UNIT – IV

Geometry

CHAPTER

7 TRIANGLES

Syllabus

➢ *Definitions, examples, counter examples of similar triangles.*

1. *(Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.*
2. *(Motivate) If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.*
3. *(Motivate) If in two triangles, the corresponding angles are equal, then their corresponding sides are proportional and the triangles are similar.*
4. *(Motivate) If the corresponding sides of two triangles are proportional, then their corresponding angles are equal and the two triangles are similar.*
5. *(Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including this angle are proportional, then the two triangles are similar.*
6. *(Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on each side of the perpendicular are similar to each other and to the whole triangle.*
7. *(Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.*
8. *(Prove) In a right triangle, the square on the hypotenuse is equal to the sum of squares of the other two sides.*
9. *(Prove) In a triangle, if the square of one side is equal to the sum of the squares on the other two sides, then the angles opposite to the first side is a right angle.*

Chapter Analysis

List of Topics	2016			2017			2018
	Delhi	Outside Delhi	Foreign	Delhi	Outside Delhi	Foreign	Delhi & Outside Delhi
Question based on Area of Triangle	Summative Assessment-I						1 Q (1 M) 1 Q (3 M) 1 Q (4 M)
Question based on Proving Properties of Triangle							OR 1 Q (4 M)

Revision Notes

➢ A triangle is one of the basic shapes of geometry. It is a polygon with 3 sides and 3 vertices/corners.

➢ Two figures are said to be congruent if they have the same shape and the same size.

➢ Those figures which have the same shape but not necessarily the same size are called similar figures. Hence, we can say that all congruent figures are similar but all similar figures are not congruent.

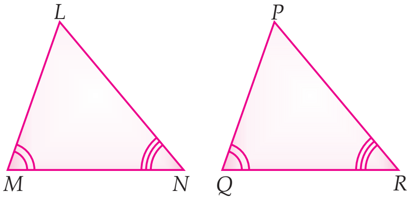
➢ **Similarity of Triangles :** Two triangles are similar, if :

(i) their corresponding sides are proportional.

(ii) their corresponding angles are equal.

If ΔABC and ΔDEF are similar, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.

- **Criteria for Similarity of Triangles :**



In ΔLMN and ΔPQR, if

(i) $\angle L = \angle P$, $\angle M = \angle Q$ and $\angle N = \angle R$ **(ii)** $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$,

then $\Delta LMN \sim \Delta PQR$.

(i) ***AAA*-Criterion :** In two triangles, if corresponding angles are equal, then the triangles are similar and hence their corresponding sides are in the same ratio.

Remark : If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, *AAA* similarity criterion can also be stated as follows :

***AA*-Criterion :** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

(ii) ***SSS*-Criterion :** In two triangles if the sides of one triangle are proportional to the sides of another triangle, then the two triangles are similar and hence corresponding angles are equal.

(iii) ***SAS*-Criterion :** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

Some theorems based on similarity of triangles :

(i) If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. It is known as '**Basic Proportionality Theorem**' or '**Thales Theorem**'.

(ii) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the **Converse of Basic Proportionality Theorem.**

(iii) If two triangles are similar, then the ratio of areas of these triangles is equal to the ratio of squares of their corresponding sides.

- **Theorems Based on Right Angled Triangles :**

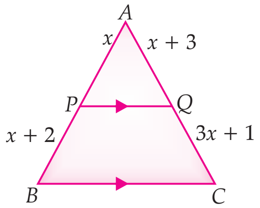
(i) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.

(ii) In a right angled triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides. It is known as **Pythagoras Theorem.**

- In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.
- Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the median of the triangle.
- Three times of the square of any side of an equilateral triangle is equal to four times the square of the altitude.

How it is done on GREENBOARD?

Q. In the given triangle, find x if PQ || BC



Sol. : Step I : Given that PQ || BC

Hence by using Basic Proportionality theorem

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Step II :

$$\frac{x}{x+2} = \frac{x+3}{3x+1}$$

or, $x(3x+1) = (x+2)(x+3)$

or, $3x^2 + x = x^2 + 5x + 6$

or, $2x^2 - 4x - 6 = 0$

or, $x^2 - 2x - 3 = 0$

or, $(x-3)(x+1) = 0$

or, $x = 3$ or $x = -1$

Since, side of a triangle is not negative.

or, $x = 3$.

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. ***ABC* and *BDE* are two equilateral triangles such that *D* is the mid-point of *BC*. Ratio of the areas of triangles *ABC* and *BDE* is :**

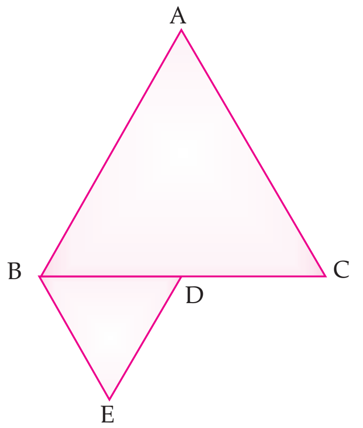
(a) 2 : 1 **(b)** 1 : 2

(c) 4 : 1 **(d)** 1 : 4

C [NCERT Exemp.]

Sol. Correct option : (c)

Explanation :



$$\frac{ar\Delta ABC}{ar\Delta BDE} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{(BD)^2} = \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2} = \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

Q. 2. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio :

(a) 2 : 3 **(b)** 4 : 9

(c) 81 : 16 **(d)** 16 : 81

U [NCERT Exemp.]

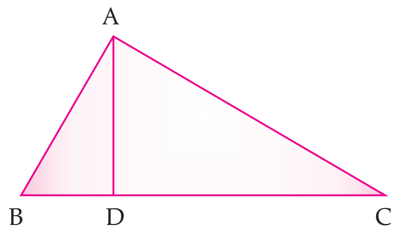
Sol. Correct option : (d)

Explanation : We know that the ratio of the areas of the triangles will be equal to the square of the ratio of the corresponding sides of the triangles.

Thus, required ratio of the areas of the two triangles

$$= \left(\frac{4}{9}\right)^2 = \frac{16}{81}.$$

Q. 3. In the figure given below, $\angle BAC = 90°$ and $AD \perp BC$. Then :



(a) $BD \times CD = BC^2$ **(b)** $AB \times AC = BC^2$

(c) $BD \times CD = AD^2$ **(d)** $AB \times AC = AD^2$

U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : In ΔABD and ΔCAD, $\angle ADB = \angle ADC = 90°$ and $\angle ABD = \angle CAD = \theta$. By *AA* Similarity, we get, $\Delta ABD \sim \Delta CAD$

$$\Rightarrow \frac{AD}{BD} = \frac{CD}{AD} \Rightarrow BD \times CD = AD^2$$

Q. 4. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) $BC \times EF = AC \times FD$ **(b)** $AB \times EF = AC \times DE$

(c) $BC \times DE = AB \times EF$ **(d)** $BC \times DE = AB \times FD$

U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Since $\Delta ABC \sim \Delta EDF$, then we get $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$.

From first two, $AB \times EF = AC \times DE$. Option (b) is correct.

From last two, $BC \times EF = AC \times FD$. Option (a) is correct.

From first and last, $BC \times DE = AB \times FD$. Option (d) is correct.

Thus, option (c) is incorrect.

Q. 5. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then :

(a) $\Delta PQR \sim \Delta CAB$ **(b)** $\Delta PQR \sim \Delta ABC$

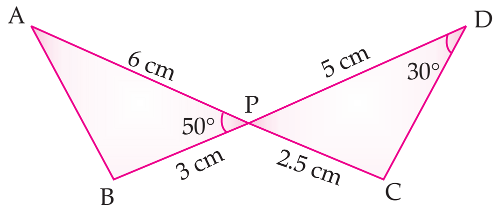
(c) $\Delta CBA \sim \Delta PQR$ **(d)** $\Delta BCA \sim \Delta PQR$

U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Given that, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, by SSS similarity, we get $\Delta PQR \sim \Delta CAB$.

Q. 6. In the figure given below, two line segments *AC* and *BD* intersect each other at the point *P* such that *PA* = 6 cm, *PB* = 3 cm, *PC* = 2.5 cm, *PD* = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to :



(a) 50° **(b)** 30°

(d) 60° **(d)** 100°

A + U[NCERT Exemp.]

Sol. Correct option : (d)

Explanation : In the given figure, $\frac{PA}{PB}=\frac{6}{3}=2$ and $\frac{PD}{PC}=\frac{5}{2.5}=2$. Thus $\frac{PA}{PB}=\frac{PD}{PC}$ and $\angle APB=\angle DPC$.

By *SAS* similarity, we get $\Delta APB \sim \Delta DPC$. Hence, $\angle PBA=100°$.

Q. 7. If in two triangles *DEF* and *PQR*, $\angle D=\angle Q$ and $\angle R=\angle E$, then which of the following is not true?

(a) $\frac{EF}{PR}=\frac{DF}{PQ}$ (b) $\frac{DE}{PQ}=\frac{FE}{RP}$

(c) $\frac{DE}{QR}=\frac{DF}{PQ}$ (d) $\frac{EF}{RP}=\frac{DE}{QR}$

U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In ΔDEF and ΔPQR, $\angle D=\angle Q$ and $\angle R=\angle E$. By *AA* similarity, we get $\Delta DEF \sim \Delta QRP$.

Hence, $\frac{DE}{QR}=\frac{EF}{RP}=\frac{DF}{QP}$. Option (b) is incorrect.

Q. 8. In triangles *ABC* and *DEF*, $\angle B=\angle E$, $\angle F=\angle C$ and *AB* = 3*DE*. Then, the two triangles are :

(a) congruent but not similar

(b) similar but not congruent

(c) neither congruent nor similar

(d) congruent as well as similar

U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In ΔABC and ΔDEF, $\angle B=\angle E$ and $\angle F=\angle C$. By *AA* similarity, we get $\Delta ABC \sim \Delta DEF$. Thus, the triangles are similar but not congruent.

Q. 9. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR}=\frac{1}{3}$. Then, $\frac{ar\,\Delta PRQ}{ar\,\Delta BCA}$ is equal to :

(a) 9 (b) 3

(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

A + R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Since $\Delta ABC \sim \Delta PQR$, we have

$$\frac{ar\Delta PRQ}{ar\Delta BCA}=\left(\frac{QR}{BC}\right)^2=\left(\frac{3}{1}\right)^2=9.$$

Q. 10. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A=30°$, $\angle C=50°$, *AB* = 5 cm, *AC* = 8 cm and *DF* = 7.5 cm. Then, the following is true :

(a) DE = 12 cm, $\angle F=50°$

(b) DE = 12 cm, $\angle F=100°$

(c) EF = 12 cm, $\angle D=100°$

(d) EF = 12 cm, $\angle D=30°$

U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Since $\Delta ABC \sim \Delta DFE$, we have

$\angle A=\angle D=30°$, $\angle B=\angle F=100°$, $\angle C=\angle E=50°$,

And $\frac{AB}{DF}=\frac{AC}{DE}=\frac{BC}{FE} \Rightarrow DE=\frac{AC\times DF}{AB}=\frac{8\times 7.5}{5}=12$.

Q. 11. If in triangles *ABC* and *DEF*, $\frac{AB}{DE}=\frac{BC}{FD}$ then they will be similar, when :

(a) $\angle B=\angle E$ (b) $\angle A=\angle D$

(c) $\angle B=\angle D$ (d) $\angle A=\angle F$

U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : In ΔABC and ΔDEF, we have $\frac{AB}{DE}=\frac{BC}{FD} \Rightarrow \frac{AB}{ED}=\frac{BC}{DF}$. To be similar of ΔABC and ΔDEF, we must have $\angle B=\angle D$.

Q. 12. If $\Delta ABC \sim \Delta QRP$, $\frac{ar\Delta ABC}{ar\Delta PQR}=\frac{9}{4}$, *AB* = 18 cm and *BC* = 15 cm, then *PR* is equal to

(a) 10 cm (b) 12 cm

(c) $\frac{20}{3}$ cm (d) 8 cm

U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Since $\Delta ABC \sim \Delta QRP$, we have

$$\frac{AB}{QR}=\frac{BC}{RP}=\frac{AC}{QP} \text{ and } \frac{ar\Delta ABC}{ar\Delta PQR}=\left(\frac{AB}{QR}\right)^2=\left(\frac{BC}{RP}\right)^2=\left(\frac{AC}{QP}\right)^2$$

Now,

$$\frac{ar\Delta ABC}{ar\Delta PQR}=\left(\frac{AB}{QR}\right)^2 \Rightarrow \frac{9}{4}=\left(\frac{18}{QR}\right)^2 \Rightarrow \frac{3}{2}=\frac{18}{QR}$$

$\Rightarrow QR$ = 12 cm

Thus, $\frac{AB}{QR}=\frac{BC}{RP} \Rightarrow PR=\frac{BC\times QR}{AB}=\frac{15\times 12}{18}=10$ cm.

Q. 13. Tick the correct answer and justify : In ΔABC, *AB* = $6\sqrt{3}$ cm, *AC* = 12 cm and *BC* = 6 cm. The angle B is :

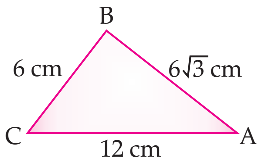
(a) 120° **(b) 60°**

(c) 90° **(d) 45°**

U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation :



Given that, $AB=6\sqrt{3}$ cm, *AC* = 2 cm, *BC* = 3 cm

It can be observed that,

$AB^2=108$ cm, $AC^2=144$ cm, $BC^2=36$ cm

Where,

$AB^2+BC^2=AC^2$

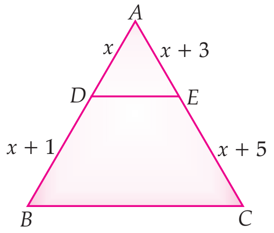
The given triangle ΔABC is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle at B.

So, angle B is 90°.

[B] Very Short Answer Type Questions :

AI Q. 1. In ΔABC, $DE \mid\mid BC$, find the value of x.



U **[Board Term-1, 2016, Set-O4YP6G7]**

Sol. As $\quad DE \mid\mid BC$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\text{or,} \quad \frac{x}{x+1} = \frac{x+3}{x+5}$$

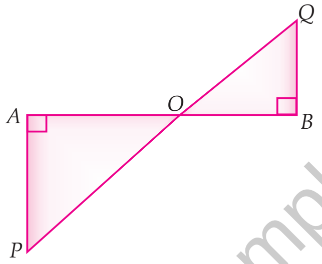
$$\text{or,} \quad x^2 + 5x = x^2 + 4x + 3$$

$$\text{or,} \quad x = 3$$ **1**

[CBSE Marking Scheme, 2016]

Q. 2. In the given figure, if $\angle A = 90°$, $\angle B = 90°$, $OB = 4.5$ cm, $OA = 6$ cm and $AP = 4$ cm, then find QB.

U **[Board Term-1, 2015, Set–DDE-E]**



Sol. In ΔPAO and ΔQBO,

$\angle A = \angle B = 90°$ (Given)

$\angle POA = \angle QOB$ (Vertically Opposite Angle)

Since, $\Delta PAO \sim \Delta QBO$, (by AA)

$$\text{Then,} \quad \frac{OA}{OB} = \frac{PA}{QB}$$

$$\text{or,} \quad \frac{6}{4.5} = \frac{4}{QB}$$

$$\text{or,} \quad QB = \frac{4 \times 4.5}{6}$$

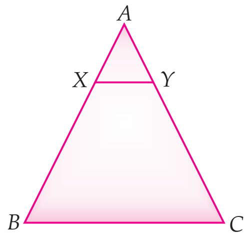
$$\therefore \quad QB = 3 \text{ cm}$$ **1**

Q. 3. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

U **[Board Term-1, 2016, Set-MV98HN3]**

[Board Term-1, 2015, Set-CJTOQ]

Sol.



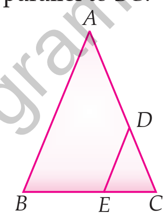
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9 \quad \text{(Given)}$$

$$\text{Then,} \quad \frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

$$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC. **1**

Q. 4.

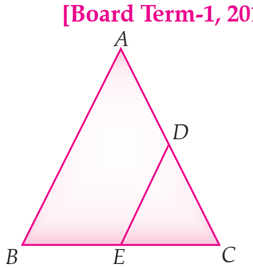


In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE \mid\mid AB$, $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$. Then, find x.

A **[Board Term-1, 2016, Set-LGRKEGO]**

OR

In the figure of ΔABC, $DE \mid\mid AB$. If $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$, then find the value of x. **[Board Term-1, 2015, Set-DDE-M]**



Sol. $$\frac{CD}{AD} = \frac{CE}{BE} \text{ or, } \frac{x+3}{2x} = \frac{x}{2x-1}$$ ½

$$\text{or,} \quad 5x = 3 \text{ or, } x = \frac{3}{5}$$ ½

[CBSE Marking Scheme, 2016]

Detailed Answer :

In ABC, $DE \mid\mid AB$ (Given)

$$\text{Then,} \quad \frac{CD}{CA} = \frac{CE}{CB}$$

$$\text{or,} \quad \frac{CD}{CD+AD} = \frac{CE}{CE+BE}$$

$$\text{or,} \quad \frac{x+3}{x+3+2x} = \frac{x}{x+2x-1}$$ ½

$$\text{or,} \quad \frac{x+3}{3x+3} = \frac{x}{3x-1}$$

$$\begin{aligned}
\text{or,} \quad (x+3)(3x-1) &= x(3x+3)\\
\text{or,} \quad 3x^2 - x + 9x - 3 &= 3x^2 + 3x\\
\text{or,} \quad 8x - 3 &= 3x\\
\text{or,} \quad 8x - 3x &= 3\\
\text{or,} \quad 5x &= 3\\
\therefore \quad x &= \frac{3}{5} \qquad \tfrac{1}{2}
\end{aligned}$$

Q. 5. Are two triangles having corresponding sides equal are similar.

R **[Board Term-1, 2015, Set-FHN8MGD]**

Sol. Yes, Two triangles having equal corresponding sides are congruent and all congruent Δs have equal angles, hence they are similar too. **1**

Q. 6. If ratio of corresponding sides of two similar triangles is 5 : 6, then find ratio of their areas.

R **[Board Term-1, 2015, Set-WJQZQBN]**

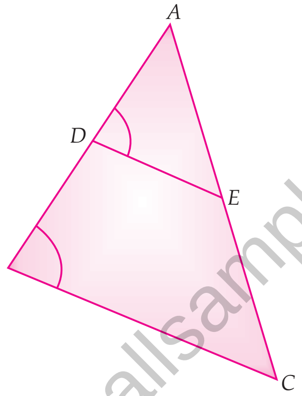
Sol. Let the triangles be ΔABC and ΔDEF. ½

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$= 25 : 36 \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2015]

Q. 7. In figure, if $AD = 6$ cm, $DB = 9$ cm, $AE = 8$ cm and $EC = 12$ cm and $\angle ADE = 48°$. Find $\angle ABC$.



U **[CBSE SQP 2018-19]**

Sol.

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ and } DE \,||\, BC \qquad \tfrac{1}{2}$$

$$\Rightarrow \quad \angle ADE = \angle ABC = 48° \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2018]

Q. 8. Given, $\Delta ABC \sim \Delta PQR$, if $\dfrac{AB}{PQ} = \dfrac{1}{3}$, then find $\dfrac{ar\ \Delta ABC}{ar\ \Delta PQR}$

[CBSE Delhi/Outside Delhi Set, 2018]

Sol.

$$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \frac{AB^2}{PQ^2}$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

[CBSE Marking Scheme, 2018] 1

Detailed Answers :

$\Delta ABC \sim \Delta PQR$ (Given)

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{3}$$

$$\text{and} \quad \frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \left(\frac{AB}{PQ}\right)^2$$

$$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \frac{1}{9} \qquad 1$$

AI **Q. 9. If $\Delta ABC \sim \Delta QRP$, $\dfrac{ar(\Delta ABC)}{ar(\Delta QRP)} = \dfrac{9}{4}$, and $BC = 15$ cm, then find PR. [CBSE Compt. Set I, II, III, 2018]**

Sol.

$$\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2$$

$$\Rightarrow \quad \frac{9}{4} = \left(\frac{15}{PR}\right)^2 \Rightarrow PR = 10 \text{ cm} \qquad 1$$

[CBSE Marking Scheme, 2018]

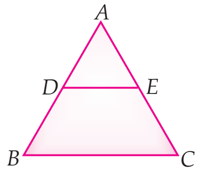
Commonly Made Error

- Some candidates take the ratio of area $\dfrac{\Delta ABC}{\Delta QRP} = \dfrac{BC}{RP}$ instead of $\dfrac{BC^2}{RP^2}$ hence get the incorrect value of PR.

Answering Tip

- Candidates should know that the ratio of the area of two similar triangular are equal to the ratio of the squares of any two corresponding sides.

Q. 10. In given figure, $DE \,||\, BC$. If $AD = 3$ cm, $DB = 4$ cm and $AE = 6$ cm, then find EC.



U **[Board Term-1, 2016, Set-ORDAWEZ]**

Sol. Since $DE \,||\, BC$

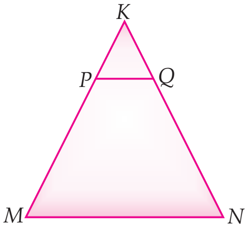
$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\text{or,} \quad \frac{3}{4} = \frac{6}{EC}$$

$$\therefore \quad EC = 8 \text{ cm} \qquad 1$$

[CBSE Marking Scheme, 2016]

Q. 11. In the figure, PQ is parallel to MN. If $\dfrac{KP}{PM} = \dfrac{4}{13}$ and $KN = 20.4$ cm, then find KQ. A



Sol. $\because$ $PQ \,||\, MN$

So, $\dfrac{KP}{PM} = \dfrac{KQ}{QN}$ (By BPT)

or, $\dfrac{KP}{PM} = \dfrac{KQ}{KN - KQ}$

or, $\dfrac{4}{13} = \dfrac{KQ}{20.4 - KQ}$

or, $4 \times 20.4 - 4\,KQ = 13\,KQ$

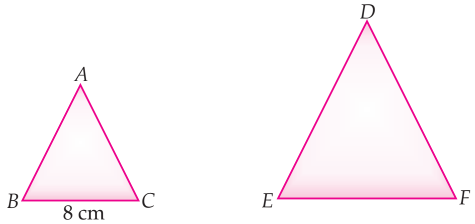
or, $17\,KQ = 4 \times 20.4$

$\therefore$ $KQ = \dfrac{20.4 \times 4}{17} = 4.8$ cm. **1**

Q. 12. If triangle *ABC* is similar to triangle *DEF* such that 2*AB* =*DE* and *BC* = 8 cm, then find *EF*. A

Sol. Given, $2AB = DE$ and $BC = 8$ cm

$\because$ $\Delta ABC \sim \Delta DEF$

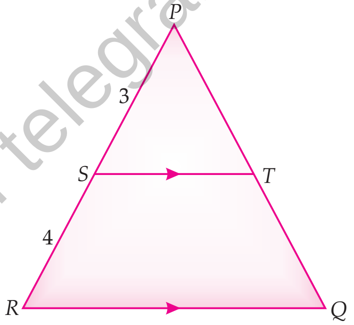


So, $\dfrac{AB}{BC} = \dfrac{DE}{EF}$

or, $\dfrac{AB}{8} = \dfrac{2AB}{EF}$

$\therefore$ $EF = 2 \times 8 = 16$ cm. **1**

Q. 13. In the given figure, *ST* || *RQ*, *PS* = 3 cm and SR = 4 cm. Find the ratio of the area of Δ*PST* to the area of Δ*PRQ*. A **[Sample Question Paper 2017]**



Sol. $PS = 3$ cm, $SR = 4$ cm and $ST \parallel RQ$.

PR = PS + SR

= 3 + 4 = 7 cm

$$\frac{\text{ar } \Delta PST}{\text{ar } \Delta PQR} = \frac{PS^2}{PR^2} = \frac{3^2}{7^2} = \frac{9}{49}$$

Hence, required ratio = 9 : 49. **1**

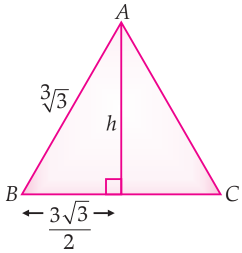
Short Answer Type Questions-I (2 marks each)

Q. 1. In an equilateral triangle of side $3\sqrt{3}$ cm, find the length of the altitude.

U **[Board Term-1, 2016, Set-MV98HN3]**

[Board Term-1, 2015, Set-DDE-E]

Sol.



1

$$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

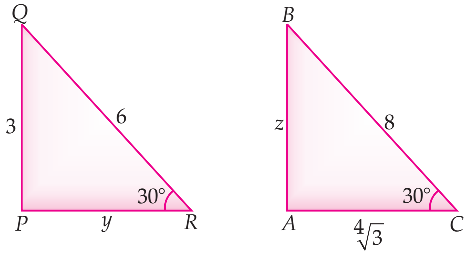
or, $27 = h^2 + \dfrac{27}{4}$

or, $h^2 = 27 - \dfrac{27}{4}$

or, $h^2 = \dfrac{81}{4}$

$\therefore$ $h = \dfrac{9}{2} = 4.5$ cm **1**

Q. 2. In the given figure, Δ*ABC* ~ Δ*PQR*. Find the value of *y* + *z*. A **[Board Term-1, 2015, Set–CJTOQ]**



Sol. Given, $\Delta ABC \sim \Delta PQR$ (Given)

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

or, $$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

or, $$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

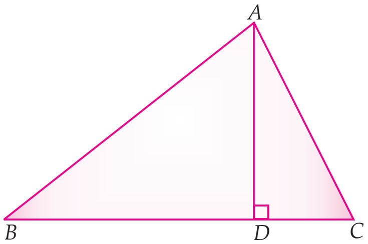
or, $$z = \frac{8\times 3}{6} \text{ and } y = \frac{4\sqrt{3}\times 6}{8}$$

$\therefore$ $$z = 4 \text{ and } y = 3\sqrt{3}$$

$\therefore$ $$y + z = 3\sqrt{3} + 4$$ **2**

Q. 3. In ΔABC, $AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

A [Board Term-1, 2015, Set–FHN8MGD]



Sol. Given, $AD^2 = BD \times CD$

or, $$\frac{AD}{CD} = \frac{BD}{AD}$$ ½

$\therefore$ $\Delta ADC \sim \Delta BDA$ (by SAS; $\because \angle D = 90°$)

or, $\angle BAD = \angle ACD$;

$\angle DAC = \angle DBA$

(Corresponding angles of similar triangles) ½

$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180°$

or, $2\angle BAD + 2\angle DAC = 180°$ ½

or, $\angle BAD + \angle DAC = 90°$

$\therefore$ $\angle A = 90°$ Hence proved. ½

Q. 4. Find the altitude of an equilateral triangle, when each of its side is 'a' cm.

A [Board Term-1, 2016, Set-O4YP6G7]

Sol. Try yourself similar to Q. No. 1 in SATQ-I

Q. 5. Let $\Delta ABC \sim \Delta DEF$. If $ar\,(\Delta ABC) = 100$ cm², $ar\,(\Delta DEF) = 196$ cm² and $DE = 7$, then find AB.

A [Board Term-1, 2015, Set–DDE–M]

Sol. $\Delta ABC \sim \Delta DEF$ (Given)

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$$ **1**

or, $$\frac{100}{196} = \frac{AB^2}{(7)^2}$$

or, $$\frac{100}{196} = \frac{AB^2}{49}$$

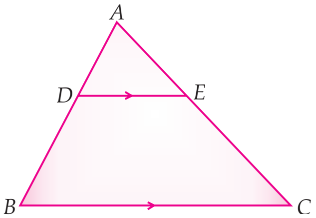
or, $$AB^2 = \frac{49\times 100}{196}$$

or, $$AB^2 = 25$$

$\therefore$ $$AB = 5 \text{ cm}$$ **1**

AI **Q. 6. In the given figure, $DE \parallel BC$. If $AD = 1.5$ cm, $BD = 2AD$, then find $\dfrac{ar(\Delta ADE)}{ar(\text{trapezium } BCED)}$.**

A [Board Term-1, 2013, Set FFC]



Sol. Given, $AD = 1.5$ cm, $\therefore BD = 3$ cm and $AB = AD + BD = 1.5 + 3.0 = 4.5$ cm.

In triangles, ADE and ABC,

$\angle A$ is common and $DE \parallel BC$

Then, $\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

(Corresponding angles) **1**

$\therefore$ $\Delta ADE \sim \Delta ABC$, ($AA$ similarity)

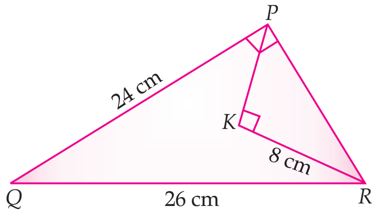
$\therefore$ $$\frac{ar\,(\Delta ADE)}{ar\,(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9}$$

or, $$\frac{ar(\Delta ADE)}{ar(\Delta ABC) - ar(\Delta ADE)} = \frac{1}{9-1}$$ **1**

$\therefore$ $$\frac{ar(\Delta ADE)}{ar(\text{trapezium } BCED)} = \frac{1}{8}$$

Q. 7. In the given triangle PQR, $\angle QPR = 90°$, $PQ = 24$ cm and $QR = 26$ cm and in ΔPKR, $\angle PKR = 90°$ and $KR = 8$ cm, find PK.

A [Board Term-1, 2012, Set-21]



Sol. According to the question,

Given, $\angle QPR = 90°$

$\therefore$ $QR^2 = QP^2 + PR^2$ **1**

$\therefore$ $PR = \sqrt{26^2 - 24^2}$

$= \sqrt{100} = 10$ cm

$\angle PKR = 90°$ (Given)

$\therefore$ $PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$

$= \sqrt{36} = 6$ cm. **1**

Q. 8. The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the perimeter P_2 of ΔDEF. Are the two triangles similar ? If yes, find $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta DEF)}$

A [Board Term-1, 2012, Set-39]

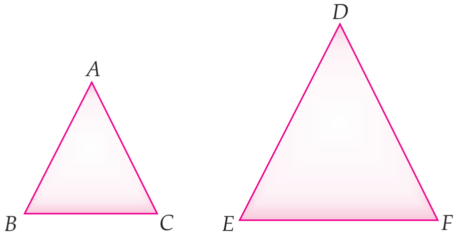
Sol. In ΔABC and ΔDEF,

$AB = 3DE$ and $AC = 3DF$

or, $$\frac{AB}{DE} = 3 \text{ and } \frac{AC}{DF} = 3;$$

$$P_1 = 3P_2$$

or, $$BC = 3EF$$

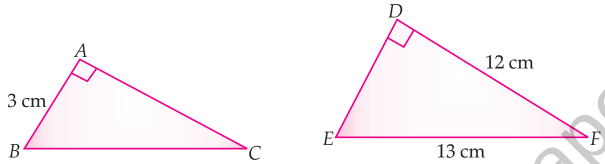


or, $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$$

or, $\Delta ABC \sim \Delta DEF$ (*SSS* Criterion) **1**

or, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9. \quad \textbf{1}$$

Q. 9. Given, $\Delta ABC \sim \Delta DEF$, find $\dfrac{\text{ar } \Delta ABC}{\text{ar } \Delta DEF}$.



A **[Board Term-1, 2016, Set-ORDAWEZ]**

Sol. In ΔDEF,

$$DE = \sqrt{(13)^2 - (12)^2}$$

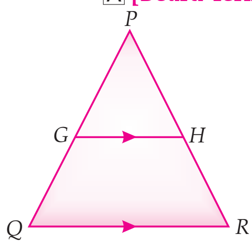
$$= \sqrt{169 - 144} = \sqrt{25} = 5 \quad \textbf{1}$$

$$\frac{\text{ar } \Delta ABC}{\text{ar } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25} \quad \textbf{1}$$

[CBSE Marking Scheme, 2016]

Q. 10. In the given figure, *G* is the mid-point of the side *PQ* of ΔPQR and $GH || QR$. Prove that *H* is the mid-point of the side *PR* of the triangle *PQR*.

A **[Board Term-1, 2012, Set-43]**



Sol. Since, *G* is the mid-point of *PQ*.

$\therefore$ $$PG = GQ$$

or, $$\frac{PG}{GQ} = 1 \quad \textbf{1}$$

According to the question,

$$GH \,||\, QR$$

$\therefore$ $$\frac{PG}{GQ} = \frac{PH}{HR} \quad \text{(By BPT)}$$

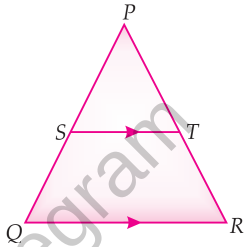
or, $$1 = \frac{PH}{HR}$$

$\therefore$ $PH = HR$. **Hence Proved. 1**

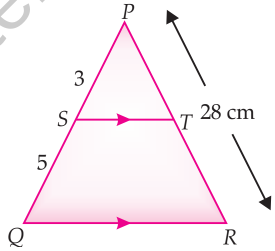
Hence, *H* is the mid-point of *PR*.

Q. 11. In the given figure, in a triangle *PQR*, $ST || QR$ and $\dfrac{PS}{SQ} = \dfrac{3}{5}$ and $PR = 28$ cm, find *PT*.

A **[Board Term-1, 2012, Set-64; Set-I 2011]**



Sol. According to the question,



$$ST \,||\, QR$$

$\therefore$ $$\frac{PS}{PQ} = \frac{PT}{PR} \quad \text{(By BPT)}$$

Given, $$\frac{PS}{SQ} = \frac{3}{5} \text{ and } PR = 28 \text{ cm} \quad \textbf{1}$$

$$PQ = 3 + 5 = 8 \text{ cm}$$

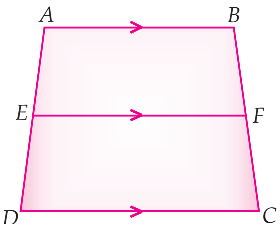
or, $$\frac{PS}{PQ} = \frac{PT}{PR}$$

or, $$\frac{PS}{PQ} = \frac{PT}{PR}$$

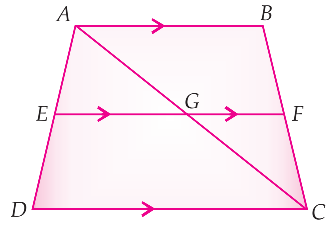
$\therefore$ $$PT = \frac{3 \times 28}{8} = 10{\cdot}5 \text{ cm.} \quad \textbf{1}$$

Q. 12. In the given figure, if *ABCD* is a trapezium in which $AB \,||\, CD \,||\, EF$, then prove that $\dfrac{AE}{ED} = \dfrac{BF}{FC}$.

U **[Board Term-1, 2012, Set-25]**



Sol. Draw AC intersecting EF at G.



In ΔCAB, $GF \mid\mid AB$

or, $\dfrac{AG}{CG} = \dfrac{BF}{FC}$ (By BPT)...(i) **1**

In ΔADC, $EG \mid\mid DC$

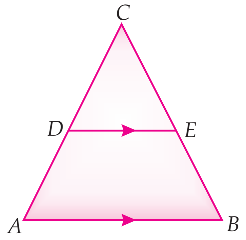
or, $\dfrac{AE}{ED} = \dfrac{AG}{CG}$ (By BPT)...(ii)

From equations (i) and (ii),

$\dfrac{AE}{ED} = \dfrac{BF}{FC}$. **Hence Proved. 1**

Q. 13. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE \mid\mid AB$.

U **[Board Term-1, 2012, Set-63]**



Sol. In ΔCAB, $\angle A = \angle B$ (Given)

$\therefore$ $AC = CB$

(By isosceles triangle property) **1**

But, $AD = BE$ (Given) ...(i)

or, $AC - AD = CB - BE$

$\therefore$ $CD = CE$ ...(ii)

Dividing equation (ii) by (i),

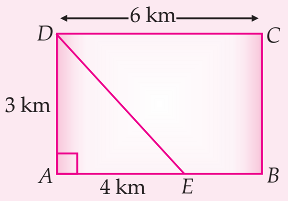
$\dfrac{CD}{AD} = \dfrac{CE}{BE}$

By converse of BPT,

$DE \mid\mid AB$. **1**

Q. 14. In a rectangle $ABCD$, E is a point on AB such that $AE = \dfrac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE. A **[Board Term-1, 2016, Set–LGRKEGO]**

Sol.



1

Given, $AE = \dfrac{2}{3}AB = \dfrac{2}{3}\times 6 = 4$ km

In right triangle ADE,

$DE^2 = (3)^2 + (4)^2$

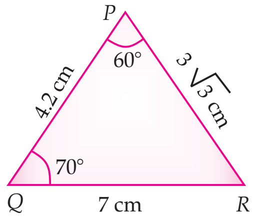
or, $DE^2 = 25$

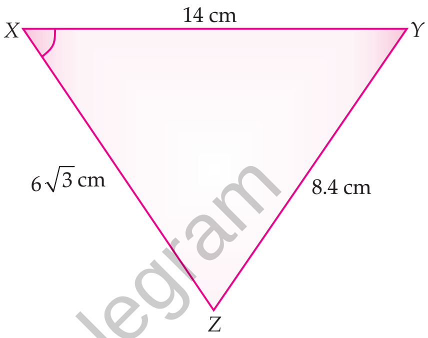
$\therefore$ $DE = 5$ km. **1**

[CBSE Marking Scheme, 2016]

Q. 15. In the given figures, find the measure of $\angle X$.

A **[Board Term-1, 2012, Set-38]**





Sol. From given figures,

$\dfrac{PQ}{ZY} = \dfrac{4.2}{8.4} = \dfrac{1}{2}$;

$\dfrac{PR}{ZX} = \dfrac{3\sqrt{3}}{6\sqrt{3}} = \dfrac{1}{2}$;

$\dfrac{QR}{YX} = \dfrac{7}{14} = \dfrac{1}{2}$

or, $\dfrac{PQ}{ZY} = \dfrac{PR}{ZX} = \dfrac{QR}{YX}$ **1**

or, $\Delta PQR \sim \Delta ZYX$ (By SSS)

$\therefore$ $\angle X = \angle R$

$\therefore$ $\angle X = 180° - (60° + 70°) = 50°$ **1**

$\therefore$ $\angle X = 50°$

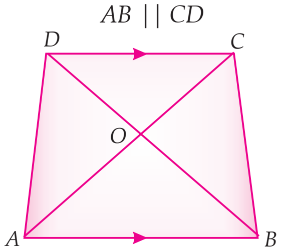
Q. 16. $ABCD$ is a trapezium in which $AB \mid\mid DC$ and its diagonals intersect each other at the point O.

Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

U **[Board Term-1, 2012, Set-66]**

Sol. Proof : In ΔAOB and ΔCOD,

$AB \mid\mid CD$



$\therefore$ $\angle OAB = \angle DCO$ and $\angle OBA = \angle ODC$

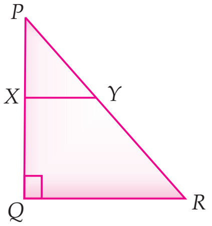
(Alternate angles) **1**

$\therefore$ $\Delta AOB \sim \Delta COD$ (AA similarity)

$\therefore$ $\dfrac{AO}{CO} = \dfrac{BO}{DO}$

(Corresponding sides of similar triangles) or $\dfrac{AO}{BO} = \dfrac{CO}{DO}$. **1**

Q. 17. In the given figure, PQR is a right angled triangle at Q and XY || QR. If PQ = 6 cm, PY = 4 cm and $PX : XQ$ = 1 : 2. Calculate the lengths of PR and QR. A **[Board Term-1, 2012, Set-44]**



Sol. Since, $XY \,||\, QR$

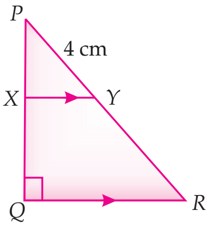
$$\therefore \quad \frac{PX}{XQ} = \frac{PY}{YR} \quad \text{(By } BPT\text{)}$$

$$\text{or,} \quad \frac{1}{2} = \frac{PY}{PR - PY}$$

$$\text{or,} \quad \frac{1}{2} = \frac{4}{PR - 4} \quad \mathbf{1}$$

$$\text{or,} \quad PR - 4 = 8$$

$$\text{or,} \quad PR = 12 \text{ cm}$$



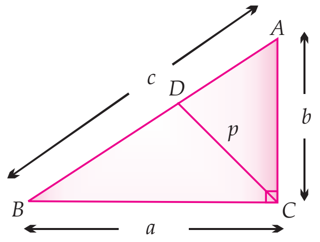
$\therefore$ In right ΔPQR, $QR^2 = PR^2 - PQ^2$

$$= 12^2 - 6^2$$

$$= 144 - 36 = 108$$

$$\therefore \quad QR = 6\sqrt{3} \text{ cm} \quad \mathbf{1}$$

AI **Q. 18. ABC is a right triangle, right angled at C. Let $BC = a$, $CA = b$, $AB = c$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.** A **[Board Term-1, 2012, Set-65]**



Sol. Let $CD \perp AB$,

then $CD = p$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} \quad \mathbf{1}$$

$$\text{or,} \quad \text{Area of } \Delta ABC = \frac{1}{2} \times AB \times CD = \frac{1}{2}\, cp$$

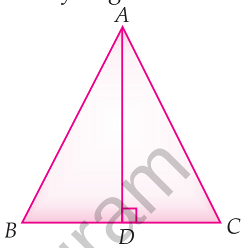
$$\text{Also,} \quad \text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2}\, ab$$

$$\frac{1}{2}\, cp = \frac{1}{2}\, ab$$

or, $cp = ab$. **Hence proved. 1**

Q. 19. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$. U **[Board Term-1, 2012, Set-40]**

Sol. In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2 \quad \mathbf{1}$$

$$\text{or,} \quad BC^2 = AD^2 + BD^2,$$

(as $AB = BC = CA$)

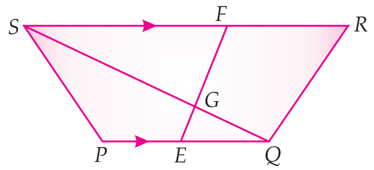
$$\text{or,} \quad (2BD)^2 = AD^2 + BD^2,$$

($\perp$ is the median in an equilateral Δ)

$$4BD^2 - BD^2 = AD^2$$

$\therefore$ $3BD^2 = AD^2$. **Hence proved. 1**

Q. 20. In the figure, $PQRS$ is a trapezium in which PQ || RS. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



U **[Board Term-1, 2016, Set–O4YP6G7]**

Sol. In ΔGEQ and ΔGFS

$$\angle EGQ = \angle FGS \quad \text{(vert. opp. angles)}$$

$$\angle EQG = \angle FSG \quad \text{(alt. angles)}$$

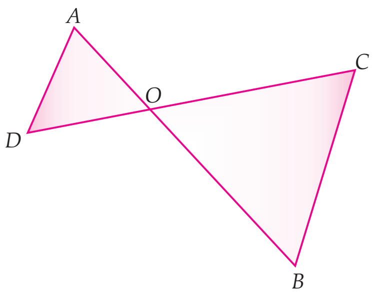
$$\therefore \quad \Delta GEQ \sim \Delta GFS \quad (AA \text{ similarity}) \ \mathbf{1}$$

$$\text{or,} \quad \frac{EQ}{FS} = \frac{GQ}{GS}$$

$$\text{or,} \quad EQ \times GS = GQ \times FS. \quad \mathbf{1}$$

[CBSE Marking Scheme, 2016]

Q. 21. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$. U **[Board Term-1, 2012, Set-71]**



Sol. Since, $OA \times OB = OC \times OD$

$\therefore \quad \dfrac{OA}{OD} = \dfrac{OC}{OB}$ **1**

$\angle AOD = \angle COB$

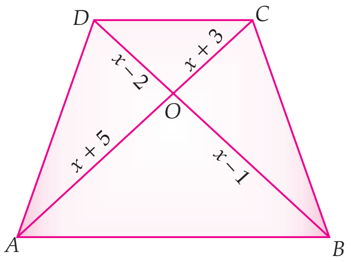
(vertically opposite angles)

$\therefore \quad \Delta AOD \sim \Delta COB$ (*SAS* similarity)

$\therefore \quad \angle A = \angle C$ and $\angle B = \angle D$. **1**

(Corresponding angles of similar triangles)

Q. 22. In the given figure, if $AB \parallel DC$, find the value of x. [A] **[Board Term-1, 2012, Set-35]**



Sol. Since, the diagonals of a trapezium divide each other proportionally, so we have

$$\frac{OA}{OC} = \frac{BO}{OD}$$

or, $\quad \dfrac{x+5}{x+3} = \dfrac{x-1}{x-2}$ **1**

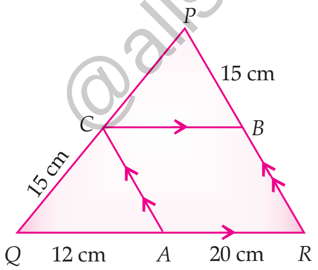
or, $\quad (x+5)(x-2) = (x-1)(x+3)$

or, $\quad x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$

or, $\quad 3x - 2x = 10 - 3$

$\therefore \quad x = 7.$ **1**

Q. 23. In the given figure, $CB \parallel QR$ and $CA \parallel PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR. [A] **[Board Term-1, 2012, Set-55]**



Sol. In ΔPQR, $\quad CA \parallel PR$

$\therefore \quad \dfrac{PC}{CQ} = \dfrac{RA}{AQ}$ (By BPT)

or, $\quad \dfrac{PC}{15} = \dfrac{20}{12}$

$\therefore \quad PC = \dfrac{15 \times 20}{12} = 25$ cm **1**

In ΔPQR, $\quad CB \parallel QR$

$\therefore \quad \dfrac{PC}{CQ} = \dfrac{PB}{BR}$ (By BPT)

or, $\quad \dfrac{25}{15} = \dfrac{15}{BR}$

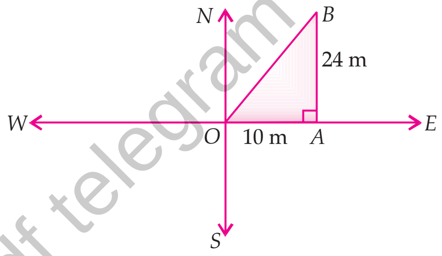
$\therefore \quad BR = \dfrac{15 \times 15}{25} = 9$ cm. **1**

Q. 24. A man steadily goes 10 m due east and then 24 m due north.

(i) Find the distance from the starting point.

(ii) Which mathematical concept is used in his problem ? [A]

Sol. (i) Let the initial position of the man be at O and his final position be B. Since, the man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. ½



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

or, $\quad OB^2 = (10)^2 + (24)^2$

$= 100 + 576$

$= 676$

or, $\quad OB = \sqrt{676}$

$= 26$ m **1**

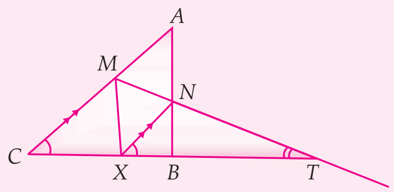
Hence, the man is at a distance of 26 m from the starting point.

(ii) Pythagoras Theorem ½

Q. 25. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$.

[CBSE Comptt. Set-I, II, III, 2018]

Sol. $\Delta TXN \sim \Delta TCM$



$\Rightarrow \quad \dfrac{TX}{TC} = \dfrac{XN}{CM} = \dfrac{TN}{TM}$

$\Rightarrow \quad TX \times TM = TC \times TN$ ...(i) ½

Again, $\Delta TBN \sim \Delta TXM$

$\Rightarrow \quad \dfrac{TB}{TX} = \dfrac{BN}{XM} = \dfrac{TN}{TM}$

$\Rightarrow \quad TM = \dfrac{TN \times TX}{TB}$ ...(ii) ½

Using *(ii)* in *(i)*, we get

$$\Rightarrow \quad TX^2 = \frac{TN}{TB} = TC \times TN$$

$$\Rightarrow \quad TX^2 = TC \times TB \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2018]

Q. 26. There are three villages A, B and C such that distance from A to B is 7 km from B to C is 5 km and C to A is 8 km. The gram-pradhan wants to dig, a well in such a way that the distance of the well from each village is equal.

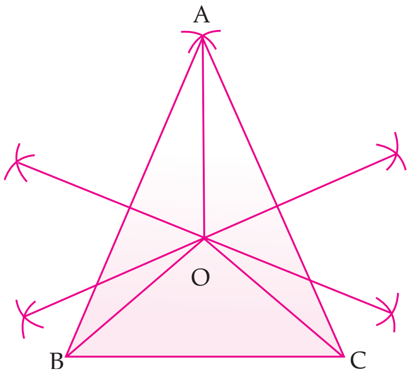
(ii) Which mathematical concept is used to solve the above question ?

(ii) What should be the location of the well ? AE

Sol. Distance from village A to B = 7 km

Distance from village B to C = 5 km

and distance from village C to A = 8 km



(i) Triangle **1**

(ii) Location of the well be will at the circumcenter of the triangle. **1**

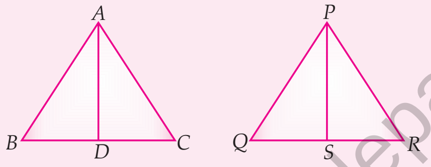
Short Answer Type Questions-II

(3 marks each)

Q. 1. If $\Delta ABC \sim \Delta PQR$ and AD and PS are bisectors of corresponding angles A and P, then prove that $\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \frac{AD^2}{PS^2}$.

U **[Board Term-1, 2016, Set-MV98HN3]**

Sol.



$$\Delta ABC \sim \Delta PQR$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \qquad \mathbf{1}$$

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \frac{AB^2}{PQ^2} \qquad ...(i)$$

$$\angle A = \angle P$$

or, $$\frac{1}{2}\angle A = \frac{1}{2}\angle P$$

or, $$\angle BAD = \angle QPS$$

$$\Delta BAD \sim \Delta QPS \qquad (AA \text{ similarity}) \ \mathbf{1}$$

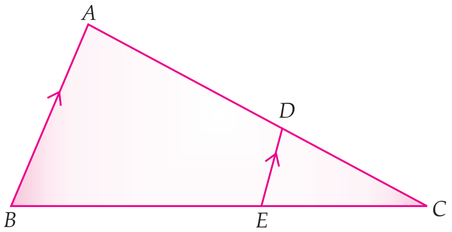
$$\frac{BA}{QP} = \frac{AD}{PS} \qquad ...(ii)$$

By eqs. (i) and (ii),

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \frac{AD^2}{PS^2} \qquad \textbf{Hence Proved. 1}$$

[CBSE Marking Scheme, 2016]

Q. 2. In given figure, D is a point on AC such that $AD = 2CD$, also $DE || AB$.



Find : $\frac{ar\,(\Delta DCE)}{ar\,(\Delta ACB)}$

U **[Board Term-1, 2015, Set–FHN8MGD]**

Sol. Given $AD = 2CD$

In ΔCDE and ΔCAB

$$\angle C = \angle C \qquad \text{(Common)}$$

$$\angle CDE = \angle CAB$$

(Corresponding angles)

$\therefore$ $\Delta CDE \sim \Delta CAB$ (By AA similarity rule)

Now $$\frac{ar\,(\Delta DCE)}{ar\,(\Delta ACB)} = \frac{CD^2}{CA^2} = \frac{CD^2}{(AD+DC)^2}$$

or, $$\frac{ar\,(\Delta DCE)}{ar\,(\Delta ACB)} = \frac{CD^2}{(3CD)^2} = \frac{1}{9} \qquad \mathbf{3}$$

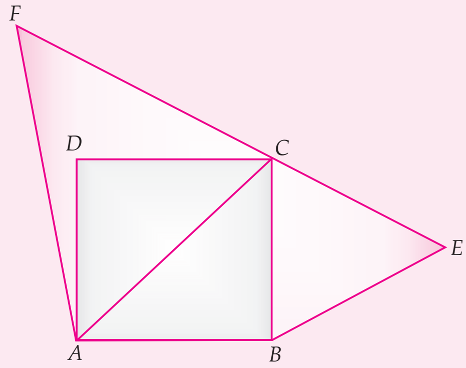
[CBSE Marking Scheme, 2015]

Q. 3. Prove that area of the equilateral triangle described on the side of a square is half of the area of the equilateral triangle described on its diagonal.

A **[Board Term-1, 2015, Set–WJQZQBN]**

[CBSE Delhi/Outside Delhi Set-2018]

Sol.



Let the side of the square be 'a' units

$\therefore$ $$AC^2 = a^2 + a^2 = 2a^2$$

$\Rightarrow$ $$AC = \sqrt{2}a \text{ units} \qquad \mathbf{1}$$

Area of equilateral triangle $\Delta BCE = \frac{\sqrt{3}}{4} a^2$ sq.u ½

Area of equilateral triangle

$$\Delta ACF = \frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2 = \frac{\sqrt{3}}{2}a^2 \text{ sq.u } \mathbf{1}$$

Area of $\Delta BCE = \frac{1}{2} Ar\ \Delta ACF$ ½

[CBSE Marking Scheme, 2018]

Q. 4. In a trapezium *ABCD*, diagonals *AC* and *BD* intersect at *O*. If *AB* = 3*CD*, then find ratio of areas of triangles *COD* and *AOB*.

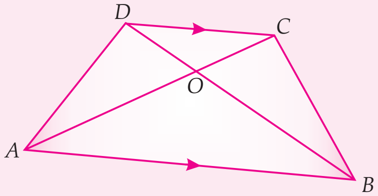
U **[Board Term-1, 2015, Set–FHN8MGD]**

Sol. $\Delta AOB \sim \Delta COD$ (*AA* similarity)

$$\frac{\text{ar}(\Delta COD)}{\text{ar}(\Delta AOB)} = \frac{CD^2}{AB^2} \quad \mathbf{1}$$

$$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9} \quad \mathbf{1}$$

Ratio = 1 : 9



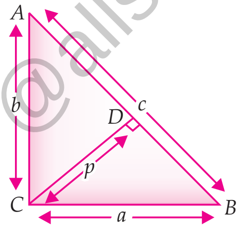
1

[CBSE Marking Scheme, 2015]

AI **Q. 5. ΔABC is a right angled at *C*. If *p* is the length of the perpendicular from *C* to *AB* and *a*, *b*, *c* are the lengths of the sides opposite $\angle A$, $\angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.**

A **[Board Term-1, 2016, Set-O4YP6G7]**

Sol.



In ΔACB and ΔCDB

$\angle ACB = \angle CDB = 90°$

$\angle B = \angle B$ (common)

$\therefore$ $\Delta ACB \sim \Delta CDB$ (by *AA* Similarity) **1**

or, $\frac{b}{p} = \frac{c}{a}$

or, $\frac{1}{p} = \frac{c}{ab}$ **1**

Squaring on both sides,

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

or, $\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$ $[\because c^2 = a^2+b^2]$

$\therefore$ $\frac{1}{p^2} = \frac{1}{a^2}+\frac{1}{b^2}$ **Hence Proved. 1**

Q. 6. In ΔABC, $DE || BC$. If $AD = x + 2$, $DB = 3x + 16$, $AE = x$ and $EC = 3x + 5$, then find *x*.

C + A **[Board Term-1, 2015, Set–DDE-E]**

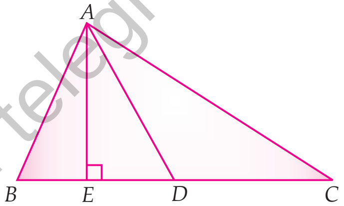
Sol. Try yourself, Similar to Q. No. 1 in VSATQ.

AI **Q. 7. If in ΔABC, *AD* is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.**

A **[Board Term-1, 2015, Set–DDE-E]**

Sol. To prove :

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}(BC)^2$$



Draw $AE \perp BC$

In ΔABE,

$AB^2 = AE^2 + BE^2$ (Pythagoras theorem)

or, $AB^2 = AD^2 - DE^2 + (BD - DE)^2$

$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\therefore$ $AB^2 = AD^2 + BD^2 - 2BD \times DE$ ....(i) **1**

In ΔAEC,

$AC^2 = AE^2 + EC^2$

or, $AC^2 = (AD^2 - ED^2) + (ED + DC)^2$

$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$

or, $AC^2 = AD^2 + CD^2 + 2ED \times CD$

or, $AC^2 = AD^2 + DC^2 + 2DC \times DE$ ....(ii) **1**

Adding eqns. (i) & (ii),

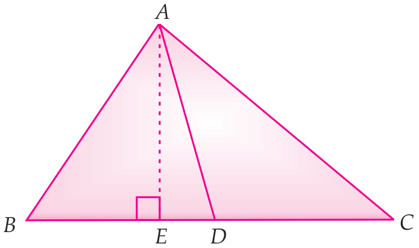
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ $(\because BD = DC)$

$$= \left[2AD^2 + 2\left(\frac{1}{2}BC\right)^2\right]$$

$$= 2AD^2 + \frac{1}{2}BC^2 \quad (\text{as } BD = \frac{1}{2}BC)\ \mathbf{1}$$

Hence Proved.

Q. 8. In ΔABC, if *AD* is the median, then show that $AB^2+AC^2=2(AD^2+BD^2)$.

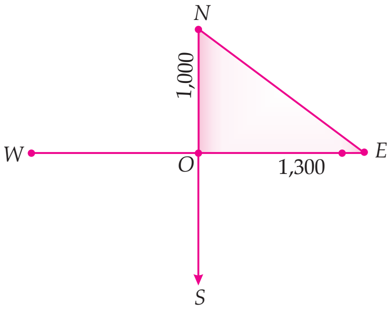


A **[CBSE SQP, 2018-19]**

Sol. Try yourself, Similar to Q. No. 7 in SATQ-II.

Q. 9. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours. A [Board Term-1, 2015, Set–DDE-E]

Sol.



Distance covered by first aeroplane due North after two hours = 500 × 2 = 1,000 km. **1**

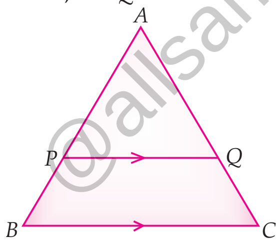
Distance covered by second aeroplane due East after two hours = 650 × 2 = 1,300 km. **1**

Distance between two aeroplanes after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$
$$= \sqrt{(1000)^2 + (1300)^2}$$
$$= \sqrt{1000000 + 1690000}$$
$$= \sqrt{2690000}$$
$$= 1640.12 \text{ km}$$ **1**

Q. 10. *ABC* is a triangle, *PQ* is the line segment intersecting *AB* in *P* and *AC* in *Q* such that *PQ* || *BC* and divides Δ*ABC* into two parts, equal in area, find *BP* : *AB*. U [Board Term-1, 2012, LK-59]

Sol. Here, Since $PQ \parallel BC$ and PQ divides ΔABC into two equal parts : So, $\Delta APQ \sim \Delta ABC$



$$\therefore \quad \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{AP^2}{AB^2}$$

or, $$\frac{1}{2} = \frac{AP^2}{AB^2}$$

or, $$\frac{1}{\sqrt{2}} = \frac{AB - BP}{AB} \quad (\because AB = AP + BP)$$

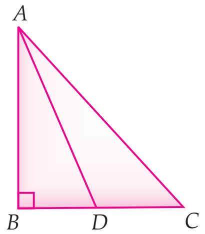
or, $$\frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$ **1**

or, $$1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

or, $$\frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\therefore \quad BP : AB = (\sqrt{2} - 1) : \sqrt{2}$$ **2**

AI **Q. 11. In the given figure, *ABC* is a right angled triangle at ∠*B* = 90°. *D* is the mid-point of *BC*. Show that $AC^2 = AD^2 + 3CD^2$.** U [Board Term-1, 2016, Set-ORDAWEZ 2011, Set-60]



Sol. Given,

$$BD = CD = \frac{BC}{2}$$ **1**

or, $$BC = 2BD$$

Using Pythagoras theorem in the right Δ*ABC*, we have

$$AC^2 = AB^2 + BC^2$$
$$= AB^2 + 4BD^2$$ **1**
$$= (AB^2 + BD^2) + 3BD^2$$
$$AC^2 = AD^2 + 3CD^2. \ [\because BD = CD]$$ **1**

Q. 12. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium. A [Board Term-1, 2011, Set-39]

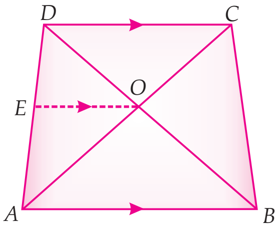
Sol. Proof :

In quadrilateral *ABCD*,

$$\frac{AO}{BO} = \frac{CO}{DO} \quad \text{(Given)}$$

or, $$\frac{AO}{CO} = \frac{BO}{DO} \quad \text{...(i)}$$

Draw $EO \parallel AB$ on



In Δ*ABD*, $EO \parallel AB$ (By construction)

$$\therefore \quad \frac{AE}{ED} = \frac{BO}{DO}$$ (By BPT) ...(ii) **1**

From eqns. (i) and (ii),

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In Δ*ADC*, $$\frac{AE}{ED} = \frac{AO}{CO}$$

or, $EO \parallel DC$ (Converse of BPT) **1**

$EO \parallel AB$ (Construction)

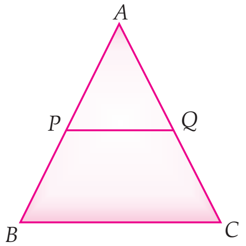
$\therefore$ $AB \parallel DC$

or, In quad. *ABCD*, $AB \parallel DC$ **1**

Hence, *ABCD* is a trapezium. **Hence Proved**

Q. 13. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm, find BC.

A [Board Term-1, 2011, Set-40]



Sol. Here, $\dfrac{AP}{AB} = \dfrac{3.5}{10.5} = \dfrac{1}{3}$

or, $\dfrac{AQ}{AC} = \dfrac{3}{9} = \dfrac{1}{3}$ **1**

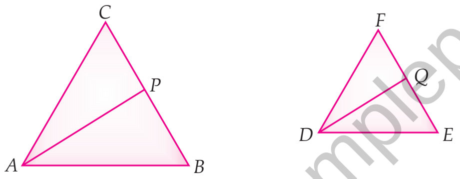
In ΔABC, $\dfrac{AP}{AB} = \dfrac{AQ}{AC}$ and $\angle A$ is common **1**

or, $\Delta APQ \sim \Delta ABC$ (By SAS)

$\therefore$ $\dfrac{AP}{AB} = \dfrac{PQ}{BC}$

or, $\dfrac{1}{3} = \dfrac{4.5}{BC}$ or, BC = 13.5 cm. **1**

Q. 14. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



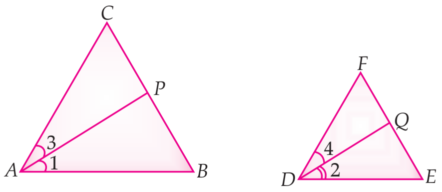
Prove that :

(i) $\dfrac{AP}{DQ} = \dfrac{AB}{DE}$

(ii) $\Delta CAP \sim \Delta FDQ$.

U [Board Term-1, 2016, Set-LGRKEGO]

Sol.



(i) Here, $\Delta ABC \sim \Delta DEF$

$\therefore$ $\angle A = \angle D$ (corresponding angles)

and $2\angle 1 = 2\angle 2$

or, $\angle 1 = \angle 2$

Also $\angle B = \angle E$ (corresponding angles)

Since, $\Delta APB \sim \Delta DQE$ **1**

Hence, $\dfrac{AP}{DQ} = \dfrac{AB}{DE}$ **1**

(ii) $\because$ $\Delta ABC \sim \Delta DEF$

$\therefore$ $\angle A = \angle D$

and $\angle C = \angle F$

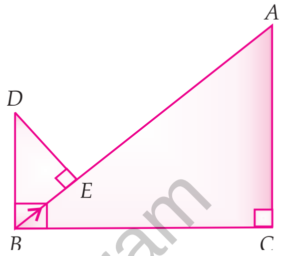
Hence, $2\angle 3 = 2\angle 4$ or, $\angle 3 = \angle 4$ **1**

$\therefore$ $\Delta CAP \sim \Delta FDQ$ (By AA similarity)

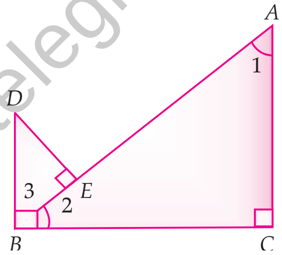
Hence Proved.

Q. 15. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\dfrac{BE}{DE} = \dfrac{AC}{BC}$

U [Board Term-1, 2011, Set-40]



Sol. Given : $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.



To prove : $\dfrac{BE}{DE} = \dfrac{AC}{BC}$ **1**

Proof : In ΔABC,

$\angle 1 + \angle 2 = 90°$ [$\angle C = 90°$] **1**

But $\angle 2 + \angle 3 = 90°$ (Given)

or, $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

$\angle 1 = \angle 3$ (Proved)

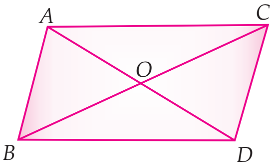
$\angle ACB = \angle DEB = 90°$ (Given)

$\therefore$ $\Delta ABC \sim \Delta BDE$ (By AA Similarity)

or, $\dfrac{AC}{BC} = \dfrac{BE}{DE}$. **Hence Proved. 1**

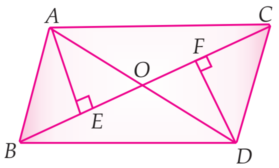
Q. 16. In the given figure, ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$

U [Board Term-1, 2011, Set-40]
[Board Term-1, 2016, Set-ORDAWEZ]



Sol. To prove : $\dfrac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \dfrac{AO}{DO}$ ½

Construction : Draw $AE \perp BC$ and $DF \perp BC$.



Proof : ½

In ΔAOE and ΔDOF,

$\angle AOE = \angle DOF$

(Vertically opposite angles)

$\angle AEO = \angle DFO = 90°$ (Construction)

or, $\Delta AOE \sim \Delta DOF$ (By *AA* Similarity)

$\therefore \quad \dfrac{AO}{DO} = \dfrac{AE}{DF}$ ...(i) **1**

Now, $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta DBC)} = \dfrac{\frac{1}{2}\times BC \times AE}{\frac{1}{2}\times BC\times DF}$

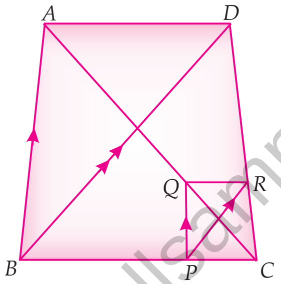
$= \dfrac{AE}{DF}$ ½

$= \dfrac{AO}{DO}$ [From equation (i)] ½

Hence Proved.

Q. 17. In the given figure, two triangles *ABC* and *DBC* lie on the same side of *BC* such that *PQ* || *BA* and *PR* || *BD*. Prove that *QR* || *AD*.

U **[Board Term-1, 2011, Set -21]**



Sol. Given, in ΔABC, $PQ \parallel AB$ and $PR \parallel BD$

$\therefore \quad \dfrac{BP}{PC} = \dfrac{AQ}{QC}$ (By BPT) ...(i) **1**

Again, in ΔBCD, $PR \parallel BD$

or, $\dfrac{BP}{PC} = \dfrac{DR}{RC}$ (By BPT) ...(ii) **1**

From equations (i) and (ii),

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

or, $QR \parallel AD$ (By converse of BPT)

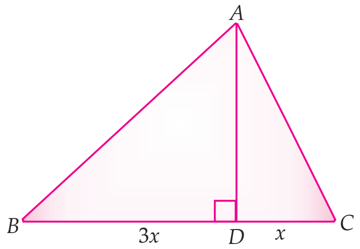
Hence proved. 1

AI **Q. 18. The perpendicular *AD* on the base *BC* of a ΔABC intersects *BC* at *D* so that *DB* = 3*CD*. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.**

U **[Board Term-1, 2011, Set-44, 60, 2012, 2016 Se-39, Set-NH$_3$]**

Sol. Given, in ΔADB, $AB^2 = AD^2 + BD^2$ ...(i)

(Pythagoras Theorem)



In ΔADC, $AC^2 = AD^2 + CD^2$ ...(ii)

(Pythagoras theorem)

Subtracting eqn. (ii) from eqn. (i),

$AB^2 - AC^2 = BD^2 - CD^2$ **1**

or, $= \left(\dfrac{3}{4}BC\right)^2 - \left(\dfrac{1}{4}BC\right)^2$

or, $= \dfrac{9}{16}BC^2 - \dfrac{1}{16}BC^2 = \dfrac{BC^2}{2}$ **1**

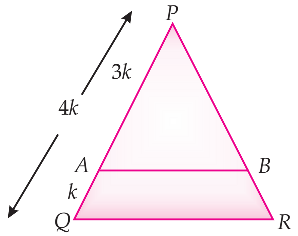
$\therefore \quad 2(AB^2 - AC^2) = BC^2$

$\therefore \quad 2(AB)^2 = 2AC^2 + BC^2$. **Hence proved. 1**

Q. 19. In the given figure, $\dfrac{PA}{AQ} = \dfrac{PB}{BR} = 3$. If the area of ΔPQR is 32 cm², then find the area of the quadrilateral *AQRB*. A **[Board Term-1, 2011, Set-44]**

Sol. We have, $\Delta PQR \sim \Delta PAB$

$\left(\because \angle P \text{ is common and } \dfrac{PA}{PQ} = \dfrac{PB}{PR}\right)$ **1**



or, $\dfrac{\text{area }(\Delta PQR)}{\text{area }(\Delta PAB)} = \dfrac{PQ^2}{PA^2}$

or, $\dfrac{32}{\text{area }(\Delta PAB)} = \dfrac{(4k)^2}{(3k)^2} = \dfrac{16k^2}{9k^2}$ **1**

or, Area of $\Delta PAB = 18$ cm²

$\therefore$ Area of quadrilateral *AQRB*

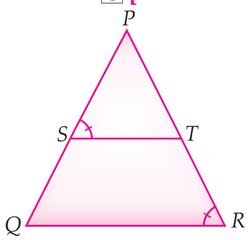
$= $ area of $\Delta PQR -$ area of ΔPAB

$= 32 - 18$

$= 14$ cm² **1**

Q. 20. In the given figure, $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that *PQR* is an isosceles triangle.

U **[Board Term-1, 2011, Set-74]**



Sol. Given, $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

and $\angle PST = \angle PRQ$

To prove : PQR is an isosceles triangle.

Proof : $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

By converse of B.P.T.,

$ST \,||\, QR$ **1**

$\therefore \quad \angle PST = \angle PQR$

(Corresponding angles) **1**

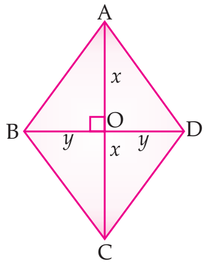
and $\angle PST = \angle PRQ$ (Given)

$\therefore \quad \angle PQR = \angle PRQ$

So, ΔPQR is an isosceles triangle. **Hence proved. 1**

Q. 21. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals. A **[Board Term-1, 2011, Set-21]**

Sol. Let, $ABCD$ is a rhombus and diagonals of a rhombus bisect each other at 90°



$\therefore \quad AO = OC \Rightarrow AO^2 = OC^2$

$BO = OD \Rightarrow BO^2 = OD^2$

and $\angle AOB = 90°$

$\therefore \quad AB^2 = OA^2 + BO^2 = x^2 + y^2$ **1**

Similarly, $AD^2 = x^2 + y^2 = BC^2 = CD^2$ **1**

$\therefore \; AB^2 + BC^2 + CD^2 + DA^2 = 4AO^2 + 4DO^2$

$= (2AO)^2 + (2DO)^2$

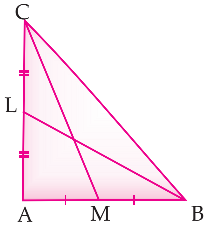
$= (2x)^2 + (2y)^2$

$\therefore \; AB^2 + BC^2 + CD^2 + AD^2$

$= AC^2 + BD^2$

Hence proved. 1

AI **Q. 22. In the given figure, BL and CM are medians of ΔABC, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.** U **[Board Term-1, 2011, Set-74]**



Sol. Given, ΔABC, right angled at A and BL and CM are medians.

To prove :

$4(BL^2 + CM^2) = 5BC^2$

Proof : In ΔABL,

$BL^2 = AB^2 + AL^2$ **1**

$= AB^2 + \left(\dfrac{AC}{2}\right)^2$ (BL is median)

In ΔACM, $CM^2 = AC^2 + AM^2$

$= AC^2 + \left(\dfrac{AB}{2}\right)^2$

(CM is median) **1**

$BL^2 + CM^2 = AB^2 + AC^2 + \dfrac{AC^2}{4} + \dfrac{AB^2}{4}$

or, $4(BL^2 + CM^2) = 5AB^2 + 5AC^2$

$= 5(AB^2 + AC^2)$

$= 5BC^2.$ **Hence proved. 1**

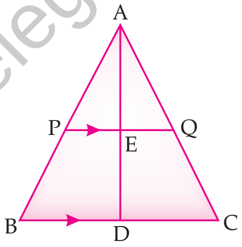
Q. 23. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ \,||\, BC$. Prove that the median AD bisects PQ. U **[Board Term-1, 2011, Set-70]**

Sol. Suppose the median AD intersects PQ at E.

Given, $PQ \,||\, BC$

Then, $\angle APE = \angle B$ and $\angle AQE = \angle C$

(Corresponding angles)



So, in ΔAPE and ΔABD,

$\angle APE = \angle ABD$

$\angle PAE = \angle BAD$ (common) **1**

$\therefore \quad \Delta APE \sim \Delta ABD$

or, $\dfrac{PE}{BD} = \dfrac{AE}{AD}$ ...(i) **1**

Similarly, $\Delta AQE \sim \Delta ACD$

or, $\dfrac{QE}{CD} = \dfrac{AE}{AD}$ ...(ii) ½

From eqns. (i) and (ii),

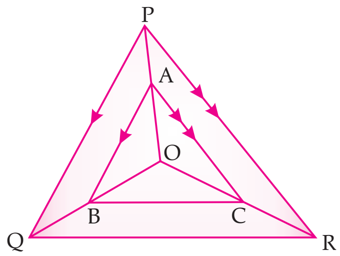
$\dfrac{PE}{BD} = \dfrac{QE}{CD}$

or, $\dfrac{PE}{BD} = \dfrac{QE}{BD}$, as $CD = BD$

or, $PE = QE$

Hence, AD bisects PQ. **Hence proved.** ½

Q. 24. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB \,||\, PQ$ and $AC \,||\, PR$. Prove that $BC \,||\, QR$. U **[Board Term-1, 2012, Set-66]**



Sol. Proof : In ΔPOQ, $AB \parallel PQ$, (Given)

$\therefore \quad \dfrac{AO}{AP} = \dfrac{OB}{BQ}$ ...(i) (BPT) **1**

In ΔOPR, $AC \parallel PR$, (Given)

$\therefore \quad \dfrac{OA}{AP} = \dfrac{OC}{CR}$ ...(ii) (BPT) **1**

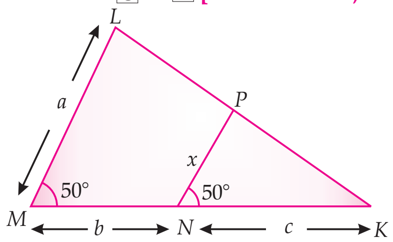
From equations (i) and (ii),

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$\therefore \quad BC \parallel QR$, (By converse of BPT)**1**

Hence Proved.

Q. 25. In the given figure, find the value of x in terms of a, b and c. C + A **[Board Term-1, 2012, Set-52]**



Sol. In triangles LMK and PNK,

$\angle M = \angle N = 50°$ (Given) **1**

$\angle K = \angle K$ (Common)

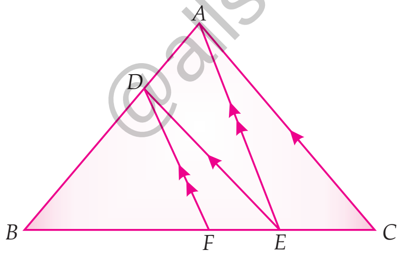
$\therefore \quad \Delta LMK \sim \Delta PNK$ (AA similarity)

or, $\dfrac{LM}{PN} = \dfrac{KM}{KN}$ **1**

or, $\dfrac{a}{x} = \dfrac{b+c}{c}$

$\therefore \quad x = \dfrac{ac}{b+c}$ **1**

Q. 26. In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\dfrac{BE}{FE} = \dfrac{BE}{EC}$. U **[Board Term-1, 2012, Set-66]**



Sol. In ΔABC, $DE \parallel AC$, (Given)

$\therefore \quad \dfrac{BD}{DA} = \dfrac{BE}{EC}$ (BPT) ...(i) **1**

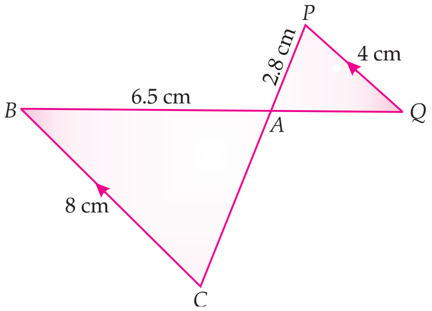
In ΔABE, $DF \parallel AE$, (Given)

$\therefore \quad \dfrac{BD}{DA} = \dfrac{BF}{FE}$ (BPT) ...(ii) **1**

From (i) and (ii), we have

$\dfrac{BF}{FE} = \dfrac{BE}{EC}$. **Hence proved. 1**

Q. 27. In the given figure, $BC \parallel PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, $AP = 2.8$ cm. Find CA and AQ. A **[Board Term-1, 2012, Set-66]**



Sol. In ΔABC and ΔAPQ,

$AB = 6{\cdot}5$ cm, $BC = 8$ cm, $PQ = 4$ cm

and $AP = 2{\cdot}8$ cm

$BC \parallel PQ$ (Given)

$\therefore \quad \angle CBA = \angle AQP$ (Alternate angles)

$\angle BAC = \angle PAQ$

(Vertically opposite angles)

$\therefore \quad \Delta ABC \sim \Delta AQP$ (AA Similarity) **1**

or, $\dfrac{AB}{AQ} = \dfrac{BC}{QP} = \dfrac{AC}{AP}$ **1**

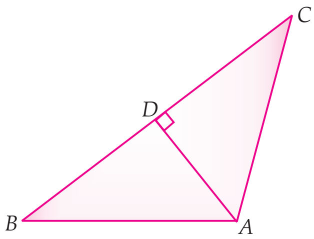
or, $\dfrac{6.5}{AQ} = \dfrac{8}{4} = \dfrac{AC}{2.8}$

or, $AQ = \dfrac{6.5}{2} = 3.25$ cm,

and $AC = 2 \times 2.8 = 5.6$ cm. **1**

Q. 28. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.

U **[Board Term-1, 2012, Set-25]**



Sol. In right ΔADC, $AC^2 = AD^2 + CD^2$ ...(i)

In right ΔADB, $AB^2 = AD^2 + BD^2$ ...(ii) **1**

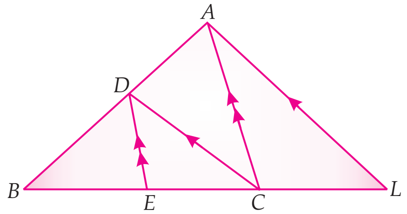
Subtracting eqn. (i) from eqn. (ii),

$AB^2 - AC^2 = BD^2 - CD^2$ **1**

or, $AB^2 + CD^2 = AC^2 + BD^2$.

Hence proved. 1

Q. 29. In the given figure, $CD \parallel LA$ and $DE \parallel AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm. A **[Board Term-1, 2012, Set–39]**



Sol. Given, in ΔABC, $DE \parallel AC$ and $BE = 4$ cm, $EC = 2$ cm (Given)

Then, $\dfrac{BD}{DA} = \dfrac{BE}{EC}$ ....(i) (BPT) **1**

In ΔABL, $\quad DC \,||\, AL$ (Given)

Then, $\quad \dfrac{BD}{DA} = \dfrac{BC}{CL}$ ...(ii) (BPT) **1**

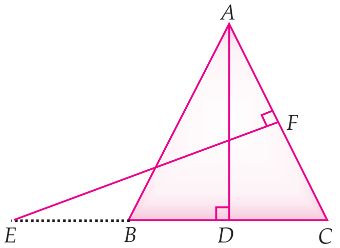
From equations (i) and (ii),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

or, $\quad \dfrac{4}{2} = \dfrac{6}{CL}$

or, $\quad CL = 3$ cm. **1**

Q. 30. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC. Prove that ΔABD is similar to ΔCEF. A [Board Term-1 2012, Set-60]



Sol. In ΔABD and ΔCEF, given

$AB = AC$

Then, $\quad \angle ABC = \angle ACB$ **1**

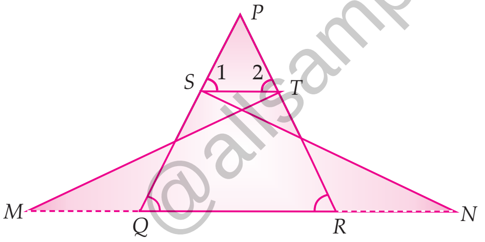
or $\quad \angle ABD = \angle ECF$

$\angle ADB = \angle EFC,$ (each 90°) **1**

$\therefore \quad \Delta ABD \sim \Delta ECF.$ (AA Similarity) **1**

Hence Proved.

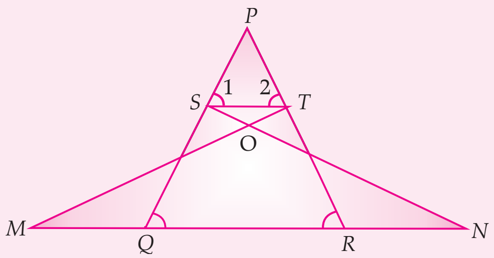
Q. 31. In figure $\angle 1 = \angle 2$ and $\Delta NSQ \cong \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$.



A [Board Term-1 2012, Set-21]

[SQP 2018-2019]

Sol.



$\angle 1 = \angle 2$

$\Rightarrow \quad PT = PS$ ....(i) **1**

$\Delta NSQ \cong \Delta MTR$

$\Rightarrow \quad \angle NQS = \angle MRT$

$\Rightarrow \quad \angle PQR = \angle PRQ$

$\Rightarrow \quad PR = PQ$ ....(ii) **1**

From (i) and (ii), $\quad \dfrac{PT}{PR} = \dfrac{PS}{PQ}$

Also, $\quad \angle TPS = \angle RPQ$ (common)

$\Rightarrow \quad \Delta PTS \sim \Delta PRQ$ **1**

[CBSE Marking Scheme, 2018]

Q. 32. If the area of two similar triangles are equal, then prove that they are congruent.

A [Board Term-1, 2012, Set-35]

[CBSE Delhi/OD Set-2018]

Sol. Let, $\quad \Delta ABC \sim \Delta PQR$

$\therefore \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$ **1**

Given $\quad$ ar (ΔABC) = ar (ΔPQR)

$\Rightarrow \quad \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$ **1**

$\Rightarrow \quad AB = PQ$, $BC = QR$ and $AC = PR$

$\Rightarrow$ Therefore, $\quad \Delta ABC \cong \Delta PQR$ **1**

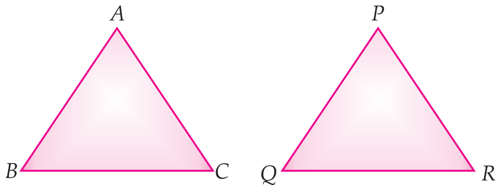
(SSS Congruence Rule)

[CBSE Marking Scheme, 2018]

Detailed Answer :

Given : $\quad \Delta ABC \sim \Delta PQR,$

and $\quad$ ar ΔABC = ar ΔPQR



To prove : $\quad \Delta ABC \cong \Delta PQR$

Proof : $\quad \Delta ABC \sim \Delta PQR$ (Given)

or, $\quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$ ...(i)

Also $\quad$ ar (ΔABC) = ar (ΔPQR) (Given)

or, $\quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$ **1**

From equation (i), we have

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$$

or, $\quad \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP} = 1$ **1**

or, $\quad AB = PQ,$

$BC = QR$

and $\quad CA = RP$

$\Delta ABC \cong \Delta PQR$ (SSS) **1**

Commonly Made Error

- Most candidates are not able to prove $\Delta ABC \cong \Delta PQR$

Answering Tip

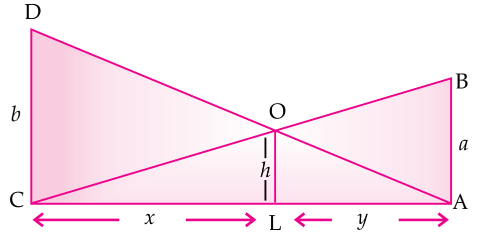
- Candidates should know about SSS - criteria for Congruence of triangles.

Q. 33. Two trees of height a and b are p metre apart.

(i) Prove that the height of the point of intersection of the lines joining the top of each tree to the foot of the opposite trees is given by $\frac{ab}{a+b}$ m.

(ii) Which mathematical concept is used in this problem ? $\boxed{\text{AE}}$

Sol. (i) Let AB and CD be the two trees of height a and b metre such that the trees are p metre apart *i.e.*, $AC = p$. Let the lines AD and BC meet at O such that $OL = h$ m.



½

Let CL be x and LA be y,

then $x + y = p$

In ΔABC and ΔLOC,

$$\angle CAB = \angle CLO \quad \text{(each 90°)}$$

$$\angle C = \angle C \quad \text{(Common)}$$

$$\therefore \quad \Delta CAB \sim \Delta CLO \quad (AA \text{ similarity})$$

or, $$\frac{CA}{CL} = \frac{AB}{LO}$$

or, $$\frac{p}{x} = \frac{a}{h}$$

or, $$x = \frac{ph}{a} \quad \text{...(i) ½}$$

In ΔALO and ΔACD,

$$\angle ALO = \angle ACD \quad \text{(each 90°)}$$

$$\angle A = \angle A \quad \text{(Common)}$$

$$\therefore \quad \angle ALO \sim \angle ACD \quad (AA \text{ similarity})$$

or, $$\frac{AL}{AC} = \frac{OL}{DC}$$

or, $$\frac{y}{p} = \frac{h}{b}$$

or, $$y = \frac{ph}{d} \quad \text{...(ii) ½}$$

Adding eqns. (i) and (ii),

$$x + y = \frac{ph}{a} + \frac{ph}{b}$$

or, $$p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$$

or, $$\frac{1}{h} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

$$\therefore \quad h = \frac{ab}{a+b} \text{ m.} \quad ½$$

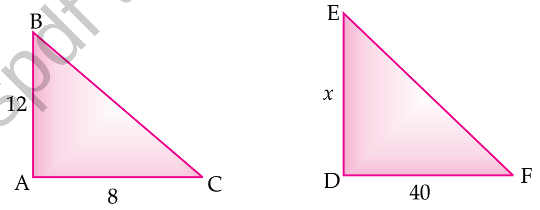
(ii) Similar Triangles. **1**

Q. 34. A vertical row of trees 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground.

(i) Determine the height of the tower.

(ii) Which mathematical concept is used in this problem ? $\boxed{\text{A}}$; $\boxed{\text{E}}$

Sol. (i) Let AB be the vertical tree and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Join BC and EF. Let $DE= x$.



We have $AB = 12$ m

$AC = 8$ m

and $DF = 40$ m **1**

In ΔABC and ΔDEF,

$$\angle A = \angle D = 90°$$

Since, the sun casts equal angle at the same time

$$\angle C = \angle F$$

Therefore by AA criterion of similarity, we have

$$\Delta ABC \sim \Delta DEF$$

or, $$\frac{AB}{DE} = \frac{AC}{DF}$$

or, $$\frac{12}{x} = \frac{8}{40}$$

or, $$x = \frac{12 \times 40}{8}$$

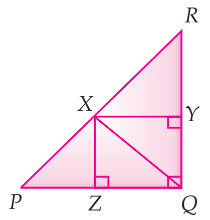
or, $x = 60$ m. or, height of tower $=60$ m. **1**

(ii) Similar Triangles. **1**

Long Answer Type Questions

(4 marks each)

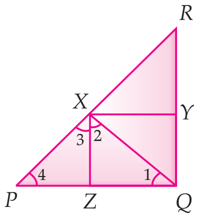
Q. 1.



ΔPQR is right angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.

$\boxed{\text{U}}$ **[Board Term-1, 2015, Set–MV98HN3]**

Sol.



1

Here, $RQ \perp PQ$ and $XZ \perp PQ$

or, $XZ \parallel YQ$

$\therefore$ Similarly, $XY \parallel ZQ$

$XYQZ$ is a rectangle. $(\because \angle PQR = 90°)$ **1**

In ΔXZQ, $\angle 1 + \angle 2 = 90°$ ...(i)

and in ΔPZX, $\angle 3 + \angle 4 = 90°$ ...(ii)

$XQ \perp PR$ or, $\angle 2 + \angle 3 = 90°$ ...(iii)

By eqs. (i) and (iii), we get ½

$$\angle 1 = \angle 3$$

By eqs. (ii) and (iii), we get ½

$$\angle 2 = \angle 4$$

$\therefore$ $\Delta PZX \sim \Delta XZQ$ (*AA* similarity) **1**

$$\therefore \quad \frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

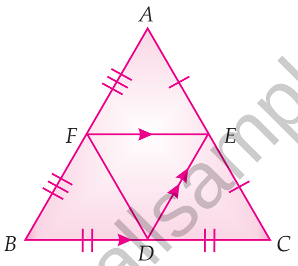
Thus, $XZ^2 = PZ \times ZQ$ **1**

Hence Proved.

AI **Q. 2. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of ar (ΔDEF) to ar (ΔABC).**

A **[Board Term-1, 2015, Set–DDE-M]**

Sol.



½

In ΔABC, given that F, E and D are the mid-points of AB, AC and BC respectively.

Hence, $FE \parallel BC$, $DE \parallel AB$ and $DF \parallel AC$. ½

By mid-point theorem,

If $DE \parallel BA$

then $DE \parallel BF$

and if $FE \parallel BC$

then $FE \parallel BD$ **1**

$\therefore$ $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of parallelogram divides it into two equal Areas.

Hence $ar\ (\Delta BDF) = ar\ (\Delta DEF)$ ...(i)

Similarly $ar\ (\Delta CDE) = ar\ (\Delta DEF)$ ...(ii)

or $(\Delta AFE) = ar\ (\Delta DEF)$ ...(iii)

or $(\Delta DEF) = ar\ (\Delta DEF)$ ...(iv)

On adding eqns. (i), (ii), (iii) and (iv),

$ar\ (\Delta BDF) + ar\ (\Delta CDE) + ar\ (\Delta AFE) + ar\ (\Delta DEF)$ **1**

$= 4ar\ (\Delta DEF)$

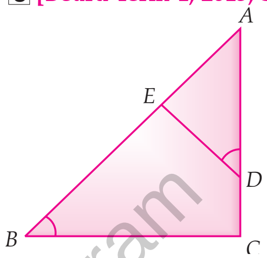
or, $ar\ (\Delta ABC) = 4ar\ (\Delta DEF)$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$ **1**

Q. 3. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.

Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.

U **[Board Term-1, 2015, Set–WJQZQBN]**



Sol. In ΔADE and ΔABC,

$\angle A = \angle A$ (Common)

$\angle ADE = \angle ABC$ (Given)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (*AA* similarity) **1**

or, $\dfrac{AD}{AB} = \dfrac{DE}{BC}$ **1**

or, $\dfrac{AD}{AE+BE} = \dfrac{DE}{BC}$ **1**

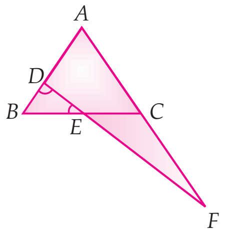
or, $\dfrac{7.6}{7.2+4.2} = \dfrac{DE}{8.4}$

or, $DE = \dfrac{7.6 \times 8.4}{11.4}$

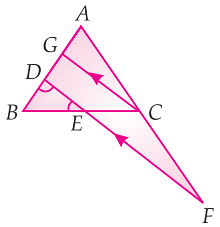
$= 5.6$ cm. **1**

Q. 4. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\dfrac{AF}{CF} = \dfrac{AD}{BE}$.

U **[Board Term-1, 2015, Set–WJQZQBN]**



Sol.



½

Construction : Draw $CG \parallel FD$

Given $\angle BED = \angle BDE$ ½

or, $BE = BD = EC$ (i)

(Given that E is the mid-point of BC)

In ΔBCG, $DE \parallel GC$

or, $\dfrac{BD}{DG} = \dfrac{BE}{EC} = 1$ (from (i)) ½ or, $BD = DG = EC = BE$ [using (i)]

In ΔADF, $CG \parallel FD$

or, $$\frac{AG}{GD} = \frac{AC}{CF}$$ (By BPT) ½

By adding 1 on both sides,

$$\frac{AG}{GD}+1 = \frac{AC}{CF}+1$$ 1

or, $$\frac{AD}{GD} = \frac{AF}{CF}$$

$\therefore$ $$\frac{AF}{CF} = \frac{AD}{BE}$$ [using (i)] 1

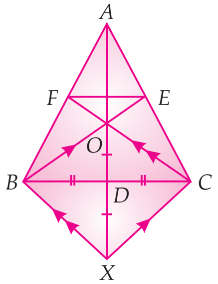
Hence Proved.

Q. 5. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(i) $EF \parallel BC$

(ii) $AO : AX = AF : AB$



A [Board Term-1, 2015, Set–O4YP6G7]

Sol. (i) Since, BC and OX bisect each other.

So, $BXCO$ is a parallelogram then $BE \parallel XC$ and $BX \parallel CF$.

In ΔABX, by B.P.T.,

$$\frac{AF}{FB} = \frac{AO}{OX}$$ ...(i) ½

In ΔAXC,

$$\frac{AE}{EC} = \frac{AO}{OX}$$ ...(ii) ½

Eqn. (i) and (ii) gives,

$$\frac{AF}{FB} = \frac{AE}{EC}$$ 1

So by converse of B.P.T.,

$$EF \parallel BC$$

(ii) Given $$\frac{OX}{OA} = \frac{FB}{AF}$$ 1

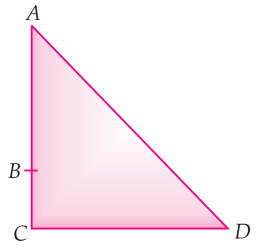
Adding 1 on both sides

$$\frac{AX}{OA} = \frac{AB}{AF}$$

or $OA : AX = AF : AB$ **Hence Proved. 1**

Q. 6. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x$, $BC = h$ and $CD = d$, then find x (in terms of h and d).

C + U [Board Term-1, 2015, Set–FHN8MGD]



Sol. Given, $AB + AD = BC + CD$

$AD = BC + CD - AB$

or, $AD = h + d - x$ 1

In rt ΔACD,

$AD^2 = AC^2 + DC^2$

or, $(h + d - x)^2 = (x + h)^2 + d^2$

or, $(h + d - x)^2 - (x + h)^2 = d^2$ 1

or, $(h + d - x - x - h)(h + d - x + x + h) = d^2$

or, $(d - 2x)(2h + d) = d^2$

or, $2hd + d^2 - 4hx - 2xd = d^2$

or, $2hd = 4hx + 2xd$

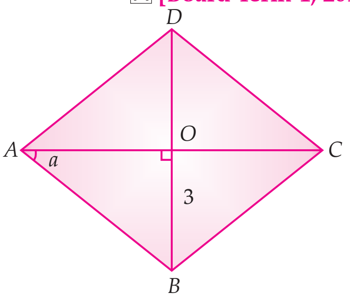
$= 2(2h + d)x$

or, $$x = \frac{hd}{2h+d}$$ 2

[CBSE Marking Scheme, 2015]

Q. 7. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.

A [Board Term-1, 2013, Set FFC]



Sol. Given, $$\cos\alpha = \frac{2}{3}$$

and $OB = 3$ cm

In ΔAOB, $$\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$$ 1

Let $OA = 2x$ and $AB = 3x$.

In ΔAOB,

$AB^2 = AO^2 + OB^2$

or, $(3x)^2 = (2x)^2 + (3)^2$

or, $9x^2 = 4x^2 + 9$ 1

or, $5x^2 = 9$

$\therefore$ $$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and $$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$ 1

So, diagonal $BD = 2 \times OB = 2 \times 3 = 6$ cm ½ and $\quad AC = 2AO$

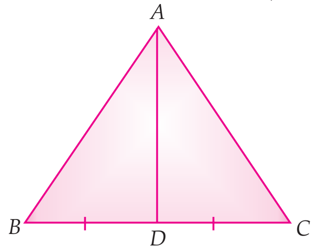
$$= 2 \times \frac{6}{\sqrt{5}}$$

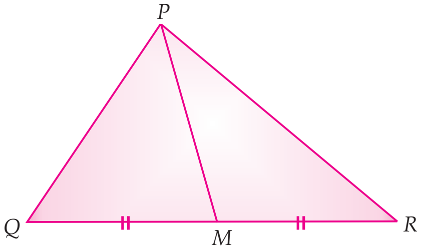
$$= \frac{12}{\sqrt{5}} \text{ cm.} \qquad \tfrac{1}{2}$$

Q. 8. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, proved that $\Delta ABC \sim \Delta PQR$.

A **[Board Term-1, 2013 FFC; 2012, Set-48]**

Sol. Proof : In ΔABC, $\quad BC = 2BD \quad$ (AD is the median)





and in $\Delta PQR \quad QR = 2QM$ (PM is the median) **1**

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM} \qquad \mathbf{1}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

$\therefore \quad \Delta ABD \sim \Delta PQM \quad$ (SSS Similarity)

or, $\quad \angle B = \angle Q$, $\quad$ (By CPST) **1**

In ΔABC and ΔPQR,

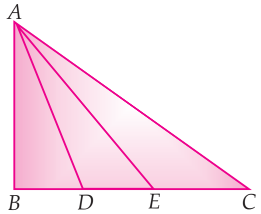
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

and $\quad \angle B = \angle Q \quad$ (SAS Similarity)

$\therefore \quad \Delta ABC \sim \Delta PQR$. $\quad$ **Hence proved. 1**

AI **Q. 9. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.**

U **[Board Term-1, 2013, LK-59]**



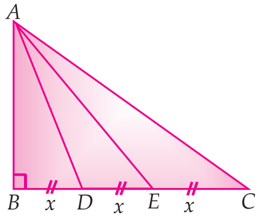
Sol. Let $\quad BD = DE = EC$ be x

$BE = 2x$

and $\quad BC = 3x$

Now, in ΔABE,

$$AE^2 = AB^2 + BE^2$$
$$= AB^2 + 4x^2, \qquad \text{...(i) } \mathbf{1}$$



In ΔABC,

$$AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$$

In ΔADB,

and $\quad AD^2 = AB^2 + BD^2 = AB^2 + x^2 \quad$ **1**

Now, on multiplying (i) by 8,

$$8AE^2 = 8AB^2 + 32x^2 \qquad \text{...(ii)}$$

and $\quad 3AC^2 + 5AD^2 = 3(AB^2 + 9x^2) + 5(AB^2 + x^2)$ **1**

$$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$$
$$= 8AB^2 + 32x^2$$
$$= 8(AB^2 + 4x^2)$$

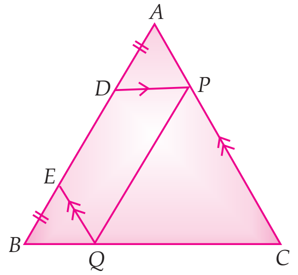
$\therefore \quad 3AC^2 + 5AD^2 = 8AE^2$. $\quad$ [From eqns. (i)]

Hence proved. 1

Q. 10. Let ABC be a triangle and D and E be two points on side AB such that $AD = BE$. If $DP \,||\, BC$ and $EQ \,||\, AC$, then prove that $PQ \,||\, AB$.

U **[Board Term-1, 2012, Set-44]**

Sol. In ΔABC, $\quad DP \,||\, BC$



$$\frac{AD}{DB} = \frac{AP}{PC}, \qquad (BPT) \text{ ...(i)}$$

Similarly, in ΔABC,

$$EQ \,||\, AC$$

or, $$\frac{BQ}{QC} = \frac{BE}{EA} \qquad \text{...(ii) } \mathbf{1}$$

From figure, $\quad EA = AD + DE$

(Given, $BE = AD$)

$$= BE + ED \qquad \mathbf{1}$$
$$= BD$$

Then, eqn. (ii) becomes, $$\frac{BQ}{QC} = \frac{AD}{BD} \qquad \text{...(iii)}$$

From (i) and (iii), we get **1**

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

$\therefore$ By converse of *BPT*,

$PQ \,||\, AB$. **Hence proved 1**

Q. 11. Prove that ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

A **[Board Term-1, 2015, Set-FHN8MGD]**

[Sample Question Paper 2017]

[CBSE Comptt. Set-I, II, III-2018]

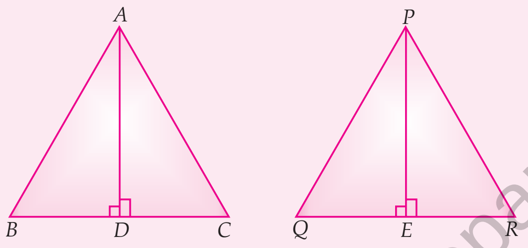
Sol. Given, $\Delta ABC \sim \Delta PQR$

To Prove : $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2$

$$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Construction : Draw $AD \perp BC$ and $PE \perp QR$.

Proof : $\Delta ABC \sim \Delta PQR$



1

$\Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ and

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad ...(i)$$

($\because$ Similar triangles are equiangular and their corresponding are proportional)

In ΔADB and ΔPEQ,

$\angle B = \angle Q$ [From (i)]

$\angle ADB = \angle PEQ$ [each 90°]

$\therefore \quad \Delta ADB \sim \Delta PEQ$ (*AA* similarity)

or, $\dfrac{AD}{PE} = \dfrac{AB}{PQ}$ ...(ii) **1**

(Corresponding sides of similar triangles)

From eqs. (i) and eq. (ii),

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE} \quad ...(iii)$$

Now $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \dfrac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times QR\times PE}$

$$= \left(\frac{BC}{QR}\right)\times\left(\frac{AD}{PE}\right)$$

$$= \frac{BC}{QR}\times\frac{BC}{QR}$$

or, $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ ...(iv) **1**

[from eq. (iii)]

From eq (iii) and eq (iv),

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2015]

Q. 12. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. A **[CBSE Delhi/OD Set-2018]**

[SQP, 2018-19]

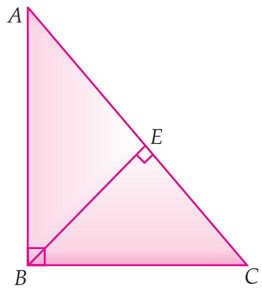
OR

Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus *ABCD*, 4 $AB^2 = AC^2 + BD^2$.

A **[Board Term-1, 2015 Set CJTOQ]**

[Sample Question Paper 2017]

Sol.



½

Given : $AB \perp BC$

Construction : Draw $BE \perp AC$ ½

To Prove : $AB^2 + BC^2 = AC^2$ ½

Proof : In ΔAEB and ΔABC

$\angle A = \angle A$ (Common)

$\angle E = \angle B$ (each 90°)

$\Delta AEB \sim \Delta ABC$ (By *AA* similarity)

or, $\dfrac{AE}{AB} = \dfrac{AB}{AC}$

or, $AB^2 = AE \times AC$ ....(i) ½

Now, in ΔCEB and ΔCBA,

$\angle C = \angle C$ (Common)

$\angle E = \angle B$ (each 90°)

$\Delta CEB \sim \Delta CBA$ (By *AA* similarity)

or, $\dfrac{CE}{BC} = \dfrac{BC}{AC}$

or, $BC^2 = CE \times AC$ ...(ii)

On adding eqns. (i) and (ii),

$AB^2 + BC^2 = AE \times AC + CE \times AC$

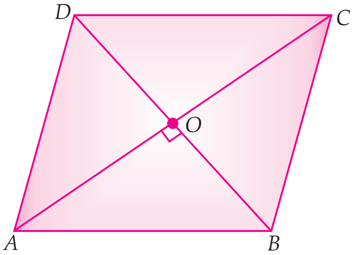
or, $AB^2 + BC^2 = AC\,(AE + CE)$

or, $AB^2 + BC^2 = AC \times AC$

$\therefore \quad AB^2 + BC^2 = AC^2$ **Hence proved. 2**

OR

We have already proved $AB^2 + BC^2 = AC^2$ in above part now.



Given : $ABCD$ is a rhombus. **1**

Construction : Draw diagonals AC and BD

$\therefore \quad AO = OC = \frac{1}{2} AC$

and $\quad BO = OD = \frac{1}{2} BD$ **1**

$AC \perp BD$ [$\because \square$ $ABCD$ is rhombus]

To Prove : $4\,AB^2 = AC^2 + BD^2$

Proof : $\angle AOB = 90°$ (Diagonal of rhombus bisect each other at right angle)

$AB^2 = OA^2 + OB^2$

or, $\quad AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$ **1**

$AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$

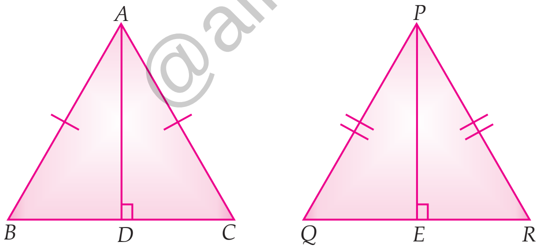
$\therefore \quad 4AB^2 = AC^2 + BD^2$ **1**

Hence proved.

Q. 13. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio 16 : 25, then find the ratio of their altitudes drawn from vertex to the opposite side.

U **[Board Term-1, 2015 Set CJTOQ]**

Sol.



1

Given : $\angle A = \angle P$

$\angle B = \angle Q, \angle C = \angle R$

Proof : Let $\angle A = \angle P$ be x

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

or, $\quad x° + \angle B + \angle B = 180°$ ($\angle B = \angle C$ given)

or, $\quad 2\angle B = 180° - x$

or, $\quad \angle B = \frac{180° - x}{2}$ ...(i) **1**

Now, in ΔPQR

$\angle P + \angle Q + \angle R = 180°$ (Given, $\angle Q = \angle R$)

or, $\quad x° + \angle Q + \angle Q = 180°$

or, $\quad 2\angle Q = 180° - x$

or, $\quad \angle Q = \frac{180° - x}{2}$ ...(ii)

In ΔABC and ΔPQR,

$\angle A = \angle P$ [Given]

$\angle B = \angle Q$ [from eqs. (i) and (ii)]

$\Delta ABC \sim \Delta PQR$ (AA similarity)

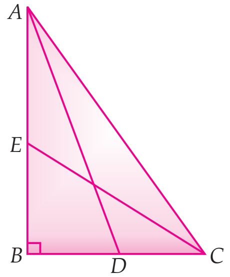
or, $\quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AD^2}{PE^2}$ **1**

or, $\quad \frac{16}{25} = \frac{AD^2}{PE^2}$

or, $\quad \frac{4}{5} = \frac{AD}{PE}$

$\therefore \quad \frac{AD}{PE} = \frac{4}{5}$ **1**

Q. 14. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE. U **[Board Term-1, 2013 FFC]**



Sol. In ΔABC, $\angle B = 90°$, AD and CE are two medians.

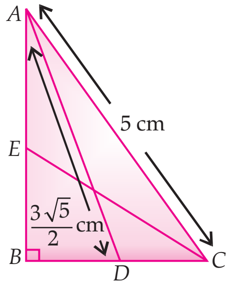
$\therefore AC^2 = AB^2 + BC^2 = (5)^2 = 25$

(By Pythagoras theorem) ....(i)

In ΔABD, $\quad AD^2 = AB^2 + BD^2$

or, $\quad \left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$

or, $\quad \frac{45}{4} = AB^2 + \frac{BC^2}{4}$ ....(ii) **1**



In ΔEBC, $\quad CE^2 = BC^2 + \frac{AB^2}{4}$ ....(iii) **1**

Subtracting equation (ii) from equation (i),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

or, $$BC^2 = \frac{55}{3}$$ ....(iv) ½

From eqn. (ii),

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

or, $$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$ ½

From eqn. (iii), $$CE^2 = \frac{55}{3} + \frac{20}{3\times 4}$$

$$= \frac{240}{12} = 20$$

$\therefore$ $$CE = \sqrt{20} = 2\sqrt{5} \text{ cm.}$$ 1

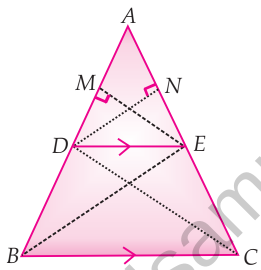
Q. 15. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it. A **[Board Term-1, 2013 FFC, 2012 Set 15]**

Sol. Given : ABC is a triangle in which $DE \parallel BC$.

To prove : $$\frac{AD}{BD} = \frac{AE}{CE}$$

Construction : Draw $DN \perp AE$ and $EM \perp AD$, Join BE and CD.

Proof : In ΔADE,



$$\text{area }(\Delta ADE) = \frac{1}{2} \times AE \times DN$$ ...(i)

In ΔDEC,

$$\text{area }(\Delta DCE) = \frac{1}{2} \times CE \times DN$$ ...(ii)

Dividing eqn. (i) by eqn. (ii),

$$\frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{\frac{1}{2}\times AE\times DN}{\frac{1}{2}\times CE\times DN}$$

or, $$\frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{AE}{CE}$$ ...(iii) 1

Now, in ΔADE,

$$\text{area }(\Delta ADE) = \frac{1}{2} \times AD \times EM$$ ...(iv)

and in ΔDEB,

$$\text{area }(\Delta DEB) = \frac{1}{2} \times EM \times BD$$ ...(v)

Dividing eqn. (iv) by eqn. (v),

$$\frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{\frac{1}{2}\times AD\times EM}{\frac{1}{2}\times BD\times EM}$$ 1

or, $$\frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AD}{BD}$$ ...(vi) 1

ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$\therefore$ area (ΔDEB) = area (ΔDEC)

From equation (iii),

$$\frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AE}{CE}$$ ...(vii)

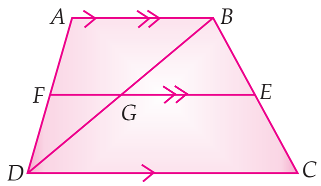
From equations (vi) and (vii),

$$\frac{AE}{CE} = \frac{AD}{BD}.$$ **Hence proved. 1**

AI **Q. 16. In a trapezium $ABCD$, $AB \parallel DC$ and $DC = 2AB$. $EF \parallel AB$, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.**

[Board Term-1, 2012 Set 65]

Sol.



In trapezium $ABCD$, $AB \parallel DC$ and $DC = 2AB$.

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

In trapezium $ABCD$, $EF \parallel AB \parallel CD$

$\therefore$ $$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC,

$\angle B = \angle B$ (Common)

$\angle BEG = \angle BCD$ (corresponding angles)

$\therefore$ $\Delta BGE \sim \Delta BDC$ [AA similarity] 1

or, $$\frac{EG}{CD} = \frac{BE}{BC}$$ ...(i)

As, $$\frac{BE}{EC} = \frac{4}{3}$$

or, $$\frac{EC}{BE} = \frac{3}{4}$$

or, $$\frac{EC}{BE} + 1 = \frac{3}{4} + 1$$ 1

or, $$\frac{EC+BE}{BE} = \frac{7}{4}$$

or, $$\frac{BC}{BE} = \frac{7}{4} \text{ or } \frac{BE}{BC} = \frac{4}{7}$$

From (i), $$\frac{EG}{CD} = \frac{4}{7}$$

or, $EG = \frac{4}{7}CD$ ...(i)

Similarly, $\Delta DGF \sim \Delta DBA$

or, $\frac{DF}{DA} = \frac{FG}{AB}$ **1**

or, $\frac{FG}{AB} = \frac{3}{7}$

or, $FG = \frac{3}{7}AB$ ...(ii)

$$\left[\because \frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding eqns. (i) and (ii),

$$EG + FG = \frac{4}{7}CD + \frac{3}{7}AB$$

or, $EF = \frac{4}{7}\times(2AB) + \frac{3}{7}AB$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$$

$\therefore$ $7EF = 11AB$. **Hence proved. 1**

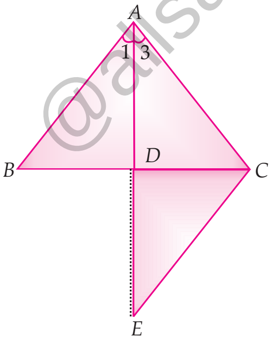
Q. 17. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$. A **[Board Term-1, 2012 Set 62]**

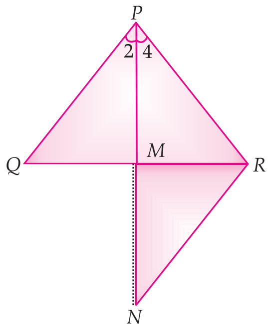
Sol. Given : In ΔABC and ΔPQR, AD and PM are their medians,

such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. Join CE and RN.





Proof : In ΔABD and ΔECD,

$AD = DE$ (By construction)

$\angle ADB = \angle EDC$ (VOA)

$BD = DC$ (AD is a median)

$\therefore$ $\Delta ABD \cong \Delta ECD$ ½

(By SAS congruency)

or, $AB = CE$ (By CPCT) ½

Similarly, $PQ = RN$ and $\angle 1 = \angle 2$

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ (Given) **1**

or, $\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$

or, $\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$ **1**

$\therefore$ By SSS similarity,

$\Delta AEC \sim \Delta PNR$

$\therefore$ $\angle 3 = \angle 4$ (By)

$\because$ $\angle 1 = \angle 2$

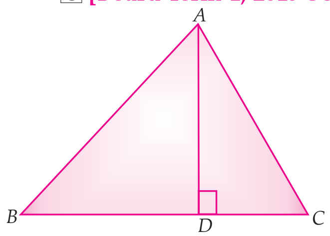
$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$

By SAS similarity,

$\Delta ABC \sim \Delta PQR$. **1**

AI **Q. 18. In ΔABC, $AD \perp BC$ and point D lies on BC such that $2\,DB = 3\,CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.**

Sol. U **[Board Term-1, 2015 Set DDE-E]**



Given : $AD \perp BC$

$2DB = 3CD$

To prove : $5\,AB^2 = 5AC^2 + BC^2$

Proof : Since, $2DB = 3CD$

or, $\frac{DB}{CD} = \frac{3}{2}$ **1**

or, Let DB be $3x$, CD be $2x$, so $BC = 5x$

In ΔADB, $\angle D = 90°$

$AB^2 = AD^2 + DB^2$

or, $AB^2 = AD^2 + (3x)^2$

or, $AB^2 = AD^2 + 9x^2$

Now $5AB^2 = 5AD^2 + 45\,x^2$

or, $5AD^2 = 5\,AB^2 - 45x^2$ ...(i) **1**

and $AC^2 = AD^2 + CD^2$

or, $AC^2 = AD^2 + (2x)^2$

or, $AC^2 = AD^2 + 4x^2$

or, $5AC^2 = 5AD^2 + 20x^2$

or, $5AD^2 = 5AC^2 - 20x^2$ ...(ii) **1**

From equation (i) & (ii)

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

or, $5AB^2 = 5AC^2 - 20x^2 + 45x^2$

or, $5AB^2 = 5AC^2 + 25x^2$

or, $5AB^2 = 5AC^2 + (5x)^2$

$\therefore$ $5AB^2 = 5AC^2 + BC^2$ [$\because$ $BC = 5x$]

Hence proved. 1

AI **Q. 19. In an equilateral triangle ΔABC, D is a point on side BC such the $BD = \frac{1}{3}BC$.**

Prove that $9(AD)^2 = 7AB^2$.

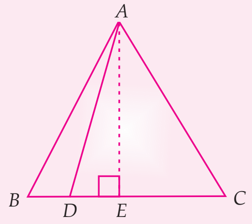
A [CBSE Delhi/OD Set-2018]
[Sample Question Paper 2017]

Sol. Draw $AE \perp BC$

$\Delta AEB \cong \Delta AEC$ (RHS congruence rule)

$\therefore \quad BE = EC = \frac{1}{2}BC = \frac{1}{2}AB$

Let, $\quad AB = BC = AC = x$



Now $\quad BE = \frac{x}{2}$ and $DE = BE - BD$

$$= \frac{x}{2} - \frac{x}{3}$$

$$= \frac{x}{6} \qquad 1$$

Now $\quad AB^2 = AE^2 + BE^2 \qquad ...(1)$

And $\quad AD^2 = AE^2 + DE^2 \qquad ...(2)$

From (1) and (2), $AB^2 - AD^2 = BE^2 - DE^2 \qquad$ **1**

$$\Rightarrow \quad x^2 - AD^2 = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2$$

$$\Rightarrow \quad AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$$

$$\Rightarrow \quad AD^2 = \frac{28}{36}x^2$$

$$\Rightarrow \quad 9AD^2 = 7AB^2.$$

Hence proved 2

[CBSE Marking Scheme, 2018]

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