CBSE Test Paper 05

Chapter 13 Surface Area and Volume

1. The shape of a glass tumbler is usually in the form of **(1)**

   a. a cylinder
   b. a sphere
   c. a cone
   d. frustum of a cone

2. The height of a cone whose base area and volume are equal numerically is **(1)**

   a. 9 units
   b. 3 units
   c. 6 units
   d. 1 unit

3. Frustum is a _____ word **(1)**

   a. Greek
   b. None of these
   c. Latin
   d. English

4. Test tube available in a chemistry lab is a combination of **(1)**

   a. a hemisphere and a cylinder
   b. a hemisphere and a cone
   c. None of these
   d. a cone and a cylinder

5. If two solid hemispheres of same base radius 'x' cm are joined together along their bases, then the CSA of the new solid formed is **(1)**

   a. $4\pi x^2\ cm^2$
   b. $5\pi x^2\ cm^2$

c. $8\pi x^2\ cm^2$
d. $6\pi x^2\ cm^2$

6. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are formed? **(1)**

7. A cubic cm of gold is drawn into a wire of 0.1 mm in diameter, find the length of the wire. **(1)**

8. Find the area of a rhombus whose diagonals are 48 cm and 20 cm long. **(1)**

9. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere. **(1)**

10. A joker's cap is in the form of a right circular cone of base radius 7 cm and the slant height is 25 cm. Find the area of the cap. **(1)**

11. One iron solid is a cuboid of dimensions 30 cm $\times$ 30 cm $\times$ 42.6 cm. It is melted and cubes each of side 3 cm are moulded from it. Find the number of cubes formed. **(2)**

12. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed. **2**

13. In a cylindrical vessel of diameter 24 cm, filled up with sufficient quantity of water, a solid spherical ball of radius 6 cm is completely immersed. Find the increase in height of water level. **(2)**

14. Four right circular cylindrical vessels each having diameter 21 cm and height 38 cm are full of ice cream. The ice cream is to be filled in cones of height 12 cm and diameter 7 cm having a hemispherical shape on the top. Find the total number of such cones which can be filled with ice cream. **(3)**

15. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed. **(3)**

16. A toy is in the form of a cone surmounted on a hemisphere of a common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. $\left[\text{Use } \pi = \frac{22}{7}\right]$ **(3)**

17. A cone of height 24 cm has a curved surface area of 550 cm^2. Find its volume. **(3)**

18. Due to heavy floods in a state , thousands were rendered homeless . 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share their whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m , with conical upper part of same radius but of height 2.1 m . If the canvas used to make the tents cost 120 Rupees / sq. metre , then find the amount shared by each school to set up tents . What value is generated by the above problem? **(4)**

19. A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment. **(4)**

20. The sides of a triangle are in the ratio 5:12:13, and its perimeter is 150 m. Find the area of the triangle. **(4)**

CBSE Test Paper 05

Chapter 13 Surface Area and Volume

Solution

1. d. frustum of a cone

   Explanation: The shape of a glass tumbler is usually in the form of the frustum of a cone.

2. b. 3 units

   Explanation: According to the question,

   Base area of cone = Volume of cone

   $\Rightarrow \pi r^2 = \frac{1}{3}\pi r^2 h$

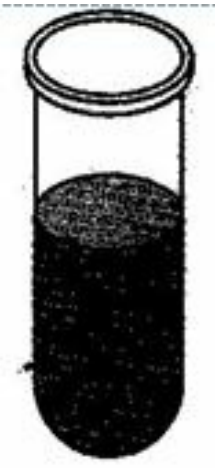
   $\Rightarrow h = 3$ units

3. c. Latin

   Explanation: Frustum is a Latin word, which means piece or bit.

4. a. a hemisphere and a cylinder

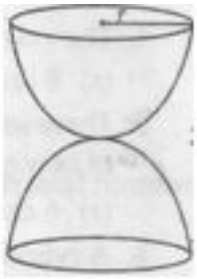
   Explanation: Test tube available in a chemistry lab is a combination of a hemisphere and a cylinder.



5. a. $4\pi x^2\ cm^2$

   Explanation: If two solid hemispheres of same base radius ‘x’ cm are joined

together along their bases, then the CSA of the new solid formed is $4\pi r^2 = 4\pi x^2$.



6. Number of spheres $= \frac{\text{Vdume of the cone}}{\text{Volume of each sphere}}$
   $= \frac{\frac{1}{3}\pi^2 h}{\frac{4}{3}\pi R^3}$
   $= \frac{(12)^2(24)}{4\times(2)^3}$
   = 108

7. Given,
   Volume of gold = 1 cm^3 = 1000 mm^3 .....(1)
   Diameter of wire = 0.1 mm
   Radius of wire = $\frac{0.1}{2}$ = 0.05 mm
   Let, the length of the wire = h mm
   According to the question,
   Volume of wire = Volume of gold
   $\Rightarrow \pi r^2 h$ = 1000 .(Since, wire is cylindrical & from (1))
   $\Rightarrow \pi \times (0.05)^2 \times h$ = 1000
   $\Rightarrow \frac{22}{7} \times 0.0025 \times h$ = 1000
   $\Rightarrow h = \frac{1000\times 7}{22\times 0.0025}$ mm
   $\Rightarrow$ h = 127272.72 mm
   $\Rightarrow$ h = 127.27 m(approx.)

8. Area of rhombus = $\frac{1}{2}\times$ Product of diagonals
   = $\frac{1}{2} \times 48 \times 20$ = 480 cm^2

9. A sphere with maximum volume that can be cut out from a solid hemisphere will be the one whose diameter will be equal to that of the radius of the hemisphere
   i.e, radius of the sphere = 1/2 radius of hemisphere = 3 cm

$V = \frac{4}{3} \times \frac{22}{7} \times 3^3 cm^3$

$= \frac{88}{21} \times 27 cm^3$ = 113.14cm^3

10. Base radius of Joker's cap = 7 cm

    Slant height = 25 cm

    Curved Surface area = $\pi rl = \frac{22}{7} \times 7 \times 25$ = 550 cm^2

11. Let n be the number of cones.

    Volume of a cuboid = $l \times b \times h = 30 \times 30 \times 42.6$ cm^3

    Volume of one cube = 3^3

    Volume of cuboid = n(volume of one cube)

    $30 \times 30 \times 42.6 = 3 \times 3 \times 3 \times n$

    $\Rightarrow n = \frac{30 \times 30 \times 42.6}{3 \times 3 \times 3}$ = 1420

12. When two identical cones are joined base to base, the total surface area of new solid becomes equal to the sum of curved surface areas of both the cones.

    So, total surface area of solid = πrl + πrl = 2πrl

    In two cones, r = 8 cm, h = 15 cm

    Now, $l^2 = r^2 + h^2 = 8^2 + 15^2 = 64 + 225 = 289$

    $\Rightarrow l^2 = (17)^2$

    $\Rightarrow$ l = 17 cm

    $\therefore$ Total surface area of solid = 2πrl

    = 2 × π × 8 × 17

    = 272π cm^2

    = 854.857 cm^2

    Hence, the surface area of new solid = 854.857 cm^2.

13. Diameter of cylindrical vessel = 24 cm

    Radius of cylindrical vessel = $\frac{24}{2}$ = 12 cm

    Let height of water level rise = h cm

    Radius of ball = 6 cm

    According to the question

    Volume of spherical ball = Volume of water rise in cylinder

$\Rightarrow \quad \frac{4}{3}\pi(6)^3 = \pi(12)^2 \times h$

$\Rightarrow \quad \frac{4}{3} \times 6 \times 6 \times 6 = 12 \times 12 \times h$

$\Rightarrow \quad h = \frac{4\times6\times6\times6}{3\times12\times12}$ = 2 cm

14. According to question it is given that

Diameter of cylinder = 21 cm,

Therefore, radius of cylinder = 21/2 cm

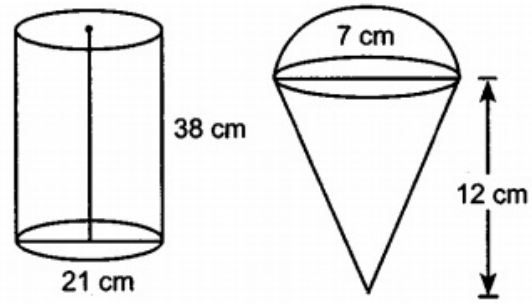
Height of cylinder = 38 cm.

Therefore, Volume of four cylindrical vessels

$= 4 \times \pi\left(\frac{21}{2}\right)^2 \times 38\text{cm}^3$

Dimensions of cone with hemispherical end:

Diameter of base of cone = diameter of hemisphere = 7 cm



Height of cone = 12cm

Volume of one ice cream cone = volume of conical part + volume of hemispherical part

$= \frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \times 12 + \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$

$= \frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \left[12 + 2 \times \frac{7}{2}\right]$

$= \frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \times 19$

Total number of ice cream cones

$= \frac{\text{volume of four cylindrical vessels}}{\text{volume of one ice cream cone}}$

$= \frac{4\pi\left(\frac{21}{2}\right)^2 \times 38}{\frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \times 19} = \frac{4\times3\times441\times2}{49}$ = 216

Hence, 216 cones can be filled.

15. Diameter of sphere = 16 cm

$\therefore \quad \text{radius } = 8\text{cm}$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 8 \times 8 \times 8\text{cm}^3$

Radius of cone = 4cm and height of cone 8 cm

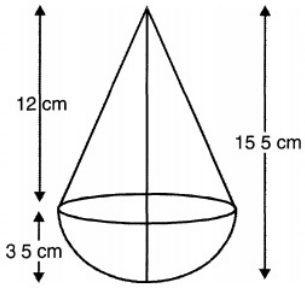
Volume of each cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 4 \times 4 \times 8\text{cm}^3$

Let, number of cones recasted be n

$\therefore \quad n = \frac{\text{Volume of Sphere}}{\text{Volume of One Cone}} = \frac{\frac{4}{3}\times\pi\times8\times8\times8}{\frac{1}{3}\times\pi\times4\times4\times8} = 16$

Hence, number of recasted cones = 16

16. It is given that a toy is in the form of a cone surmounted on a hemisphere of a common base of diameter 7 cm. If the height of the toy is 15.5 cm, we have to find the total surface area of the toy.



radius, r = 3.5 cm

height, h = 12 cm

Slant height of cone, $l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 3.5^2} = 12.5$

Total surface area of the toy = Surface area of hemisphere + Curved surface area of cone

$= 2\pi r^2 + \pi r l$

$= \pi r\left(2r + l\right)$

$= \frac{22}{7} \times \frac{7}{2}\left(2 \times \frac{7}{2} + 12.5\right)$

$= 11 \times 19.5$

$= 214.5\text{cm}^2$

17. Height of the cone = 24 cm

Let radius be r cm and slant height be l cm.

$\therefore \quad l = \sqrt{h^2 + r^2} \Rightarrow l = \sqrt{24^2 + r^2}$

$\Rightarrow \quad l = \sqrt{576 + r^2}$

Now, curved surface area = 550 cm^2

$\Rightarrow \quad \pi r l = 550$

$\Rightarrow \quad \frac{22}{7} \times r \times \sqrt{576 + r^2} = 550$

$\Rightarrow \quad r \times \sqrt{576 + r^2} = 175$

On squaring both sides, we get

$r^2(576 + r^2) = 30625$

$\Rightarrow r^4 + 576r^2 - 30625 = 0$

$\Rightarrow r^4 + 625r^2 - 49r^2 - 30625 = 0$

$\Rightarrow r^2(r^2 + 625) - 49(r^2 + 625) = 0$

$\Rightarrow (r^2 + 625)(r^2 - 49) = 0$

$\Rightarrow r^2 = -625$ or $r^2 = 49$

rejecting $r^2 = -625$

we get, $r^2 = 49 \Rightarrow r = 7\text{cm}$

Now, volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

= 1232 cm^3

18. Radius of cylindrical part and cone part of tent, r = 2.8m

height of cylindrical part of tent, H = 3.5m

height of conical part of tent, h = 2.1m

Number of tents to be made = 1500

Cost of canvas for making tent = 120 rupees/sq.m

Number of school = 50

Canvas required for one tent = Curved surafce area of cylinder + Curved surface of cone = $2\pi rh + \pi rl$

where, $l = \sqrt{h^2 + r^2}$

$l = \sqrt{(2.1)^2 + (2.8)^2}$

$\therefore l = 3.5$

$\therefore$ Canvas required for one tent = $92.4\ sq.m$

So, canvas required for 1500 tents = 1500 × canvas required for one tent.

= 92.4 × 1500 = 1,38,600 sq.m

Thus, cost of the canvas = 120 × canvas required for 1500 tents = Rs 1,66,32,000

Hence, amount shared by each school$= \frac{cost}{50} = \frac{16632000}{50} = Rs\ 332640$

Values schools feel their responsibility for helping government in its efforts to face natural calamities like floods, etc. And, also want their students to be aware of the

causes of such natural calamities and how we could help in such situation.

19. According to the question, A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment
The volume of soil taken out from the well
$= \pi^2 rh$
$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14\text{m}^3$
The radius of embankment with well $= \frac{3}{2} + 5 = \frac{13}{2}m$
Let the height of embankment be x
$\therefore$ The volume of soil used in embankment be x
$\pi \left(R^2 - r^2\right) x = \pi r^2 h$
$\Rightarrow \frac{22}{7}\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] x = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14$
$\Rightarrow \quad \frac{160}{4} x = \frac{3}{2} \times \frac{3}{2} \times 14$
$\Rightarrow \quad x = \frac{3 \times 3 \times 14 \times 4}{2 \times 2 \times 160}$ = 0.7875 m
Hence the height of embankment = 78.75 cm

20. According to question given sides are in the ratio of 5 : 12 : 13
On dividing 150 m in the ratio 5 : 12 : 13, we get
Length of one side $= \left(150 \times \frac{5}{30}\right) m = 25m$
Length of the second side = $\left(150 \times \frac{12}{30}\right) \text{m} = 60\text{m}$
Length of third side = $\left(150 \times \frac{13}{30}\right) m = 65m$
Let a = 25 m, b = 60 m, c = 65 m
Then, $s = \frac{1}{2}(25 + 60 + 65)m = 75m$
Now (s - a) = 75 cm - 25 cm = 50 cm
(s - b) = 75 cm - 60 cm = 15 cm
(s - c) = 75 cm - 65 cm = 10 cm
Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{75 \times 50 \times 15 \times 10}\text{m}^2$ =750 m^2
Hence, area of the triangle = 750 m^2