10. Circles

Exercise 10.1

1. Question

Fill in the blanks:

(i) The common point of a tangent and the circle is called……….. .

(ii) A circle may have ……….. parallel tangents.

(iii) A tangent to a circle intersects it in ……… points(s).

(iv) A line intersecting a circle in two points is called a ……….. .

(v) The angle between tangent at a point on a circle and the radius through the point is ……….. .

Answer

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(ii) A circle may have two parallel tangents

(iii) A tangent to a circle intersects it in one point.

(iv) Secant is a line intersecting a circle in two points

(v) The angle between tangent at a point on a circle and the radius through the point is 90°

2. Question

How many tangents can a circle have?

Answer

A circle can have infinite tangents.

3. Question

O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB.

Answer

$$OB^2 = 8^2 + 15^2$$
$$OB = \sqrt{64 + 225}$$
$$OB = 17cm$$

4. Question

If the tangent at a point P to a circle with centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm. Find the radius of the circle.

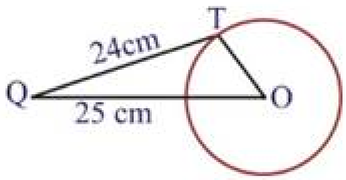
Answer

since QT is a tangent to the circle at T and OT is radius,

Therefore OT perpendicular QT

It is given that OQ=25 cm and QT= 24 cm

By Pythagoras theorem we have



$OB^2 = OA^2 + AB^2$

$OQ^2 = QT^2 + OT^2$

$OT^2 = 25^2 - 24^2$

$= 49 \times 1 = 49$

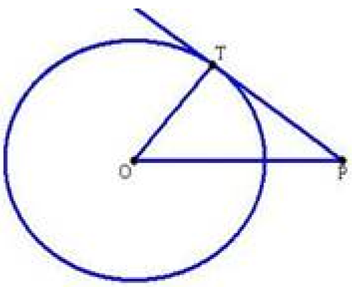
$OT = 7$

Exercise 10.2

1. Question

If PT is a tangent at T to a circle whose centre is O and OP = 17 cm, OT = 8 cm, find the length of the tangent segment PT.

Answer



Given that O is the center of the circle and OP =17 cm and the radius of the circle OT=8cm.

We need to find the length of the segment PT.

The line PT is the tangent line to the circle at the point T, the line through the centre is perpendicular to PT.

$OP^2 = OT^2 + PT^2$

$17^2 = 8^2 + PT^2$

$PT^2 = 289 - 64$

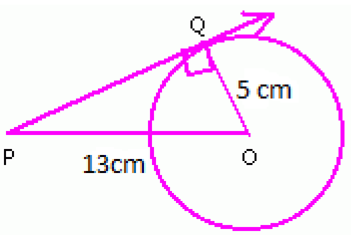
$= 225$

$PT^2 = \sqrt{225} = 15$

2. Question

Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.

Answer



Given: PQ is a tangent to the circle intersect at OP=13cm and OQ=5 cm

Proof: In right triangle OQP

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{169 - 25} = 12cm$$

Therefore the length of the tangent from the point is 12cm

3. Question

A point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Answer

Let PQ be a tangent to the circle from point P and OQ be the radius at the point of contact.

$\therefore \quad \angle OQP = 90°$

$\Rightarrow \quad OP^2 = OQ^2 + PQ^2$

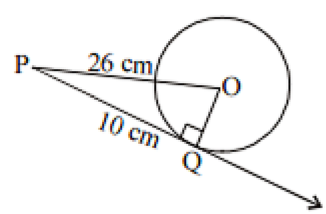
$(\because$ using Pythagoras theoren.

$\Rightarrow \quad OQ^2 = OP^2 - PQ^2 = 26^2 - 10^2$

$= (26 + 10)(26 - 10) = 36 \times 16$

$\Rightarrow \quad OQ = 6 \times 4 = 24$

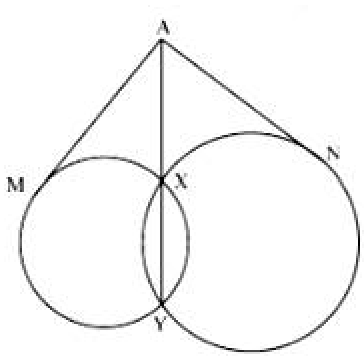
$\therefore$ Radius of the circle = **24 cm.**



4. Question

If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

Answer



Let the two circle intersect at a point X and Y , XY is the common chord.

Suppose A is a point on their common chord and AM and AN be the tangent drawn from A to the circle

AM is the tangent and AXY is a secant.

$AM^2 = AX \times AY$ ....(i)

AN is the tangent and AXY is the secant.

$AN^2 = AX \times AY$ ....(i)

Therefore, from equations (i) and (ii), we get,

AM = AN.

5. Question

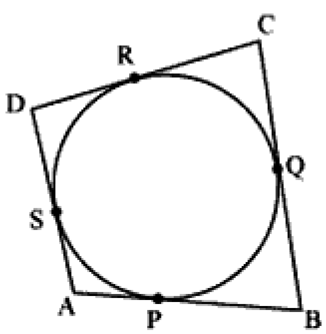
If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.

Answer

Given: the sides of a quadrilateral touch a circle

To prove: the sum of a pair of opposite sides is equal to the sum of the other pair.

Proof:



From the theoram which states that the lengths of the two tangents drawn from an external point to a circle are equalFrom points A the tangents drawn are AP and AS,AP = AS .... (1)From points B the tangents drawn are BP and BQ,BP = BQ ..... (2)From points D the tangents drawn are DR and DS,DR = DS ....(3)From points C the tangents drawn are CR and CQ,CR = CQ ..... (4)Add 1,2,3 and 4 to getAP+BP+DR+CR = AS+BQ+DS+CQ(AP+BP)+(DR+CR)= (AS+DS)+ (BQ+CQ)AB+ DC = AD + BC

Hence proved

6. Question

If AB, AC, PQ are tangents in Fig. 10.51 and AB = 5 cm, find the perimeter of ΔAPQ.

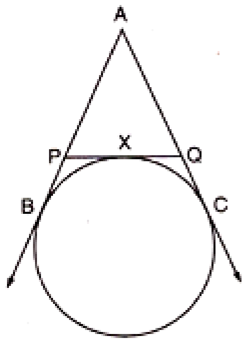


Fig. 10.51

Answer

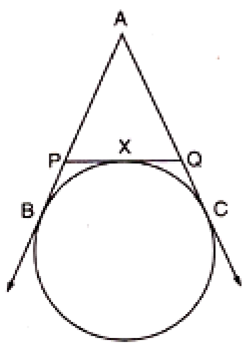


Fig. 10.51

Given: AB and Ac are tangent to the circle with centre O

PQ is tangent to the circle at X which intersect AB and Ac in P and Q

To find : Perimeter of triangle APQ

Proof:

$$\mathrm{AB} = \mathrm{AC}$$
$$QC = QX$$
$$PB = PX$$
$$AB = AC = 5cm$$
$$Perimeter\ of\ \Delta APQ = AQ + QP + AP$$
$$= AC + AB = 5 + 5 = 10cm$$

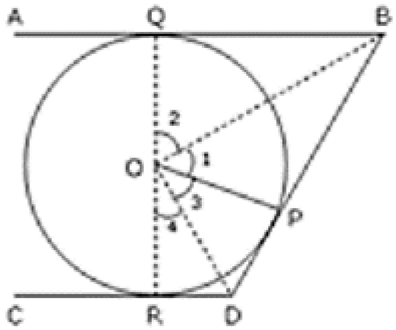
7. Question

Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Answer

Three tangent AB,CD and BD of a circle such as AB and CD are two parallel tangent BD intercept an angle BOD at the centre.

To Prove: $\angle BOD = 90^{\circ}$



$Construction: Join\ OQ\ and\ OR$

$Proof: OP \perp BD$

$In\ rtght\ angle\ \triangle OQB\ and\ OPB.$

$\triangle s\ OQB \cong OPB.$

$Since\ OQ = OP, \angle Q = \angle P\ and\ OB\ is\ common$

$So\ \angle 1 = \angle 2$

$Simillarly\ in\ \triangle s\ ORD\ and\ OPD\ we\ have\ \angle 3 = \angle 4$

$\angle BOD = \angle 1 + \angle 3$

$= \frac{1}{2}(\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}(180^\circ) = 90^\circ$

8. Question

In Fig. 10.52, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30^0$, find $m\angle PRS$.

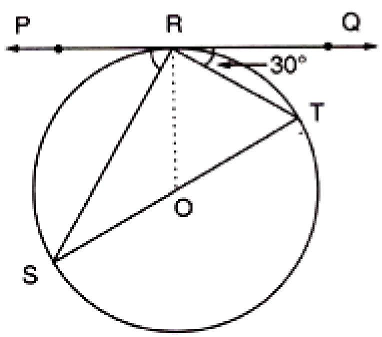


Fig. 10.52

Answer

$Given\ that, \angle TRQ = 30^\circ$

$Since, ST\ is\ a\ diameter\ and\ angle\ in\ a\ semi-circle\ is\ at\ rt.\ angle$

$Therefore, \angle SRT = 90^\circ$

$Now,$

$\angle TRQ + \angle SRT + \angle PRS = 180^\circ$

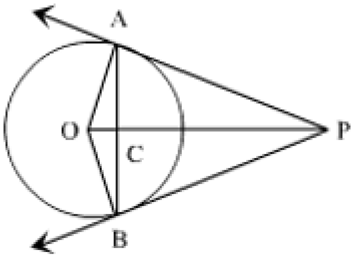
$30^\circ + 90^\circ + \angle PRS = 180^\circ$

$\angle PRS = 60^\circ$

9. Question

If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60^{o}$. Find the length of chord AB.

Answer



Given: PA and PB are tangent of a circle PA= 10 cm and angle APB= 60^{o}

Let O be the center of the given circle and C be the point of intersection of OP and AB

In triangle PAC and triangle PBC

PA = PB (tangent from an external point are equal)

APC =BPC (tangent from an external point are equally inclined to the segment joining center to the point)

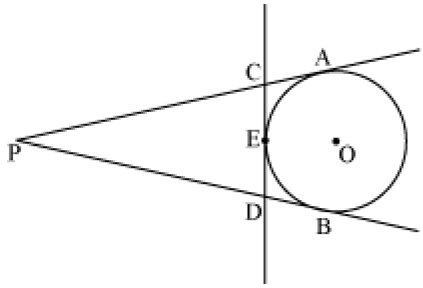
PC =PC (common)

$$\triangle PAC \cong \triangle PBC \,(by\, SAS)$$
$$AC = BC$$
$$\angle APB = \angle APC + \angle BPC$$
$$\angle APC = \frac{1}{2}\angle APB = \frac{1}{2}\times 60^{0} = 30^{0}$$
$$\angle APC + \angle BCP = 180^{o} = 90^{o}$$
$$In\, rt\, triangle\, ACP$$
$$\sin 30^{o} = \frac{AC}{AP}$$
$$\frac{1}{2} = \frac{AC}{10} = 5cm$$
$$AC = BC$$
$$AB = 5 + 5 = 10\,cm$$

10. Question

From an external point P, tangents PA are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of $\triangle PCD$.

Answer



Given: PA and PB are tangent to the circle with centre O

CD is tangent to the circle at E which intersect PA and PB in C and D

To find : Perimeter of triangle PCD

Proof:

$$PA = PB$$
$$CA = DE$$
$$DB = DE$$
$$PA = PB = 14cm$$
$$Perimeter\ of\ \Delta PCD = PC + CD + PD$$
$$= PA + PB = 14 + 14 = 28cm$$

11. Question

In Fig. 10.53, ABC is a right triangle right-angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of its incircle.

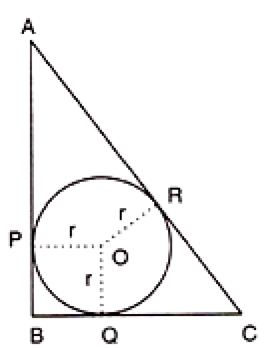
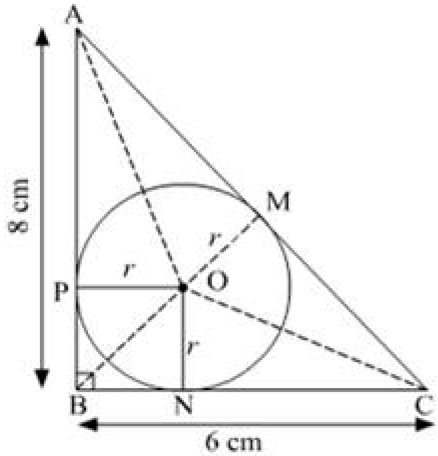


Fig. 10.53

Answer

Let ABC be the right angled triangle such that angle B=90^{o}, BC=6cm, AB= 8cm. Let O be the centre and r be the radius of the in circle.



AB, BC and CA are tangent to the circle at P,N and M

OP=ON=OM=r (radius of the circle)

$$Area\ of\ \triangle ABC = \frac{1}{2} \times 6 \times 8 = 24\ cm^2$$

By Pythagoras theorem,

$$CA^2 = AB^2 + BC^2$$

$$CA^2 = 8^2 + 6^2$$

$$CA = 10\ cm$$

$$Area\ of\ \triangle ABC = Area\ of\ \triangle OAB + Area\ of\ \triangle OBC + Area\ of\ \triangle OCA$$

$$24 = \frac{1}{2} r \times AB + \frac{1}{2} r \times BC + \frac{1}{2} r \times CA$$

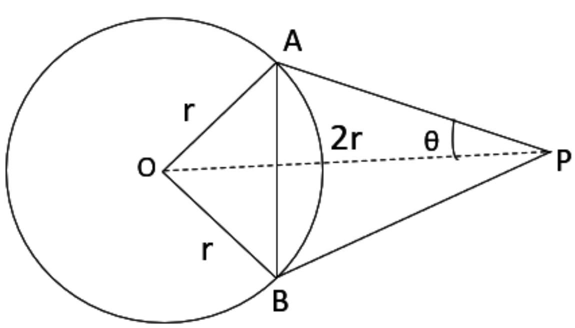
$$r = \frac{2 \times 24}{AB + BC + CA}$$

$$r = \frac{48}{8+6+10} = \frac{48}{24} = 2\ cm$$

12. Question

From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that $\triangle APB$ is equilateral.

Answer



AP is the tangent to the circle,

According to the theorem which states that tangent to a circle is perpendicular to the radius through the point of contact.

⇒ $OA \perp AP$

∠OAP = 90°

Also $OB \perp BP$

⇒ ∠OBP = 90°

In Δ OAP sin θ =perpendicular/hypotenuse

$$\sin\angle OPA = \frac{r}{2r} = \frac{1}{2}$$

As

$$sin\ 30^{\circ} = \frac{1}{2}$$

$$\Rightarrow \angle OPA = 30^{\circ}$$

Similarly ∠OPB = 30°Now ∠APB =∠OPA + ∠OPB

= 30°+30°

= 60° .... (1)

In Δ PAB,As PA and PB are drawn from external point P,By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal

.⇒ PA=PBAlso ∠PAB = ∠PBA .... (2)

As ∠PAB + ∠PBA + ∠APB = 180° (sum of angles of triangle)

∠PAB + ∠PBA = 180° - ∠APB

∠PAB + ∠PBA = 180° - 60°

⇒ 2∠PAB = 120°

⇒ ∠PAB = 60° .... (3)

From 1 and 2 and 3,∠PAB = ∠PBA = ∠APB = 60°Hence ΔPAB is an equilateral triangle.

13. Question

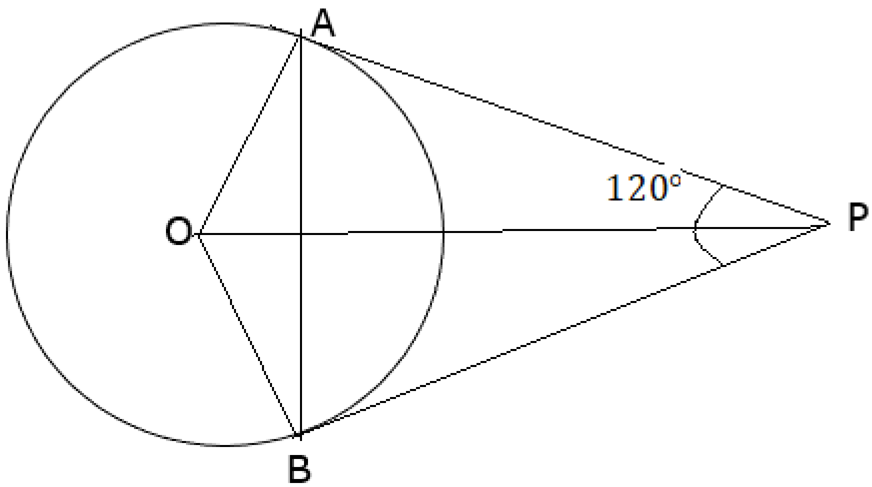
Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^{0}$. Prove that OP = 2 AP.

Answer

Given: Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^{0}$.

To prove: OP = 2 AP

Proof: Construct the figure according to the conditions given.



HereIn triangle OAP and OBP,PA = PB (Length of Tangents from external point are equal)

OA = OB (Radii of same circle)

OP = OP (common)

Δ OAP ~ Δ OBP (By SSS criterion)

∠OPA = ∠OPB = 60^o.

In Triangle OAP , ∠OAP = 90^o (By theoram which states that tangent to a circle is perpendicular to the radius through the point of contact)We know in a right angle triangle

$$\sin\theta = \frac{perpendicular}{hypotenuse}$$

⇒ sin 60^o= AP / OP , i.e 1/2 = AP / OP

So,OP = 2 AP

Hence proved.

14. Question

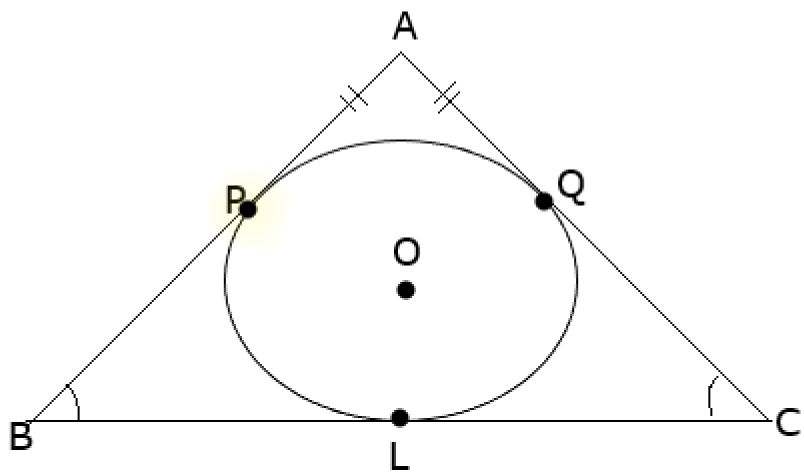
If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L, prove that L bisect BC.

Answer

Given: If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L.

To prove: L bisect BC.

Proof: Construct the figure according to given condition.



AB = AC (given)From the theorem which states that the lengths of two tangents drawn from external point to a circle are equal. ..... (1) As tangents AP and AQ are drawn from the external point A.AP = AQ Also,AB = AC⇒ AP + PB = AQ + QC⇒ AP + PB = AP + QC⇒ PB = QCFrom (1) as tangents BP and BL are drawn from external point B,And tangents CQ and CL are drawn from external point C.⇒ BP = BL ..... (3) CQ = CL ...... (4)As we have proved PB = QCFrom 3 and 4BL = CL⇒ L bisects BC.Hence proved.

15. Question

In Fig. 10.54, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.

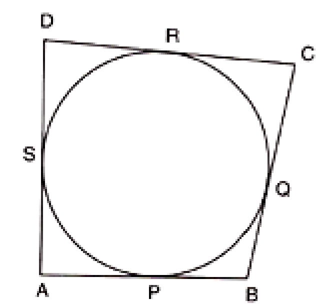


Fig. 10.54

Answer

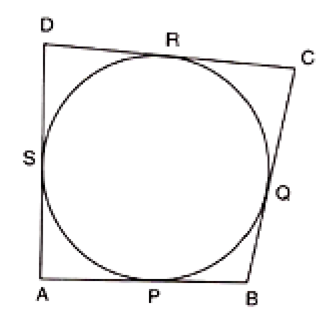


Fig. 10.54

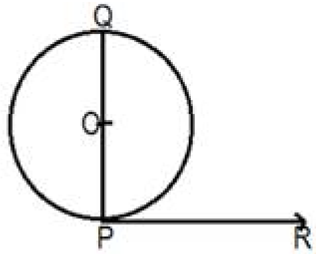
$Here,$
$AP = AS$
$Let,\ AP = AS = X$
$Simillarly, BP = BQ$
$CQ = CR$
$RD = DS$
$\text{Sin}\,ce, AP = X$
$\Rightarrow BP = AB - AP = 6 - X$
$Now,\ BP = BQ = 6 - X$
$CQ = BC - BQ = 7 - (6 - X)$
$= 1 + X$
$now, CQ = CR = 1 + X$
$RD = CD - CR = 4 - (1 + X)$
$= 3 - X$
$RD = DS = 3 - X$
$AD = AS + SD$
$X + 3 - X = 3$
$AD = 3cm$

$Here,$
$AP = AS$
$Let,\ AP = AS = X$
$Simillarly, BP = BQ$
$CQ = CR$
$RD = DS$
$\text{Sin}\,ce, AP = X$
$\Rightarrow BP = AB - AP = 6 - X$
$Now,\ BP = BQ = 6 - X$
$CQ = BC - BQ = 7 - (6 - X)$
$= 1 + X$
$now, CQ = CR = 1 + X$
$RD = CD - CR = 4 - (1 + X)$
$= 3 - X$
$RD = DS = 3 - X$
$AD = AS + SD$
$X + 3 - X = 3$
$AD = 3cm$

16. Question

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

Answer



Draw a circle with centre O, draw a tangent PR touching circle at P.Draw QP perpendicular to RP at a point P, QP lies in the circle.Now,∠OPR = 90ºAlso, ∠QPR = 90º Therefore,∠OPR = ∠QPRThis is possible only when O lies on QP.Hence, it is proved that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

17. Question

In fig. 10.55, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^0$.

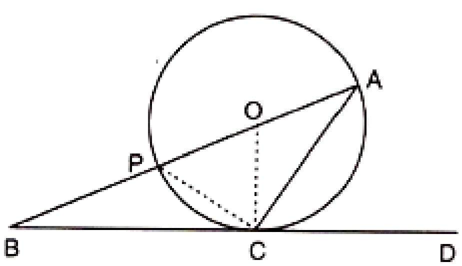


Fig. 10.55

Answer

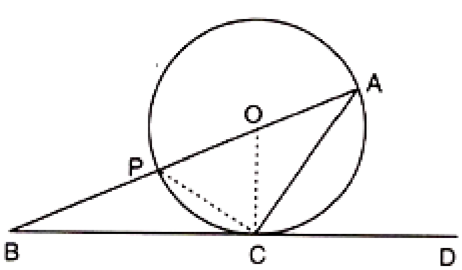


Fig. 10.55

Given: In the above figure, O is the centre of the circle and BCD is tangent to it at C.To prove: ∠BAC + ∠ACD = 90°Proof:

In ΔOAC

OA = OC   [radii of same circle]

⇒ ∠OCA = ∠OAC   [angles opposite to equal sides are equal]

⇒ ∠OCA = ∠BAC   [1]

Also,

OC ⊥ BD   [Tangent at any point on a circle is perpendicular to the radius through point of contact]

⇒ ∠OCD = 90°

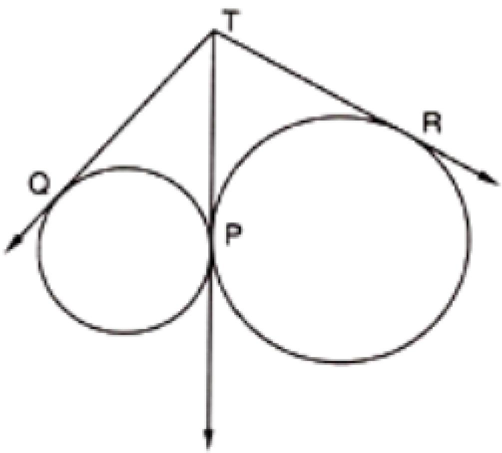
⇒ ∠OCA + ∠ACD = 90°

⇒ ∠BAC + ∠ACD = 90°   [From 1]

Hence Proved

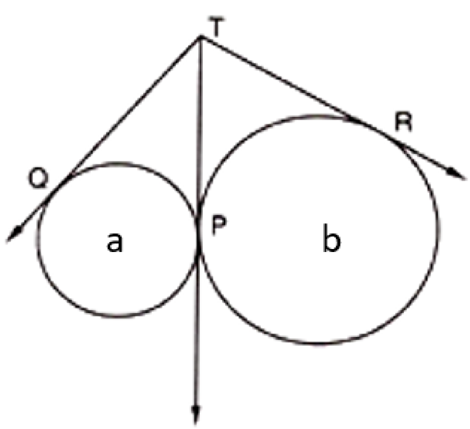
18. Question

Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR



Answer

Let us label two circles as 'a' and 'b'



As TQ and TP are tangents to circle a,And TP and TR are tangents to circle b.By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal.

TQ=TP ...(1)TP=TR ...(2)From 1 and 2,TQ=TRHence proved

19. Question

In Fig 10.57, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^{0}$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius r of the circle.

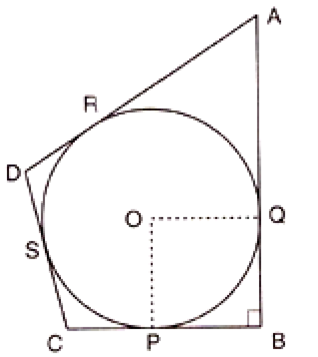
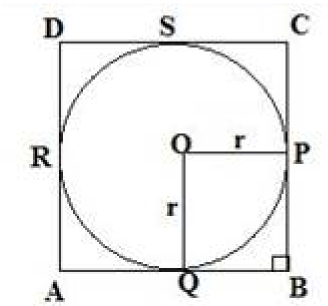


Fig. 10.57

Answer



Given: ABCD is a quadrilateral in which

$\angle B = 90^\circ$

$AD = 23cm, DS = 5cm \; and \; AB = 29cm$

Let the radius of the incircle be r cm.

AD = DS = 5cm (tangent from an external point)

Since AD=23 cm

So,

AR + RD + AD

AR + 5 = 23 cm

AR = 18 cm ------(i)

and AQ = AR

since AR = 18 cm

So, AQ + QB = AB

Now OP and OQ are radius of the circle. So from tangent P and Q

$\angle OPB = \angle OQB = 90^\circ$

$OPBQ \; is \; a \; square$

$OP = QB$

$RAdius \; of \; the \; circle = 11cm$

20. Question

In Fig. 10.58, there are two concentric circles with centre O of radii 5 cm and 3 cm. from an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

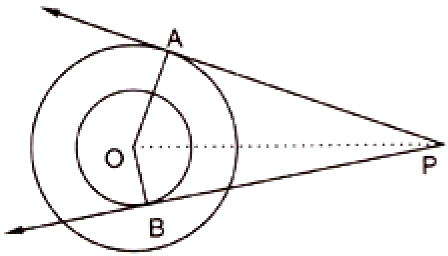
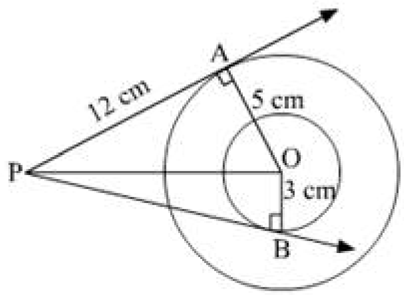


Fig. 10.58

Answer



PA and PB are the tangent drawn from the external point P to outer and inner circle respectively

$\angle OAP = 2\ \angle OBP = 90^{\circ}$

Given $OA = 5\,cm, OB = 3cm$ and $AP = 12cm$

In $\triangle OAP$

$OP^2 = (12cm)^2 + (5cm)^2 = 169cm^2$

$OP = 13cm$

IN $\triangle OBP$,

$PB^2 = OP^2 - OB^2$

$PB^2 = (13cm)^2 - (3cm)^2 = 160cm^2$

$PB = 4\sqrt{10}cm$

Thus the length of $PB = 4\sqrt{10}cm$

21. Question

In Fig. 10.59, AB is a cord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

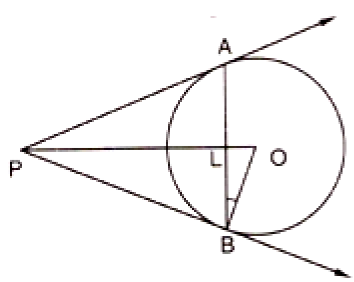
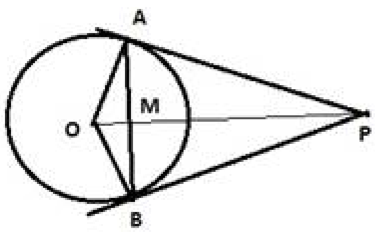


Fig. 10.59

Answer



OA=10 cm

As we know Perpendicular from centre to the chord bisects the chord.

So AM=MB=8cm

Using Pythagoras theorem in triangle AOM

$$OM = \sqrt{10^2 - 8^2} = 6\,cm$$

$$\tan \angle AOM = \frac{8}{6} = \frac{4}{3}$$

$$Now\,in\,\triangle OAP$$

$$\tan \angle AOM = \frac{PA}{OA}$$

$$PA = \frac{40}{3}$$

22. Question

In Fig. 10.60, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at A. Prove that PL + LM = PN + MN.

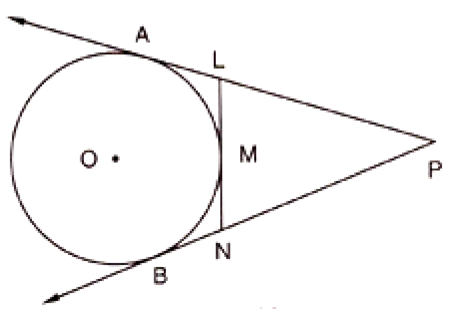


Fig. 10.60

Answer

$$PA = PB ------(i)$$

As tangent drawn from external points to a circle are equal in length

$$PL + AL = PN + BN ------(ii)$$

PLA and PNB are the two tengent which are equal

$$AL = ML\ and\ BN = MN -------(iii)$$

$$From\,(ii)\ and\ (iii)$$

$$PL = ML$$

23. Question

In Fig. 10.61, BDC is a tangent to the given circle at point D such that BD = 30 cm and CD = 7 cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle. Calculate (i) AF (ii) radius of the circle.

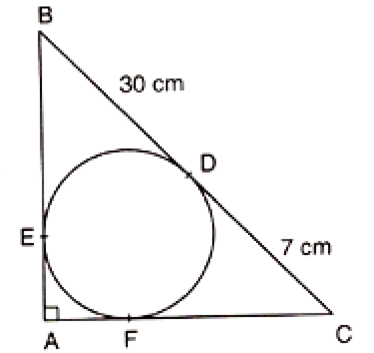


Fig. 10.61

Answer

Given : AB, BC and AC are tangents to the circle at E, D and F.

BD = 30 cm and DC = 7 cm and ∠BAC = 90°

Recall that tangents drawn from an exterior point to a circle are equal in length

Hence BE = BD = 30 cm

Also FC = DC = 7 cm

Let AE = AF = x → (1)

Then AB = BE + AE = (30 + x)

AC = AF + FC = (7 + x)

BC = BD + DC = 30 + 7 = 37 cm

Consider right Δ ABC, by Pythagoras theorem we have

$BC^2 = AB^2 + AC^2$

⇒ $(37)^2 = (30 + x)^2 + (7 + x)^2$

⇒ $1369 = 900 + 60x + x^2 + 49 + 14x + x^2$

⇒ $2x^2 + 74x + 949 - 1369 = 0$

⇒ $2x^2 + 74x - 420 = 0$

⇒ $x^2 + 37x - 210 = 0$

⇒ $x^2 + 42x - 5x - 210 = 0$

⇒ $x (x + 42) - 5 (x + 42) = 0$

⇒ $(x - 5) (x + 42) = 0$

⇒ $(x - 5) = 0$ or $(x + 42) = 0$

⇒ x = 5 or x = − 42

⇒ x = 5 [Since x cannot be negative]

∴ AF = 5 cm [From (1)]

Therefore AB =30 +x = 30 + 5 = 35 cm

AC = 7 + x = 7 + 5 = 12 cm

Let ‘O’ be the centre of the circle and ‘r’ the radius of the circle.

Join point O, F; points O, D and points O, E.

From the figure,

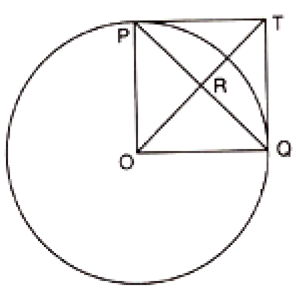
Area of (ΔABC) = Area (ΔAOB) + Area (ΔBOC) + Area (ΔAOC)

∴ r = 5

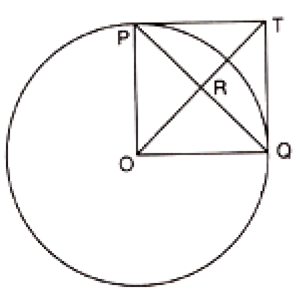
Thus the radius of the circle is 5 cm

24. Question

In Fig. 10.62, $PO \perp OQ$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.



Answer



To prove: PQ and OT are the right bisectors.

Proof: To prove PQ and OT are the right bisectors,

We need to prove ∠PRT= ∠TRQ=∠QRO=∠ORP = 90^o

As it is given that $PO \perp OQ$,

⇒ ∠POQ = 90^o

In Δ POT and Δ OQT

OP=OQ (Radius)∠OPT = ∠OQT = 90^{o} (Tangent to a circle at a point is perpendicular to the radius through the point of contact)OT=OT (common)

∴ Δ POT ≅ Δ OQTThus PT=OQ (BY C.P.C.T) ..... (1) Now in Δ PRT and Δ ORQ ∠TPR = ∠OQR (alternate angles)∠PTO = ∠TOQ (alternate angles)PT=OQ (from (1))∴ Δ PRT ≅ Δ ORQThus TQ = OP (By C.P.C.T 0Hence PT=TQ=OQ=OPThus it is a square,⇒ The diagnols bisect at 90^{o}.

Hence proved

25. Question

In Fig. 10.63, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^{0}$. Prove that OA = 2AB.

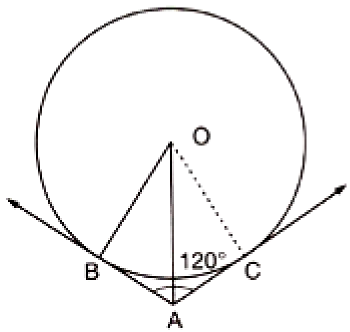
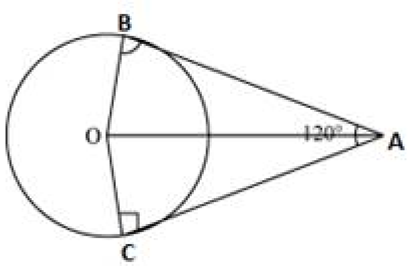


Fig. 10.63

Answer



It can be clearly show that OA bisects angle CAB.

$\angle OBC = \angle OBD = 60^{o}$

$In \triangle OAB$

$\angle OAB = 60^{o}, \angle OBA = 90^{o}$

$\angle BOA + \angle OAB + \angle OBA = 180^{o}$

$\angle BOA = 180^{o} - 150^{o} = 30^{o}$

$\sin(\angle BOA) = \frac{AB}{AO}$

$\sin 30^{o} = \frac{AB}{AO}$

$\frac{1}{2} = \frac{AB}{AO}$

$AO = 2AB$

26. Question

In Fig. 10.64, BC is a tangent to the circle O. OE bisects AP. Prove that $\triangle AEO \sim \triangle ABC$.

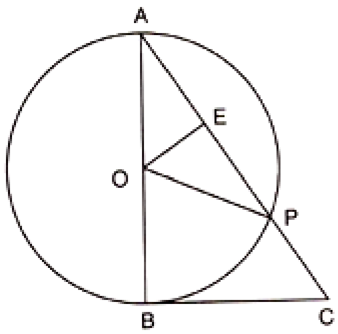
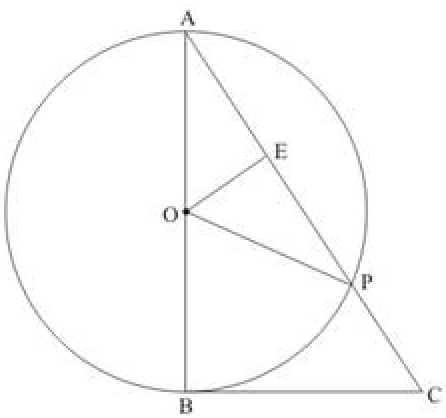


Fig. 10.64

Answer



Triangle AOP is an isosceles triangle because OA=OP as they are the radius of the circle. We know that radius of the circle is always perpendicular to the tangent at the point of contact.

Here OB is the radius and BC is the tangent and B is the point of contact, Therefore

$\angle ABC = 90^o$

Also from the property of isoceles triangle we have found that

$\angle OEA = 90^o$

Therefore,

$\angle ABC = \angle OEA$

$\angle A$ *is common angle to both triangle*

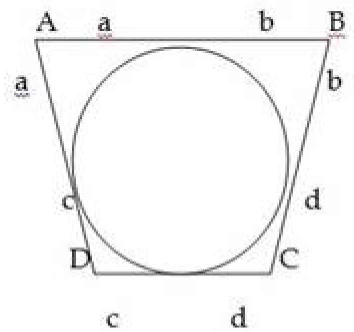
THerefore, from AA postulates of similar triangle

$\triangle AOE \sim \triangle ABC$

27. Question

The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm and 7 cm respectively. Determine the length of the fourth side.

Answer



Given

$AB = 7, BC = 5, CD = 4$

Length of the tangent drawn from an external point to the circle are equal

Therefore

$AB + CD = BC + AD$

$7 + 4 = 5 + AD$

$AD = 11 - 5 = 6$

28. Question

In Fig. 10.65, common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS.

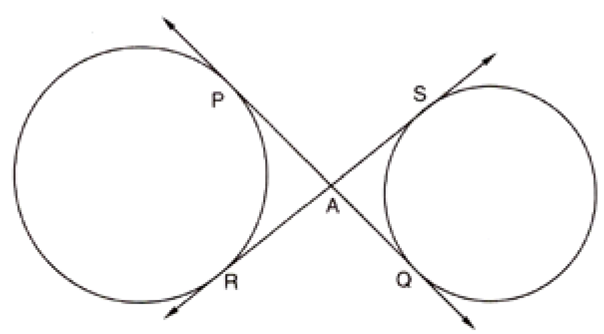
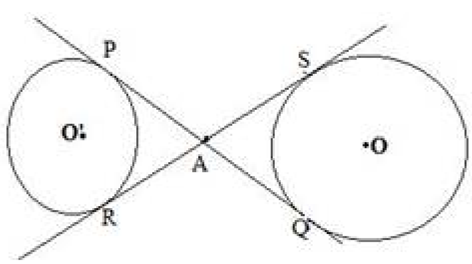


Fig. 10.65

Answer



Given: PQ and RS are the two common tangent to the two circle

To Proof: A is the point of intersection of PQ and RS

We know that , length of two tangent drawn from an exterior point to acirclr are equal.

Therefore

PA = RA----------------- (i)

QA = SA ------------------- (ii)

Adding two equations we get

PA + QA =RA + SA

PQ =RS (proved)

29. Question

Equal circles with centres O and O′ touch each other at X. OO′ produced to meet a circle with centre O′, at A. AC is tangent to the circle whose centre is O. O′ D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.

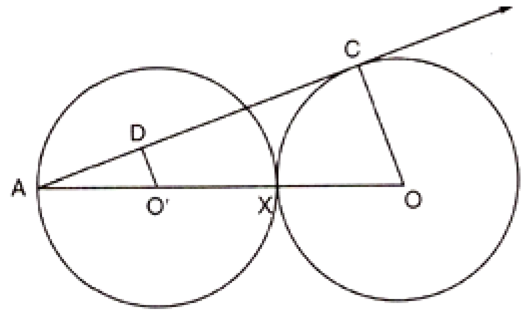


Fig. 10.66

Answer

We know that ∠ADO′ = 90° (since O′D is perpendicular to AC)

As we know radius is perpendicular to the tangent.

So, OC ⊥ AC

⇒ ∠ACO = 90°

In ΔADO′ and ΔACO,

∠ADO′ = ∠ACO (each 90°)

∠DAO = ∠CAO (common)

By AA criteria,

ΔADO′ ~ ΔACO

As we know corresponding sides of a triangle are in ratio. $\frac{AO'}{AO} = \frac{DO'}{CO}$

AO = AO′ + O′X + OX

As radii of two circles are equal.

⇒ AO = AO′ + AO′ + AO′

= 3 AO′

$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3}$ $\frac{DO'}{CO} = \frac{AO'}{3AO} = \frac{1}{3}$ $\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

30. Question

In Fig. 10.67, OQ:PQ=3:4 and perimeter of ΔPOQ = 60 cm. Determine PQ, OR and OP.

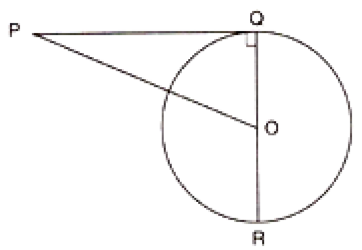
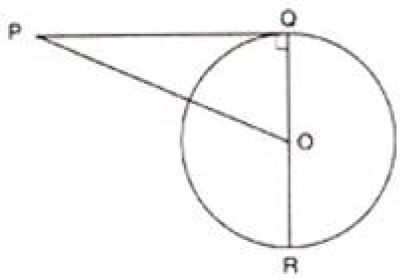


Fig. 10.67

Answer



Given that OQ: PQ=3:4

Let ratio coefficient =x, so

OQ=3x and PQ=4x

We know that a tangent to a circle is perpendicular to the radius at the point of tangency

So

$\angle OQP = 90^\circ$

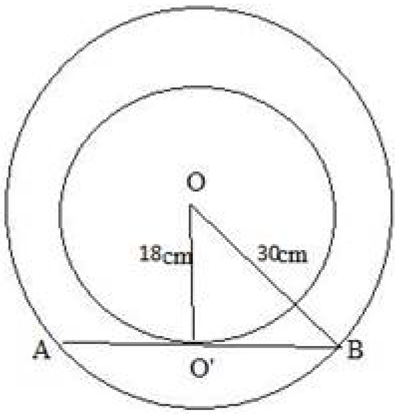
Then applying Pythagoras theorem in triangle POQ

$$OP^2 = OQ^2 + PQ^2$$
$$OP^2 = (3x)^2 + (4x)^2$$
$$OP^2 = 9x^2 + 16x^2$$
$$OP^2 = 25x^2$$
$$OP = 5x$$
$$Perimeter\ of\ a\ triangle\ POQ\ is = 60\,cm,\ So$$
$$3x + 4x + 5x = 60$$
$$12x = 60$$
$$x = 5$$
$$So,$$
$$OQ = 3x = 15cm$$
$$PQ = 4x = 20cm$$
$$OP = 5x = 25cm$$
$$QR = 2(OQ) = 2 \times 15 = 30\,cm$$

31. Question

Two concentric circles are of diameters 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer



In the diagram AB is the chord touching the smaller circle. We have the right angled triangle OO'B

By Pythagoras theorem

$$O'B = \sqrt{(30)^2 + (18)^2} = 24\,cm$$

Now since the chord of the larger circle which touches the smaller circle is bisected at the point of contact

We have

AB= 2 × 24= 48 cm

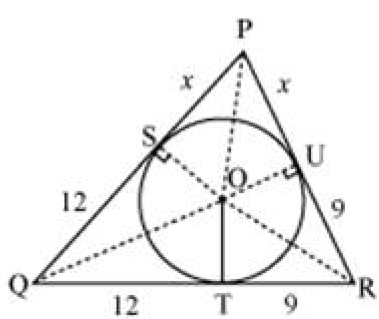
So ans is 18 cm.

32. Question

A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of ΔPQR is 336 cm^2, find the sides PQ and PR.

Answer

Let PQ and PR touch the circle at points S and U respectively. Join O with P, Q, R, S and U



We have OS = OT = OU = 6cm

QT = 12 cm and TR = 9cm

QR= QT +TR = 12cm + 9cm = 21 cm

Now QT = QS=12 cm (tangent from the same point)

TR =RU=9cm

Let PS =PU =x cm

Then PQ=PS+SQ- (12 + x)cm and PR = PU+RU= (9+x)cm

It is clear that

$$ar(\triangle OQR)+ar(\triangle OPR)+ar(\triangle OPQ)=ar(\triangle PQR)$$
$$\frac{1}{2}\times QR\times OT+\frac{1}{2}\times PR\times OU+\frac{1}{2}\times PQ\times OS=189cm^2$$
$$\frac{1}{2}\times(12+x)\times 6+\frac{1}{2}\times(9+x)\times 6+\frac{1}{2}\times 21\times 6=189$$
$$\frac{1}{2}\times 6(12+x+9+x+21)=189$$
$$3(42+2x)=189$$
$$42+2x=63$$
$$x=\frac{21}{2}=10.5$$
Thus PQ $=(12+10.5)$ cm $=22.5$ cm and PR $=(9+10.5)$ cm $=19.5$ cm

33. Question

In Fig. 10.68, a $\triangle ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of $\triangle ABC$ is 84 cm^2.

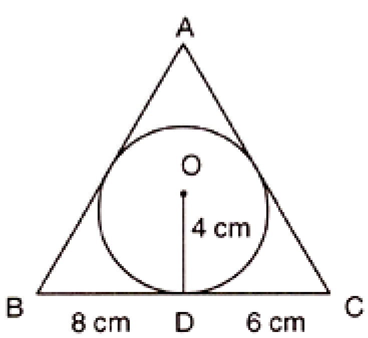
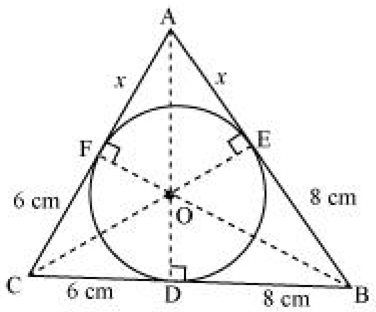


Fig. 10.68

Answer



Firstly consider that the given circle will touch the given circle will touch the sides AB and AC of the triangle at a point E and F respectively.

Let AF=x

Now in triangle ABC

CF = CD=6cm

(Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point C)

BE = BD =8cm (Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point B)

AE = AF =X

Now AB= AE + EB =x + 8

Also BC = BD+ DC = 8+6 =14 and CA= CF+FA = 6+ x

Now we get all side of the triangle and its area can be find by using hero’s formula

$$Semi-perimeter = s = \frac{28+2x}{2} = 14+x$$

$$Area\ of\ \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(14+x)\{(14+x)-14\}\{(14+x)(6+x)\}\{(14+x)-(8+x)\}}$$

$$= 4\sqrt{3(14x+x^2)}$$

$$Ar\ of\ \triangle OBC = \frac{1}{2}\times 4\times 14 = 28$$

$$Ar\ of\ \triangle OCA = \frac{1}{2}\times 4\times(6+x) = 12+2x$$

$$Ar\ of\ \triangle OAB = \frac{1}{2}\times 4\times(8+x) = 16+2x$$

$$Area\ of\ \triangle ABC = Ar\ of\ \triangle OBC + Ar\ of\ \triangle OCA + Ar\ of\ \triangle OAB$$

$$4\sqrt{3(14x+x^2)} = 28+12+2x+16+2x$$

$$\sqrt{3(14x+x^2)} = 14+x$$

Squarring both side and solving we get

$$x(x+14)-7(x+14)=0$$

$$or\ x=-14\ and\ 7$$

$$x=-14\ is\ not\ possible$$

$$so\ x=7$$

$$hence\ AB = 7+8 = 15\ cm$$

$$CA = 6+7 = 13cm$$

34. Question

In Fig. 10.69, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^{0}$, find $\angle ATQ$.

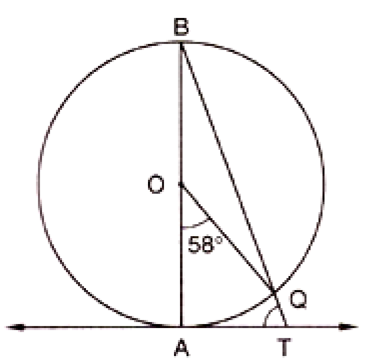


Fig. 10.69

Answer

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58 = 29^{\circ}$$

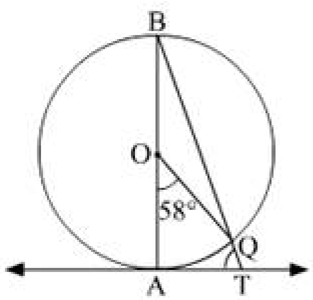
$\angle A = 90^{\circ}$

So

$\angle BAT + \angle ABT + \angle ATQ = 180$

$\angle ATQ = 180 - 90 + 29 = 61^{\circ}$

$\angle ATQ = 61^{\circ}$



35. Question

In Fig. 10.70, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30^{o}. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

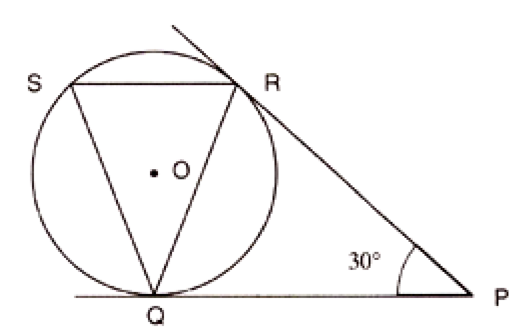


Fig. 10.70

Answer

As we know that the tangents drawn from an external point to a circle are equal.

Therefore, PQ = PR

Also, from the figure, PQR is an isosceles triangle, because PQ = PR

Therefore, ∠RQP =∠QRP (Because the corresponding angles of the equal sides of the isosceles triangle are equal)And, from the angle sum property of a triangle,

∠RQP + ∠QRP + ∠RPQ = 180^{o}

∠RQP + ∠RQP + ∠RPQ = 180^{o}

2∠RQP +∠RPQ = 180^{o}

2∠RQP +30^{o} = 180^{o}

$2\angle RQP = 180^o - 30^o$

$2\angle RQP = 150^o$

$\angle RQP = 150^o/2$

Therefore, $\angle RQP = 75^o$

SR || QP and QR is a transversal

∵ $\angle SRQ = \angle PQR$ …[Alternate interior angle]

∴ $\angle SRQ = 75°$

⇒ $\angle ORP = 90°$…[Tangent is Perpendicular to the radius through the point of contact]

$\angle ORP = \angle ORQ + \angle QRP$

⇒ $90° = \angle ORQ + 75°$

⇒ $\angle ORQ = 15°$

Similarly, $\angle RQO = 15°$

In Δ QOR,

$\angle QOR + \angle QRO + \angle OQR = 180°$

⇒ $\angle QOR + 15° + 15° = 180°$

⇒ $\angle QOR = 150°$

⇒ $\angle QSR = \angle QOR/2$

⇒ $\angle QSR = 150°/2 = 75°$

In ΔRSQ,

$\angle RSQ + \angle QRS + \angle RQS = 180°$

⇒ $75° + 75° + \angle RQS = 180°$

$\angle RQS = 30^o$

CCE - Formative Assessment

1. Question

In Fig. 10.72, PA and PB are tangents to the circle drawn from an external point P. CD are a third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length PC?

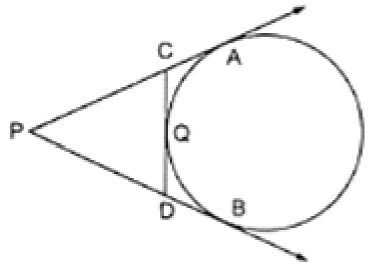


Fig. 10.72

Answer

Given:

PB = 10 cm

CQ = 2 cm

<u>*Property:*</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Using the above property,

PA = PB = 10 cm (tangent from P)

And,

CA = CQ= 10 cm (tangent from C)

Now,

PC = PA – CA

= 10 cm – 2 cm

= 8 cm

Hence, PC = 8 cm

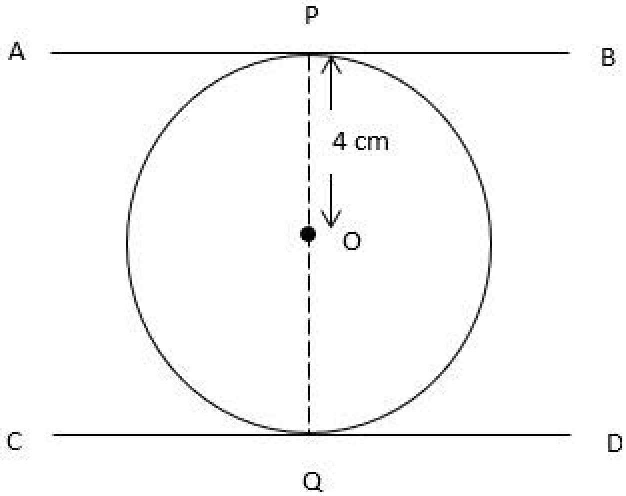
2. Question

What is the distance between two parallel tangents of a circle of radius 4 cm?

Answer

Given:

Radius of circle (say PO) = 4 cm



Let AB ∥ CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

Property: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

PQ = 2 × PO

= 2 × 4 cm

= 8 cm

Hence, Distance between tangents = 8 cm

3. Question

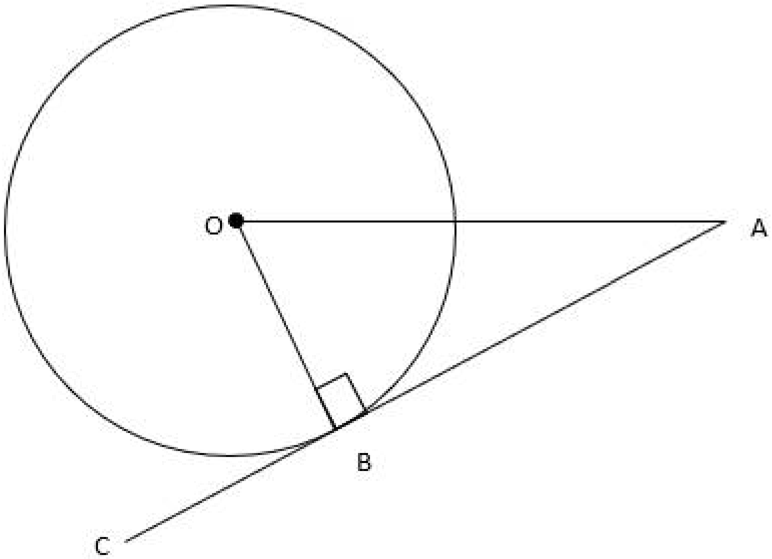
The length of tangent from a point A at a distance of 5 cm from the center of the circle is 4 cm. What is the radius of the circle?

Answer

Given:

OA (say) = 5 cm

AB = 4 cm



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

<u>*Property:*</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, ΔAOB is right-angled at ∠OBA.

Therefore, by Pythagoras Theorem,

$AB^2 + OB^2 = AO^2$

$\Rightarrow OB^2 = AO^2 - AB^2$

$\Rightarrow OB = \sqrt{(AO^2 - AB^2)}$

$\Rightarrow OB = \sqrt{(5^2 - 4^2)}$

$\Rightarrow OB = \sqrt{(25 - 16)}$

$\Rightarrow OB = \sqrt{9}$

$\Rightarrow OB = 3$ cm

Hence, Radius = 3 cm

4. Question

Two tangents TP and TQ are drawn from an external point T to a circle with center O as shown in Fig. 10.73. If they are inclined to each other at an angle of 100°, then what is the value of ∠ POQ?

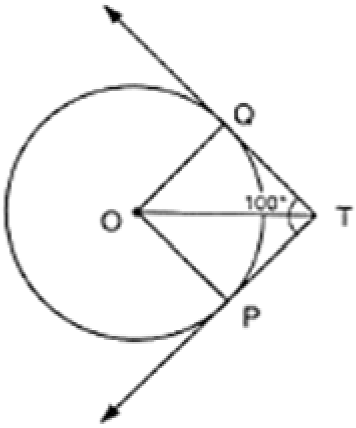


Fig. 10.73

Answer

Given:

∠QTP = 100°

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

∠OPT = 90° and ∠OQT = 90°

And,

By property 2,

∠QTP + ∠OPT + ∠OQT + ∠POQ = 360°

⇒ ∠POQ = 360° – ∠QTP + ∠OPT + ∠OQT

⇒ ∠POQ = 360° – 100° + 90° + 90°

⇒ ∠POQ = 360° – 280°

⇒ ∠POQ = 80°

Hence, ∠POQ = 80°

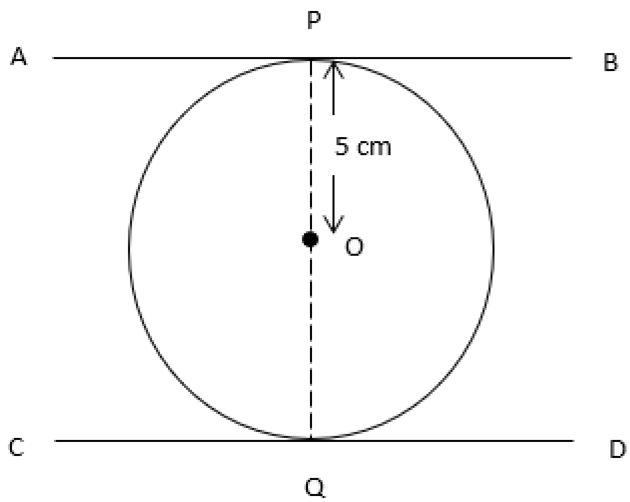
5. Question

What the distance between two parallel tangents to a circle of radius 5 cm?

Answer

Given:

Radius of circle (say PO) = 5 cm



Let AB ∥ CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

PQ = 2 × PO

= 2 × 5 cm

= 10 cm

Hence, Distance between tangents = 10 cm

6. Question

In Q. No. 1, if PB = 10 cm, what is the perimeter of Δ PCD?

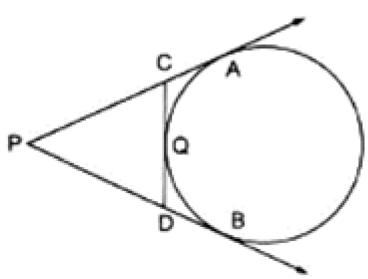


Fig. 10.72

Answer

Given:

PB = 10 cm

CQ = 2 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

PA = PB = 10 cm (tangent from P)

DB = DQ= 10 cm (tangent from D)

And,

CA = CQ= 10 cm (tangent from C)

Now,

Perimeter of ΔPCD = PC + CD + DP

= PC + CQ + QD + DP

= PC + CA + DB + PD [∵CA = CQ and DB = DQ]

= PA + PB [∵PA = PC + CA and PB = PD + BD]

= 10 cm + 10 cm

= 20 cm

Hence, Perimeter of ΔPCD = 20 cm

7. Question

In Fig. 10.74, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.

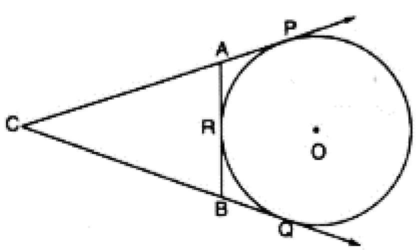


Fig. 10.74

Answer

Given:

CP = 11 cm

BC = 7 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

CP = CQ = 11 cm (tangent from C)

BQ = BR (tangent from B)

And,

AP = AR (tangent from A)

Now,

BR = BQ = CQ – CB

= 11 cm – 7 cm

= 4 cm

Hence, BR = 4 cm

8. Question

In Fig. 10.75, Δ ABC is circumscribing a circle. Find the length of BC.

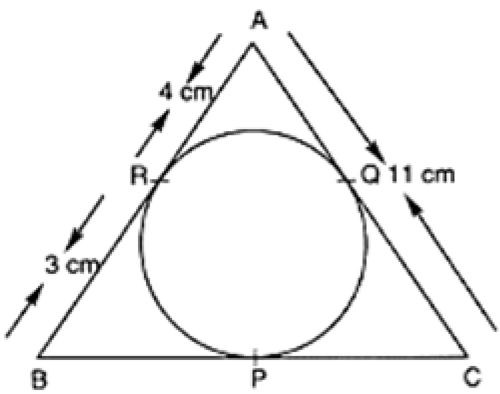


Fig. 10.75

Answer

Given:

AR = 4 cm

BR = 3 cm

AC = 11 cm

<u>*Property:*</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Using the above property,

AR = AQ = 4 cm (tangent from A)

BR= BP (tangent from B)

And,

CP = CQ (tangent from C)

Also,

CQ = CA – AQ = 11 cm – 4 cm = 7 cm

Now,

BC = BP + PC

= BR + CQ [∵ BR = BP and CP = CQ = 7 cm]

= 3 cm + 7 cm

= 10 cm

Hence, BC = 10 cm

9. Question

In Fig. 10.76, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.

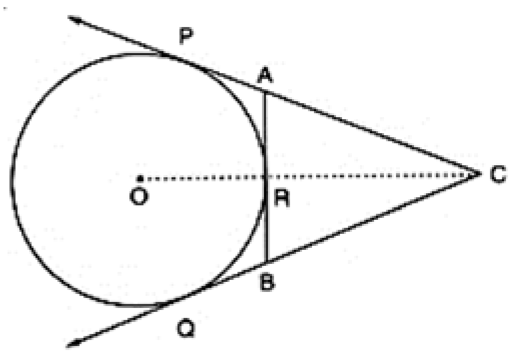


Fig. 10.76

[Hint: We have, CP = 11 cm

∴ CP = CQ = CQ = 11 cm

Now, BR = BQ [Tangents drawn from B)

⟹ BQ = 4 cm

∴ BC = CQ - BQ = (11 - 4)cm = 7 cm

Answer

Given:

BR = 4 cm

CP = 11 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

BR = BQ = 4 cm (tangent from B)

And,

CP = CQ = 11 cm (tangent from C)

Now,

BC = CQ – BQ

= 11 cm – 4 cm

=7 cm

Hence, BC = 7 cm

10. Question

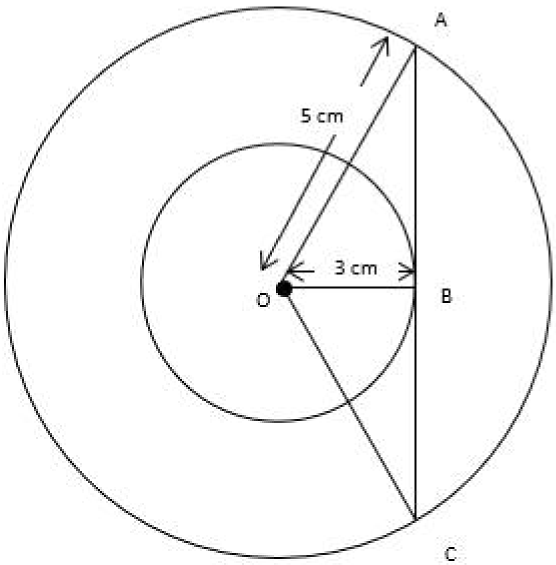
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer

Given:

AO (say) = CO (say) = 5 cm

BO (say) = 3 cm



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠OBA and ΔCOB is right-angled at ∠OBC.

Therefore,

By Pythagoras Theorem in ΔAOB,

$AB^2 + OB^2 = AO^2$

$\Rightarrow AB^2 = AO^2 - OB^2$

$\Rightarrow AB = \sqrt{(AO^2 - OB^2)}$

$\Rightarrow AB = \sqrt{(5^2 - 3^2)}$

$\Rightarrow AB = \sqrt{(25 - 9)}$

$\Rightarrow AB = \sqrt{16}$

⇒ AB= 4 cm

Similarly,

By Pythagoras Theorem in ΔCOB,

$AB^2 + OB^2 = CO^2$

⇒ $CB^2 = CO^2 - OB^2$

⇒ CB= $\sqrt{(CO^2 - OB^2)}$

⇒ CB= $\sqrt{(5^2 - 3^2)}$

⇒ CB= $\sqrt{(25 - 9)}$

⇒ CB = $\sqrt{16}$

⇒ CB= 4 cm

Now,

AC = AB + BC

= 4 cm + 4 cm

= 8 cm

Hence, Length of chord = 8 cm

11. Question

In Fig. 10.77, PA and PB are tangents to the circle with centre O such that ∠APB = 50°. Write the measure of ∠OAB

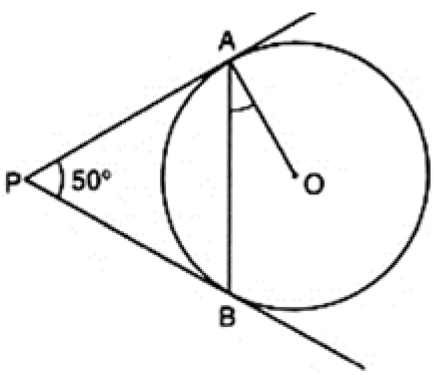


Fig. 10.77

Answer

Given:

∠APB = 50°

<u>Property 1:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a triangle = 180°.

By property 1,

AP = BP (tangent from P)

Therefore, ∠PAB = ∠PBA

Now,

By property 3 in ΔPAB,

∠PAB + ∠PBA + ∠APB = 180°

⇒ ∠PAB + ∠PBA = 180° – ∠APB

⇒ ∠PAB + ∠PBA = 180° – 50°

⇒ ∠PAB + ∠PBA = 130°

$$\Rightarrow \angle PAB = \angle PBA = \frac{130°}{2} = 65°$$

By property 2,

∠PAO = 90°

Now,

∠PAO = ∠PAB + ∠OAB

⇒ ∠OAB = ∠PAO – ∠PAB

⇒ ∠OAB = 90° – 65° = 25°

Hence, ∠OAB = 25°

1. Question

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12 cm. Length PQ is cm

A. 12 cm

B. 13 cm

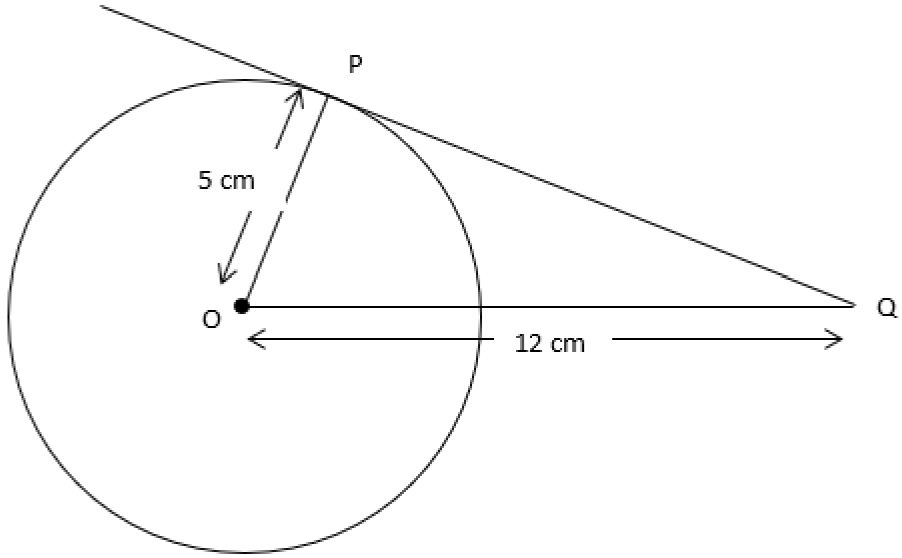
C. 8.5 cm

D. $\sqrt{119}$ cm

Answer

Given:

OQ = 12 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at ∠OPQ.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OP^2 + PQ^2 = OQ^2$

⇒ $PQ^2 = OQ^2 - OP^2$

⇒ $PQ = \sqrt{(OQ^2 - OP^2)}$

⇒ $PQ = \sqrt{(12^2 - 5^2)}$

⇒ $PQ = \sqrt{(144 - 25)}$

⇒ $PQ = \sqrt{119}$ cm

Hence, $PQ = \sqrt{119}$ cm

2. Question

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

A. 7 cm

B. 12 cm

C. 15 cm

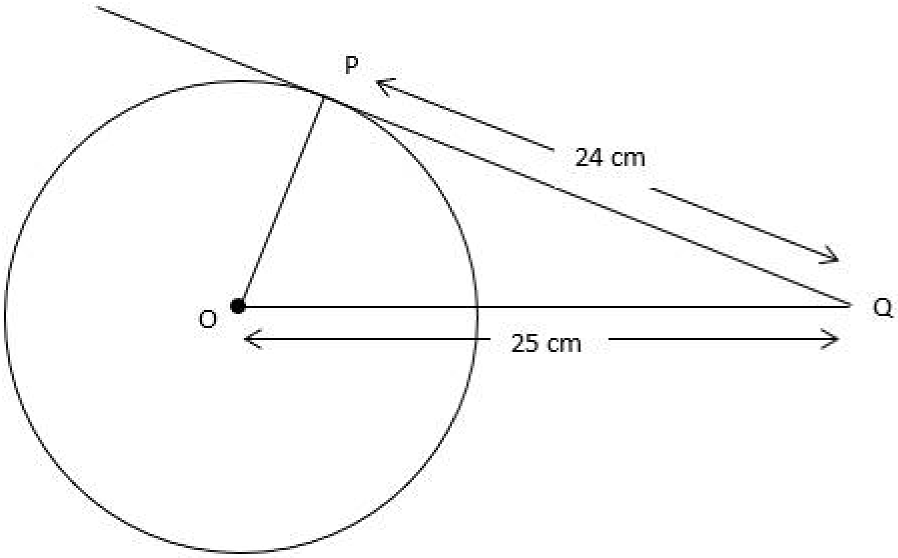
D. 24.5 cm

Answer

Given:

OQ = 25 cm

PQ = 24 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at ∠OPQ.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OP^2 + PQ^2 = OQ^2$

$\Rightarrow OP^2 = OQ^2 - PQ^2$

$\Rightarrow OP = \sqrt{(OQ^2 - PQ^2)}$

$\Rightarrow OP = \sqrt{(25^2 - 24^2)}$

$\Rightarrow OP = \sqrt{(625 - 576)}$

$\Rightarrow OP = \sqrt{49}$ cm

$\Rightarrow OP = 7$ cm

Hence, OP = 7 cm

3. Question

The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

A. $\sqrt{7}$ cm

B. 7 cm

C. 5 cm

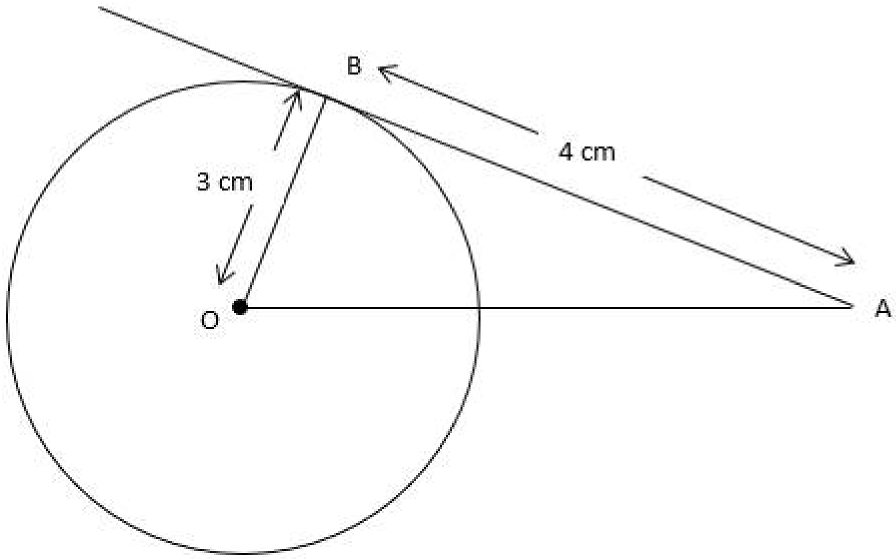
D. 25 cm

Answer

Given:

AB (say) = 4 cm

Radius (OB) = 3 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠ABO.

Therefore,

By Pythagoras Theorem in ΔPOQ,

$OA^2 = OB^2 + BA^2$

⇒ $OA = \sqrt{(OB^2 + BA^2)}$

⇒ $OA = \sqrt{(3^2 + 4^2)}$

⇒ $OA = \sqrt{(9 + 16)}$

⇒ $OA = \sqrt{25}$ cm

⇒ OA = 5 cm

Hence, distance of A from center = 5 cm

4. Question

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° then ∠POA is equal to

A. 50°

B. 60°

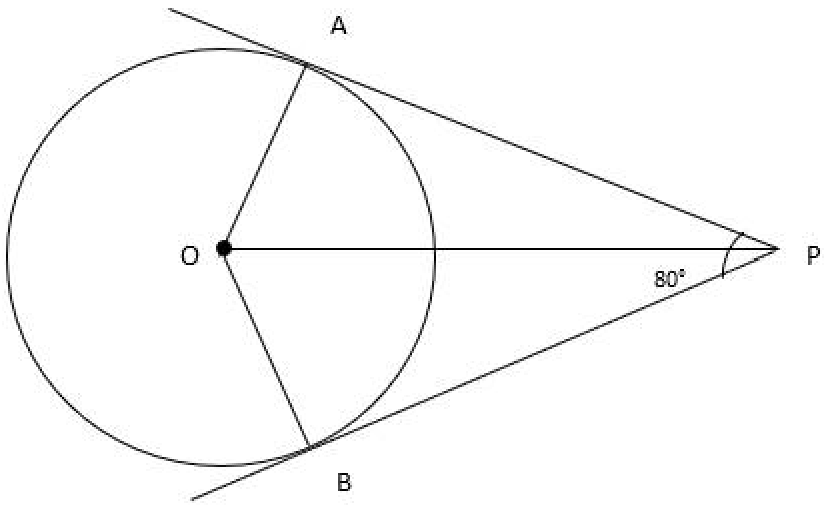
C. 70°

D. 80°

Answer

Given:

∠APB = 80°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

<u>Property 3:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1,

∠PAO = 90°

∠PBO = 90°

By property 2,

∠APB + ∠PAO + ∠PBO + ∠AOB = 360°

⇒ ∠AOB = 360° - ∠APB + ∠PAO + ∠PBO

⇒ ∠AOB = 360° - (80° + 90° + 90°)

⇒ ∠AOB = 360° - 260°

⇒ ∠AOB = 100°

Now, in ΔPOA and ΔPOB

OA = OB [∵ radius of circle]

PA = PB [By property 3 (tangent from P)]

OP = OP [∵ common]

∴ By SSS congruency,

ΔPOA ≅ ΔPOB

Hence, by CPCTC

∠POA = ∠POB

Now,

∠AOB = 100°

⇒ ∠POA + ∠POB = 100° [∵∠AOB = ∠POA + ∠POB]

⇒ ∠POA + ∠POA = 100° [∵∠POA = ∠POB]

⇒ 2∠POA = 100°

$$\Rightarrow \angle \mathrm{POA} = \frac{100°}{2}$$

⇒ ∠POA = 50°

Hence, ∠POA = 50°

5. Question

If TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then, ∠PTQ is equal to

A. 60°

B. 70°

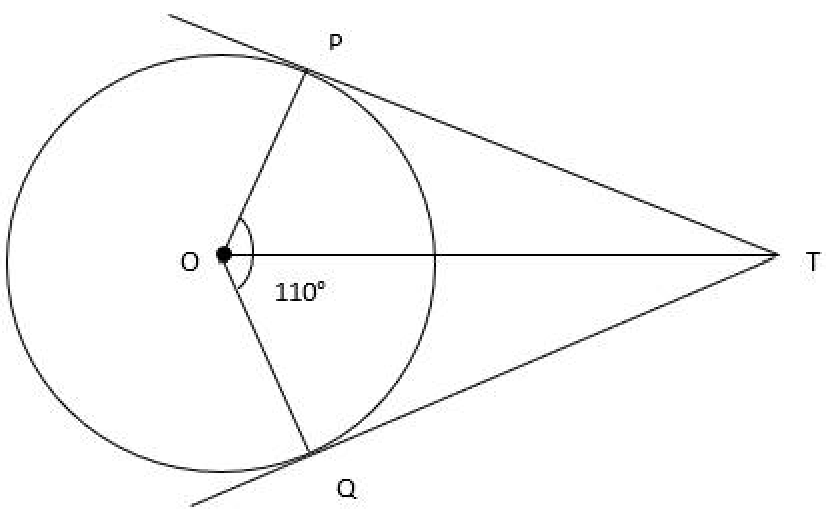
C. 80°

D. 90°

Answer

Given:

∠POQ = 110°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

∠TPO = 90°

∠TQO = 90°

By property 2,

∠POQ + ∠ TPO + ∠ TQO + ∠PTQ = 360°

⇒ ∠PTQ = 360° - ∠POQ + ∠ TPO + ∠ TQO

⇒ ∠PTQ = 360° - (110° + 90° + 90°)

⇒ ∠PTQ = 360° - 290°

⇒ ∠PTQ = 70°

Hence, ∠PTQ = 70°

6. Question

PQ is a tangent to a circle with centre 0 at the point P. If A Δ OPQ is an isosceles triangle, then ∠OQP is equal to

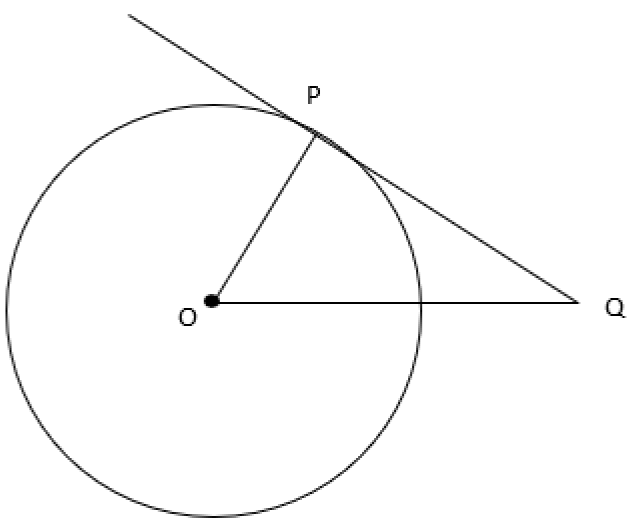
A. 30°

B. 45°

C. 60°

D. 90°

Answer



Property 1: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

Property 2: *Sum of all angles of a triangle = 180°.*

By property 1, ΔPOQ is right-angled at ∠OPQ (i.e., ∠OPQ = 90°).

∵ ΔPOQ is an isosceles triangle

∴ ∠POQ = ∠OQP

By property 2,

∠POQ + ∠OQP + ∠QPO = 180°

⇒ ∠POQ + ∠OQP = 180° - ∠QPO

⇒ ∠POQ + ∠OQP = 180° - 90°

⇒ ∠POQ + ∠OQP = 180° - 90°

⇒ ∠POQ + ∠OQP = 90°

⇒ ∠OQP + ∠OQP = 90° [∵∠POQ = ∠OQP]

⇒ 2∠OQP = 90°

$\Rightarrow \angle OQP = \frac{90°}{2}$ ⇒ ∠OQP = 45°

Hence, ∠OQP = 45°

7. Question

Two equal circles touch each other externally at C and AB is a common tangent to the circles. Then, ∠ACB =

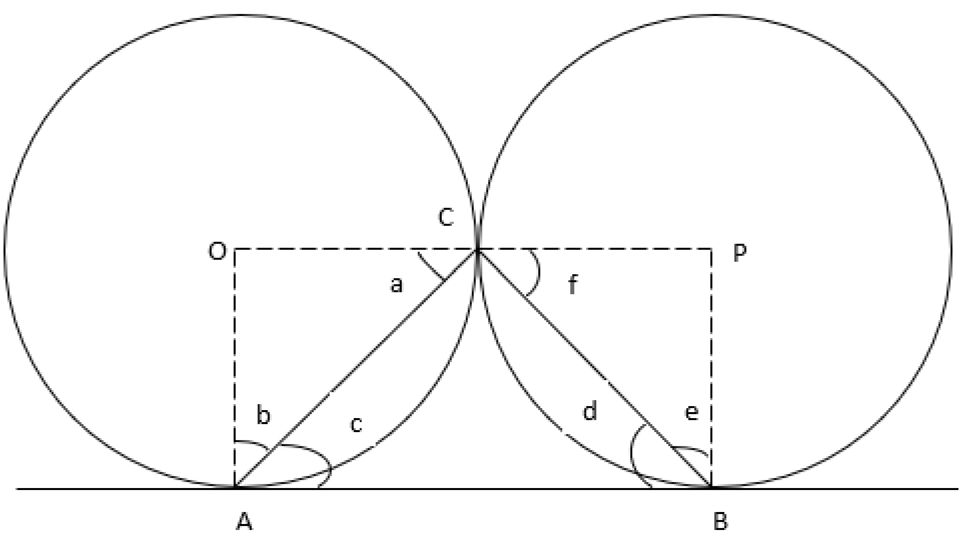
A. 60°

B. 45°

C. 30°

D. 90°

Answer



Property 1: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

Property 2: *Sum of all angles of a straight line = 180°.*

Property 3: *Sum of all angles of a triangle = 180°.*

By property 1, ΔOAB is right-angled at $\angle OAB$ (i.e., $\angle OAB = 90°$) and ΔPBA is right-angled at $\angle PBA$ (i.e., $\angle PBA = 90°$)

Clearly,

$\angle b + \angle c = \angle OAB$

$\Rightarrow \angle b + \angle c = 90°$

$\Rightarrow \angle b = 90° - \angle c$

Similarly,

$\angle d + \angle e = \angle PBA$

$\Rightarrow \angle d + \angle e = 90°$

$\Rightarrow \angle e = 90° - \angle d$

Now,

$\angle a = \angle b = 90° - \angle c$ [$\because$ OA = OC (Radius)]

And,

$\angle e = \angle f = 90° - \angle d$ [$\because$ PB = PC (Radius)]

By property 2,

$\angle a + \angle f + \angle ACB = 180°$

$\Rightarrow \angle ACB = 180° - \angle a - \angle f$

$\Rightarrow \angle ACB = 180° - (90° - \angle c) - (90° - \angle d)$

$\Rightarrow \angle ACB = 180° - 90° + \angle c - 90° + \angle d$

$\Rightarrow \angle ACB = \angle c + \angle d$

Now, in ΔACB

By property 3,

$\angle ACB + \angle c + \angle d = 180°$

$\Rightarrow \angle ACB + \angle ACB = 180°$ [$\because \angle ACB = \angle c + \angle d$]

$\Rightarrow 2\angle ACB = 180°$

$$\Rightarrow \angle ACB = \frac{180°}{2}$$

$\Rightarrow \angle ACB = 90°$

Hence, $\angle ACB = 90°$

8. Question

ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is

A. 1 cm

B. 2 cm

C. 3 cm

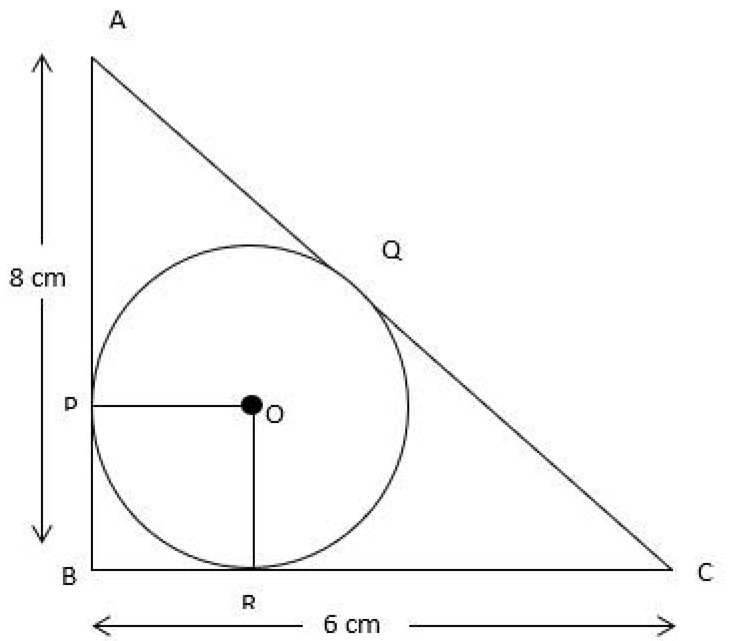
D. 4 cm

Answer

Given:

BC = 6 cm

AB = 8 cm



<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

AP = AQ (Tangent from A)

BP = BR (Tangent from B)

CR = CQ (Tangent from C)

∵ ABC is a right-angled triangle, ∴ by Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 8^2 + 6^2$

$\Rightarrow AC^2 = 64 + 36$

$\Rightarrow AC^2 = 100$

$\Rightarrow AC = \sqrt{100}$

$\Rightarrow$ AC = 10 cm

Clearly,

AQ + QC = AC = 10 cm

$\Rightarrow$ AP + RC = 10 cm [∵ AQ = AP and QC = RC]

Also,

AB + BC = 8 cm + 6 cm = 14 cm

$\Rightarrow$ AP + PB + BR + RC = 14 cm [∵ AB = AP + PB and BC = BR + RC]

$\Rightarrow$ AP + RC + PB + BR = 14 cm

$\Rightarrow$ 10 cm + BR + BR = 14 cm [∵ AP + RC = 10 cm and PB = BR]

$\Rightarrow$ 10 cm + 2BR = 14 cm

$\Rightarrow$ 2BR = 14 cm – 10 cm = 4 cm

$$\Rightarrow BR = \frac{4}{2}\text{cm}$$

$\Rightarrow$ BR = 2 cm

Now,

$\angle BPO = 90°$ [By property 3]

$\angle BRO = 90°$ [By property 3]

$\angle PBM = 90°$ [Given]

Now by property 2,

$\angle BPO + \angle BRO + \angle PBM + \angle ROP = 360°$

$\Rightarrow \angle ROP = 360° - \angle BPO + \angle BRO + \angle PBM$

$\Rightarrow \angle ROP = 360° - (90° + 90° + 90°)$

$\Rightarrow \angle ROP = 360° - 270°$

$\Rightarrow \angle ROP = 90°$

Now, ∵ $\angle ROP = 90°$ and BP = BR which are adjacent sides

∴ Quadrilateral PBRO is a square

⇒ PO = BR = 2 cm

Hence, Radius = 2 cm

9. Question

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120° , then ∠OPQ is

A. 60°

B. 45°

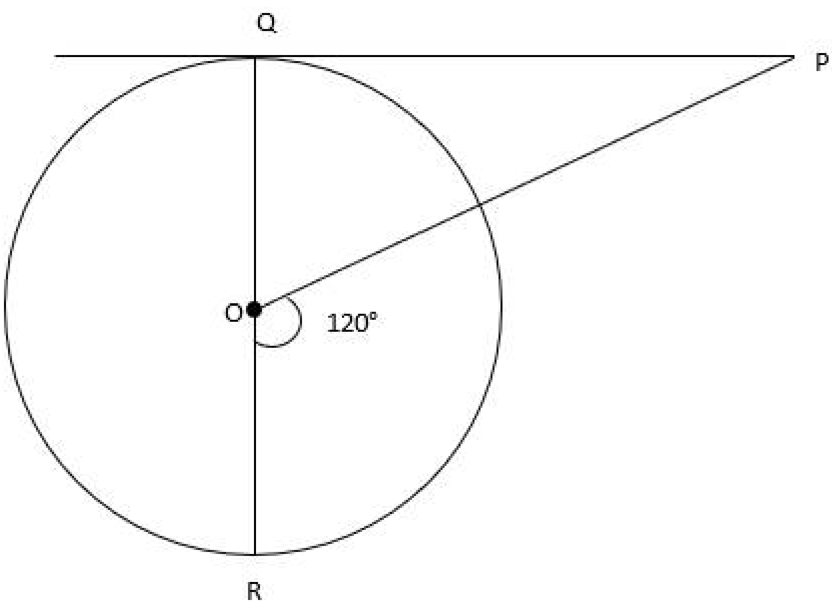
C. 30°

D. 90°

Answer

Given:

∠POR = 120°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a straight line = 180°.

<u>Property 3:</u> Sum of all angles of a triangle = 180°.

By property 1,

∠PQO = 90°

By property 2,

∠POQ + ∠POR = 180°

⇒ ∠POQ + 120° = 180°

⇒ ∠POQ = 180° - 120°

⇒ ∠POQ = 60°

Now by property 3 in ΔOPQ,

∠POQ + ∠PQO + ∠OPQ = 180°

⇒ ∠OPQ = 180° - ∠POQ + ∠PQO

⇒ ∠OPQ = 180° - (60° + 90°)

⇒ ∠OPQ = 180° - 150°

⇒ ∠OPQ = 30°

Hence, ∠OPQ = 30°

10. Question

If four sides of a quadrilateral ABCD are tangential to a circle, then

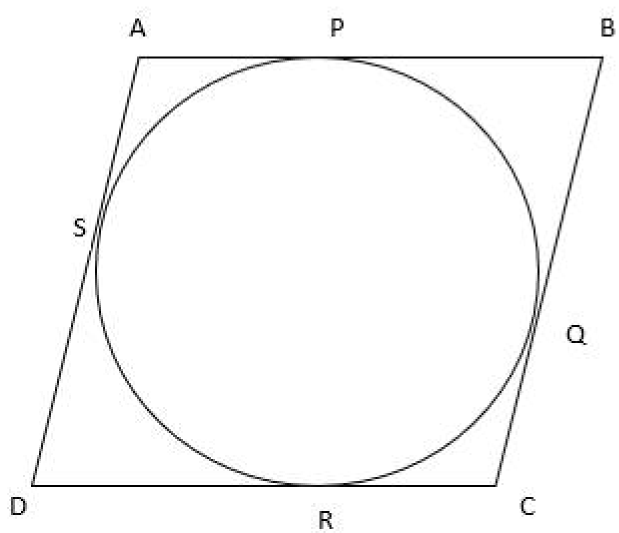
A. AC + AD = BD + CD

B. AB + CD = BC + AD

C. AB + CD = AC + BC

D. AC + AD = BC + DB

Answer



Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By the above property,

AP = AS (tangent from A)

BP = BQ (tangent from B)

CR = CQ (tangent from C)

DR = DS (tangent from D)

Now we add above 4 equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ AB + CD = AD + BC

[∵ AP + BP = AB

CR + DR = CD

AS + DS = AD

BQ + CQ = BC]

Hence, the right option is AB + CD = AD + BC

11. Question

The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

A. √7 cm

B. 2√7 cm

C. 10 cm

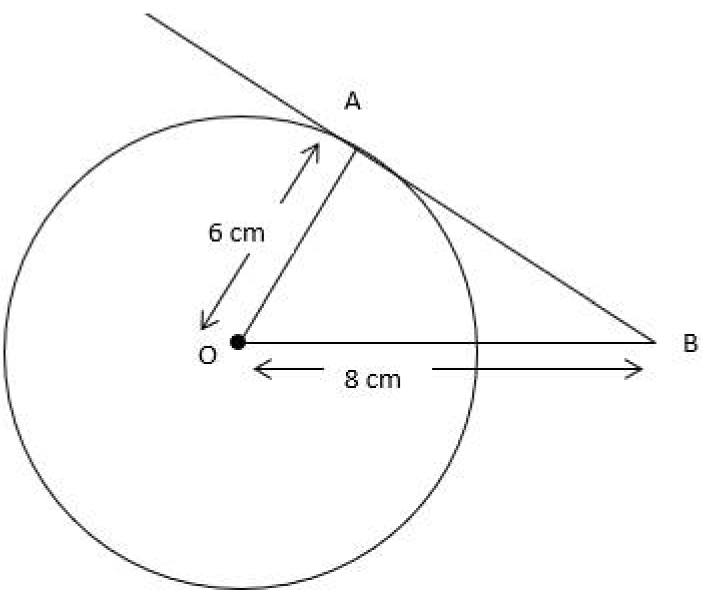
D. 5 cm

Answer

Given:

OA = 6 cm

OB = 8 cm



Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at ∠OAB (i.e., ∠OAB = 90°).

Therefore by Pythagoras theorem,

$OA^2 + AB^2 = OB^2$

$\Rightarrow AB^2 = OB^2 - OA^2$

$\Rightarrow AB^2 = 8^2 - 6^2$

$\Rightarrow AB^2 = 64 - 36$

$\Rightarrow AB^2 = 28$

$\Rightarrow AB = \sqrt{28}$

$\Rightarrow AB = 2\sqrt{7}$

Hence, length of tangent is $2\sqrt{7}$ cm.

12. Question

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to

A. 4 cm

B. 6 cm

C. 8 cm

D. 12 cm

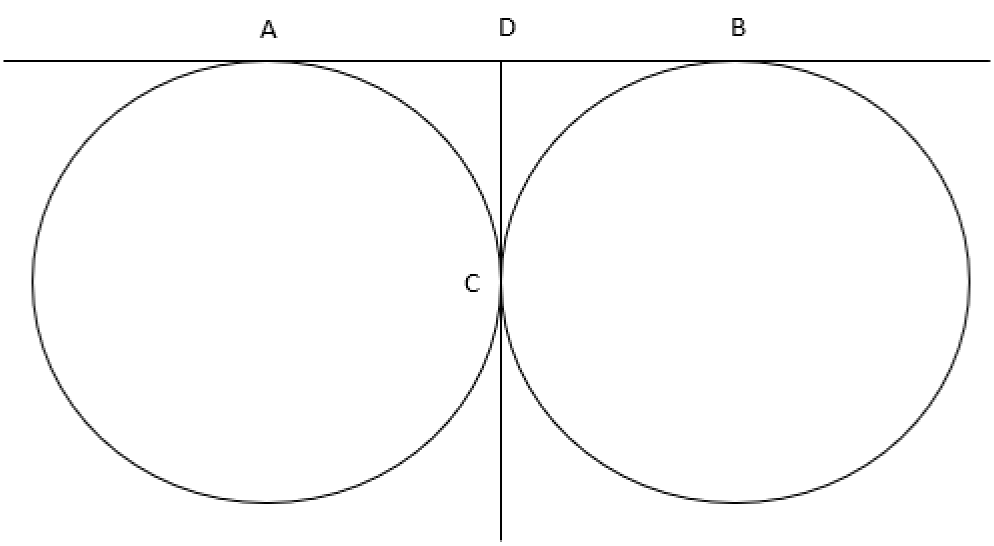
Answer

Given:

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm

To find: length of AB

Solution:



<u>Property:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Since c is the external point to the circles and two common tangents touch each other at c.

By the above property,

AD = BD = CD = 4 cm (tangent from D)

Now clearly,

AB = AD + BD

⇒ AB = AD + BD

⇒ AB = 4 cm + 4 cm

⇒ AB = 8 cm

Hence, AB = 8 cm

13. Question

In Fig. 10.78, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,

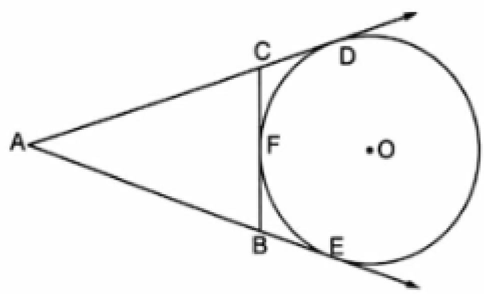


Fig. 10.78

A. AD = AB + BC + CA

B. 2AD = AB + BC + CA

C. 3AD = AB + BC + CA

D. 4AD = AB + BC + CA

Answer

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AE = AD (tangent from A)

AB = AC (tangent from A)

CD = CF (tangent from C)

BF = BE (tangent from B)

Now adding the above equations,

AB + BC + CA = AB + BF + FC + CA

⇒ AB + BC + CA = AB + BE + CD + CA

⇒ AB + BC + CA = AE + AD [∵ AE = AB + BE and AD = AC + CD]

⇒ AB + BC + CA = AD + AD [∵ AD = AE]

⇒ AB + BC + CA = 2AD

Hence, 2AD = AB + BC + CA

14. Question

In Fig. 10.79, RQ is a tangent to the circle with centre O. If SQ = 6 cm and QR = 4 cm, then OR =

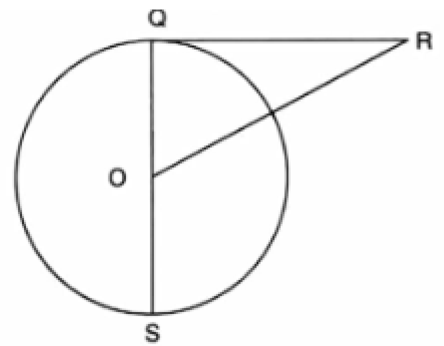


Fig. 10.79

A. 8 cm

B. 3 cm

C. 2.5 cm

D. 5 cm

Answer

Given:

SQ = 6 cm

QR = 4 cm

<u>*Property:*</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, ΔROQ is right-angled at ∠OQR (i.e., ∠OQR = 90°).

Diameter QS = 6 cm

$$\text{Radius} = \frac{\text{Diameter}}{2}$$

$$\Rightarrow \text{Radius} = \frac{6\text{ cm}}{2}$$

⇒Radius (OQ) = 3 cm

Now by Pythagoras theorem,

$OR^2 = OQ^2 + QR^2$

$\Rightarrow OR^2 = 3^2 + 4^2$

$\Rightarrow OR^2 = 9 + 16$

$\Rightarrow OR^2 = 25$

$\Rightarrow OR = \sqrt{25}$

$\Rightarrow OR = 5$ cm

Hence, OR= 5 cm.

15. Question

In Fig. 10.80, the perimeter of ΔABC is

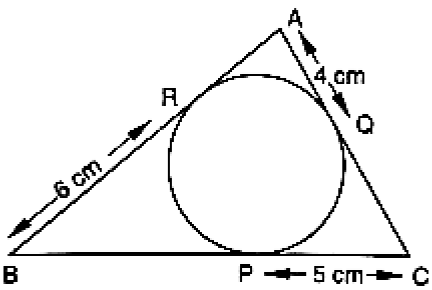


Fig. 10.80

A. 30 cm

B. 60 cm

C. 45 cm

D. 15 cm

Answer

Given:

AQ = 4 cm

BR = 6 cm

PC = 5 cm

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By the above property,

AR = AQ = 4 cm (tangent from A)

BR = BP = 6 cm (tangent from B)

CP = CQ = 5 cm (tangent from C)

Now,

Perimeter of ΔABC = AB + BC + CA

⇒ Perimeter of ΔABC = AR + RB + BP + PC + CQ + QA

[∵ AB = AR + RB

BC = BP + PC

CA = CQ + QA]

⇒ Perimeter of ΔABC = 4 cm + 6 cm + 6 cm + 5 cm + 5 cm + 4 cm

⇒ Perimeter of ΔABC = 30 cm

Hence, Perimeter of ΔABC = 30 cm

16. Question

In Fig. 10.81, AP is a tangent to the circle with centre O such that OP = 4 cm and ∠OPA = 30°. Then, AP =

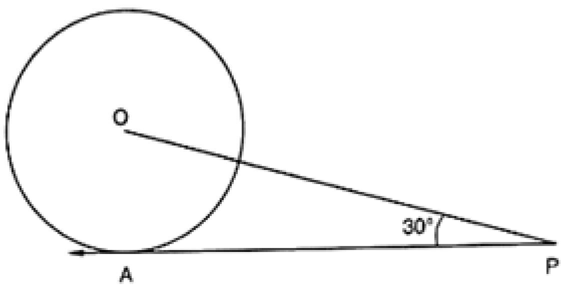


Fig. 10.81

A. 2√2 cm

B. 2 cm

C. 2√3cm

D. 3√2 cm

Answer

Given:

OP = 4 cm

∠OPA = 30°

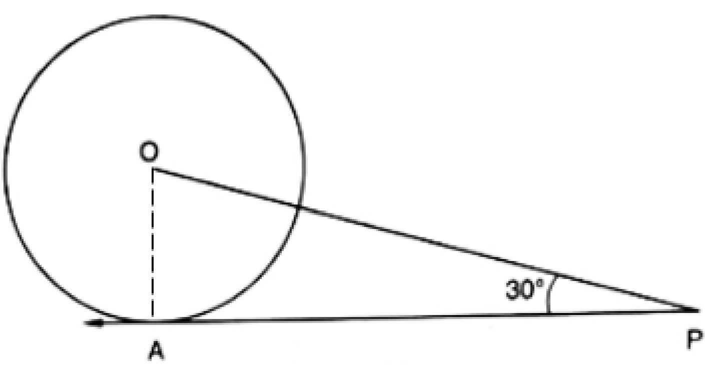


Fig. 10.81

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOA is right-angled at ∠OAP (i.e., ∠OAP = 90°).

Now we know that,

$$\text{Cos}\ \theta = \frac{\text{Base}}{\text{Hypotnuse}}$$

Therefore,

$$\text{Cos}\ \angle P = \frac{AP}{OP}$$

$$\Rightarrow \text{Cos}\ 30° = \frac{AP}{4}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{4}$$

$$\Rightarrow AP = \frac{4 \times \sqrt{3}}{2}$$

⇒ AP = 2√3 cm

Hence, AP = 2√3 cm

17. Question

AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If OA = 15 cm, then AP + AQ =

A. 12 cm

B. 18 cm

C. 24cm

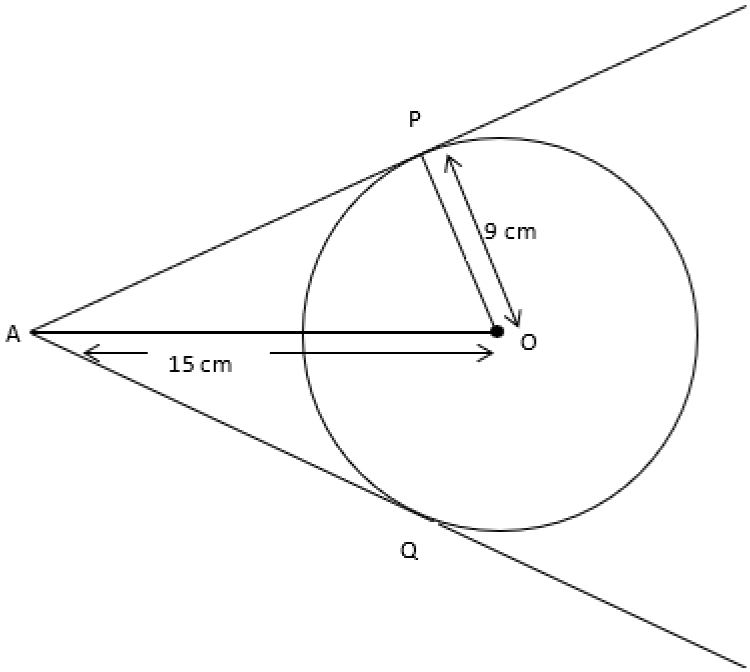
D. 36 cm

Answer

Given:

Radius = 9 cm

OA = 15 cm



Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AQ (tangent from A)

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOA is right-angled at ∠OAP (i.e., ∠OPA = 90°).

Therefore by Pythagoras theorem,

$AP^2 + PO^2 = AO^2$

$\Rightarrow AP^2 = AO^2 - PO^2$

$\Rightarrow AP^2 = 15^2 - 9^2$

$\Rightarrow AP^2 = 225 - 81$

$\Rightarrow AP^2 = 144$

$\Rightarrow AP = \sqrt{144}$

$\Rightarrow AP = 12$

AP + AQ = 12 cm + 12 cm = 24 cm

Hence, AP + AQ = 24 cm

18. Question

At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is

A. 5 cm

B. 6 cm

C. 7 cm

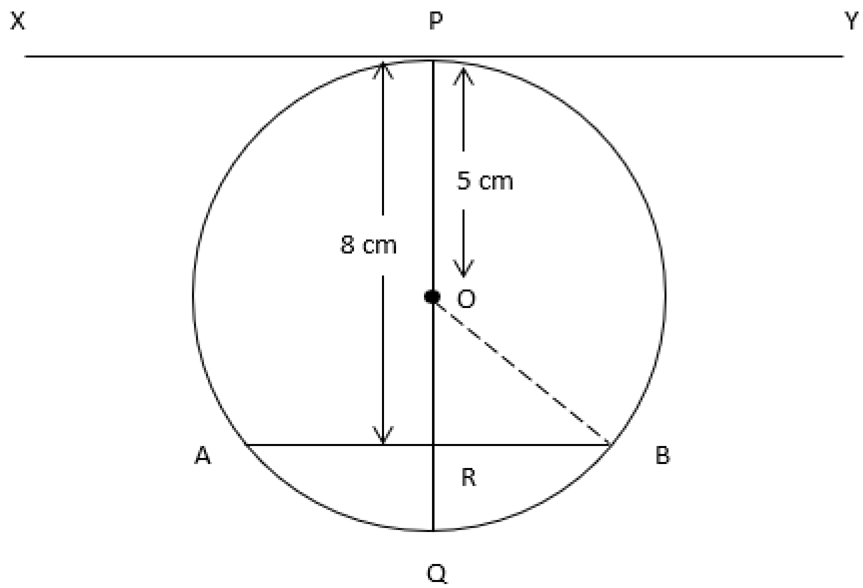
D. 8 cm

Answer

Given:

Radius = OP = 5 cm

Distance of AB and XY = 8 cm



∵ Distance of AB and XY = 8 cm

And AB is parallel to XY

∴ PR = 8 cm

Join OB

Now,

OB = OP = 5 cm [radius]

Also,

OR = PR − PO

⇒ OR = 8 cm − 5 cm

⇒ OR = 3 cm

∴ By Pythagoras theorem in ΔORB,

$OB^2 = OR^2 + RB^2$

⇒ $5^2 = 3^2 + RB^2$

⇒ $RB^2 = 5^2 - 3^2$

⇒ $RB^2 = 25 - 9$

⇒ $RB^2 = 16$

⇒ RB = 4

Now,

AB = AR + RB

⇒ AB = 2RB

⇒ AB = 2 × 4

⇒ AB = 8 cm

Hence, Length of chord = 8 cm

19. Question

If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then ∠OPT + ∠POT =

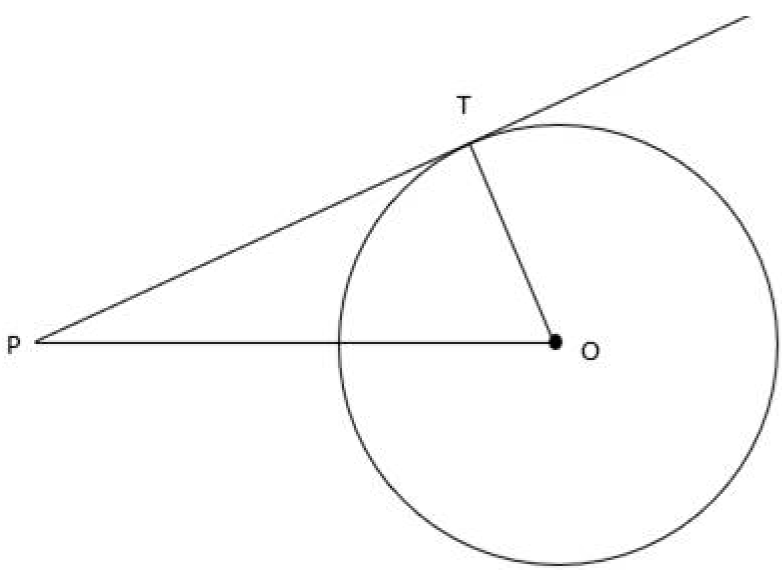
A. 30°

B. 60°

C. 90°

D. 180°

Answer



<u>Property 1:</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

<u>Property 2:</u> *Sum of all angles of a triangle = 180°*

By property 1, ΔPTO is right-angled at ∠OTP (i.e., ∠OTP = 90°).

By property 2,

∠OTP + ∠POT + ∠TPO = 180°

⇒ 90° + ∠POT + ∠TPO = 180°

⇒ ∠POT + ∠TPO = 180° - 90°

⇒ ∠POT + ∠TPO = 90°

Hence, ∠POT + ∠TPO = 90°

20. Question

In the adjacent figure, if AB = 12 cm, BC = 8 cm and AC = 10 cm, then AD =

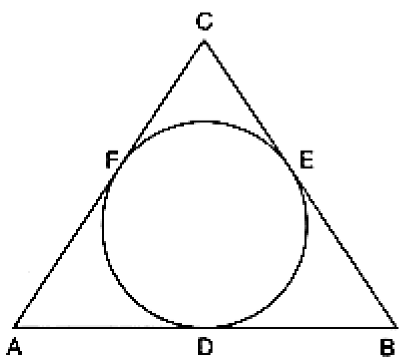


Fig. 10.82

A. 5 cm

B. 4 cm

C. 6 cm

D. 7 cm

Answer

Given:

AB = 12 cm

BC = 8 cm

AC = 10 cm

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By the above property,

AD = AF (tangent from A)

BD = BE (tangent from B)

CF = CE (tangent from C)

Clearly,

AB = AD + DB = 12 cm

BC = BE + EC = 8 cm

AC = AF + FC = 10 cm

Now,

AB − BC = 12 cm − 8 cm

⇒ (AD + DB) − (BE + EC) = 12 cm − 8 cm

⇒ AD + DB − BE − EC = 12 cm − 8 cm

⇒ AD + BE − BE − CF = 12 cm − 8 cm [∵ DB = BE and CF = CE]

⇒ AD − CF = 12 cm − 8 cm

⇒ AD − (10 cm − AF) = 12 cm − 8 cm [∵AF + FC = 10 cm ⇒ FC = 10 cm − AF]

⇒ AD − (10 cm − AF) = 4 cm

⇒ AD − 10 cm + AF = 4 cm

⇒ AD + AD = 4 cm + 10 cm [∵ AD = AF]

⇒ 2AD = 14 cm

$$\Rightarrow AD = \frac{14\text{ cm}}{2}$$

⇒ AD = 7 cm

Hence, AD = 7 cm

21. Question

In Fig. 10.83, if AP = PB, then

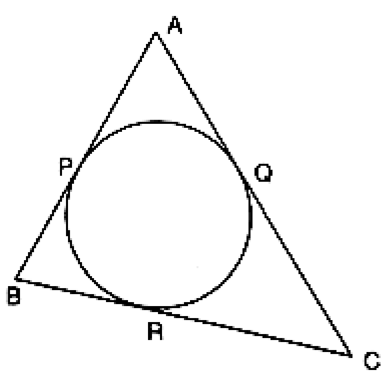


Fig. 10.83

A. AC = AB

B. AC = BC

C. AQ = QC

D. AB = BC

Answer

Given:

AP = PB

Property: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By the above property,

AP = AQ (tangent from A)

BR = BP (tangent from B)

CQ = CR (tangent from C)

Clearly,

AP = BP = BR

AQ = AP = BR

Now,

AQ + QC = BR + RC

⇒ AC = BC [∵AC = AQ + QC and BC = BR + RC]

Hence, AC = BC

22. Question

In Fig. 10.84, if AP = 10 cm, then BP =

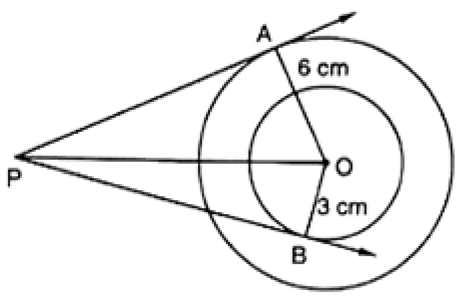


Fig. 10.84

A. $\sqrt{109}$ cm

B. $\sqrt{127}$ cm

C. $\sqrt{119}$ cm

D. $\sqrt{109}$ cm

Answer

Given:

AP = 10 cm

OA = 6 cm

OB = 3 cm

<u>Property :</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPAO is right-angled at ∠PAO (i.e., ∠PAO = 90°) and ΔPBO is right-angled at ∠PBO (i.e., ∠PBO = 90°).

Therefore by Pythagoras theorem in ΔPAO,

$OP^2 = OA^2 + AP^2$

$\Rightarrow OP^2 = 6^2 + 10^2$

$\Rightarrow OP^2 = 36 + 100$

$\Rightarrow OP = \sqrt{136}$

Now by Pythagoras theorem in ΔPBO,

$OP^2 = OB^2 + BP^2$

$BP^2 = OP^2 - OB^2$

$\Rightarrow BP^2 = (\sqrt{136})^2 - 3^2$

$\Rightarrow BP^2 = 136 - 9$

$\Rightarrow BP = \sqrt{127}$

Hence, BP= $\sqrt{127}$ cm

23. Question

In Fig. 10.85, if PR is tangent to the circle at P and Q is the centre of the circle, then ∠POQ =

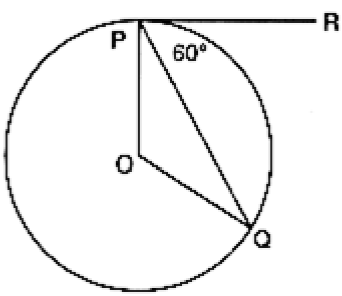


Fig. 10.85

A. 110°

B. 100°

C. 120°

D. 90°

Answer

Given:

∠RPQ = 60°

<u>Property 1:</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

<u>Property 2:</u> *Sum of all angles of a triangle = 180°.*

By property 1, ΔOPR is right-angled at ∠OPR (i.e., ∠OPR = 90°).

OP = OQ [∵ radius of circle]

∴ ∠OPQ = ∠OQP = 30°

Now by property 2,

∠OPQ + ∠OQP + ∠POQ = 180°

⇒ 30° + 30° + ∠POQ = 180°

⇒ 60° + ∠POQ = 180°

⇒ ∠POQ = 180° - 60°

⇒ ∠POQ = 120°

Hence, ⇒ ∠POQ = 120°

24. Question

In Fig. 10.86, if quadrilateral PQRS circumscribes a circle, then PD + QB =

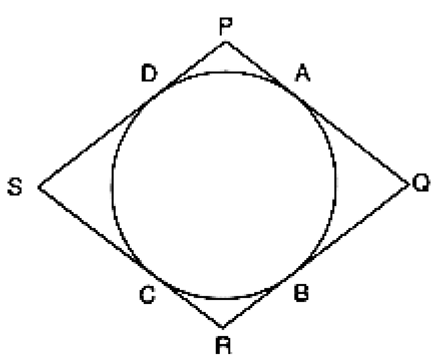


Fig. 10.86

A. PQ

B. QR

C. PR

D. PS

Answer

<u>Property:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By the above property,

PD = PA (tangent from P)

QB = QA (tangent from Q)

RC = RB (tangent from R)

SC = SD (tangent from S)

Now,

PD + QB = PA + QA

⇒ PD + QB = PQ [∵PQ = PA + QA]

Hence, PD + QB = PQ

25. Question

In Fig. 10.87, two equal circles touch each other at T, if QP = 4.5 cm, then QR =

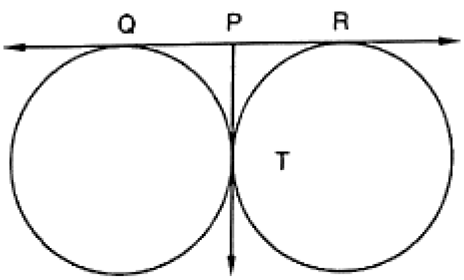


Fig. 10.87

A. 9 cm

B. 18 cm

C. 15 cm

D. 13.5 cm

Answer

Given:

QP = 4.5 cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

PQ = PT = PR = 4.5 cm (tangent from P)

Now,

QR = PQ + PR

QR = PQ + PQ [∵PQ = PR]

QR = 2PQ

QR = 2 × 4.5 cm

QR = 9 cm

Hence, QR = 9 cm

26. Question

In Fig. 10.88, APB is a tangent to a circle with centre O at point P. If ∠QPB = 500, then the measure of ∠POQ is

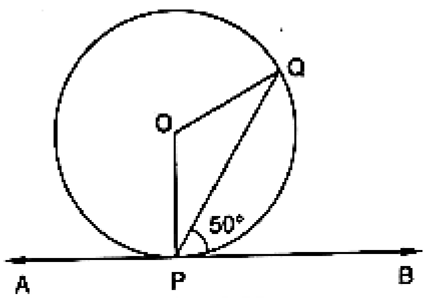


Fig. 10.88

A. 100°

B. 120°

C. 140°

D. 150°

Answer

Given:

∠QPB =50°

<u>Property 1:</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

<u>Property 2:</u> *Sum of all angles of a triangle = 180°.*

By property 1, ΔOPB is right-angled at ∠OPB (i.e., ∠OPB = 90°).

∠OPQ = ∠OPB – ∠QPB

⇒ ∠OPQ = 90° – 50° = 40°

And,

∠OPQ = ∠OQP [∵ OP = OQ (radius of circle)]

Now by property 2,

∠OPQ + ∠OQP + ∠POQ = 180°

⇒ 40° + 40° + ∠POQ = 180°

⇒ 80° + ∠POQ = 180°

⇒ ∠POQ = 180° - 80°

⇒ ∠POQ = 100°

Hence, ⇒ ∠POQ = 100°

27. Question

In Fig. 10.89, if tangents PA and PB are drawn to a circle such that ∠APS = 30° and chord AC is drawn parallel to the tangent PB, then ∠ABC =

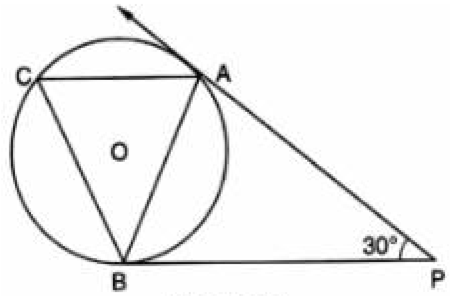


Fig. 10.89

A. 60°

B. 90°

C. 30°

D. None of these

Answer

Given:

∠APB = 30°

<u>Property 1:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

<u>Property 2:</u> *Sum of all angles of a triangle = 180°*

By property 1,

PA = PB (tangent from P)

And,

∠PAB = ∠PBA [∵PA = PB]

By property 2,

∠PAB + ∠PBA + ∠APB = 180°

⇒ ∠PAB + ∠PBA + 30° = 180°

⇒ ∠PAB + ∠PBA = 180° - 30°

⇒ ∠ PAB + ∠ PBA = 150°

⇒ ∠ PBA + ∠ PBA = 150° [∵∠PAB = ∠PBA]

⇒ 2∠PBA = 150°

$\Rightarrow \angle PBA = \dfrac{150°}{2}$

⇒ ∠PBA = 75°

Now,

∠PBA = ∠CAB = 75° [Alternate angles]

∠PBA = ∠ACB = 75° [Alternate segment theorem]

Again by property 2,

∠CAB + ∠ACB + ∠CBA = 180°

⇒ 75° + 75° + ∠CBA = 180°

⇒ 150° + ∠CBA = 180°

⇒ ∠CBA = 180° - 150°

⇒ ∠CBA = 30°

Hence, ∠CBA = 30°

28. Question

In Fig. 10.90, PR =

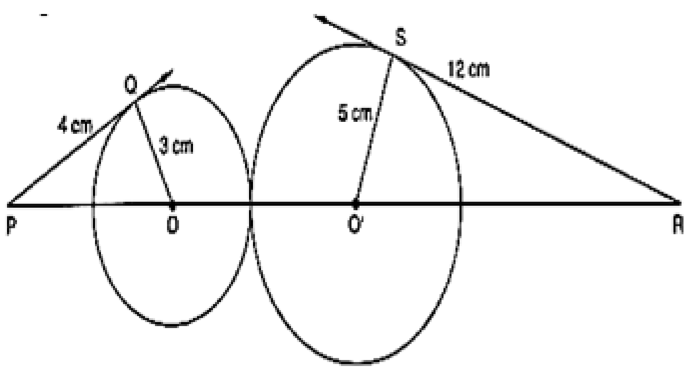


Fig. 10.90

A. 20 cm

B. 26 cm

C. 24 cm

D. 28 cm

Answer

Given:

QP = 4 cm

OQ = 3 cm

SR = 12 cm

SO’ = 5 cm

Property: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

By above property, ΔOPQ is right-angled at ∠OQP (i.e., ∠OQP = 90°) and ΔO’SR is right-angled at ∠O’SR (i.e., ∠O’SR = 90°).

By Pythagoras theorem in ΔOPQ,

$OP^2 = QP^2 + OQ^2$

⇒ $OP^2 = 4^2 + 3^2$

⇒ $OP^2 = 16 + 9$

⇒ $OP^2 = 25$

⇒ $OP = \sqrt{25}$

⇒ OP = 5 cm

By Pythagoras theorem in ΔO’SR,

$O'R^2 = SR^2 + O'S^2$

⇒ $O'R^2 = 12^2 + 5^2$

⇒ $O'R^2 = 144 + 25$

⇒ $O'R^2 = 169$

⇒ $O'R = \sqrt{169}$

⇒ $O'R^2 = 13$ cm

Now,

PR = PO + ON + NO’ + O’R

⇒ PR = 5 cm + 3 cm + 5 cm + 13 cm

⇒ PR = 26 cm

Hence, PR = 26 cm

29. Question

Two circles of same radii r and centres O and O' touch each other at P as shown in Fig. 10.91. If 00' is produced to meet the circle C (O', r) at A and AT is a tangent to the circle C(O, r) such that O'Q ⊥ AT. Then AO: AO' =

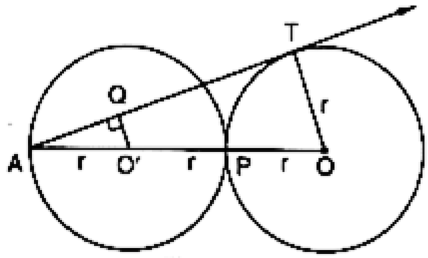


Fig. 10.91

A. 3/2

B. 2

C. 3

D. 1/4

Answer

Given:

AO′ = r

O′P = r

PO = r

AO = AO′ + O′P + PO

⇒ AO = r + r + r

⇒ AO = 3r

Now,

$$\frac{AO}{AO'} = \frac{3r}{r} = 3$$

Hence, AO: AO′ = 3

30. Question

Two concentric circles of radii 3 cm and 5 cm are given. Then length of chord BC which touches the inner circle at P is equal to

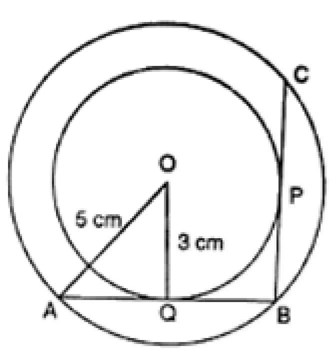


Fig. 10.92

A. 4 cm

B. 6 cm

C. 8 cm

D. 10 cm

Answer

Given:

OA = 5 cm

OQ = 3 cm

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, ΔOAQ is right-angled at ∠OQA (i.e., ∠OQA = 90°).

By Pythagoras theorem in ΔOAQ,

$OA^2 = QA^2 + OQ^2$

$\Rightarrow QA^2 = OA^2 - OQ^2$

$\Rightarrow QA^2 = 5^2 - 3^2$

$\Rightarrow QA^2 = 25^2 - 9^2$

$\Rightarrow QA^2 = 16$

$\Rightarrow QA = \sqrt{16}$

⇒ QA = 4 cm

By property 2,

BQ = BP (tangent from B)

And,

AQ = BQ = 4 cm [∵ Q is midpoint of AB]

PB = PC = 4 cm [∵ P is midpoint of BC]

Now,

BC = BP + PC

⇒ BC = 4 cm + 4 cm

⇒ BC = 8 cm

Hence, BC = 8 cm

31. Question

In Fig. 10.93, there are two concentric circles with centre O. PR and PQS are tangents to the inner circle from point plying on the outer circle. If PR = 7.5 cm, then PS is equal to

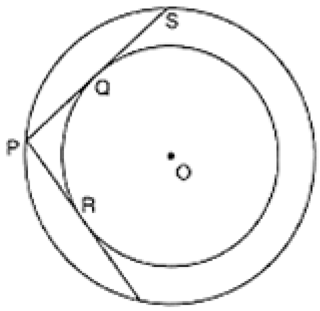


Fig. 10.93

A. 10 cm

B. 12 cm

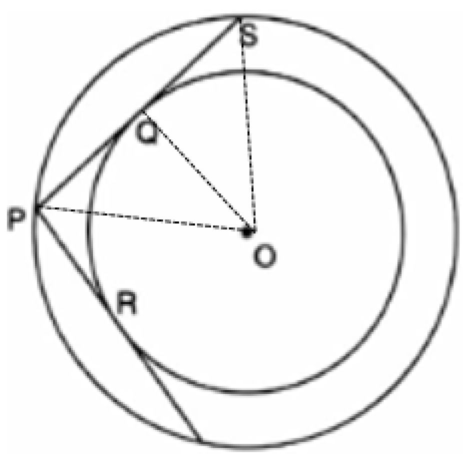
C. 15 cm

D. 18 cm

Answer

Given:

PR = 7.5 cm



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, ΔOSQ is right-angled at ∠OQS (i.e., ∠OQS = 90°) and ΔOPQ is right-angled at ∠OQP (i.e., ∠OQP = 90°).

∴ OQ ⊥ PS

∵ PO = OS [radius of circle]

∴ ΔPOS is an isosceles triangle

Now,

∵ ΔPOS is an isosceles triangle and OQ is perpendicular to its base

∴ OQ bisects PS

i.e., PQ = QS

By property 2,

PR = PQ = 7.5 cm (tangent from P)

Now,

PS = PQ + QS

⇒ PS = PQ + PQ [∵ PQ = QS]

⇒ PS = 7.5 cm + 7.5 cm

⇒ PS = 15 cm

Hence, PS = 15 cm

32. Question

In Fig. 10.94, if AB = 8 cm and PE = 3 cm, then AE =

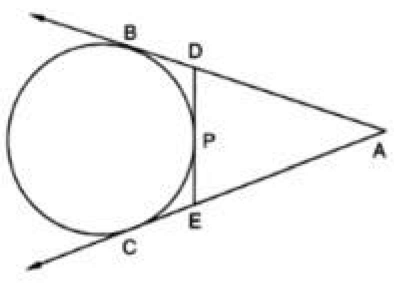


Fig. 10.94

A. 11 cm

B. 7 cm

C. 5 cm

D. 3 cm

Answer

Given:

AB = 8 cm

PE = 3 cm

<u>Property:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By above property,

AB = AC = 8 cm (tangent from A)

PE = CE = 3 cm (tangent from E)

Now,

AE = AC − CE

⇒ AE = 8 cm − 3 cm

⇒ AE = 5 cm

Hence, AE = 5 cm

33. Question

In Fig. 10.95, PQ and PR are tangents drawn from P to a circle with centre O. If ∠OPQ = 35°, then

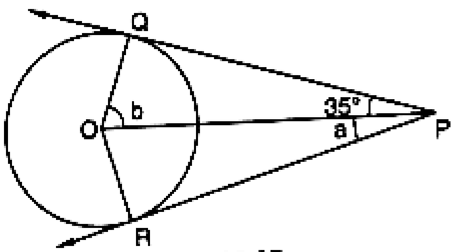


Fig. 10.95

A. a = 30°, b = 60°

B. a = 35°, b = 55°

C. a = 40°, b = 50°

D. a = 45°, b = 45°

Answer

Given:

∠OPQ = 35°

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: The sum of all angles of a triangle = 180°.

By property 1,

QP = QR (tangent from Q)

By property 2, ΔOPQ is right-angled at ∠OQP (i.e., ∠OQP = 90°) and ΔORP is right-angled at ∠ORP (i.e., ∠ORP = 90°).

∴ OQ ⊥ QP

OR⊥ RP

Now,

∠OQP = ∠ORP = 90° [Property 1]

QP = QR [Property 2]

OP = OP [Given]

∴ ΔOPQ ≅ ΔOPR By SAS

Hence, ∠OPQ = ∠OPR = 35° By CPCTC

i.e. ∠a = 35°

By property 3,

∠OQP + ∠OPQ + ∠QOP = 180°

⇒ 90° + 35° + ∠QOP = 180°

⇒ 125° + ∠QOP = 180°

⇒ ∠QOP = 180° - 125°

⇒ ∠QOP = 55°

i.e. ∠b = 55°

Hence, ∠a = 35° and ∠b = 55°

34. Question

In Fig. 10.96, if TP and TQ are tangents drawn from an external point T to a circle with centre O such that ∠TQP = 60°, then ∠OPQ =

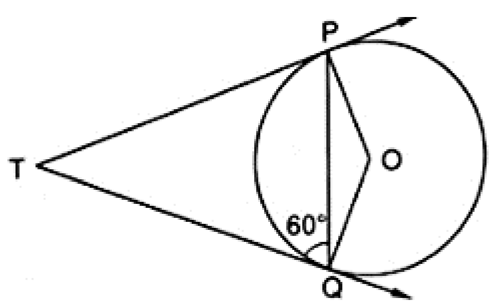


Fig. 10.96

A. 25°

B. 30°

C. 40°

D. 60°

Answer

Given:

∠TQP = 60°

<u>*Property 1:*</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By property 1,

TP = TQ (tangent from T)

⇒ ∠TPQ = ∠TQP = 60°

By property 2, ΔOPT is right-angled at ∠OPT (i.e., ∠OPT = 90°) and ΔOQT is right-angled at ∠OQT (i.e., ∠OQT = 90°).

Now,

∠OPQ = ∠OPT − ∠TPQ

⇒ ∠OPQ = 90° − 60°

⇒ ∠OPQ = 30°

Hence, ∠OPQ = 30°

35. Question

In Fig. 10.97, the sides AB, BC and CA of triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11cm, then length of BC is

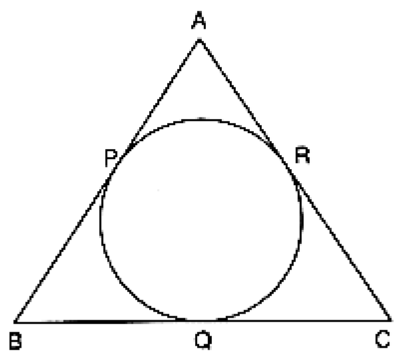


Fig. 10.97

A. 11 cm

B. 10 cm

C. 14 cm

D. 15 cm

Answer

Given:

PA = 4 cm

BP = 3 cm

AC = 11cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

AP = AR = 4 cm (tangent from A)

BP = BQ = 3 cm (tangent from B)

QC = RC (tangent from C)

Clearly,

RC = AC – AR

⇒ RC = 11 cm – 4 cm

⇒ RC = 7 cm

Now,

BC = BQ + QC

⇒ BC = BQ + RC [∵ QC = RC]

⇒ BC = 3 cm + 7 cm

⇒ BC = 10 cm

Hence, BC = 10 cm

36. Question

In Fig. 10.98, a circle touches the side DF of AEDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of ΔEDF is

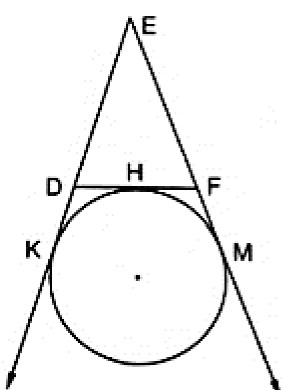


Fig. 10.98

A. 18 cm

B. 13.5 cm

C. 12 cm

D. 9 cm

Answer

Given:

EK = 9 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

EM = EK = 9 cm (tangent from E)

DK = DH (tangent from D)

FM = FH (tangent from F)

Now,

Perimeter of ΔEDF = ED + DF + FE

⇒ Perimeter of ΔEDF = (EK − KD) + (DH + HF) + (EM − MF)

[∵ED = EK − KD

DF = DH + HF

FE = EM − MF]

⇒ Perimeter of ΔEDF = EK − KD + DH + HF + EM − MF

⇒ Perimeter of ΔEDF = EK − DH + DH + HF + EM − HF [∵DK = DH and FM = FH]

⇒ Perimeter of ΔEDF = EK + EM

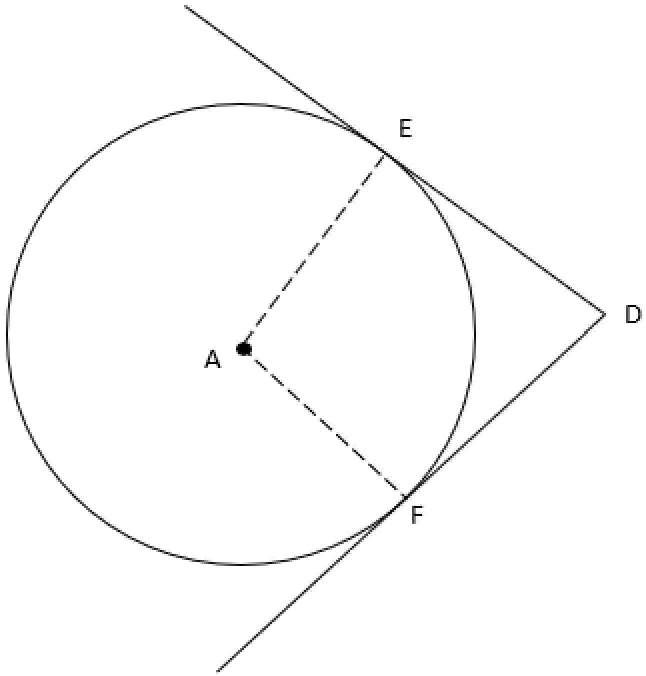
⇒ Perimeter of ΔEDF = 9 cm + 9 cm

⇒ Perimeter of ΔEDF = 18 cm

Hence, Perimeter of ΔEDF = 18 cm

37. Question

In Fig, DE and DF are tangents from an external point D to a circle with centre A. If DE = 5 cm and DE ⊥ DF, then the radius of the circle is



A. 3 cm

B. 5 cm

C. 4 cm

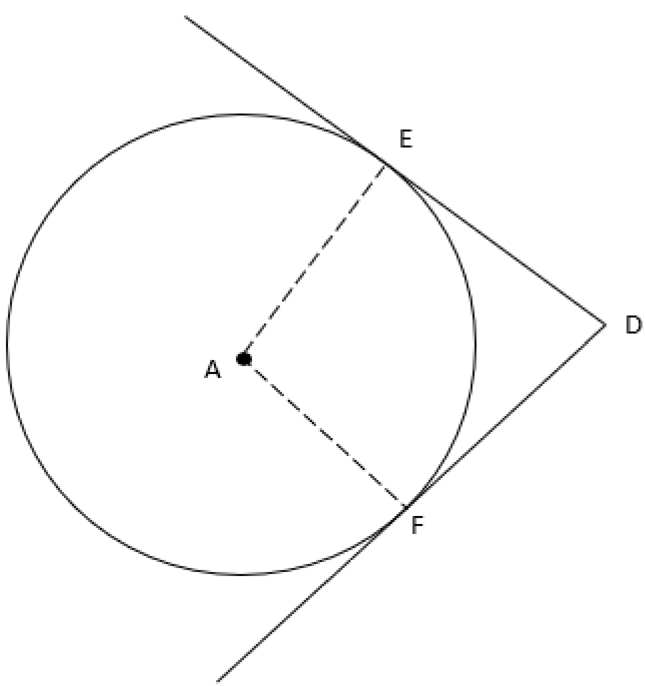
D. 6 cm

Answer

Given:

DE = 5 cm

DE $\perp$ DF



Join AE and AF

<u>Property 1:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

<u>Property 2:</u> *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

<u>Property 3:</u> *Sum of all angles of a quadrilateral = 360°.*

By property 1,

EF = ED = 5 cm(tangent from E)

And,

AE = AF [radius]

By property 2, ∠AED = 90° and ∠AFD = 90°.

Also,

∠EDF = 90° [∵ ED⊥EF]

By property 3,

∠AED + ∠AFD + ∠EDF + ∠EAF = 360°

⇒ 90° + 90° + 90° + ∠EAF = 360°

⇒ ∠EAF = 360° - (90° + 90° + 90°)

⇒ ∠EAF = 360° - 270°

⇒ ∠EAF = 90°

∵ All angles are equal and adjacent sides are equal ∴ AEDF is a square.

Hence, all sides are equal

⇒ AE = AF = ED = EF = 5 cm

Hence, Radius of circle = 5 cm

38. Question

In Fig. 10.100, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches sides BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23 cm, ∠B = 90° and DS = 5 cm, then the radius of the circle (in cm) is

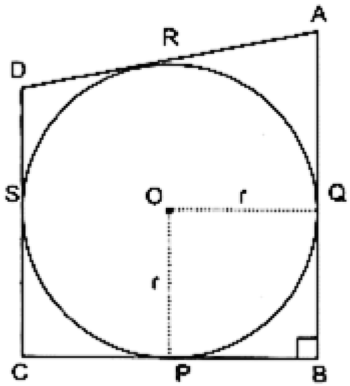


Fig. 10.100

A. 11

B. 18

C. 6

D. 15

Answer

Given:

AB = 29 cm

AD = 23 cm

∠B = 90°

DS = 5 cm

Property 1: *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

Property 2: *The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.*

Property 3: *Sum of all angles of a quadrilateral = 360°.*

By property 1,

BP = BQ (tangent from B)

DS = DR = 5 cm (tangent from D)

AR = AQ (tangent from A)

Also,

OQ = OP (radius)

By property 2, ΔOQB is right-angled at ∠OQB (i.e., ∠OQB = 90°) and ΔOPB is right-angled at ∠OPB (i.e., ∠OPB = 90°).

Now by property 3,

∠PBC + ∠BQO + ∠QOP + ∠OPB = 360°

⇒ 90° + 90° + ∠QOP + 90°= 360°

⇒ 270° + ∠QOP = 360°

⇒ ∠QOP = 360° - 270°

⇒ ∠QOP = 90°

∵ adjacent sides (i.e., BP = BQ and OQ = OP) are equal and all angles are 90°

∴ quadrilateral OPBQ is a square

Now,

AD = 23 cm

⇒ AR + RD = 23 cm [∵ AD = AR + RD]

⇒ AR + 5 cm = 23 cm

⇒ AR = 23 cm – 5 cm

⇒ AR = 18 cm

⇒ AQ = AR = 18 cm

Now,

AB = 29 cm

⇒ AQ + QB = 29 cm [∵ AD = AR + RD]

⇒ 18 cm + QB = 29 cm

⇒ QB = 29 cm – 18 cm

⇒ QB = 11 cm

∵ OPBQ is a square

∴ OP = BQ = 11 cm

Hence, radius = 11 cm

39. Question

In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is

A. 4

B. 3

C. 2

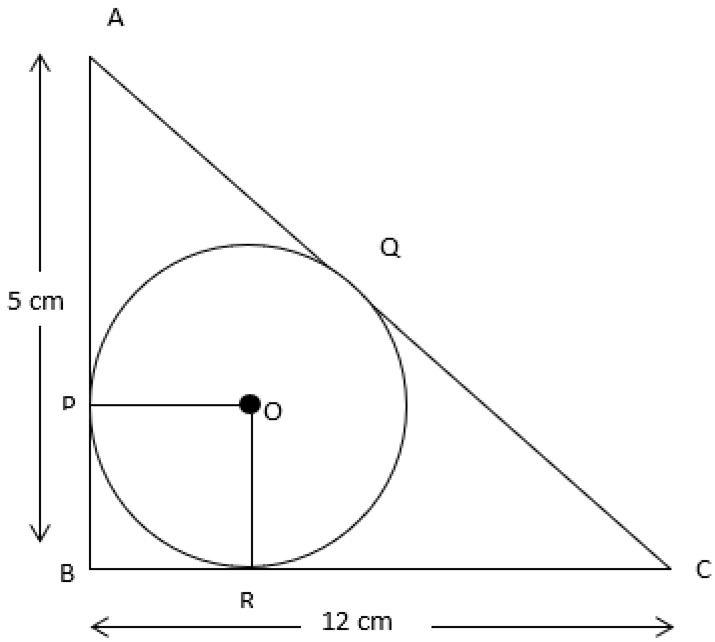
D. 1

Answer

Given:

BC = 12 cm

AB = 5 cm



<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

AP = AQ (Tangent from A)

BP = BR (Tangent from B)

CR = CQ (Tangent from C)

∵ ABC is a right-angled triangle, ∴ by Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 5^2 + 12^2$

$\Rightarrow AC^2 = 25 + 144$

$\Rightarrow AC^2 = 169$

$\Rightarrow AC = \sqrt{169}$

$\Rightarrow$ AC = 13 cm

Clearly,

AQ + QC = AC = 13 cm

$\Rightarrow$ AP + RC = 13 cm [∵ AQ = AP and QC = RC]

Also,

AB + BC = 5 cm + 12 cm= 17 cm

⇒ AP + PB + BR + RC = 17 cm [∵ AB = AP + PB and BC = BR + RC]

⇒ AP + RC + PB + BR = 17 cm

⇒ 13 cm + BR + BR = 17 cm [∵ AP + RC = 10 cm and PB = BR]

⇒ 13 cm + 2BR = 17 cm

⇒ 2BR = 17 cm – 13 cm= 4 cm

$\Rightarrow \mathrm{BR} = \frac{4}{2}\mathrm{cm}$

⇒ BR = 2 cm

Now,

∠BPO = 90° [By property 2]

∠BRO = 90° [By property 2]

∠PBM = 90° [Given]

Now by property 3,

∠BPO + ∠BRO + ∠PBM + ∠ROP = 360°

⇒ ∠ROP = 360° - (∠BPO + ∠BRO + ∠PBM)

⇒ ∠ROP = 360° - (90° + 90° + 90°)

⇒ ∠ROP = 360° - 270°

⇒ ∠ROP = 90°

Now, ∵ ∠ROP = 90° and BP = BR which are adjacent sides

∴ Quadrilateral PBRO is a square

⇒ PO = BR = 2 cm

Hence, Radius = 2 cm

40. Question

Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. The value of ∠APB is

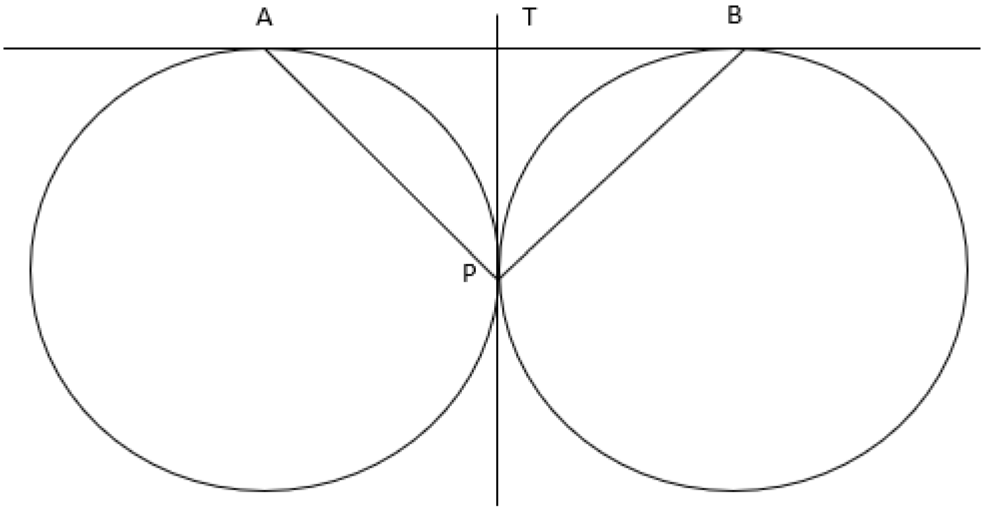
A. 30°

B. 45°

C. 60°

D. 90°

Answer



Draw a tangent from a point T on B to P.

<u>*Property 1:*</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

<u>*Property 2:*</u> *Sum of all angles of a triangle = 180°.*

By property 1,

TA = TP (tangent from T)

TB = TP (tangent from T)

Now in ΔATP,

TA = TP

∴ ∠APT = ∠PAT

And in ΔBTP,

TB = TP

∴ ∠BPT = ∠PBT

By property 2,

∠APB + ∠PBA + ∠PAB = 180°

⇒ ∠APB + ∠PBT + ∠PAT = 180°

⇒ ∠APB + ∠BPT + ∠ APT = 180° [∵ ∠APT = ∠PAT and ∠BPT = ∠PBT]

⇒ ∠APB + ∠APB = 180° [∵∠APB = ∠BPT + ∠APT]

⇒ 2∠APB = 180°

$$\Rightarrow \angle APB = \frac{180°}{2}$$

⇒ ∠APB = 90°

Hence, ∠APB = 90°

41. Question

In Fig. 10.101, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 46°, then ∠QOR equals

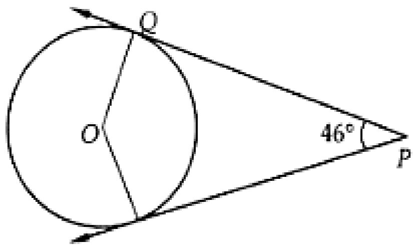


Fig. 10.101

A. 67°

B. 134°

C. 44°

D. 46°

Answer

Given:

∠QPR = 46°

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360°.

By property 1, ΔOQP is right-angled at ∠OQP (i.e., ∠OQP = 90°) and ΔORP is right-angled at ∠ORP (i.e., ∠ORP = 90°).

Now by property 2,

∠OQP + ∠ORP + ∠QOR + ∠QPR = 360°

⇒ ∠QOR = 360° - (∠OQP + ∠ORP + ∠QPR)

⇒ ∠ROP = 360° - (90° + 90° + 46°)

⇒ ∠ROP = 360° - 226°

⇒ ∠ROP = 134°

Hence, ∠ROP = 134°

42. Question

In Fig. 10.102, QR is a common tangent to the given circles touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR (in cm) is

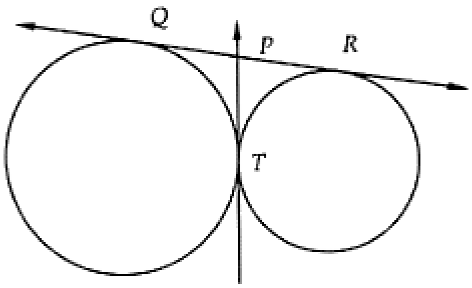


Fig. 10.102

A. 3.8

B. 7.6

C. 5.7

D. 1.9

Answer

Given:

PT = 3.8 cm

<u>Property 1:</u> *If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.*

By property 1,

PQ = PT (Tangent from P)

PR = PT (Tangent from P)

Now,

QR = PQ + PR

⇒ QR = PT + PT

⇒ QR = 3.8 cm + 3.8 cm

⇒ QR = 7.6 cm

Hence, QR = 7.6 cm

43. Question

In Fig. 10.103, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS =5 cm, then x=

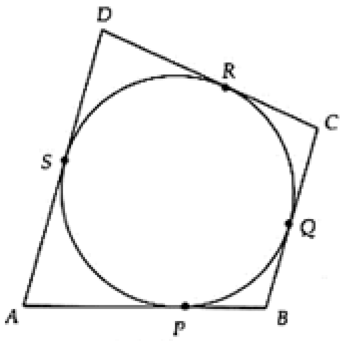


Fig. 10.103

A. 10

B. 9

C. 8

D. 7

Answer

Given:

AB = x cm

BC = 7 cm

CR = 3 cm

AS =5 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AS (tangent from A)

BP = BQ (tangent from B)

CR = CQ (tangent from C)

DR = DS (tangent from D)

Clearly,

QB = CB − CQ

⇒ QB = CB − CR [∵ CQ = CR]

⇒ QB = 7 cm − 3 cm

⇒ QB = 4 cm

Now,

AB = AP + PB

⇒ AB = AS + QB

⇒ AB = 5 cm + 4 cm

⇒ AB = 9 cm

⇒ AB = x = 9 cm

Hence, x = 9 cm