG and CG. Is $\frac{AG}{CG} = \frac{BG}{GD} = \frac{AB}{DC}$? and Is AG = BG, DG = CG?

9. Sum of the two opposite angles =?
$\angle ADC + \angle ABC = ?$
$\angle DAB + \angle DCB = ?$



3.10 Family of quadrilateral



From the above figure, study the mutual relations of the family of quadrilateral. Some of them are–

(i) Any trapezium, parallelogram, rectangle, rhombus, square are called quadrilateral.

(ii) Any parallelogram, rectangle, rhombus or square are called trapezium.

(iii) Any rectangle, square or rhombus is also a parallelogram.

(iv) A square is both a rectangle or a rhombus.

(v) All quadrilaterals are not trapezium.

(vi) All trapezium may not be parallelogram.

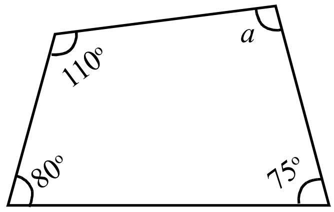
Try to find other relatons.

Example 12 : Name the quadrilaterals whose diagonals
(i) bisect each other
(ii) are perpendicular bisect of each other
(iii) are equal.

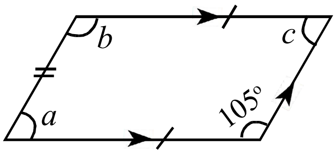
Answer : (i) parallelogram, square, rectangle, rhombus.
(ii) rhombus, square
(iii) rectangle, square.

Exercise 3.2

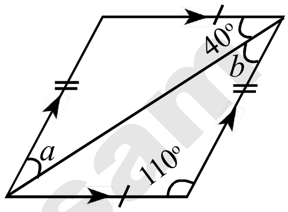
1. Find the unknown quantity (side/angle) of the following quadrilateral. Here the arrowmark (→) given in the opposite sides denotes that the pair of lines are parallel.



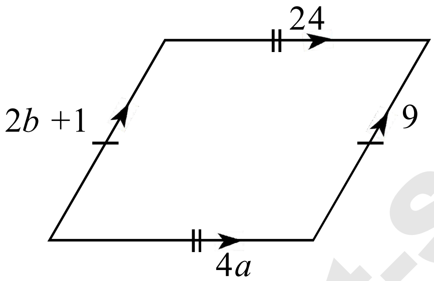
(i)



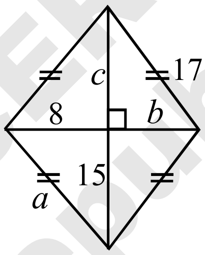
(ii)



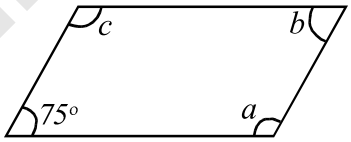
(iii)



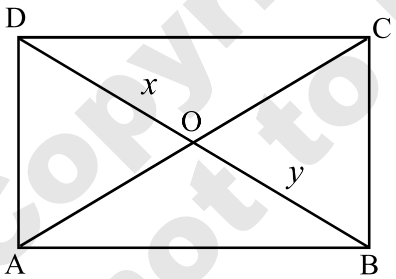
(iv)



(v)

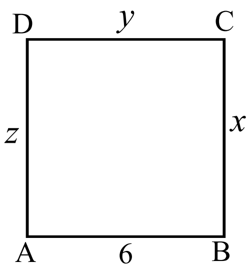


(vi)



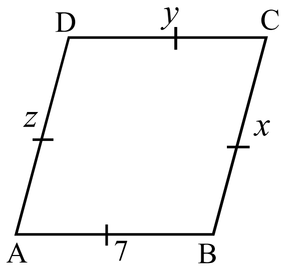
AC = 10
OA = x = ?
OB = y = ?

(vii)



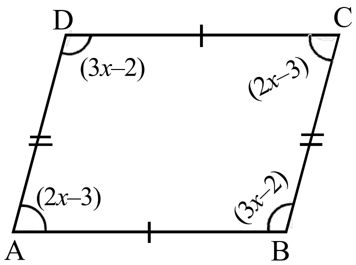
$x + y + z = ?$

(viii)

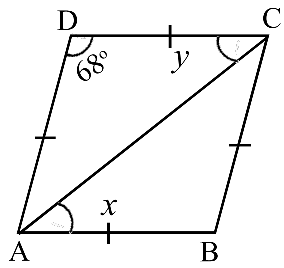


$x + y + z = ?$

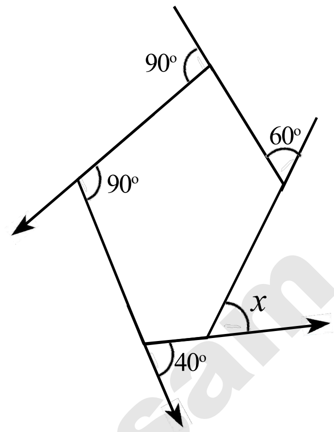
(ix)



(x)

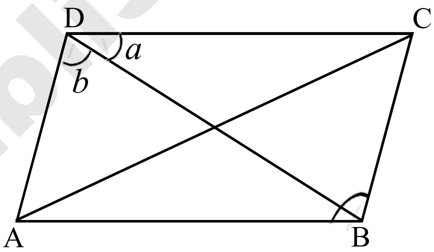


(xi)



(xii)

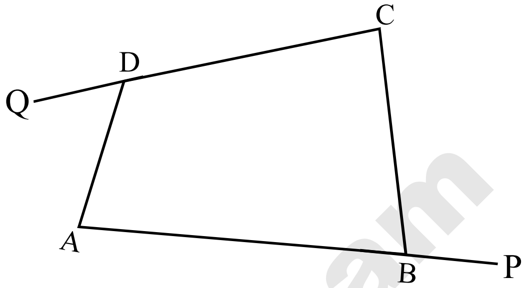
2. Measure of two adjacent angles of a parallelogram are in the ratio 4 : 5. Find the angles of the parallelogram.
3. The measures of two adjacent angles of a parallelogram are such that one is 30^o more then the other, find the measure of each angle of the parallelogram.

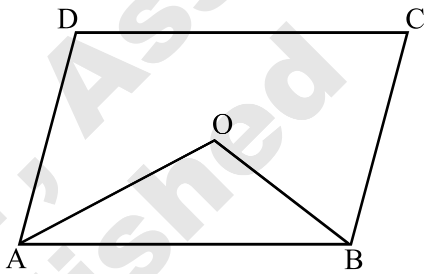


4. AC and BD are diagonals of the parallelogram ABCD. $a = 37^o$ and $\angle ABC = 120^o$. Find the measure of b.
5. In the rectangle ABCD, if the length of the diagonal AC is $6x - 2$ and the length of the diagonal BD is $4x + 2$, then find x and the length of AC.
6. The two diagonals of the rectangle intersect at O. The length of AC and BD are $3x + 1$ and $8x - 24$. Find x. Find AO and BO.
7. In the square ABCD, $\angle A = 4x + 30^0$, find x.
8. In the rectangle ABCD, AB = $2x + 5$, BC = 20 and AD = $3x + 5$. Find the sum of four sides of the rectangle.
9. In the parallelogram ABCD, $\angle B = 2x + 10^0$ and $\angle D = 3x - 13^0$, find all angles of the parallelogram.
10. If the sum of four sides of a square is 36 cm. Find the length of each sides of the square.
11. If in the rhombus ABCD, AB = $3x + 4$ and BC = $2x + 7$, then find the length of DC and AD.
12. The diagonals AC and BD of a parallelogram ABCD are bisected at the point E. If AE = $10x$ find AC if $x = 3$.

13. Perpendiculars AE and CF are drawn on the diagonal BD of the parallelogram ABCD. Show that $\Delta AED \cong \Delta CFB$.

14. The sides $\overline{AB}$ and $\overline{CD}$ of the quadrilateral are extended to the point P and Q respectively. Prove that $\angle ADQ + \angle CBP = \angle A + \angle C$





15. The bisectors of the $\angle A$ and $\angle B$ of the parallelogram ABCD meet at O. Find $\angle AOB$.

16. Find the angles of a quadrilateral if the four angles of the quadrilateral are in the ratio 3 : 4 : 5 : 6.

What we have learnt

1. A polygon is a simple closed figure in plane made up of finite number of line segments.
2. The diagonals of a polygon are the line segments connecting two non-consecutive vertices.
3. No interior angle of a convex polygon is 180^0 or greater than 180^0. But at least one interior angle of a concave polygon greater than 180^0.
4. At least one diagonal of a concave polygon lies outside the polygon but all diagonals of convex polygon lie inside the polygon.
5. An irregular polygon has sides of unequal length and angles of unequal measure. On the other hand a regular polygon has sides of equal length and angles of equal measure.
6. Sum of the measures interior angles of a polygon of n sides = $(2n - 4) \times 90^0$
7. Each interior angle of a regular polygon of n sides = $\left(\frac{2n-4}{n}\right) \times 90^o$

8. Sum of the measures of exterior angles of a polygon is 360°.
9. Trapezium is a quadrilateral with a pair of parallel sides.
10. Parallelogram is a quadrilateral whose both pairs of opposite sides are parallel.
11. **Poperties of a parallelogram**
    (i) Opposite sides are equal and parallel.
    (ii) Opposite angles are equal.
    (iii) Adjacent angles are supplementary.
    (iv) The digonals bisect each other at the point of their intersection.
12. A rectangle is a parallelogram with equal angles.
13. **Properties of rectangle**
    (i) All four properties of parallelogram.
    (ii) Each of the angles is a right angle.
    (iii) Diagonals are equal.
14. Rhombus is a parallelogram with sides of equal length.
15. **Properties of rhombus**
    (i) All properties of parallelogram.
    (ii) All sides are of equal length.
    (iii) The diagonals of a rhombus are perpendicular bisectors of each other.
16. A square is a parallelogram with all equal sides and all equal angles.
17. **Properties of square**
    (i) All the properties of parallelogram, rhombus, rectangle.
    (ii) The diagonals of a square are perpendicular besectors of each other.
    (iii) Diagonals are equal.

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