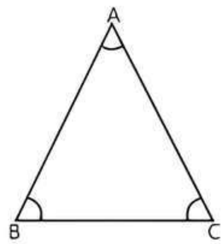
7 Triangles

Fastrack Revision

- **Triangle:** It is closed figure formed by three intersecting lines. A triangle has three sides, three angles and three vertices. In △ABC, AB, BC and CA are sides; ∠A, ∠B and ∠C are angles; A, B and C are vertices.



- **Congruence of Triangles:** Congruent means equal in all aspects or the figures with same shapes and sizes. The two triangles are said to be congruent, if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle. Congruence is denoted by the symbol '≅'.
- **Criteria for Congruence of Triangles:**
  1. **SSS (Side-Side-Side) Congruence:** If three sides of a triangle are equal to the three sides of another triangle, then the two triangles satisfy the SSS congruency.
  2. **SAS (Side-Angle-Side) Congruence:** If two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then the two triangles satisfy the SAS congruency.
  3. **ASA (Angle-Side-Angle) Congruence:** If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the triangles satisfy the ASA congruency.
  4. **RHS (Right-Hand-Side) Congruence:** If the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles satisfy the RHS congruency.
- **Some Properties of a Triangle:**
  1. The sum of three angles of a triangle is 180°.
  2. Angles opposite to equal sides of a triangle are equal.
  3. Sides opposite to equal angles of a triangle are equal.
  4. Each angle of an equilateral triangle is of 60°.
  5. The altitude drawn from vertex, it bisects perpendicularly the base of an equilateral and isosceles triangle.

Knowledge BOOSTER

1. *If △ABC and △PQR are congruent, then we have to careful for writing in symbol i.e., △ABC ≅ △PQR, hence we do not write as △ABC ≅ △QPR or △ACB ≅ △PQR etc.*
2. *CPCT is a short form of writing corresponding parts of congruent triangles.*
3. *Two geometric figures are said to be congruent, if they are equal in all respects.*
4. *Every triangle is congruent to itself.*
5. *If any two pairs of angles and one pair of corresponding sides are equal, then the two triangles are congruent. This may be called as AAS congruence rule.*
6. *The medians of an equilateral triangle are equal.*

Practice Exercise

Multiple Choice Questions

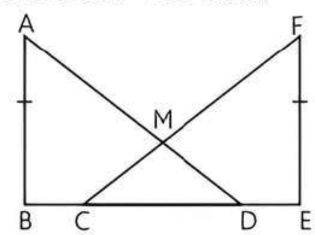
Q 1. Which of the following is not a criterion for congruence of triangles?

a. SAS b. ASA c. SSA d. SSS

Q 2. If AB = QR, BC = RP and CA = PQ, then:

a. △ABC ≅ △PQR b. △CBA ≅ △PRQ
c. △BAC ≅ △RPQ d. △PQR ≅ △BCA

Q 3. In the given figure, AB ⊥ BE and EF ⊥ BE. Also BC = DE and AB = EF. Then:



a. △ABD ≅ △EFC b. △ABD ≅ △CEF
c. △ABD ≅ △FEC d. △ABD ≅ △CMD

Q 4. It is given that $\Delta ABC \cong \Delta FDE$ and AB = 5 cm, $\angle A + \angle B + \angle C = 180°$, $\angle B = 40°$ and $\angle A = 80°$. Then which of the following is true?

a. DF = 5 cm, $\angle F = 60°$
b. DF = 5 cm, $\angle E = 60°$
c. DE = 5 cm, $\angle E = 60°$
d. DE = 5 cm, $\angle D = 40°$

Q 5. In triangles ABC and DEF, AB = FD and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if:

a. BC = EF b. AC = DE
c. AC = EF d. BC = DE

Q 6. If ΔABC; D, E and F are the mid-points of BC, CA and AB respectively; then ΔDEF is not congruent to the following triangle:

a. ABC b. BDF
c. AFE d. CDE

Q 7. In ΔABC, AB = AC and $\angle B = 50°$. Then $\angle C$ is equal to:

a. 40° b. 50°
c. 80° d. 130°

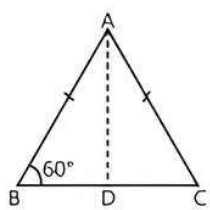
Q 8. In ΔABC, $\angle A + \angle B + \angle C = 180°$, BC = AB and $\angle B = 80°$. Then $\angle A$ is equal to:

a. 80° b. 40°
c. 50° d. 100°

Q 9. In ΔPQR, $\angle R = \angle P$ and QR = 4 cm and PR = 5 cm. Then the length of PQ is:

a. 4 cm b. 5 cm
c. 2 cm d. 2.5 cm

Q 10. In the given figure, AB = AC, AD is the median and its bisect $\angle A$, then $\angle BAD$ is equal to: (use $\angle A + \angle B + \angle C = 180°$)

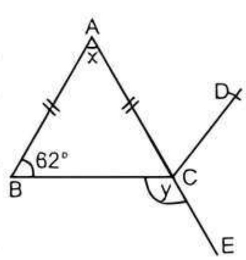


a. 15° b. 30°
c. 20° d. 40°

Q 11. In triangles ABC and PQR, AB = AC, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are:

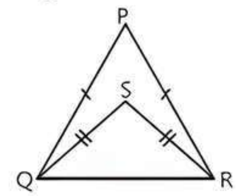
a. isosceles but not congruent
b. isosceles and congruent
c. congruent but not isosceles
d. neither congruent nor isosceles

Q 12. In the adjoining figure, ABC is an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x is: (Use the sum of all angles of a triangle is 180°).



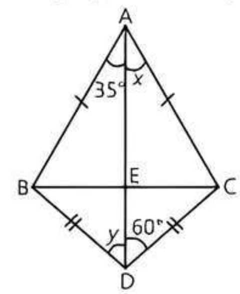
a. 56° b. 120°
c. 122° d. 76°

Q 13. In the given figure, the ratio of $\angle PQS : \angle PRS$ is:



a. 1 : 1 b. 1 : 2
c. 2 : 1 d. 1 : 3

Q 14. In the adjoining figure, x and y are:



a. $x = 30°, y = 65°$ b. $x = 35°, y = 60°$
c. $x = 35°, y = 65°$ d. $x = y = 45°$

Assertion & Reason Type Questions

Directions (Q. Nos. 15-19): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 15. Assertion (A): If we draw two triangles with angles 40°, 60° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle, then two triangles are not congruent.

Reason (R): If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

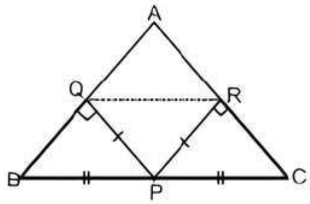
Q 16. Assertion (A): In triangles ABC and PQR, if $\angle A = \angle P$, $\angle C = \angle R$ and AC = PR, then the two triangles are congruent by ASA congruence.

Reason (R): If two angles and included side of a triangle are equal to the corresponding angles and side of the other triangle, then the triangles are congruent by ASA congruence.

Q 17. Assertion (A): In ΔPQR, $PQ = QR$ and $\angle R = 75°$, then $\angle P$ is $52°$.

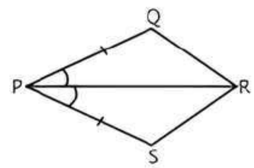
Reason (R): In a triangle, angles opposite to equal sides are equal.

Q 18. Assertion (A): In ΔABC, P is the mid-point of BC. If $PQ \perp AB$ and $PR \perp AC$, such that $PQ = PR$, then $BQ = CR$.



Reason (R): If two angles and the included side of one triangle are equal to two angles and the included side of other triangle, then two triangles are congruent by ASA rule.

Q 19. Assertion (A): In a quadrilateral PQRS, $PQ = PS$ and PR bisects $\angle P$ by SAS congruence rule.



Reason (R): Two triangles are congruent if one hypotenuse side and one of the perpendicular side of triangle is equal to the corresponding one hypotenuse side and one of the perpendicular side of the other triangle.

Fill in the Blanks Type Questions

Q 20. If two angles and the included side of one triangle are equal to two angles and the included sides of the other triangle, then the two triangles are ....

Q 21. If $\Delta ABC \cong \Delta PQR$, then side of ΔPQR equal to the side BC of ΔABC is ....

Q 22. In ΔABC, $AB = AC$, $\angle A + \angle B + \angle C = 180°$ and $\angle A = 50°$, the measure of $\angle B$ is ....

Q 23. In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then $\Delta ABC \cong$ ....

Q 24. In ΔABC, if $\angle C = \angle A$, then $AB =$ ....

True/False Type Questions

Q 25. If $\Delta ABC \cong \Delta PQR$, then $\angle B = \angle Q$ and $AC = PR$.

Q 26. If ABC is isosceles triangle with $AB = AC$ and BD, CE are its two medians, then $BD = CE$.

Q 27. In ΔABC, if $AB \perp BC$ and $\angle A = \angle C$, then $AB \neq AC$.

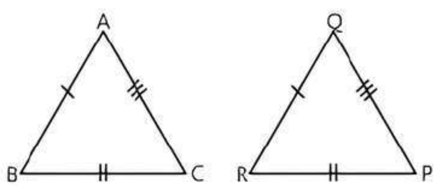
Q 28. Every triangle is not congruent to itself.

Q 29. The bisectors of two equal angles of a trinagle are equal.

Solutions

1. (c) SSA

2. (b) Given $AB = QR$, $BC = PR$ and $CA = PQ$



$\therefore$ $\Delta CBA \cong \Delta PRQ$

3. (c) In ΔABD and ΔFEC,

$AB = EF$ [Given]

$BC = DE$ [Given]

$\Rightarrow$ $BC + CD = DE + CD$

$\Rightarrow$ $BD = CE$

and $\angle ABD = \angle CEF = 90°$

TR!CK

Two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, use SAS congruence rule.

$\therefore$ $\Delta ABD \cong \Delta FEC$ [By SAS congruence]

4. (b) Given, $\Delta ABC \cong \Delta FDE$, $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$

$\because$ $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $80° + 40° + \angle C = 180°$ $\Rightarrow$ $\angle C = 60°$

Since $\Delta ABC \cong \Delta FDE$ or $\Delta FDE \cong \Delta ABC$

$\therefore$ $FD = AB = 5$ cm

$\angle E = \angle C = 60°$

5. (b) In given ΔABC and ΔDFE,

$AB = FD$

$\angle A = \angle D$

If we consider $AC = DE$, then we say that

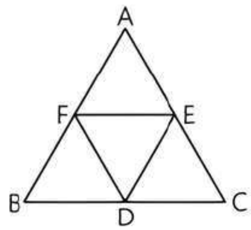
$\Delta ABC \cong \Delta DFE$ by SAS axiom.

COMMON ERR!R

Students are very careful for writing the congruency of two triangles (i.e., $\Delta ABC \cong \Delta PQR$). In this case we write all the angles corresponding to the equal angles, otherwise this will be wrong. So adequate practice is required.

6. (a) Since triangles, ΔDEF, ΔAFE, ΔCDE and ΔBDF are all congruents.

Hence, ΔDEF is not congruent to ΔABC.



7. (b) In given ΔABC, $AB = AC$ and $\angle B = 50°$

Since, $AB = AC$

TiP *Angles opposite to equal sides of a triangle are equal.*

$\Rightarrow \quad \angle C = \angle B$

$\therefore \quad \angle C = 50°$

8. (c) In given ΔABC, $\angle B = 80°$

$BC = AB$

$\Rightarrow \quad \angle A = \angle C$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\because \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 80° + \angle A = 180°$

$\Rightarrow \quad 2\angle A = 100°$

$\Rightarrow \quad \angle A = 50°$

9. (a) In given ΔPQR, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm.

Since, $\angle R = \angle P$

TiP *Sides opposite to equal angles of a triangle are equal.*

$\Rightarrow \quad PQ = QR$

$= 4$ cm

10. (b) Given $AB = AC$,

$\Rightarrow \quad \angle C = \angle B$ [$\because$ Angles opposite to equal sides of a triangle are equal]

$= 60°$

$\because \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 60° + 60° = 180°$

$\Rightarrow \quad \angle A = 60°$

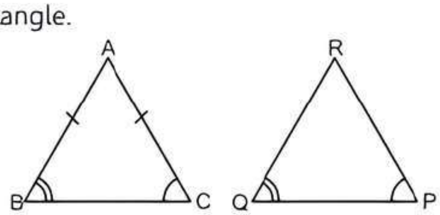
$\therefore \quad \angle BAD = \frac{1}{2}\angle A$ [$\because$ AD is the bisector of $\angle A$]

$= \frac{60°}{2} = 30°$

11. (a) In given ΔABC and ΔPQR

$AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$

Since $AB = AC$, therefore ΔABC is an isosceles triangle.



$\therefore \quad \angle C = \angle B$

$\Rightarrow \quad \angle P = \angle Q$

$\Rightarrow \Delta PQR$ is an isosceles triangle.

But it is clear that ΔABC is not congruent to ΔRQP.

12. (a) In ΔABC,

$AB = AC$ [Given]

TiP *Angles opposite to equal sides of a triangle are equal.*

$\Rightarrow \angle ACB = \angle ABC$

$= 62°$

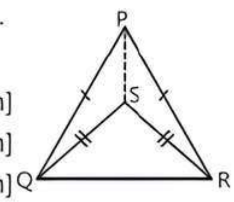
In ΔABC, use the sum of all angles of a triangle is 180°

$\therefore \quad \angle A + \angle B + \angle ACB = 180°$

$\Rightarrow \quad x + 62° + 62° = 180°$

$\Rightarrow \quad x = 180° - 124° = 56°$

13. (a) In the given figure, join PS.



In ΔPQS and ΔPRS,

$PQ = PR$ [Given]

$PS = PS$ [Common]

and $QS = RS$ [Given]

$\therefore \quad \Delta PQS \cong \Delta PRS$ by SSS congruency

$\Rightarrow \quad \angle PQS = \angle PRS$

$\Rightarrow \quad \frac{\angle PQS}{\angle PRS} = \frac{1}{1}$

14. (b) In ΔABD and ΔACD,

$AB = AC$ [Given]

$BD = CD$ [Given]

$AD = AD$ [Common]

$\therefore \quad \Delta ABD \cong \Delta ACD$ [By SSS]

$\Rightarrow \quad \angle BDA = \angle CDA$ [By CPCT]

$\Rightarrow \quad y = 60°$

and $\quad \angle BAD = \angle CAD$ [By CPCT]

$35° = x$

or $\quad x = 35°$

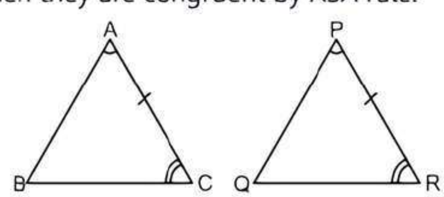
15. (a) **Assertion (A):** There is no rule of congruency when both triangles have equal corresponding angle but sides are different. So, these two triangles are not congruent.

Therefore, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

16. (a) **Assertion (A):** It is true to say that two angles and the including sides of two triangles are equal then they are congruent by ASA rule.



Reason (R): It is also true.

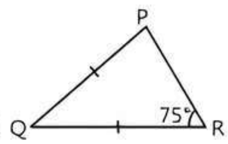
Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

17. (d) **Assertion (A):** In ΔPQR,

$PQ = QR$

$\Rightarrow \quad \angle R = \angle P = 75°$

[$\because$ Angles opposite to equal sides are equal]



So, Assertion (A) is false.

Reason (R): It is true to say that angles opposite to equal sides of triangle are equal.

Hence, Assertion (A) is false but Reason (R) is true.

18. (b) **Assertion (A):** In ΔPQB and ΔPRC,

$BP = PC$ [$\because$ P is the mid-point of BC]

$\angle Q = \angle R = 90°$

[$\because$ Angles opposite to equal sides are equal]

and $PQ = PR$ [Given]

$\therefore \quad \Delta PQB \cong \Delta PRC$ [by RHS congruence]

$\Rightarrow \quad BQ = CR$ [By CPCT]

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

19. (b) **Assertion (A):** In ΔPQR and ΔPSR,

$PQ = PS$ [Given]

$\angle RPQ = \angle RPS$ [$\because$ PR bisects $\angle P$]

$PR = PR$ [Common]

$\therefore \quad \Delta PQR \cong \Delta PSR$ [by SAS congruence rule]

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. congruent.

21. Given $\Delta ABC \cong \Delta PQR$

$\Rightarrow \quad BC = QR$ [By CPCT]

22. In given ΔABC, $AB = AC$ and $\angle A = 50°$

Since $AB = AC$

TiP

Angles opposite to equal sides of a triangle are equal.

$\Rightarrow \quad \angle C = \angle B$

$\because \quad \angle A + \angle B + \angle C = 180°$ [Given]

$\therefore \quad 50° + \angle B + \angle B = 180°$

$\Rightarrow \quad 2\angle B = 130°$

$\Rightarrow \quad \angle B = 65°.$

23. Δ EFD

24. In a ΔABC, if $\angle C = \angle A$, then sides opposite to equal angles are equal.

$\therefore \quad AB = BC$

25. True,

Given $\Delta ABC \cong \Delta PQR$,

$\therefore \quad \angle B = \angle Q$ and $AC = PR$ [By CPCT]

26. True,

Given, ΔABC is isosceles.

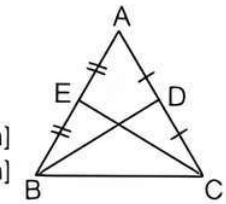
and $AB = AC$

In ΔBAD and ΔCAE

$\angle A = \angle A$ [Common]

$AB = AC$ [Given]

$AD = AE$



[$\because$ D is mid-point of AC and E is mid-point of AB

$\therefore \quad AD = DC$ and $AE = BE$

Also $AB = AC \Rightarrow AE = AD$]

$\therefore \quad \Delta BAD \cong \Delta CAE$ [By SAS congruency]

$\Rightarrow \quad BD = CE$ [By CPCT]

COMMON ERR(!)R

Students make error while applying the congruence rule. So, please be careful.

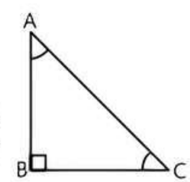
27. True,

Since $AB \perp BC$

$\therefore \quad \angle B = 90°$

In a right angled triangle, hypotenuse is always greater than any of the remaining sides.

$\therefore \quad AB \neq AC$



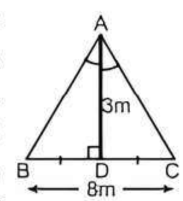
28. False

29. True

Case Study Based Questions

Case Study 1

One day, a geometry teacher came in the class and she decided to teach students about the triangle. She was considering a Mathematic problem for students; in ΔABC, AD is the perpendicular bisector of BC.



She asked the following questions from the students.

On the basis of the above information, solve the following questions.

Q 1. ΔADB is congruent to ΔADC by:

a. AAS axiom b. ASA axiom
c. RHS axiom d. SAS axiom

Q 2. ΔABC is a/an:

a. right angled triangle
b. isosceles triangle
c. equilateral triangle
d. isosceles right angled triangle

Q 3. In given figure, length of AB in ΔABC is:

a. 6 m b. 4 m
c. 5 m d. 7 m

Q 4. The area of ΔABC is:

a. 9 m^2 b. 10 m^2
c. 12 m^2 d. 14 m^2

Q 5. Which of the following axiom is not satisfy the condition of congruency?

a. RHS b. ASA
c. SSS d. SSA

Solutions

1. (d) In ΔADB and ΔADC

$AD = AD$ [Common]

TiP
Perpendicular bisector divides the line in two equal parts.

$BD = DC$

[$\because$ AD is perpendicular bisector of BC]

and $\angle ADB = \angle ADC = 90°$

$\therefore$ $\Delta ADB \cong \Delta ADC$ by SAS axiom.

So, option (d) is correct.

2. (b) $\because$ $\Delta ADB \cong \Delta ADC$,

$\therefore$ $AB = AC$ [By CPCT]

Hence, ΔABC is an isosceles triangle.

So, option (b) is correct.

3. (c) In right angled ΔADB, use Pythagoras theorem

$$AB = \sqrt{(BD)^2 + (AD)^2} = \sqrt{(4)^2 + (3)^2}$$

$$\left[\because BD = DC = \frac{8}{2} = 4 \text{ m}\right]$$

[Common side]

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ m}$$

So, option (c) is correct.

4. (c) Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$$= \frac{1}{2} \times 8 \times 3 = 12 \text{ m}^2$$

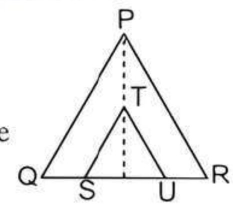
So, option (c) is correct.

5. (d) SSA axiom is not the condition of congruency.

Case Study 2

Red Pyramid (or North Pyramid) is the largest pyramid in cairo Egypt's capital. It is a rusty red pyramid made up of red limestones. The height of the pyramid is 105 m and its base is 220 m.

It is also believed that it was Egyptians first successfuly attempt at constructing a smooth side pyramid. It can be seen that two triangles ΔPQR and ΔSTU have been marked in the images of pyramid, such that ST ∥ PQ and TU ∥ PR and $\angle P = \angle T$.



On the basis of the above information, solve the following questions.

Q 1. Which of the following is not a criteria for congruency?

a. SSS b. SSA c. ASA d. RHS

Q 2. If $\angle TSU = 40°$ and TS = TU, then $\angle STU$ is:

a. 100° b. 110°
c. 120° d. 130°

Q 3. If $\angle QPR = 50°$ and QP = PR, then $\angle PQR$ is:

a. 65° b. 70°
c. 75° d. 80°

Q 4. If hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangles are congruent by axiom.

a. SSS b. RHS
c. SAS d. ASA

Q 5. Area of two congruent figures is:

a. equal b. not equal
c. (a) or (b) d. do not say anything

Solutions

1. (b) SSA is not a criteria for congruency.
So, option (b) is correct.

2. (a) In ΔTSU,

$TS = TU$ (Given)

$\Rightarrow$ $\angle U = \angle S = 40°$

(Angles opposite to equal sides are equal)

$\therefore$ $\angle T + \angle S + \angle U = 180°$

$\angle T + 40° + 40° = 180°$

$\Rightarrow$ $\angle T = 100$

So, option (a) is correct.

3. (a) In ΔPQR,

$QP = PR$ [Given]

TiP

Angles opposite to equal sides of a triangle are equal.

$\Rightarrow \quad \angle R = \angle Q$

$\because \quad \angle P + \angle Q + \angle R = 180°$

$\therefore \quad \angle P + \angle Q + \angle Q = 180°$

$50° + 2\angle Q = 180°$

$\Rightarrow \quad 2\angle Q = 130° \Rightarrow \angle Q = 65°$

So, option (a) is correct.

4. (b) RHS

So, option (b) is correct.

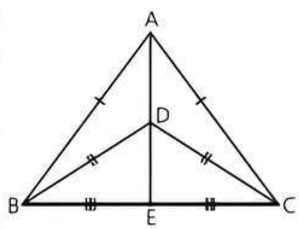
5. (a) Area of two congruent figures is always equal.

So, option (a) is correct.

Case Study 3

Sunil is a farmer who is having a triangular plot. As the land is limited with him, so he decided to divide his land in four parts so that he can use multiple cropping method in which fields are growing with different seads. The field is divided as shown below.

In this figure, ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. Line AD is extended to BC, which intersect at point E.



On the basis of the above information, solve the following questions.

Q 1. If Sunil decides to grow crop in ΔABD, then which of the other triangle is of exact shape and size?

a. $\Delta ABD \cong \Delta ACD$ b. $\Delta ABD \cong \Delta BDE$
c. $\Delta ABD \cong \Delta ACE$ d. $\Delta ACD \cong \Delta AEB$

Q 2. In ΔBDC, if $\angle B + \angle D + \angle C = 180°$, $\angle D = 70°$, then $\angle B$ is equal to:

a. 70° b. 60° c. 55° d. 80°

Q 3. If Sunil decides to grow multiple crop in ΔBDE, then which of the other triangle is of exact shape and size?

a. $\Delta BDE \cong \Delta CDE$ b. $\Delta BDE \cong \Delta CED$
c. $\Delta BED \cong \Delta CDE$ d. $\Delta CED \cong \Delta EBD$

Q 4. Which of the following pair is congruent?

a. ΔABE and ΔADC b. ΔABE and ΔACE
c. ΔAEC and ΔADB d. ΔABE and ΔABD

Q 5. Two circles with same radii but different centres, then they are:

a. not always congruent
b. always congruent
c. congruent in some special cases
d. congruent when areas are different

Solutions

1. (a) In ΔABD and ΔACD

$AB = AC$ [Given]

$BD = DC$ [Given]

$AD = AD$ [Common]

Hence, $\Delta ABD \cong \Delta ACD$. [By SSS rule]

So, option (a) is correct.

2. (c) Given $\angle D = 70°$

$\because \quad \angle B + \angle D + \angle C = 180°$

$\therefore \quad \angle B + 70° + \angle B = 180°$

$[\because BD = DC \Rightarrow \angle C = \angle B]$

$\Rightarrow \quad 2\angle B = 110°$

$\Rightarrow \quad \angle B = 55°$

So, option (c) is correct.

3. (a) In ΔBDE and ΔCDE,

$BD = CD$ [Given]

$DE = DE$ [Common]

and $BE = EC$ [Given]

TR!CK

Three sides of a triangle are equal to the three sides of another triangle, use SSS congruence rule.

$\therefore \Delta BDE \cong \Delta CDE$ [By SSS rule]

So, option (a) is correct.

4. (b) In ΔABE and ΔACE,

$AB = AC$

$BE = EC$

and $AE = AE$ [Common]

$\therefore \quad \Delta ABE \cong \Delta ACE$ [By SSS rule]

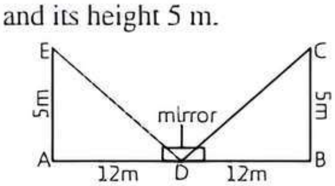
So, option (b) is correct.

5. (b) Two circles with same radii but different centres are always congruent.

So, option (b) is correct.

Case Study 4

John recently read a Mathematics experiment. He was keen to perform it on its own. He choosed a long building whose height he want to know, he placed a mirror at ground. He is standing at some distance to the building as well as mirror. John height is 5 m and the distance of John from mirror is 12 m and distance of building from mirror is also 12 m and its height 5 m.



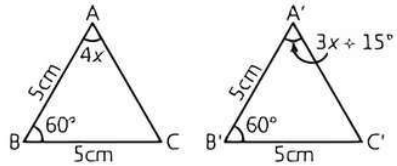
On the basis of the above information, solve the following questions.

Q 1. Write two congruent triangles formed in the given figure.

Q 2. Find the distance between top of building and mirror.

Q 3. **Find the area of ΔAED.**

Q 4. **In the given figures, find the measure of $\angle B'A'C'$.**



Solutions

1. In ΔAED and ΔBCD,

$AD = BD = 12$ m

$AE = BC = 5$ m

and $\angle A = \angle B = 90°$

$\therefore$ $\Delta AED \cong BCD$ [By SAS rule]

2. In right angled ΔEAD, use Pythagoras theorem,

$$ED = \sqrt{(AE)^2 + (AD)^2}$$

$$= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144}$$

$$= \sqrt{169} = 13 \text{ m.}$$

Hence, the distance between top of building and mirror is 13 m.

3. Area of $\Delta AED = \frac{1}{2} \times AD \times AE$

$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$$

4. In ΔABC and $\Delta A'B'C'$

$AB = A'B' = 5$ cm [Given]

$\angle B = \angle B' = 60°$ [Given]

and $BC = B'C' = 5$ cm

$\therefore$ $\Delta ABC \cong \Delta A'B'C'$ [By SAS congruence]

$\Rightarrow$ $\angle BAC = \angle B'A'C'$ [By CPCT]

$\Rightarrow$ $4x = 3x + 15°$

$\Rightarrow$ $x = 15°$

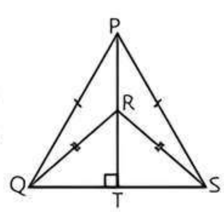
$\therefore \angle B'A'C' = 3x + 15°$

$= 3 \times 15° + 15°$ [$\because x = 15°$]

$= 45° + 15° = 60°$

Case Study 5

Kushal is a property dealer. He has been in this business from past 12 yr. He bought a triangular shaped plot. It divides the plot into four parts which is shown in the figure.



On the basis of the above information, solve the following questions.

Q 1. **If Sunil decides to sell two plots, then show that two triangles ΔPQR and ΔPSR have exact shape and size?**

Q 2. **In ΔQRS, $\angle RQS = 50°$, then find $\angle QSR$.**

Q 3. **If $\angle QPS = 60°$, then find $\angle QPT$.**

Solutions

1. In ΔPQR and ΔPSR,

$PQ = PS$ [Given]

$QR = RS$ [Given]

and $PR = PR$ [Common]

$\therefore$ $\Delta PQR \cong \Delta PSR$ [By SSS congruency]

2. In ΔQRS, $\angle RQS = 50°$

Since $QR = RS$

$\Rightarrow \angle QSR = \angle RQS = 50°$

[$\because$ angles opposite to equal sides are equal]

3. In ΔQPT and ΔSPT,

$PQ = PS$ [Given]

$PT = PT$ [Common]

and $\angle QTP = \angle STP = 90°$

TR!CK

The hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, use RHS congruence rule.

$\therefore$ $\Delta QPT \cong \Delta SPT$ [By RHS Congruence rule]

$\Rightarrow$ $\angle QPT = \angle SPT$ [By CPCT]

$\because$ $\angle QPS = 60°$

$\Rightarrow$ $2\angle QPT = 60°$ $\Rightarrow$ $\angle QPT = 30°$

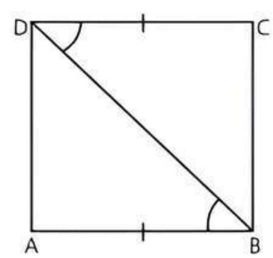
Very Short Answer Type Questions

Q 1. **It is given that $\Delta ABC \cong \Delta DEF$. Is it true to say that $AB = EF$? Justify your answer.**

Q 2. **In triangle ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that two triangles are congruent? Give reason for your answer.**

Q 3. **E and F are respectively the mid-points of equal sides AB and AC of ΔABC, show that $BF = CE$.**

Q 4. **In the figure, if $AB = DC$, $\angle ABD = \angle CDB$ which congruence rule would you apply to prove $\Delta ABD \cong \Delta CDB$?**



Q 5. **$\Delta PQR \cong \Delta ABC$, if $\angle P + \angle Q + \angle R = 180°$, $PQ = 5$ cm, $\angle Q = 40°$ and $\angle P = 80°$, calculate the value of $\angle C$.**

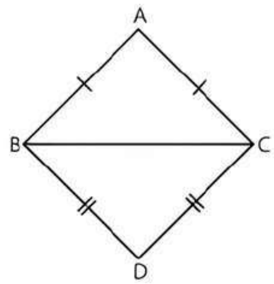
Q 6. **In ΔABC, $\angle C = \angle A$ and $BC = 6$ cm. If $AC = 5$ cm, then find the length of AB.**

Q 7. **In ΔPQR, PE is the perpendicular bisector of $\angle QPR$, then prove that $PQ = PR$.**

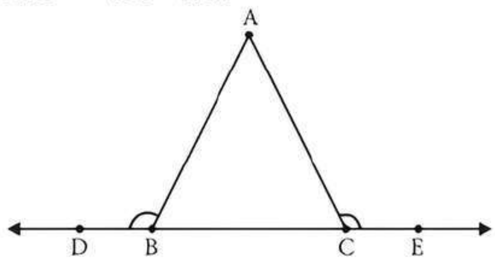
Q 8. In $\triangle PQR$, $\angle P + \angle Q + \angle R = 180°$, $\angle P = 70°$ and $\angle R = 30°$. Which angle of this triangle is the longest?

Short Answer Type-I Questions

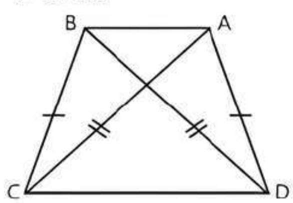
Q 1. In the figure, $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC. Prove that $\angle ABD = \angle ACD$.



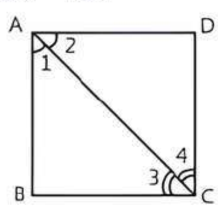
Q 2. In the figure, if $\angle ABD = \angle ACE$, then prove that AB = AC.



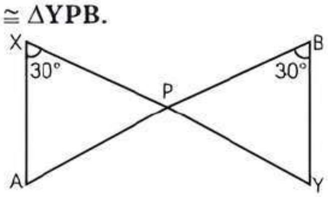
Q 3. In given figure, AD = BC and BD = AC, prove $\angle DAB = \angle CBA$.



Q 4. In given figure, if $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove that BC = CD.



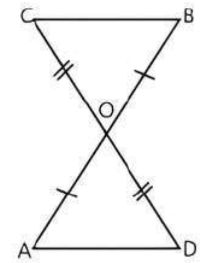
Q 5. In figure, XP = BP and AX || BY, prove that $\triangle APX \cong \triangle YPB$.



Q 6. In the figure, OA = OB and OD = OC. Show that

(i) $\triangle AOD \cong \triangle BOC$

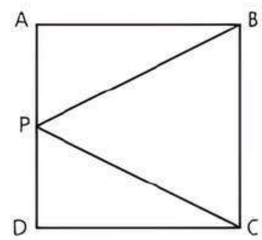
(ii) AD = BC



Q 7. In triangle ABC, D is mid-point of side AC such that $BD = \frac{1}{2}AC$ and $\angle A + \angle B + \angle C = 180°$.

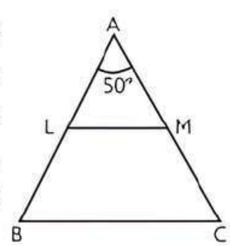
Show that $\angle ABC$ is right angle.

Q 8. In figure, ABCD is a square and P is the mid-point of AD. BP and CP are joined. Prove that $\angle PCB = \angle PBC$.



Q 9. PS is an altitude of an isosceles triangle PQR in which PQ = PR. Show that PS bisects $\angle P$.

Q 10. In the given figure, ABC is an isosceles triangle in which AB = AC and LM is parallel to BC. If $\angle A = 50°$, $\angle A + \angle B + \angle C = 180°$ find $\angle ABC$.



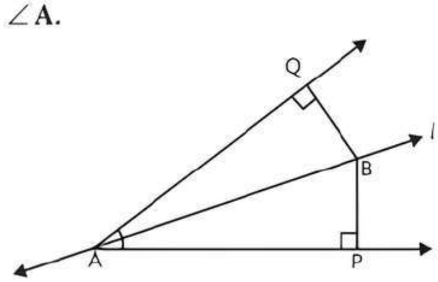
Q 11. Two lines l and m intersect at point O and P is a point on line n passing through point O such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.

Short Answer Type-II Questions

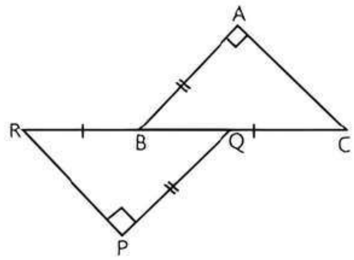
Q 1. Line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure). Show that:

(i) $\triangle APB \cong \triangle AQB$.

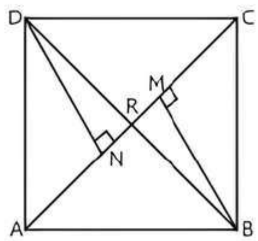
(ii) BP = BQ or B is equidistant from the arms of $\angle A$.



Q 2. In the given figure, BA ⊥ CA, RP ⊥ QP, AB = PQ and BR = CQ. Prove that PR = AC.



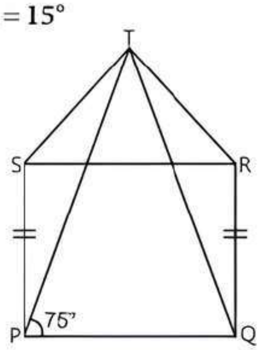
Q 3. In the given figure, BM and DN are both parallel and perpendicular to AC such that AC bisects BD and BM = DN. Prove that MR = NR.



Q 4. In the given figure, PQRS is a square and SRT is an equilateral triangle prove that:

(i) PT = QT

(ii) ∠TQR = 15°



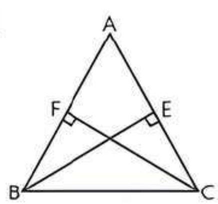
Q 5. In a right-angled triangle, if one acute angle is double the other, then prove that the hypotenuse is double the smallest side.

Q 6. In △ABC, altitudes BE and CF are equal.

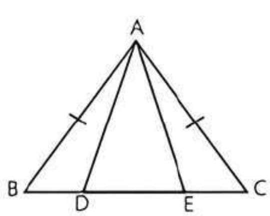
Prove the following results:

(i) △ABE ≅ △ACF

(ii) AB = AC



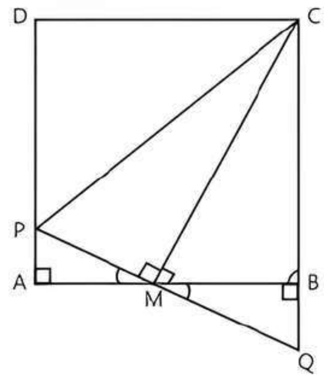
Q 7. In the given figure, ABC is an isosceles triangle with AB = AC, D and E are points on BC such that BE = CD. Show that ADE is an isosceles triangle.



Q 8. ABCD is a square. M is the mid-point of AB and PQ ⊥ CM, PQ meets AD at P, CB produced at Q. Prove that:

(i) PA = BQ

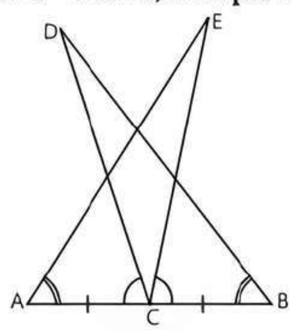
(ii) CP = AB + PA



Q 9. Triangle ABC is an isosceles triangle such that AB = AC. Side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle.

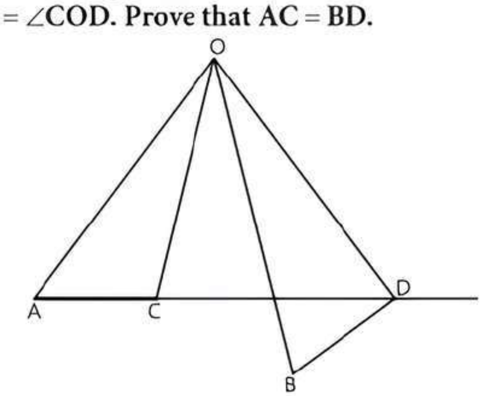
Long Answer Type Questions

Q 1. In the given figure, if AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC, then prove that AE = BD.



Q 2. In a rhombus ABCD, O is any interior point such that OA = OC. Then prove that D, O and B are collinear.

Q 3. In the figure, OA = OB, OC = OD and ∠AOB = ∠COD. Prove that AC = BD.



Q 4. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

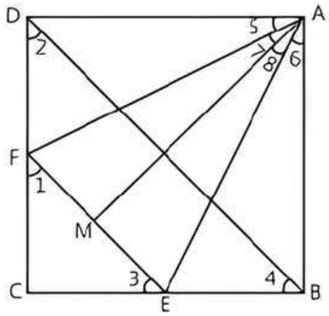
Q 5. If two isosceles triangle have a common base prove that the line joining their vertices bisects them at right angles.

Q 6. In the given figure, ABCD is a square, EF is parallel to diagonal BD, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ and EM = FM. Prove that:

(i) DF = BE.

(ii) AM bisects $\angle$BAD.

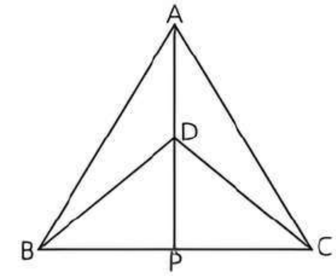
(iii) AP bisects $\angle$A as well as $\angle$D.



Q 7. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that:

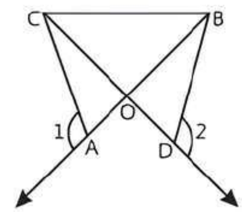
(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$



Q 8. Line segment joining mid-point M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

Q 9. In the given figure, OA = OD and $\angle 1 = \angle 2$. Prove that ΔOCB is an isosceles triangle.



Solutions

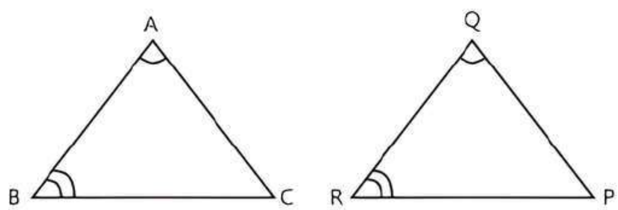
Very Short Answer Type Questions

1. No, AB and EF are not corresponding sides in triangles ABC and DEF but AB corresponds to DE. So, AB = DE.

2. In ΔABC and ΔQRP,

$\angle A = \angle Q$ [Given]

$\angle B = \angle R$ [Given]



If AB = QR, then,

ΔABC will be congruent to ΔQRP.

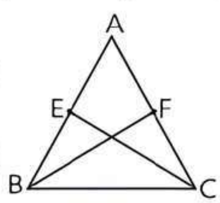
i.e., $\Delta ABC \cong \Delta QRP$ [By ASA congruence]

3. In ΔABF and ΔACE,

AB = AC [Given]

$\angle A = \angle A$ [Common]

AF = AE



$\left[\because AB = AC \Rightarrow \frac{AB}{2} = \frac{AC}{2} \Rightarrow AE = AF\right]$

$\therefore$ $\Delta ABF \cong \Delta ACE$ [By SAS congruence]

$\Rightarrow$ BF = CE [By CPCT]

4. In ΔABD and ΔCDB,

AB = DC [Given]

$\angle ABD = \angle CDB$ [Given]

BD = BD [Common]

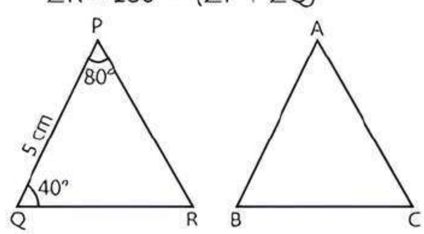
$\therefore$ By SAS congruence,

$\Delta ABD \cong \Delta CDB$

5. Given, PQ = 5 cm, $\angle Q = 40°$ and $\angle P = 80°$.

$\because$ $\angle P + \angle Q + \angle R = 180°$

$\therefore$ $\angle R = 180° - (\angle P + \angle Q)$



$\Rightarrow$ $\angle R = 180° - (80° + 40°)$

$\Rightarrow$ $\angle R = 60°$

$\because$ $\Delta PQR \cong \Delta ABC$ [Given]

$\therefore$ $\angle R = \angle C = 60°$ [By CPCT]

6. In ΔABC, $\angle C = \angle A$

TiP

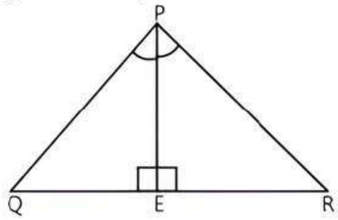
Sides opposite to equal angles of a triangle are equal.

$\Rightarrow$ AB = BC

$\Rightarrow$ AB = BC = 6 cm [Given]

Hence, the length of AB is 6 cm.

7. In ΔPEQ and ΔPER,



$\angle PEQ = \angle PER$ [Each 90°]

$PE = PE$ [Common]

$\angle QPE = \angle RPE$ [Given]

TR!CK

Two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, use ASA congruence rule.

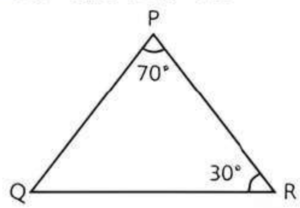
$\therefore$ $\Delta PEQ \cong \Delta PER$ [By ASA congruence]

Thus, $PQ = PR$ [By CPCT]

Hence proved

8. Given, in ΔPQR,

$\angle P = 70°$ and $\angle R = 30°$



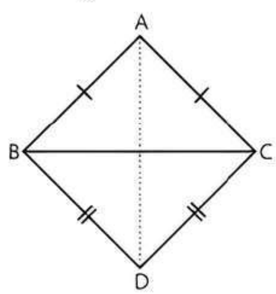
$\angle P + \angle Q + \angle R = 180°$

$\Rightarrow$ $\angle Q = 180° - (70° + 30°) = 80°$

Hence, $\angle Q$ is greatest.

Short Answer Type-I Questions

1. Join AD in the figure.



In ΔABD and ΔACD,

$AB = AC$ [Given]

$BD = CD$ [Given]

$AD = AD$ [Common]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By SSS congruence]

Thus, $\angle ABD = \angle ACD$ [By CPCT]

Hence proved

2. From figure, $\angle ABD + \angle ABE = 180°$

[Linear pair] ...(1)

Also, $\angle ACE + \angle ACD = 180°$ [Linear pair] ...(2)

$\angle ABD = \angle ACE$ [Given] ...(3)

From eqs. (1), (2) and (3), we get

$\angle ABE = \angle ACD$

or $\angle ABC = \angle ACB$

TiP

Sides opposite to equal angles of a triangle are equal.

$\Rightarrow$ $AC = AB$

Hence proved

3. In ΔDAB and ΔCBA,

$AD = BC$ [Given]

$BD = AC$ [Given]

$AB = AB$ [Common]

$\therefore$ $\Delta DAB \cong \Delta CBA$ [By SSS congruence]

Thus, $\angle DAB = \angle CBA$ [By CPCT]

Hence proved

4. In ΔABC and ΔADC,

$\angle 1 = \angle 2$ [Given]

$AC = AC$ [Common]

$\angle 3 = \angle 4$ [Given]

$\therefore$ $\Delta ABC \cong \Delta ADC$ [by ASA congruence]

Thus, $BC = CD$ [By CPCT]

Hence proved

5. Given, $AX \parallel BY$

In ΔAPX and ΔYPB,

$PX = BP$ [Given]

$\angle PXA = \angle PBY = 30°$

$\angle XPA = \angle BPY$ [Vertically opposite angles]

$\therefore$ $\Delta APX \cong \Delta YPB$ **Hence proved**

6. (i) In ΔAOD and ΔBOC,

$OA = OB$ [Given]

$\angle AOD = \angle BOC$ [Vertically opposite angles]

$OD = OC$ [Given]

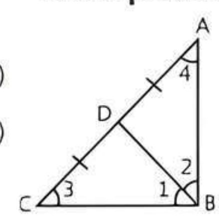
$\therefore$ $\Delta AOD \cong \Delta BOC$ [By SAS congruence]

(ii) $\because$ $\Delta AOD \cong \Delta BOC$

$\therefore$ $AD = BC$ [By CPCT]

Hence proved

7. In ΔABC, D is mid-point of AC.



$\therefore$ $AD = CD = \frac{1}{2}AC$ ...(1)

$BD = \frac{1}{2}AC$ [Given] ...(2)

From eqs. (1) and (2), we get

$AD = BD$ and $CD = BD$

TiP

Angles opposite to equal sides of a triangle are equal.

$\Rightarrow$ $\angle 2 = \angle 4$ and $\angle 1 = \angle 3$ ...(3)

In ΔABC,

$\angle ABC + \angle ACB + \angle CAB = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 1 + \angle 2 = 180°$ [From eq. (3)]

$\Rightarrow$ $2(\angle 1 + \angle 2) = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 = 90°$

$\Rightarrow$ $\angle ABC = 90°$

So, ΔABC is a right angle. **Hence proved**

8. In ΔPAB and ΔPDC,

$PA = PD$ [Given]

$AB = CD$ [Sides of a square]

$\angle PAB = \angle PDC$ [Each 90°]

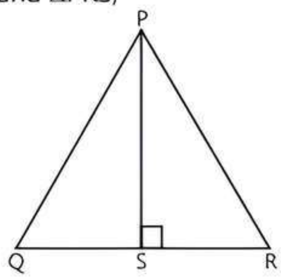
$\therefore \Delta PAB \cong \Delta PDC$ [By SAS congruence]

Thus, $PB = PC$ [By CPCT]

$\Rightarrow \angle PCB = \angle PBC$ [Angles opposite to equal sides of a triangle are equal]

Hence proved

9. In ΔPQS and ΔPRS,



$PQ = PR$ [Given]

$PS = PS$ [Common]

$\angle PSQ = \angle PSR = 90°$ [PS is altitude]

TR!CK

The hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, use RHS congruence rule.

$\therefore \Delta PQS \cong \Delta PRS$ [By RHS congruence]

Thus, $\angle QPS = \angle RPS$ [By CPCT]

Hence, PS bisects $\angle P$. **Hence proved**

10. In ΔABC, $AB = AC$ [Given]

$\therefore \angle C = \angle B$

[Angles opposite to equal sides are equal]

$\because \angle A + \angle B + \angle C = 180°$

$\Rightarrow 50° + \angle B + \angle C = 180°$

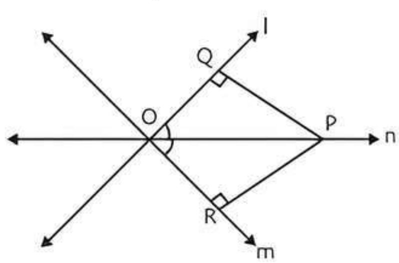
$\Rightarrow 50° + 2\angle B = 180°$

$\Rightarrow \angle B = \dfrac{180° - 50°}{2} = 65°$

$\therefore \angle B = \angle C = 65°$

Hence, $\angle ABC = 65°$

11.



In ΔOQP and ΔORP,

$\angle PQO = \angle PRO$ [Each 90°]

$OP = OP$ [Common]

$PQ = PR$ [Given]

$\therefore \Delta OQP \cong \Delta ORP$ [By RHS congruence]

Thus, $\angle POQ = \angle POR$ [By CPCT]

Hence, *n* is the bisector of $\angle QOR$.

Hence proved

Short Answer Type-II Questions

1. **Given:** l bisects $\angle A$, $BQ \perp AQ$ and $BP \perp AP$.

To Prove: (i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

Proof: (i) In ΔAPB and ΔAQB,

$\angle PAB = \angle QAB$ [l is the bisector of $\angle A$]

$\angle APB = \angle AQB$ [Each 90°]

$AB = AB$ [Common]

$\therefore \Delta APB \cong \Delta AQB$ [By AAS congruence]

(ii) $\because \Delta APB \cong \Delta AQB$

$BP = BQ$ [By CPCT]

or B is equidistant from the arms of $\angle A$.

Hence proved

2. **Given:** $BA \perp CA$, $RP \perp QP$, $AB = PQ$ and $BR = CQ$.

To Prove: $AC = PR$

Proof: Since $BR = CQ$

Adding BQ on both sides, we get

$BR + BQ = CQ + BQ$

$\Rightarrow QR = BC$ ...(1)

Now, in ΔQPR and ΔBAC

$PQ = AB$ [Given]

$\angle P = \angle A$ [Each 90°]

$QR = BC$ [From eq. (1)]

$\therefore \Delta QPR \cong \Delta BAC$ [By RHS congruence]

Thus, $PR = AC$ [By CPCT]

Hence proved

3. **Given:** $BM \perp AC$, $DN \perp AC$ and $BM = DN$.

To Prove: AC bisects BD.

Proof: In ΔBMR and ΔDNR,

$\angle N = \angle M$ [Each 90°]

$BM = DN$ [Given]

$BR = DR$ [$\because$ BD is a bisector of AC]

$\Delta BMR \cong \Delta DNR$ [By RHS congruence]

$\Rightarrow MR = NR$ [By CPCT]

4. **Given:** PQRS is a square and SRT is an equilateral triangle.

To Prove: (i) $PT = QT$

(ii) $\angle TQR = 15°$

Proof: (i) $\because$ PQRS is a square and SRT is an equilateral triangle.

$\therefore \angle PSR = 90°$ and $\angle TSR = 60°$

$\angle PSR + \angle TSR = 150°$

Similarly, $\angle QRT = 150°$

In ΔPST and ΔQRT,

$PS = QR$ [Sides of a square]

$\angle PST = \angle QRT = 150°$ [Proved above]

$ST = RT$

[Sides of an equilateral triangle]

$\therefore \Delta PST \cong \Delta QRT$ [By SAS congruence]

Thus, $PT = QT$ [By CPCT] **Hence proved**

(ii) In ΔTPQ,

$PT = QT$ [Proved above]

$\Rightarrow \angle PQT = \angle QPT = 75°$

Since PQRS is a square.

Therefore,

$\angle PQR = 90°$

$\Rightarrow \angle PQT + \angle TQR = 90°$

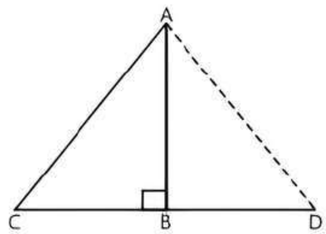
$\Rightarrow 75° + \angle TQR = 90°$

$\Rightarrow \angle TQR = 15°$

5. Let us consider a right-angled ΔABC right-angled at B.

Let $\angle CAB = x$ and $\angle ACB = 2x$.

Produce CB to D such that BD = BC



In ΔABC and ΔABD,

$AB = AB$ [Common]

$BC = BD$ [Produced]

$\angle ABC = \angle ABD$ [Each 90°]

$\therefore \Delta ABC \cong \Delta ABD$ [By SAS congruence]

$\therefore AC = AD$ and $\angle CAB = \angle DAB$ [by CPCT]

Now, $\angle CAD = x + x = 2x$

$\Rightarrow \angle CAD = \angle ACB$

TiP

Sides opposite to equal angles of a triangle are equal.

$\therefore AD = CD$

or $AD = 2BC$

$\Rightarrow AC = 2BC$

Hence, hypotenuse AC is double the smallest side BC.

6. **Given:** In ΔABC, $BE \perp AC$, $CF \perp AB$ and BE = CF

To Prove: (i) $\Delta ABE \cong \Delta ACF$ (ii) AB = AC

Proof: (i) In ΔABE and ΔACF,

$\angle AEB = \angle AFC = 90°$

$\angle BAE = \angle CAF$ [Common angle]

and $BE = CF$ [Given]

$\therefore \Delta ABE \cong \Delta ACF$ [By AAS congruence]

(ii) Since $\Delta ABE \cong \Delta ACF$

$\therefore AB = AC$ [By CPCT]

Hence proved

7. **Given:** In ΔABC, AB = AC and BE = CD.

To Prove: ΔADE is an isosceles triangle.

Proof: In ΔABC,

$AB = AC$ [Given]

$\angle C = \angle B$ [Angles opposite to equal sides of a triangle]

$BE = CD$ [Given]

$\Rightarrow BD + DE = DE + EC$

$\Rightarrow BD = EC$

In ΔABD and ΔACE,

$AB = AC$ [Given]

$\angle B = \angle C$ [Proved above]

$BD = EC$ [Proved above]

$\therefore \Delta ABD \cong \Delta ACE$ [By SAS congruence]

Thus, $\angle ADB = \angle AEC$ [By CPCT]

Also, $\angle ADB + \angle ADE = \angle AEC + \angle AED = 180°$ [Linear pair]

$\Rightarrow \angle ADE = \angle AED$ [$\because \angle ADB = \angle AEC$]

Thus, ΔADE is an isosceles triangle. **Hence proved**

8. **Given:** ABCD is a square. M is the mid-point of AB and $PQ \perp CM$.

To Prove: (i) PA = BQ

(ii) CP = AB + PA

Proof:

(i) In ΔAMP and ΔBMQ,

$\angle PAM = \angle QBM$ [Each 90°]

$AM = BM$ [$\because$ M is mid-point of AB]

$\angle AMP = \angle BMQ$ [Vertically opposite angles]

$\therefore \Delta AMP \cong \Delta BMQ$ [By ASA congruence]

Thus, PM = QM and PA = BQ [By CPCT]

Hence proved

(ii) In ΔCMP and ΔCMQ,

$CM = CM$ [Common]

$\angle PMC = \angle QMC$ [Each 90°]

$PM = QM$ [Proved above]

$\therefore \Delta CMP \cong \Delta CMQ$ [By SAS congruence]

Thus, $CP = CQ$ [By CPCT]

$\Rightarrow CP = CB + BQ$

$\Rightarrow CP = AB + BQ$ [$\because$ CB = AB]

$\Rightarrow CP = AB + PA$ [$\because$ BQ = PA]

Hence proved

9. **Given:** In ΔABC, AB = AC and AD = AB.

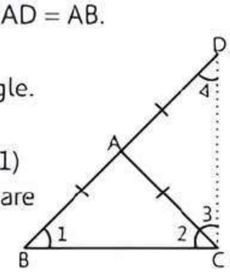
Construction: Join CD

To Prove: $\angle BCD$ is a right angle.

Proof: In ΔABC, AC = AB

$\therefore \angle 1 = \angle 2$ ...(1)

[angles opposite to equal sides are equal]



In ΔADC, $AB = AD$

$\therefore AC = AD$ [$\because$ AB = AC]

$\angle 4 = \angle 3$ ...(2)

In ΔBCD,

[Angles opposite to equal sides are equal]

Now, $\angle 1 + (\angle 2 + \angle 3) + \angle 4 = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle 2 + (\angle 2 + \angle 3) + \angle 3 = 180°$

[From eq. (1) and eq. (2)]

$\Rightarrow 2(\angle 2 + \angle 3) = 180°$

$\Rightarrow \angle 2 + \angle 3 = 90°$

$\therefore \angle BCD$ is a right angled. **Hence proved**

Long Answer Type Questions

1. **Given:** AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC.

To Prove: BD = AE

Proof: Since ∠DCA = ∠ECB

Adding ∠DCE on both sides, we get

$$\angle DCA + \angle DCE = \angle ECB + \angle DCE$$

$\Rightarrow$ ∠ECA = ∠DCB

In ΔACE and ΔBCD,

∠ECA = ∠DCB [Proved above]

∠EAC = ∠DBC [Given]

AC = BC [Given]

∴ ΔACE ≅ ΔBCD [by ASA congruence]

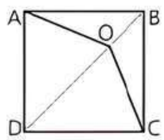
Thus, AE = BD [By CPCT]

Hence proved

2. **Given:** In rhombus ABCD, OA = OC

To Prove: D, O and B are collinear.

Construction: Join OD and OB.



Proof: In ΔAOB and ΔCOB,

AO = CO [Given]

OB = OB [Common]

AB = BC [∵ ABCD is a rhombus]

∴ ΔAOB ≅ ΔCOB [By SSS congruence]

Thus, ∠AOB = ∠COB ...(1) [By CPCT]

Similarly, ΔAOD ≅ ΔCOD

Thus, ∠AOD = ∠COD ...(2) [By CPCT]

But ∠AOD + ∠COD + ∠COB + ∠AOB = 360° [Complete angle]

$\Rightarrow$ 2(∠AOD + ∠AOB) = 360° [From eqs. (1) and (2)]

$\Rightarrow$ ∠AOD + ∠AOB = 180°

∴ D, O and B are collinear. **Hence proved**

3. **Given:** OA = OB, OC = OD and ∠AOB = ∠COD.

To Prove: AC = BD

Proof: Since ∠AOB = ∠COD

Subtracting ∠COB on both sides, we get

∠AOB – ∠COB = ∠COD – ∠COB

∠AOC = ∠BOD

Now, in ΔAOC and ΔBOD,

AO = OB [Given]

∠AOC = ∠BOD [Proved above]

OC = OD [Given]

> **TR!CK**
> *Two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, use SAS congruence rule.*

∴ ΔAOC ≅ ΔBOD [By SAS congruence]

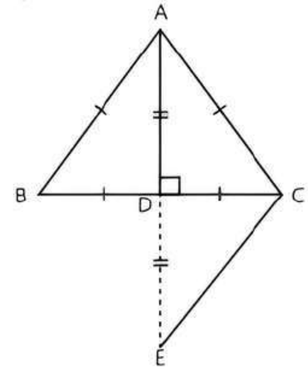
Thus, AC = BD [By CPCT]

Hence proved

4. Let us consider a ΔABC where AD bisects ∠A and side BC.

To Prove: ΔABC is isosceles triangle.

Construction: Produce AD to E such that AD = DE and then join CE.



Proof: In ΔABD and ΔECD,

BD = CD [Given]

∠ADB = ∠EDC [Vertically opposite angles]

AD = ED [By construction]

∴ ΔABD ≅ ΔECD [By SAS congruence]

Thus, AB = CE [By CPCT] ...(1)

∠BAD = ∠CED [By CPCT] ...(2)

∠BAD = ∠CAD [Given] ...(3)

From eqs. (2) and (3), we get

∠CAD = ∠CED

∴ EC = AC ...(4)

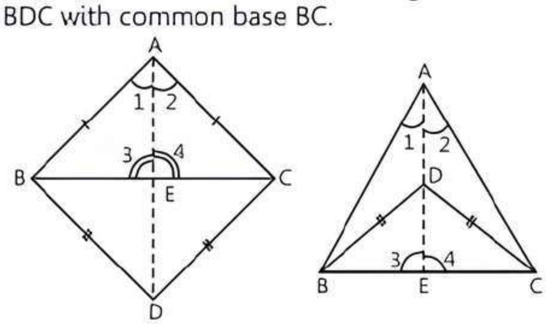
[sides opposite to equal angles are equal]

From eqs. (1) and (4), we get

AB = AC

So, the triangle is isosceles. **Hence proved**

5. Let us consider two isosceles triangles ABC and BDC with common base BC.



To Prove: AD bisects BC at right angles.

Proof: In ΔABD and ΔACD,

AB = AC [Given]

AD = AD [Common]

BD = CD [Given]

∴ ΔABD ≅ ΔACD [By SSS congruence]

Thus, ∠1 = ∠2 [By CPCT]

Now, in ΔABE and ΔACE,

AB = AC [Given]

∠1 = ∠2 [Proved above]

AE = AE [Common]

∴ ΔABE ≅ ΔACE [By SAS congruence]

Thus, BE = CE

and $\angle 3 = \angle 4$ [By CPCT]

But $\angle 3 + \angle 4 = 180°$ [Linear pair]

$\Rightarrow$ $2\angle 3 = 180°$

$\Rightarrow$ $\angle 3 = 90°$

$\therefore$ $\angle 3 = \angle 4 = 90°$

Hence, AD bisects BC at right angles.

Hence proved

6. **Given:** ABCD is a square, EF || BD and EM = FM.

To Prove: (i) DF = BE.

(ii) AM bisects $\angle$BAD.

Proof:

(i) Since EF || BD [Given]

$\Rightarrow$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Given]

$\angle 2 = \angle 4$ [Each 45°]

$\Rightarrow$ $\angle 1 = \angle 3$

CE = CF [Sides opposite to equal angles of a triangle are equal]

$\Rightarrow$ BC – CE = BC – CF [Subtracting from BC both sides]

$\Rightarrow$ BC – CE = CD – CF

[$\because$ BC = CD, sides of a square]

$\Rightarrow$ BE = DF

or DF = BE **Hence proved**

(ii) In ΔADF and ΔABE,

AD = AB [Sides of a square]

$\angle D = \angle B$ [Each 90°]

DF = BE [Proved above]

$\therefore$ $\Delta ADF \cong \Delta ABE$ [By SAS congruence]

Thus, AF = AE and $\angle 5 = \angle 6$ [By CPCT] ...(1)

In ΔAMF and ΔAME,

AF = AE [From eq. (1)]

AM = AM [Common]

FM = EM [Given]

$\therefore$ $\Delta AMF \cong \Delta AME$ [By SSS congruence]

Thus, $\angle 7 = \angle 8$ ...(2)

Adding eqs. (1) and (2), we get

$\angle 5 + \angle 7 = \angle 6 + \angle 8$

$\angle DAM = \angle BAM$

$\therefore$ AM bisects $\angle$BAD. **Hence proved**

7. **Given:** ΔABC and ΔDBC are two isosceles triangles, in which AB = AC and BD = DC.

To Prove:

(i) $\Delta ABD \cong \Delta ACD$ (ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle$A as well as $\angle$D.

Proof: (i) In ΔABD and ΔACD,

AB = AC [Given]

BD = DC [Given]

and AD = AD [Common side]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By SSS congruence]

(ii) In ΔABP and ΔACP,

AB = AC [Given]

AP = AP [Common side]

and $\angle PAB = \angle CAP$ [As $\Delta ABD \cong \Delta ACD$]

$\Rightarrow$ $\angle BAD = \angle CAD$

$\therefore$ $\Delta ABP \cong \Delta ACP$ [By SAS congruence]

(iii) As prove above,

$\Delta ABD \cong \Delta ACD$ ...(1)

$\Rightarrow$ $\angle BAD = \angle CAD$ [By CPCT]

and $\Delta ABP \cong \Delta ACP$ ...(2)

On subtracting eq. (1) from eq. (2),

$\Delta ABP - \Delta ABD \cong \Delta ACP - \Delta ACD$

$\Rightarrow$ $\Delta BDP \cong \Delta CDP$

$\therefore$ $\angle BDP = \angle CDP$ [By CPCT]

Hence, AP bisects $\angle$A as well as $\angle$D.

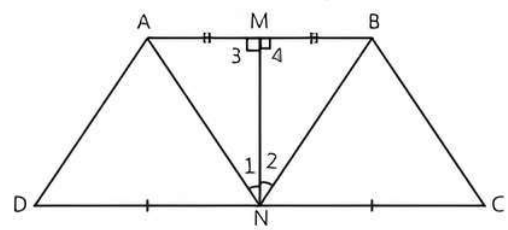
Hence proved

8. **Given:** In trapezium ABCD, AB || DC, M and N are mid-points of AB and DC respectively.

To Prove: AD = BC

Construction: Join AN, MN and BN,

Proof: In ΔAMN and ΔBMN,



AM = MB [M is the mid-point of AB]

$\angle 3 = \angle 4$ [Each 90°]

MN = MN [Common]

$\therefore$ $\Delta AMN \cong \Delta BMN$ [By SAS congruence]

Thus, $\angle 1 = \angle 2$ and AN = BN [By CPCT]

Subtracting both the sides from 90°, we get

$90° - \angle 1 = 90° - \angle 2$ [$\because$ MN $\perp$ DC]

$\Rightarrow$ $\angle AND = \angle BNC$ ...(1)

In ΔADN and ΔBCN,

AN = BN [Proved above]

$\angle AND = \angle BNC$, [From eq. (1)]

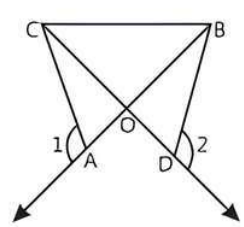
DN = NC [$\because$ N is mid-point of CD]

$\therefore$ $\Delta ADN \cong \Delta BCN$ [By SAS congruence]

Thus, AD = BC [By CPCT]

Hence proved

9. **Given:** OA = OD and $\angle 1 = \angle 2$



To Prove: ΔOCB is an isosceles triangle.

Proof: From figure,

$\angle 1 + \angle CAB = 180°$ [Linear pair]

and $\angle 2 + \angle BDC = 180°$ [Linear pair]

$\therefore$ $\angle 1 + \angle CAB = \angle 2 + \angle BDC$

But $\angle 1 = \angle 2$ [Given]

$\therefore$ $\angle CAB = \angle BDC$

In ΔOAC and ΔODB,

$\angle CAB = \angle BDC$ [Proved above]

$OA = OD$ [Given]

$\angle AOC = \angle DOB$

[Vertically opposite angles]

$\therefore$ $\Delta OAC \cong \Delta ODB$ [By ASA congruence]

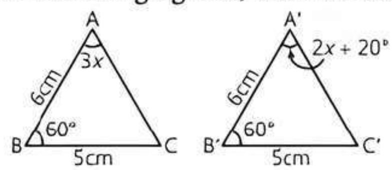
Thus, $OC = OB$ [By CPCT]

$\therefore$ ΔOCB is an isosceles triangle. **Hence proved**

Chapter Test

Multiple Choice Questions

Q 1. **In the following figures, measure of $\angle B'A'C'$ is:**



a. 50° b. 60° c. 40° d. 65°

Q 2. **In triangles ABC and RQP, if AB = AC, $\angle C = \angle P$ and $\angle B = \angle Q$, then two triangles are:**

a. isosceles but not necessarily congruent

b. isosceles and congruent

c. congruent but not isosceles

d. neither congruent nor isosceles.

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4) *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

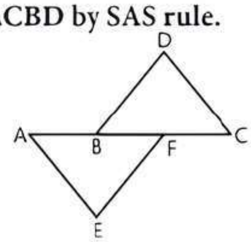
c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. **Assertion (A): In ΔABC, if AB = AC and $\angle B = 75°$, then $\angle C = 75°$.**

Reason (R): In a triangle, angles opposite to equal sides are equal.

Q 4. **Assertion (A): In the given figure, if AB = CF, EF = BD and $\angle AFE = \angle CBD$, then $\Delta AFE \cong \Delta CBD$ by SAS rule.**



Reason (R): If two sides and included angle of one triangle is equal to the two sides and included angle of the other triangle then triangles are congruent by SAS rule.

Fill in the Blanks

Q 5. **The number of triangles that can be drawn with the measure of each angle less than 60°, is ....**

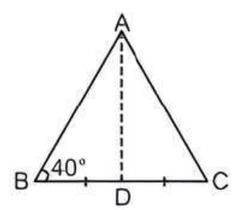
Q 6. **Triangles ABC and PQR satisfy the relations AB = QP, $\angle B = \angle P$ and BC = PR, then these triangles is congruent by ....**

True/False

Q 7. **The medians of an equilateral triangle are equal.**

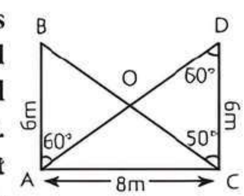
Q 8. **ABC is an isosceles triangle such that AB = AC and AD is the median to base BC.**

Then $\angle BAD = 50°$



Case Study Based Questions

Q 9. **Aryan is studying in class IX. He observe two equal height of poles of equal length AB = CD = 6m. These poles are 8m apart from each other and O is the point of intersection of the lines joining the top of each pole to the foot of opposite poles.**



On the basis of the above information, solve the following questions.

(i) **In which congruent rule, $\Delta ABC \cong \Delta CDA$?**

(ii) **Find the length of BC.**

OR

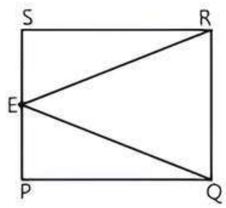
What is the area of ΔACD?

(iii) **Prove that $\Delta ABO \cong \Delta DCO$.**

Very Short Answer Type Questions

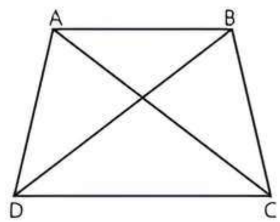
Q 10. In ΔABC and ΔPQR, it is given that $\angle A = \angle R$, $\angle C = \angle P$ and $\angle B = \angle Q$. Find either both triangles are isosceles or congruent.

Q 11. In the adjoining figure, PQRS is a square and E is the mid-point of PS and joined ER and EQ. Prove that $EQ = ER$.



Short Answer Type-I Questions

Q 12. In the adjoining figure, $AD = BC$ and $BD = CA$ prove that $\angle ADB = \angle BCA$.



Q 13. CE and BF are perpendiculars to AB and AC respectively in ΔABC such that $BE = CF$. Prove that $\angle B = \angle C$.

Short Answer Type-II Questions

Q 14. In right angled ΔABC, $\angle C = 90°$. M is the mid point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that:

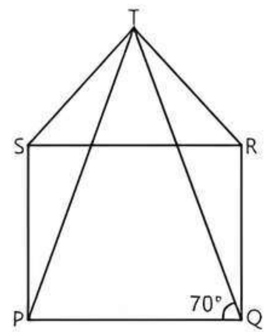
(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC = \angle ACB$

Q 15. In the adjoining figure, PQRS is a square and SRT is an equilateral triangle. Prove that

(i) $PT = QT$

(ii) $\angle TPS = 20°$



Long Answer Type Question

Q 16. If ABC is a right angled triangle such that $AB = AC$ and bisector of angle C intersects the side AB at D, then prove that $AC + AD = BC$.