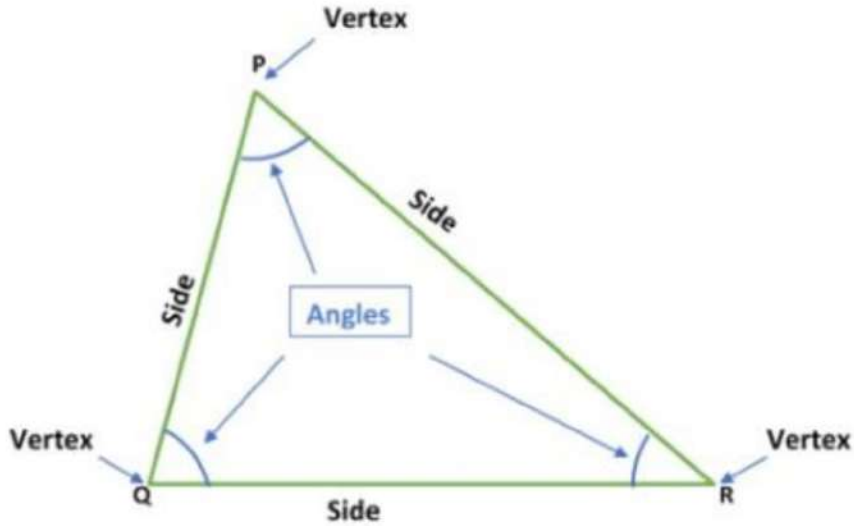
Chapter – 7

Triangles

Congruence of Triangles

Congruence of Triangles

A closed; two-dimensional figure formed by three intersecting lines is called a triangle.

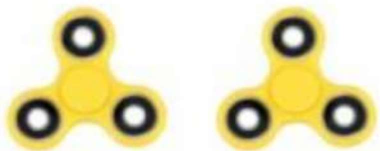


A triangle has three sides, three angles, and three vertices. For example, in the triangle PQR, PQ, QR, and RP are the three sides, ∠QPR, ∠PQR, ∠PRQ are the three angles and P, Q and R, are three vertices.

A triangle is a unique figure; it is everywhere around us. We can see the sandwiches in the shape of a triangle, traffic signals, cloth anger, set square, etc. All these are in the shape of a triangle.




Congruency of Shapes/Figures
Observe the following figures:



(I)

(II)

(III)

(IV)

In the figures shown above, each pair is identical to each other. Such figures are called congruent (they are similar and fit over one another exactly).

Congruence of Line segments

Any two-line segments are congruent if and only if their lengths are equal. For example, the two-line segments AB and CD are congruent if AB = CD. Here, AB = 3 cm and CD = 3 cm.

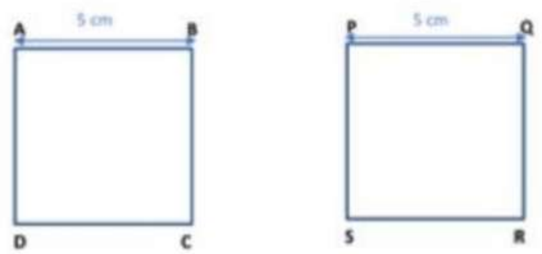
Congruence of Angles

Two angles are congruent if and only if their measures are equal. That is, two angles ABC and PQR are congruent if and only if m∠ABC = m∠PQR

Here, ∠ABC = 40° and ∠PQR = 40°. So, both angles are congruent.

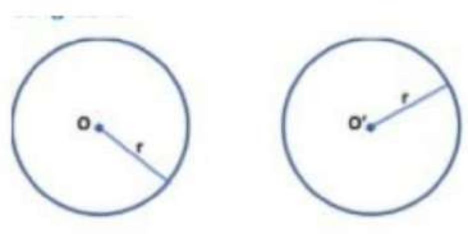
Congruence of Squares

Two squares are congruent if and only if their sides are equal. That is, two squares ABCD and PQRS are congruent if and only if the sides of ABCD is equal to the sides of PQRS.



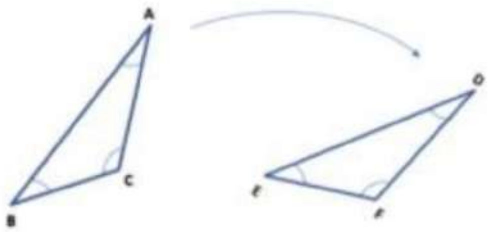
Here, the sides of both squares are equal to 5 cm. So, these squares are congruent.

Congruence of Circles
Two circles are congruent if and only if their radius is equal. In the given two circles, both have radius = r. Hence, the given circles are congruent.

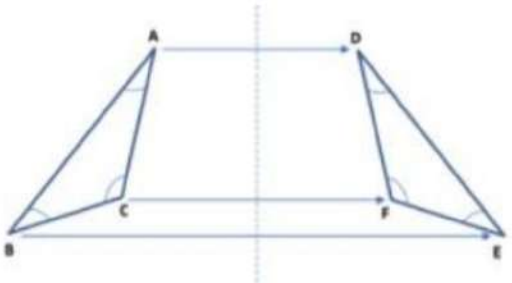


Congruence of Triangles
In transformation geometry, we can manipulate shapes in the following 3

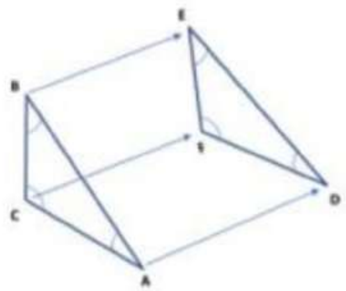
ways.
i. Rotation (Turn)



ii. Reflection (Flip)



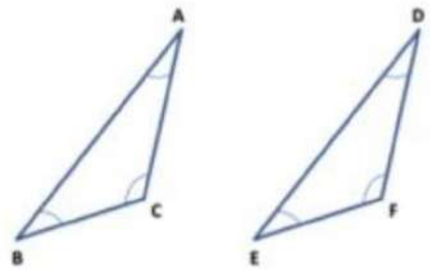
iii. Translation (Slide)



After any of these transformations (turn, flip or slide), the shape still has the same size, area, angles, and line lengths.

If a shape becomes another using the above transformations (turn, slide, and flip) then the two shapes are said to be congruent.

When we superpose Δ ABC on Δ DEF, such that it covers the other triangle completely & exactly.



In such a superposition the vertices of Δ ABC will fall on the vertices of Δ DEF, in some order.

The vertex A coincides with vertex D, vertex B coincides with vertex E and vertex C coincides with vertex F.

The side AB coincides with side DE, side AC coincides with side DF and side BC coincides with side EF.

$m\angle BAC = m\angle EDF$, $m\angle ACB = m\angle DFE$ and $m\angle CBA = m\angle FED$.

There is a one-on-one correspondence between the vertices of the two congruent triangles. That is, A corresponds to D, C corresponds to F, B corresponds to E and so on

which is written as $A \leftrightarrow D$, $C \leftrightarrow F$, $B \leftrightarrow E$.

Under this correspondence $\Delta ABC \cong \Delta DEF$; but it will not be correct to write $\Delta BCA \cong \Delta DEF$.

In congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Then, we have the following six equalities:

$AB = DE$, $BC = EF$, $AC = DF$ (Corresponding sides are congruent) $\angle BAC = \angle EDF$, $\angle ACB = \angle DFE$, $\angle CBA = \angle FED$ (Corresponding angles are congruent).

From the above discussion we obtain the following general condition for the congruence of two triangles:

- Triangles are congruent when all corresponding sides and angles are equal.

- The triangles will have equal shape and size.

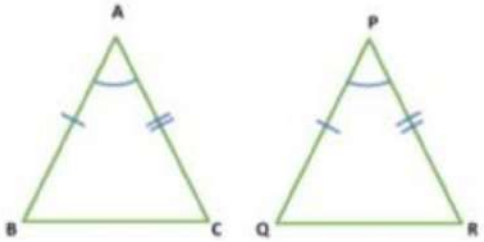
- Two triangles can be superimposed side to side and angle to angle.
- They have equal area and equal perimeter.
- Congruence is denoted by the symbol ≅.

Criteria for Congruence of Triangles

Criteria for Congruence of Triangles

Axiom 1: Side-Angle-Side (SAS) congruence rule
(An axiom is a mathematical statement which is assumed to be true without proof.)

Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.



Given: Two triangles ABC and PQR such that AB = PQ, AC = PR and ∠BAC = ∠QPR.

To prove: Δ ABC ≅ Δ PQR.

Proof: This result cannot be proved with help of previously known results. So, this rule is accepted as an axiom.

Another way to check whether the given two triangles are congruent or not, we follow a practical approach.

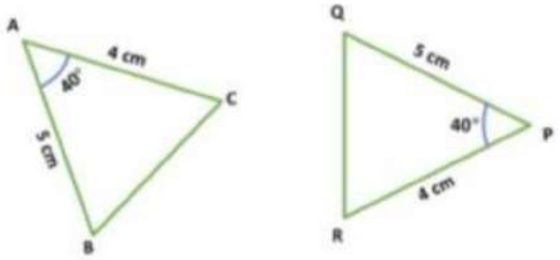
Place ΔABC over ΔPQR such that the side AB falls on side PQ, vertex A falls on vertex P and B on Q. Since ∠BAC = ∠QPR. Therefore, AC will fall on PR. But AC = PR and A

falls on P. therefore, C will fall on R. Thus, AC coincides with PR.

Now, B falls on Q and C falls on Therefore, BC coincides with QR.

Thus, Δ ABC when superposed on Δ PQR, covers it exactly. Hence, by the definition of congruence, Δ ABC ≅ Δ PQR.

Example 1: Check whether ΔABC and ΔPQR are congruent or not.



Solution: In Δ ABC and Δ PQR, we have

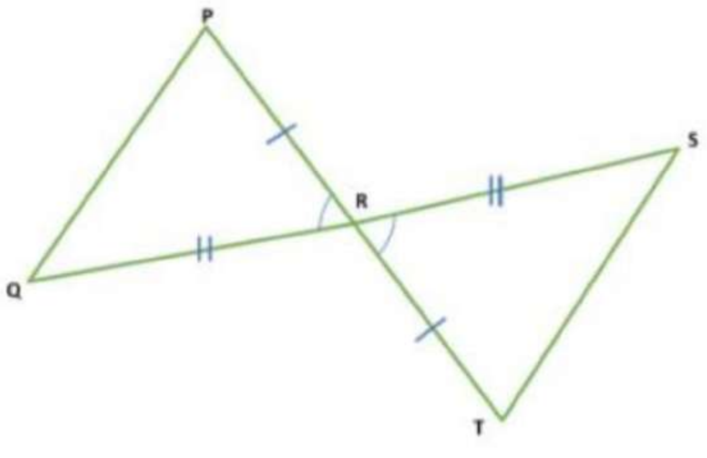
AB = PQ = 5 cm (Given)

∠ BAC = ∠QPR = 40° (Given)

AC = PR = 4 cm (Given)

Therefore, Δ ABC ≅ Δ PQR (By SAS-criterion of congruence)

Example 2: In the figure below, R is the mid-point of PT and SQ. Prove that Δ PQR ≅ Δ TSR.



Given: PR = RT and SR = RQ.

To prove: Δ PQR ≅ Δ TSR.

Proof: In Δ PQR and Δ TSR, we have

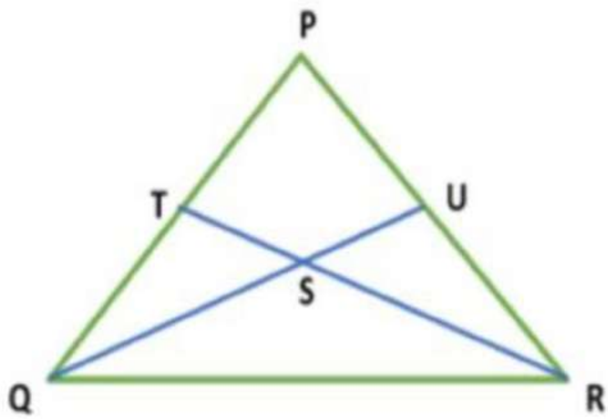
PR = TR (R is the mid-point of PT)

$\angle$PRQ = $\angle$TRS (Vertically opposite angles are equal)

QR = SR (R is the mid-point of SQ)

Therefore, Δ PQR $\cong$ Δ TSR (By SAS-criterion of congruence)

Example 3: In the figure, it is given that PT = PU and QT = RU. Prove that ΔPTR $\cong$ ΔPUQ



Given: PT = PU and QT = RU.

To prove: Δ PTR $\cong$ Δ PUQ.

Proof: We have,

$$PT = PU \quad \text{.......... (I)}$$

And, $$QT = RU \quad \text{.......... (II)}$$

Adding equation (I) and (II), We get,

$$PT + QT = PU + RU$$

$$\Rightarrow \quad PQ = PR \quad \text{.......... (III)}$$

Now, in Δ PTR and Δ PUQ, we have

$$PT = PU \text{ [Given]}$$

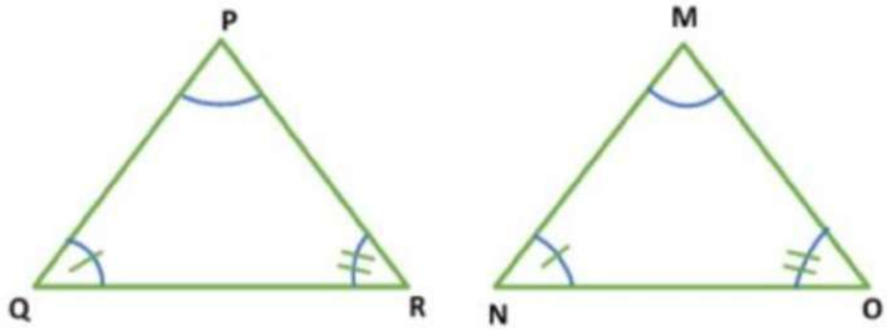
$\Rightarrow$ $\angle TPR = \angle UPQ$ [Common]

$\Rightarrow$ $PQ = PR$ [From (III)]

Therefore, $\Delta PTR \cong \Delta PUQ$ [By SAS-criterion of congruence]

Theorem 1: Angle-Side-Angle (ASA) Congruence rule Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the

included side of the other triangle.



Given: ΔPQR and ΔMNO such that $\angle PQR = \angle MNO$, $\angle PRQ = \angle MON$ and $QR = NO$.

To prove: $\Delta PQR \cong \Delta MNO$.

Proof: There are three possibilities that arise.

CASE I: When $PQ = MN$

In this case, we have

$PQ = MN$

$\angle PQR = \angle MNO$ (Given)

$QR = NO$ (Given)

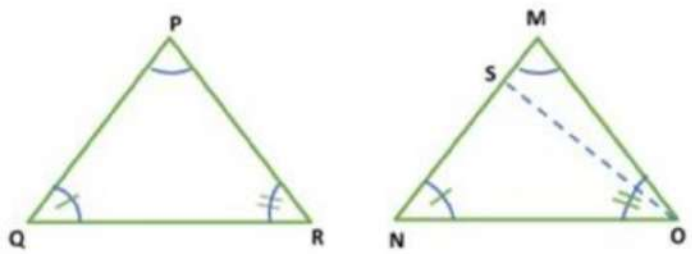
Therefore, $\Delta PQR \cong \Delta MNO$ (By SAS-criterion of congruence)

CASE II: $PQ < MN$

Construction: Join OS such that $NS = PQ$.

In Δ PQR and Δ SNO, we have

$$PQ = SN$$



$$\angle PQR = \angle MNO \quad \text{(Given)}$$

$$QR = NO \quad \text{(Given)}$$

Therefore, Δ PQR $\cong$ Δ SNO (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle PRQ = \angle SON$.

But, $\angle PRQ = \angle MON$. (Given)

This is possible only when ray SO coincides with ray MO or S coincides with M. Therefore, PQ must be equal to MN.

$\therefore$ $\angle SON = \angle MON$.

Thus, in Δ PQR and Δ MNO, we have

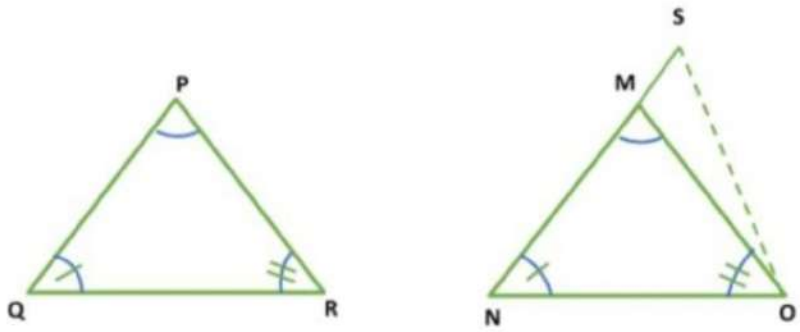
$$PQ = MN$$

$\Rightarrow$ $\angle PQR = \angle MNO$ (Given)

$\Rightarrow$ $QR = NO$ (Given)

Therefore, Δ PQR $\cong$ Δ MNO (By SAS-criterion of congruence)

CASE III: $PQ > MN$.



Construction: Join SO such that NS = PQ.

In Δ PQR and Δ SNO, we have

$$PQ = SN$$

$$\angle PQR = \angle MNO \text{ (Given)}$$

$$QR = NO \text{ (Given)}$$

Therefore, Δ PQR ≅ Δ SNO (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle PRQ = \angle SON.$

But, $\angle PRQ = \angle MON.$ (Given)

This is possible only when ray SO coincides with ray MO or S coincides with M. Therefore, PQ must be equal to MN.

$\therefore$ $\angle SON = \angle MON.$

Thus, in Δ PQR and Δ MNO, we have

$PQ = MN$

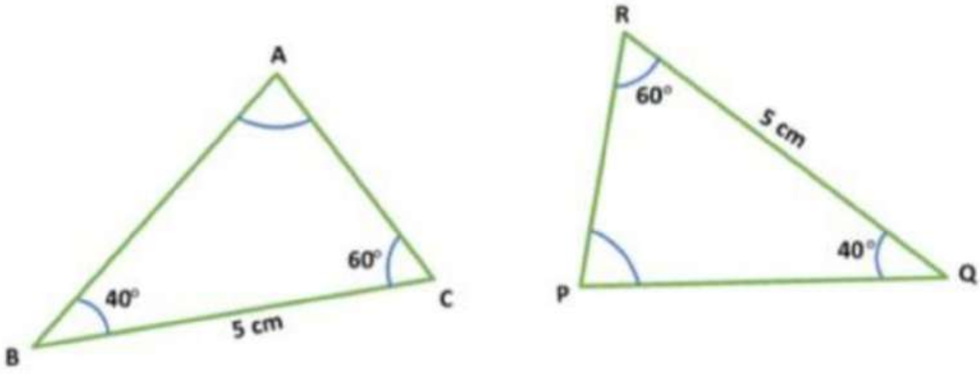
$\Rightarrow \angle PQR = \angle MNO$ (Given)

$\Rightarrow QR = NO$ (Given)

Therefore, Δ PQR ≅ Δ MNO (By SAS-criterion of congruence

Hence, in all the three cases, we have Δ PQR ≅ Δ MNO.

Example 1: Check whether $\Delta ABC \cong \Delta PQR$?



Solution: In ΔABC and ΔPQR, we have

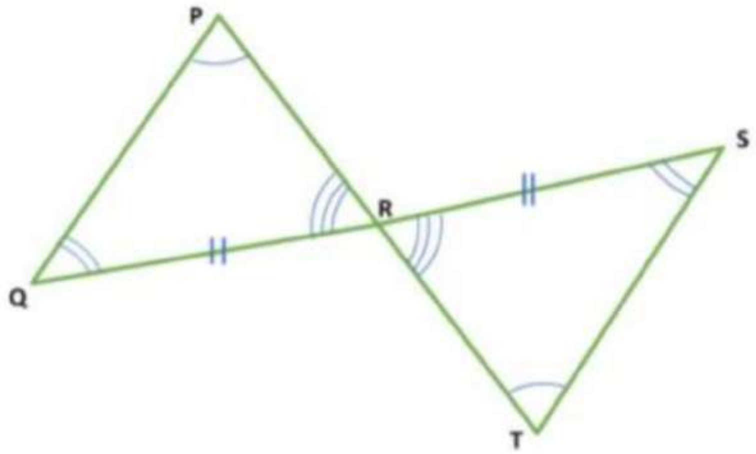
$$\angle ABC = \angle PQR = 40^{\circ} \qquad \text{(Given)}$$

$$BC = QR = 5 \text{ cm} \qquad \text{(Given)}$$

$$\angle BCA = \angle QRP = 60^{\circ} \qquad \text{(Given)}$$

Therefore, Δ ABC $\cong$ Δ PQR (By ASA-criterion of congruence)

Example 2: In the figure, ST ∥ QP and R is the mid-point of SQ, prove that R is also the mid-point of PT.



Given: ST ∥ QP and R is the mid-point of SQ.

To prove: PR = RT.

Proof: Since QP ∥ ST and transversal PT cuts them at P and T respectively.

∴ $\angle RTS = \angle RPQ$. (Alternate interior angles) .... (I)

Similarly, $\angle RST = \angle RQP$. (Alternate interior angles) .... (II)

Since PT and SQ intersect at R.

∴ $\angle QRP = \angle SRT$. (Vertically opposite angles) ....(III)

Thus, in Δ PQR and Δ SRT, we have

$\angle$ PQR = $\angle$ RST [From (II)]

⇒ QR = RS [Given]

⇒ $\angle$ QRP = $\angle$ SRT [From (III)]

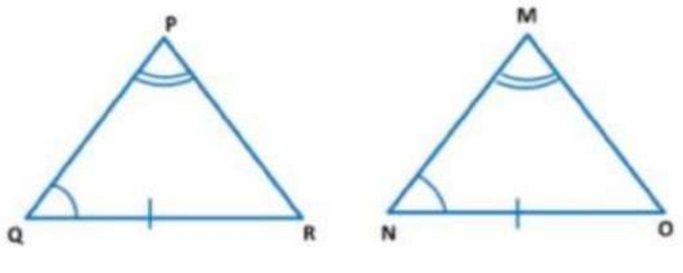
Therefore, Δ PQR ≅ Δ SRT [By ASA-criterion of congruence]

By using corresponding parts of congruent triangles

⇒ PR = RT.

Hence, R is the mid-point of PT.

Theorem 2: Angle-Angle-Side (AAS) Congruence rule If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.



Given: In ΔPQR and ΔMNO, $\angle PQR = \angle MNO$, $\angle QPR = \angle NMO$ & QR=NO.

To prove: $\Delta PQR \cong \Delta MNO$

Proof: We have,

$$\angle PQR = \angle MNO \text{ and } \angle QPR = \angle NMO$$

$\Rightarrow$ $\angle PQR + \angle QPR = \angle NMO + \angle MNO$ .... (I)

$\because$ $\angle PRQ + \angle PQR + \angle QPR = 180^\circ$ [Sum of angles of a triangle]

$\therefore$ $\angle PQR + \angle QPR = 180^\circ - \angle PRQ$ .... (II)

Similarly, $\angle NMO + \angle MNO = 180^\circ - \angle MON$ .... (III)

From equation (I), (II) & (III), we have

$$180^\circ - \angle PRQ = 180^\circ - \angle MON$$

$\Rightarrow$ $\angle PRQ = \angle MON$ .... (IV)

Now, in ΔPQR and ΔMNO, we have

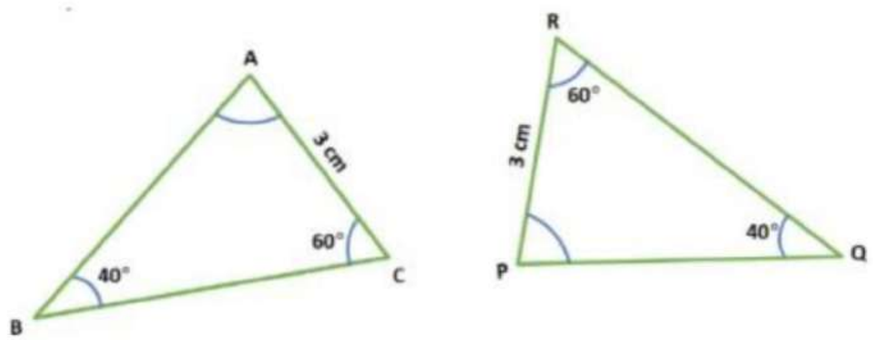
$\angle PQR = \angle MNO$ [Given]

$QR = NO$ [Given]

$\angle PRQ = \angle MON$ [From (IV)]

Therefore, $\Delta PQR \cong \Delta MNO$ [By ASA-criterion of congruence]

Example1: Check whether two triangles ABC and PQR are congruent.



Solution: In Δ ABC and Δ PQR, we have

$\angle ABC = \angle PQR = 40^{\circ}$ (Given)

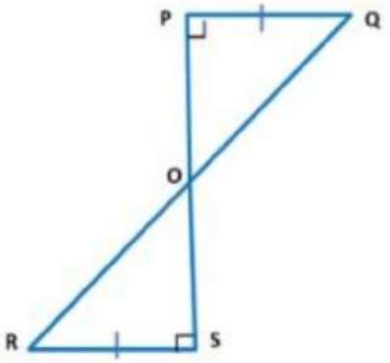
$\angle BCA = \angle QRP = 60^{\circ}$ (Given)

$AC = PR = 3$ cm (Given)

Therefore, $\Delta ABC \cong \Delta PQR$ (By AAS-criterion of congruence)

Example 2: PQ and RS are perpendiculars of equal length, to a line segment PS.

Show that QR bisects PS.



Given: $PQ = RS$, $PQ \perp PS$ and $RS \perp PS$.

To prove: $OP = OS$.

Proof: In triangles OPQ and ORS, we have

$\angle POQ = \angle SOR$ [Vertically opposite angles]

$\angle OPQ = \angle OSR$ [Each equal to 90°]

$PQ = RS$ [Given]

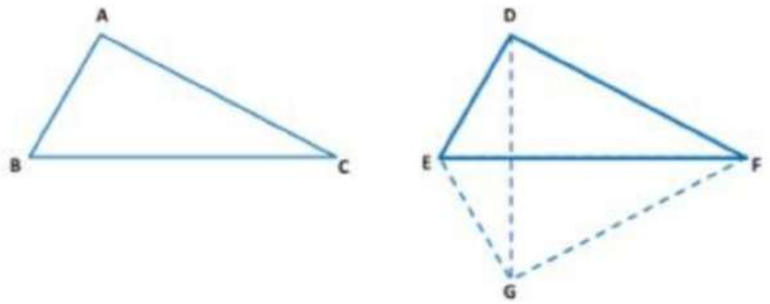
Therefore, $\Delta POQ \cong \Delta SOR$ [By AAS-criterion of congruence]

By using corresponding parts of congruent triangles

$\Rightarrow \quad OP = OS.$

Hence, O is the mid-point of PS.

Theorem 3: Side-Side-Side (SSS) Congruence rule If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.



Given that: Two Δ ABC and Δ DEF such that AB = DE, BC = EF and AC = DF.

To prove: $\Delta ABC \cong \Delta DEF$.

Construction: Suppose BC is the longest side. Draw EG such that

$\angle FEG = \angle ABC$ and EG = AB. Join GF and GD

Proof: In ΔABC and ΔGEF, we have

$BC = EF$ [Given]

$AB = GE$ [By Construction]

$\angle ABC = \angle GEF$ [By Construction]

Therefore, $\Delta ABC \cong \Delta GEF$ [By SAS-criterion of congruence]

By using corresponding parts of congruent triangles

$\Rightarrow \quad \angle BAC = \angle EGF$ and $AC = GF$

Now, AB = DE and AB = GE

$\Rightarrow \quad DE = GE$ .... (I)

Similarly, $AC = DF$ and $AC = GF$

$\Rightarrow$ $DF = GF$ .... (II)

In Δ EGD, we have

$DE = GE$ [From (I)]

Since angles opposite to equal sides of an isosceles triangle are equal.

$\Rightarrow$ $\angle EDG = \angle EGD$ .... (III)

In Δ FGD, we have

$DF = GF$ [From (II)]

Since angles opposite to equal sides of an isosceles triangle are equal.

$\Rightarrow$ $\angle FDG = \angle FGD$ .... (IV)

From (III) and (IV), we have,

$\angle EDG + \angle FDG = \angle EGD + \angle FGD$

$\Rightarrow$ $\angle EDF = \angle EGF$

But, $\angle BAC = \angle EDF$ .... (V)

In Δ ABC and Δ DEF, we have

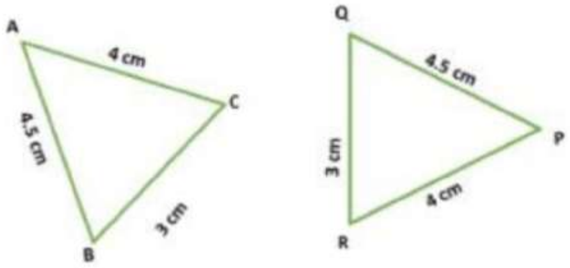
$AC = DF$ [Given]

$\Rightarrow$ $\angle BAC = \angle EDF$ [From (V)]

And, $AB = DE$ [Given]

Therefore, $\Delta ABC \cong \Delta DEF$ [By SAS-criterion of congruence]

Example 1: Check whether two triangles ABC and PQR are congruent.

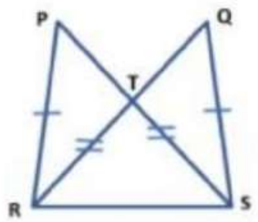


Solution: In Δ ABC and Δ PQR, we have

	BC = QR	(Given)
⇒	AB = PQ	(Given)
⇒	AC = PR	(Given)

Therefore, Δ ABC ≅ Δ PQR (By SSS-criterion of congruence)

Example 2: In the figure, it is given that PR = QS and PS = RQ. Prove that Δ SPR ≅ Δ RQS.

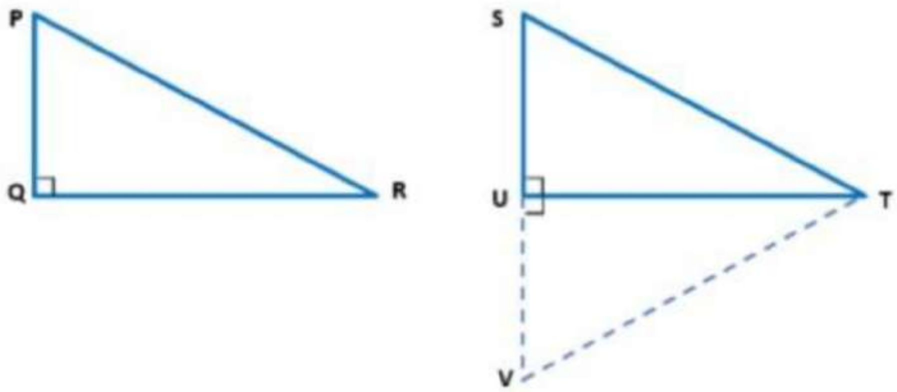


Proof : In triangles SPR and RQS, we have

	PR = QS	[Given]
⇒	PS = QR	[Given]
⇒	RS = SR	[Given]

Therefore, Δ SPR ≅ ΔRQS [By SSS-criterion of congruence]

Theorem 4: Right angle -Hypotenuse-Side (RHS) Congruence rule If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.



Given that: Two right triangles PQR and SUT in which $\angle PQR = \angle SU = 90°$, PR $= ST$, $QR = UT$.

To prove: $\Delta PQR \cong \Delta SUT$.

Construction: Produce SU to V so that $UV = PQ$. Join VT.

Proof: In Δ PQR and Δ SUT, we have

$PQ = VU$ [By construction]

$\angle PQR = \angle VUT$ [Each equal to 90°]

$QR = UT$ [Given]

Therefore, $\Delta PQR \cong \Delta SUT$ [By SAS-criterion of congruence]

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle QPR = \angle UVT$ .... (I)

And, $PR = VT$

$\Rightarrow$ $PR = ST$ .... (II)

$\therefore$ $VT = ST$

Since, angles opposite to equal sides in Δ STV are equal.

$\Rightarrow$ $\angle UST = \angle TVS$ .... (III)

From (I) and (III), we get,

$\angle QPR = \angle UST$ .... (IV)

And given that, $\angle RQP = \angle TUS$ .... (V)

Adding (IV) & (V), we get,

$\angle QPR + \angle RQP = \angle UST + \angle TUS$ .... (VI)

$\because \angle PRQ + \angle RQP + \angle QPR = 180°$

$\therefore$ $\angle QPR + \angle RQP = 180 - \angle PRQ$ .... (VII)

Similarly, $\angle UST + \angle TUS = 180° - \angle STU$ .... (VIII)

From equation (VI), (VII) & (VIII), we have

$180° - \angle PRQ = 180° - \angle STU$

$\Rightarrow$ $\angle PRQ = \angle STU.$ .... (IX)

Now, in Δ PQR and Δ SUT

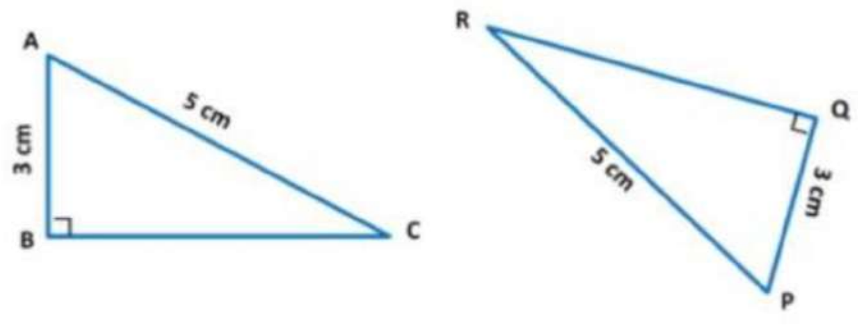
QR = UT [Given]

$\Rightarrow$ $\angle PRQ = \angle STU.$ [From (IX)]

And, PR = ST [Given]

Therefore, $\Delta PQR \cong \Delta SUT$ [By SAS-criterion of congruence]

Example 1: Check whether two triangles ABC and PQR are congruent.



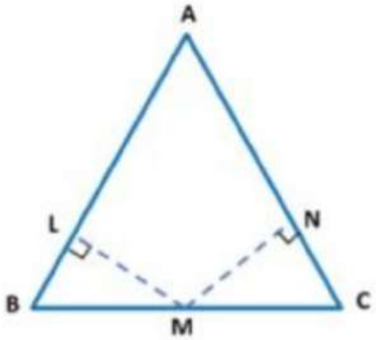
Solution: In Δ ABC and Δ PQR, we have

$\angle$ ABC = $\angle$ PQR = 90° (Given)

AC = PR (Given)
AB = PQ (Given)

Therefore, Δ ABC $\cong$ Δ PQR (By RHS-criterion of congruence)

Example 2: In the figure below, it is given that LM = MN, BM = MC, ML $\perp$ AB and MN $\perp$ AC. Prove that AB = AC.



Proof: In right-angled Δ BLM and Δ CNM, we have

BM = MC [Given]

$\Rightarrow$ LM = MN [Given]

Therefore, Δ BLM $\cong$ Δ CNM [By RHS-criterion of congruence]

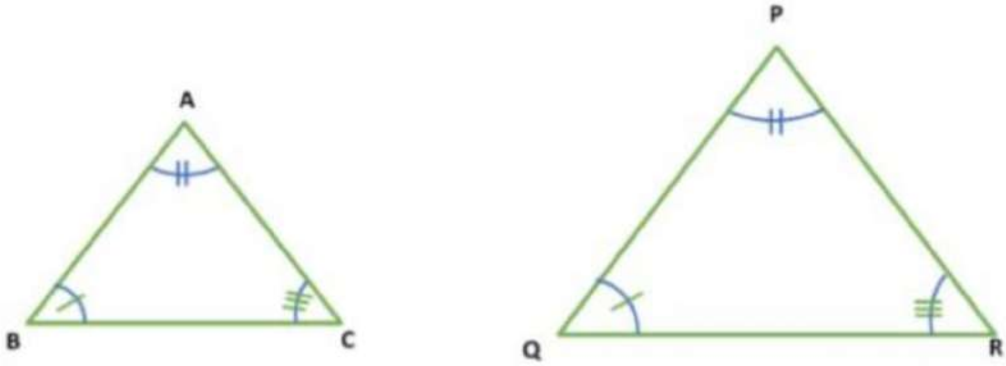
By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle$LBM = $\angle$NCM
Since, sides opposite to equal angles are equal.

Therefore, AB = AC.

Some more rules

1. Angle-Angle-Angle (AAA) Rule

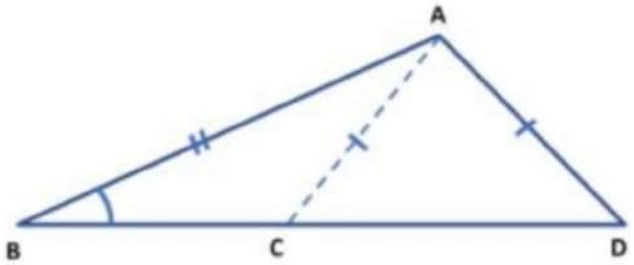


In Δ ABC and Δ PQR, we have

$\angle BAC = \angle QPR$ (Given)
$\angle ACB = \angle PRQ$ (Given)
$\angle CBA = \angle RQP$ (Given)

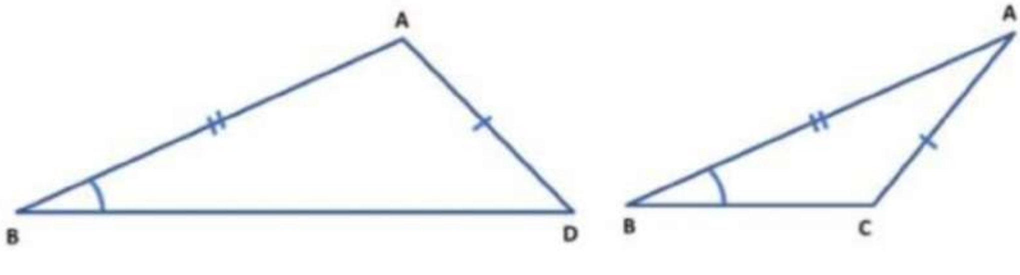
But, Δ ABC and Δ PQR are similar but not congruent because their sizes are different.

2. Angle-Side-Side (ASS or SSA) Rule



We have a triangle ABD. We draw AC such that AC = AD.

Now, consider the two triangles, Δ ABD and Δ ABC



Now, let us check whether we can use ASS criteria for the congruency of two triangles or not. In Δ ABD and Δ ABC, we have

$$\angle ABD = \angle ABC \qquad \text{(Given)}$$
$$AB = AB \qquad \text{(Common)}$$
$$AD = AC \qquad \text{(Given)}$$

So, the corresponding Angle-Side-Side of the two triangles are equal but, these two figures are different in shape.

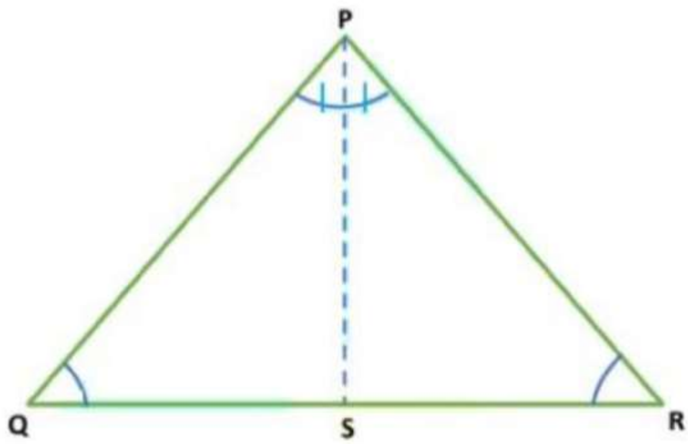
So, we can conclude that this method is not a universal method for proving triangles congruent.

Some Properties of a Triangle

Some Properties of a Triangle

Theorem 5: Angles opposite to equal sides of an isosceles triangle are equal.

Given: ΔPQR is an isosceles triangle in which PQ = PR.



To prove: ∠ PQS = ∠PRS.

Construction: Draw the bisector PS of ∠QPR which meets QR in S.

Proof: In Δ PQS and Δ PRS, we have

$$PQ = PR \qquad \text{(Given)}$$

$$\Rightarrow \qquad \angle QPS = \angle RPS \qquad \text{(By construction)}$$

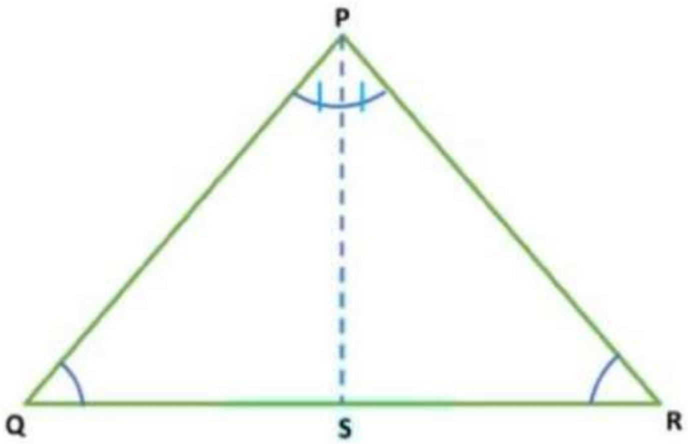
$\Rightarrow$ $PS = PS$ (Common)

Therefore, $\Delta PQS \cong \Delta PRS$ (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle PQS = \angle PRS$.

Theorem 6: The sides opposite to equal angles of a triangle are equal.



Given: In triangle PQR, $\angle PQR = \angle PRQ$.

To prove: $PQ = PR$.

Construction: Draw the bisector of $\angle QPR$ and let it meet QR at S.

Proof: In Δ PQS and Δ PRS, we have

$\angle PQR = \angle PRQ$ (Given)

$\Rightarrow$ $\angle QPS = \angle RPS$ (By construction)

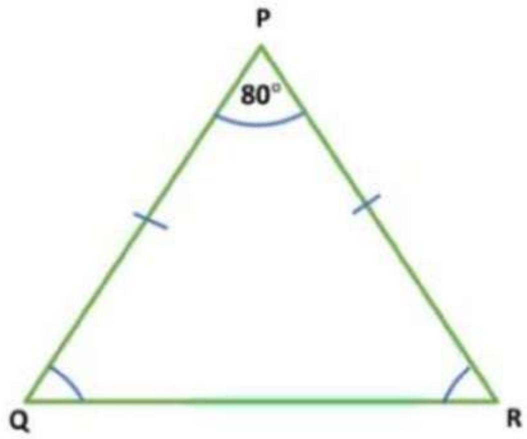
$\Rightarrow$ $PS = PS$ (Common)

Therefore, $\Delta PQS \cong \Delta PRS$ (By AAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $PQ = PR$.

Example 1: In Δ PQR, $\angle$ QPR $= 80°$ and $PQ = PR$. Find $\angle RQP$ and $\angle PRQ$.



Given that: $\angle QPR = 80°$ and $PQ = PR$.

Solution: We have

$$PQ = PR$$

Since angles opposite to equal sides are equal.

$\Rightarrow \quad \angle RQP = \angle PRQ$

In Δ PQR, We have

$$\angle QPR + \angle RQP + \angle PRQ = 180°.$$

$\Rightarrow \quad \angle QPR + \angle RQP + \angle RQP = 180°.\ (\because \angle\ RQP = \angle PRQ)$

$\Rightarrow \quad 80° + 2\ \angle RQP = 180°$

$\Rightarrow \quad 2\ \angle RQP = 180°\ \text{-}\ 80°$

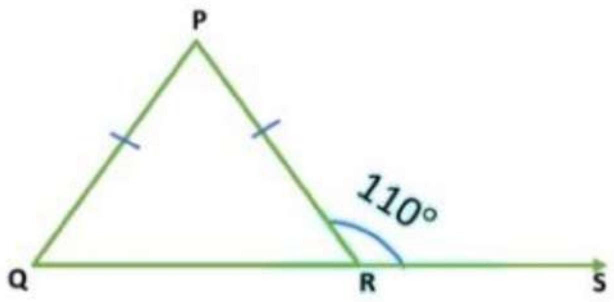
$\Rightarrow \quad 2\ \angle RQP = 100°$

$\Rightarrow \quad \angle RQP = \dfrac{100^0}{2}$

$\Rightarrow \quad \angle RQP = 50°$

Hence, $\angle RQP = \angle PRQ = 50°$

Example 2: In figure, $PQ = PR$ and $\angle PRS = 110°$. Find $\angle P$.



Solution: We have,

$$PQ = PR$$

Since angles opposite to equal sides are equal.

$\Rightarrow \quad \angle PQR = \angle PRQ.$

Now, $\quad \angle PRQ + \angle PRS = 180°$ ($\because$ Linear pair of angles)

$\Rightarrow \quad \angle PRQ + 110° = 180°$

$\Rightarrow \quad \angle PRQ = 180° - 110°$

$\Rightarrow \quad \angle PRQ = 70°$

And also, $\quad \angle PQR = 70°.$

Now, $\quad \angle QPR + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \quad \angle QPR + 70° + 70° = 180°$ ($\because \angle PQR = \angle PRQ$)

$\Rightarrow \quad \angle QPR + 140° = 180°$

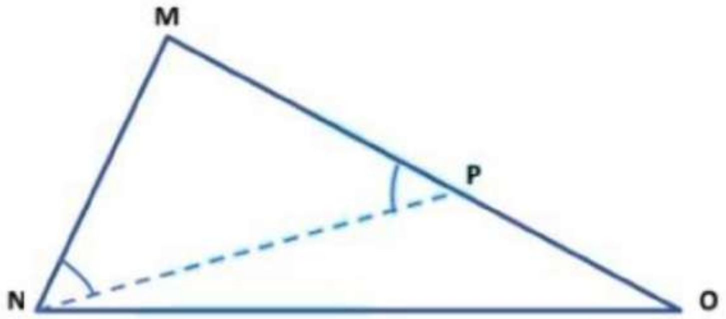
$\Rightarrow \quad \angle QPR = 180° - 140°$

$\Rightarrow \quad \angle QPR = 40°.$

Inequalities in a Triangle

Inequalities in a Triangle

Theorem 7: If two sides of a triangle are unequal, the angle opposite to the longer side is larger.



Given that: In Δ MNO, MO > MN.

To prove: $\angle$ MNO > $\angle$ MON.

Construction: Mark a point P on MO such that MN = MP. Join NP.

Proof: In Δ MNP, we have

MN = MP [By construction]

Since angles opposite to equal sides are equal.

$\Rightarrow$ $\angle$MNP = $\angle$MPN. .... (I)

Now, consider Δ NPO. We find that $\angle$MPN is the exterior angle of Δ NPO and an exterior angle is always greater than interior opposite angles.

Therefore, $\angle$MPN > $\angle$PON

$\Rightarrow$ $\angle$MPN > $\angle$MON .... (II)

From (I) and (II), we have

$\angle$MNP > $\angle$MON .... (III)

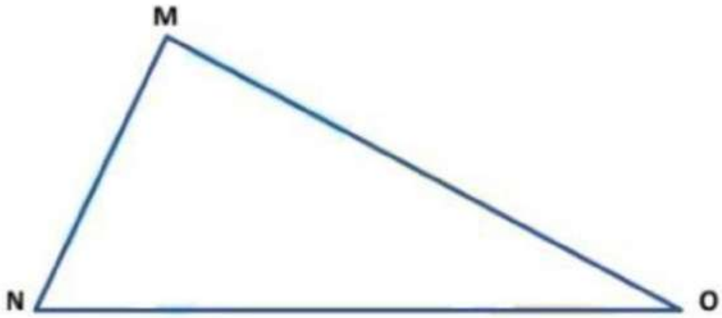
But, $\angle$MPN is a part of $\angle$ MNO.

$\therefore$ $\angle$ MNO > $\angle$MNP .... (IV)

From (III) and (IV), we get

$\angle$ MNO > $\angle$MON.

Theorem 8: In any triangle, the side opposite to the larger (greater) angle is longer.



Given that: In In Δ MNO, $\angle$ MNO $>$ $\angle$MON.

To prove: MO $>$ MN.

Proof: In Δ MNO, we have the following three possibilities.

(I) MO $=$ MN

(II) MO $<$ MN

(III) MO $>$ MN

Out of these three possibilities exactly one must be true.

CASE I: When MO $=$ MN

Since angles opposite to equal sides are equal.

$\Rightarrow \qquad \angle MNO = \angle MON$

This is a contradiction that $\angle$ MNO $>$ $\angle$MON.

$\therefore$ MO $\neq$ MN.

CASE II: When MO $<$ MN

Since the longer side has a greater angle opposite to it.

$\Rightarrow \angle MNO < \angle MON$

This also contradicts the given hypothesis that $\angle MNO > \angle MON$

Thus, we are left with the only possibilities, $MO > MN$, which must be true.

Hence, $MO > MN$.

Example: In a Δ MNO, if $\angle MON = 55°$ and $\angle ONM = 66°$, determine

the shortest and largest sides of the triangle.

Solution: We have,

$\angle MON = 55°$ and $\angle ONM = 66°$

$\therefore \angle MON + \angle ONM + \angle NMO = 180°$

$\Rightarrow \quad 55° + 66° + \angle NMO = 180°$

$\Rightarrow \quad 121° + \angle NMO = 180°$

$\Rightarrow \quad \angle NMO = 180° - 121°$

$\Rightarrow \quad \angle NMO = 59°.$

Since $\angle NMO$ and $\angle ONM$ are the smallest and largest angles respectively. Therefore, sides ON and OM are the smallest and largest sides respectively of the triangle.

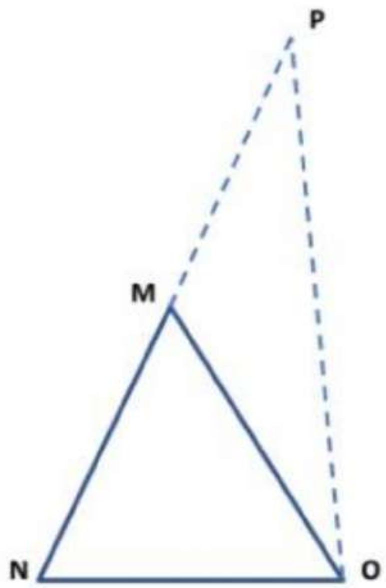
Theorem 9: The sum of any two sides of a triangle is greater than the third side.

Given: MNO is a triangle.

To prove: (I) $MN + MO > NO$

(II) $MN + NO > MO$

(III) $NO + MO > MN$



Proof: (I) Produce side NM to P such that $MP = MO$. Join PO

In Δ MOP, we have

$MP = MO$ (By construction)

Since angles opposite to equal sides are equal.

$\Rightarrow \quad \angle MOP = \angle MPO$

$\Rightarrow \quad \angle NOM + \angle MOP > \angle MPO$

$\Rightarrow \quad \angle NOP > \angle MPO$

Since side opposite to greater angle is larger.

$\therefore \quad NP > NO$

$\Rightarrow \quad MN + MP > NO$

$\Rightarrow \quad MN + MO > NO \quad [\because MO = MP \text{ by construction}]$

Thus, $MN + MO > NO$.

(II) Produce side MN to P such that $NP = NO$. Join PO.

In Δ NOP, we have

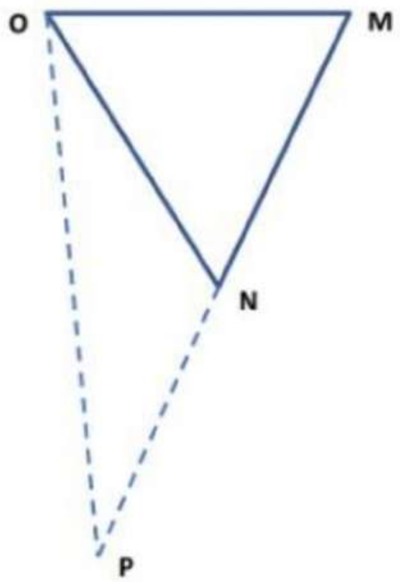
$$NP = NO \qquad \text{(By construction)}$$

Since angles opposite to equal sides are equal.

$\Rightarrow \quad \angle NOP = \angle NPO$

$\Rightarrow \quad \angle NOM + \angle NOP > \angle NPO$

$\Rightarrow \quad \angle MOP > \angle NPO$



Since side opposite to greater angle is larger.

$\therefore \quad MP > MO$

$\Rightarrow \quad MN + NP > MO$

$\Rightarrow \quad MN + NO > MO \qquad [\because NO = NP \text{ by construction}]$

Thus, $\quad MN + NO > MO.$

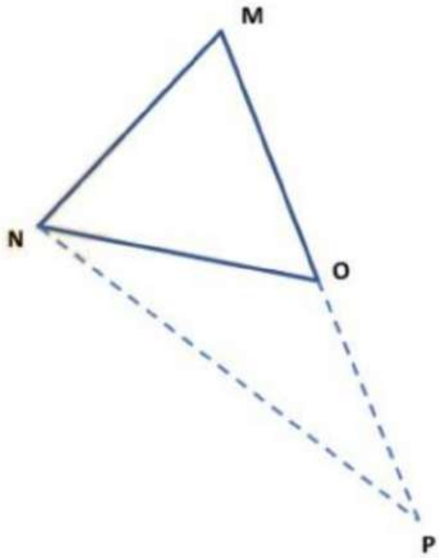
(III) Produce side MO to P such that OP = NO. Join NP

In Δ NOP, we have

$$OP = ON \qquad \text{(By construction)}$$

Since angles opposite to equal sides are equal.

$$\Rightarrow \quad \angle ONP = \angle OPN$$
$$\Rightarrow \quad \angle ONM + \angle ONP > \angle OPN$$
$$\Rightarrow \quad \angle MNP > \angle OPN$$

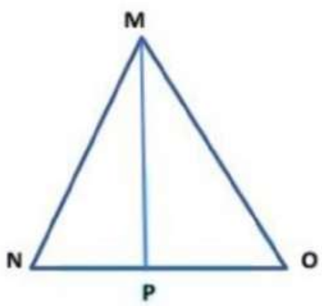


Since, side opposite to greater angle is larger.

$$\therefore \quad MP > MN$$
$$\Rightarrow \quad MO + OP > MN$$
$$\Rightarrow \quad MO + NO > MN \quad [\because NO = OP \text{ by construction}]$$
Thus, $MO + NO > MN$.

Hence, $MN + MO > NO$, $MN + NO > MO$ and $NO + MO > MN$.

Example: In Δ MNO, P is any point on the side NO. Show that $MN + MO + NO > 2\ MP$.



Solution: In Δ MNP, we have

$$MN + NP > MP \qquad \text{...............} \ (I)$$

($\because$ Sum of two sides of a triangle is greater than the third side)

Similarly, In Δ MOP, we have

$$MO + OP > MP \qquad \text{...............} \ (II)$$

Adding (I) and (II), we get

$$(MN + NP) + (MO + OP) > MP + MP$$

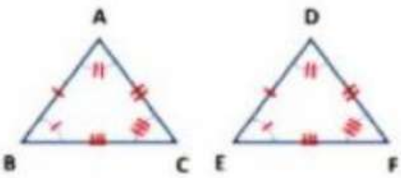
$\Rightarrow MN + (NP + OP) + MO > 2\ MP$

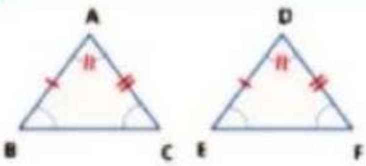
$\Rightarrow \qquad MN + NO + MO > 2\ MP.$

Summary - Triangles

Two figures are congruent, if they are of the same shape and of the same size.

Twotriangles ABC and DEF are congruent under the correspondence A $\leftrightarrow$ D, B $\leftrightarrow$ E and C $\leftrightarrow$ F, then symbolically, it is expressed as Δ ABC $\cong \Delta$ DEF.AB = DE, AC = DF, BC = EF&∠BAC = ∠EDF, ∠CBA = ∠FED, ∠ACB = ∠DFE





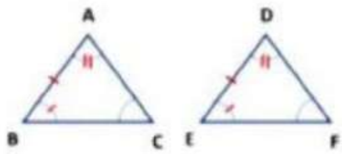
Side-Angle-Side (SAS) congruence rule

Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

AB =DE, ∠BAD = ∠EDF& AC = DF.

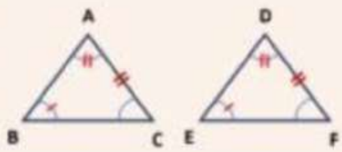
Angle-Side-Angle (ASA) Congruence rule

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle. ∠ABC = ∠DEF, AB = DE &∠BAC = ∠EDF.



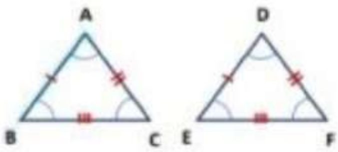
Angle-Angle-Side (AAS) Congruence rule

If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.



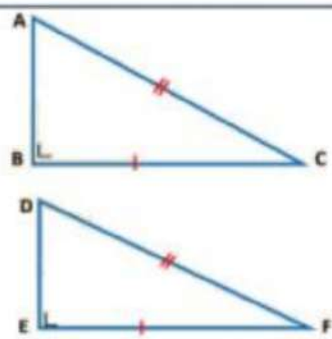
Side-Side-Side (SSS) Congruence rule

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent. AB = DE, AC = DF & BC = EF.



Right angle – Hypotenuse – Side (RHS) Congruence rule

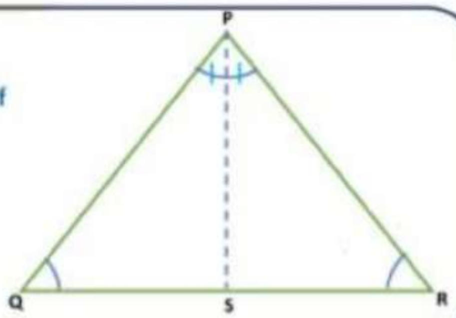
If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.∠ABC = ∠DEF, BC = EF & AC = DF



Properties of a Triangle

Angles opposite to equal sides of an isosceles triangle are equal.∠PQR = ∠PRQ

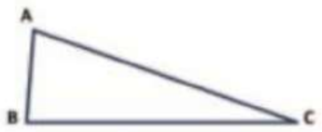
The sides opposite to equal angles of a triangle are equal.PQ = PR



Inequalities in a Triangle

If two sides of triangle ABC are unequal, the angle opposite to the longer side is larger. $\angle ABC > \angle ACB$.

Intriangle ABC, the side opposite to the larger (greater) angle is longer. AC > AB.



The sum of any two sides oftriangle ABC is greater than the third side. AB + AC > BC, AC + BC > AB & BC + AB > AC.