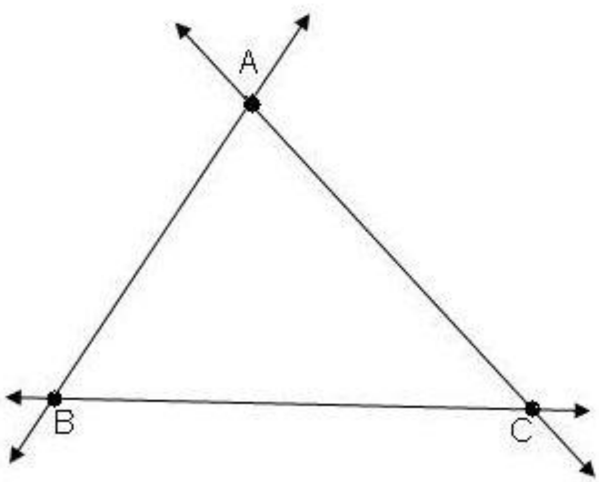
Lines And Angles

Exercise – 1.1

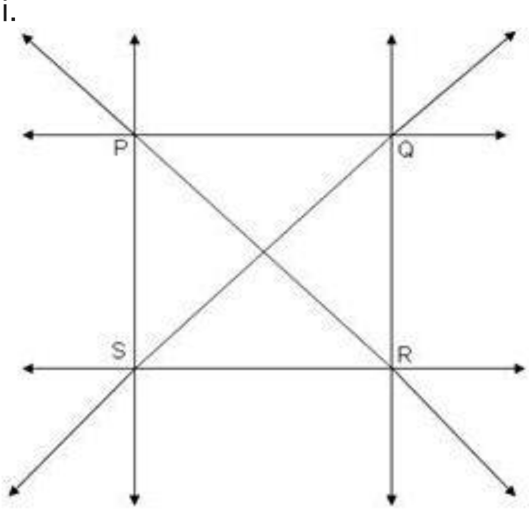
Solution 1:



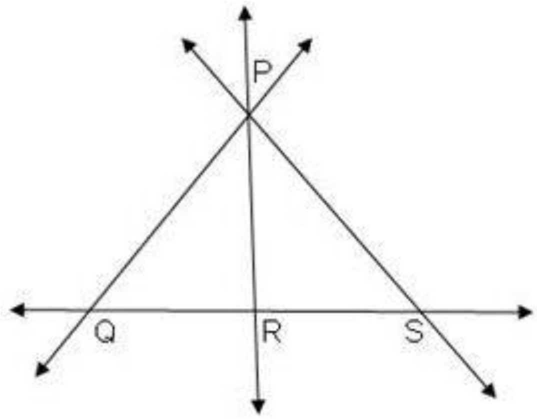
Three lines can be drawn through three non-collinear points.
Line AB, line BC and line AC are the required lines.

Solution 2:

i.



Six lines can be drawn through four given points such that no three points are collinear.
The six lines are line PQ, line QR, line RS, line SP, line PR, and line QS.
ii.
Let Q, R and S be the three collinear points.



Four lines can be drawn through given four points such that three points are collinear.
The four lines are line PQ, line PS, line PR, and line QS.

Solution 3:

The sets of collinear points are:
1. P, F, R, B
2. P, S, T, Q
3. A, R, E, Q
4. A, F, S, D
5. B, E, T, D

Solution 4:

i. Line PQ, line SR, and line DC are parallel to line AB.
ii. Yes, line AD and point R lie in the same plane.
[There is exactly one plane passing through a line and a point, not on the line (axiom)].
iii. Yes, points A, S, R, and B are coplanar (since these points lie in the same plane ASRB).
iv. Plane APSD, plane APQB and plane ABCD pass through point A.
v. Points A, S, R, B, and V.

Exercise – 1.2

Solution 1:

i. Co-ordinates of points C, S, Q, and D are -3, 4, 2 and -4 respectively.
ii. The points whose co-ordinates are 4, 5, 0, and -2 are S, T, O, and B respectively.
iii. d(Q, T)
Co-ordinate of Q is 2 and co-ordinate of T is 5, $2 < 5$
$\therefore d(Q, T) = 5 - 2 = 3$
d(E, B)

Co-ordinate of E is -5 and co-ordinate of B is -2, -5 < -2
∴d(E, B) = -2 – (-5) = -2 + 5 = 3
d(O, C)
Co-ordinate of O is 0 and co-ordinate of C is -3, -3 < O
∴d(O, C) = 0 – (-3) = 3
d(O, R)
Co-ordinate of O is 0 and co-ordinate of R is 3, 0 < 3
∴d(O, R) = 3 – 0 = 3
iv. There are two cases :
a. The point can be towards the positive side i.e. point S (∵ d(O, S) = 4 – 0 = 4)
b. The point can be towards the negative side i.e. point D (∵d(O, D) = 0 – (-4) = 4)

Solution 2:

i. Co-ordinate of point P is x = 7
Co-ordinate of point Q is y = 10
10 > 7
∴d(P, Q) = 10 – 7 = 3
∴d(P, Q) = 3
ii. Co-ordinate of point P is x = -2
Co-ordinate of point Q is y = 11
11 > -2
∴d(P, Q) = 11 – (-2) = 11 + 2 = 13
∴d(P, Q) = 13
iii. Co-ordinate of point P is x = -8
Co-ordinate of point Q is y = -3
-3 > -8
∴d(P, Q) = -3 -(-8) = -3 + 8 = 5
∴d(P, Q) = 5
iv. Co-ordinate of point P is x = 5
Co-ordinate of point Q is y = -9
5 > -9
∴d(P, Q) = 5-(-9) = 5 + 9 = 14
∴d(P, Q) = 14

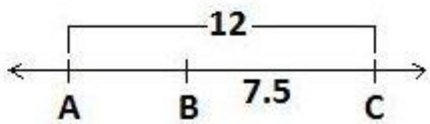
Solution 3:

If points P, Q and R are three distinct collinear points and if d(P, Q) + d(Q, R) = d(P, R), then the point Q is said to be between the points P and R. When point Q is between the points P and R, we write P – Q – R to represent the betweenness among P, Q and R.
i. d(A, B) + d(B, D) = 5 + 8 = 13
d(A, D) = 11
∴d(A, B) + d(B, D) ≠d(A,D)
∴There is no betweenness among the points A, B and D
ii. d(B, D) + d(A, D) = 6 + 5 = 11

d(A, B) = 11
∴d(B, D) + d(A, D) = d(A, B)
∴There exists a betweeness among the points A, B, and D.
The point D lies between A and B.
Hence we write, A – D – B.
iii. d(A, B) + d(B, D) = 2 + 15 = 17
d(A, D) = 17
∴d(A, B) + d(B, D) = d(A, D)
∴There exists a betweenness among the points A, B and D.
The point B lies between A and D.
Hence we write, A – B – D.

Solution 4:

i.



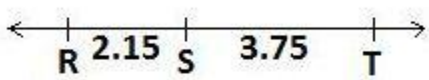
$l(AB) + l(BC) = l(AC)$ (∵ A-*B*-*C*)

$\therefore l(AB) + 7.5 = 12$

$\therefore l(AB) = 12 - 7.5$

$\therefore l(AB) = 4.5$ units

ii.

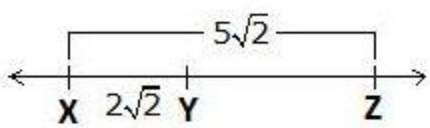


$l(RS) + l(ST) = l(RT)$ (∵ R-*S*-*T*)

$\therefore 2.15 + 3.75 = l(RT)$

$\therefore l(RT) = 5.9$ *units*

iii.



$l(XY) + l(YZ) = l(XZ)$ (Q X - Y - Z)

$2\sqrt{2} + l(YZ) = 5\sqrt{2}$

$\therefore l(YZ) = 5\sqrt{2} - 2\sqrt{2}$

$\therefore l(YZ) = 3\sqrt{2}$ units

Solution 5:

l(PL) + l(LN) = l(PN) (P-L-N)
∴ l(PL) + 5 = 11
∴ l(PL) =11 – 5
∴ l(PL) = 6 units
l(MN) + l(NR) = l(MR) (M-N-R)
∴7 + l(NR) = 13
∴ l(NR) = 13 – 7
∴ l(NR) = 6 units
l(LM) + l(MQ) = l(LQ) (L-M-Q)
∴6 + 2 = l(LQ)
∴ l(LQ) = 8 units

Solution 6:

l(AB) + l(BC) = l(AC) (A-B-C)
∴ l(AB) + 5 = 8
∴ l(AB) = 8 – 5
∴ l(AB) = 3 units … (i)
seg AC ≅ seg BD (given)
l(BD) = 8
l(BC) + l(CD) = l(BD) (B-C-D)
∴5 + l(CD) = 8
∴ l(CD) = 8 – 5
∴ l(CD) = 3 units … (ii)
seg BD ≅ seg CE (given)
l(CE) = 8
l(CD) + l(DE) = l(CE) (C-D-E)
∴3 + l(DE) = 8
∴ l(DE) = 8 – 3
∴ l(DE) = 5 units … (iii)
l(BC) = l(DE) = 5 units [from (iii) and given that l(BC) = 5]
∴seg BC ≅ seg DE
l(AB) = l(CD) = 3 units [from(i) and (ii)]
∴seg AB ≅ seg CD

Solution 7:

Co-ordinate of point P is -3
Co-ordinate of point Q is 5
5 > -3
∴d(P, Q) = 5 – (-3) = 5 + 3 = 8 units
∴ l(PQ) = 8 units

Co-ordinate of point P is -3
Co-ordinate of point R is 2
2 > -3
∴d(P, R) = 2 – (-3) = 2 + 3 = 5 units
∴ l(PR) = 5 units
Co-ordinate of point P is -3
Co-ordinate of point S is -7
-3 > -7
∴d(P, S) = -3 – (-7) = -3 + 7 = 4 units
∴ l(PS) = 4 units
Co-ordinate of point P is -3
Co-ordinate of point T is 9
9 > -3
∴d(P, T) = 9 – (-3) = 9 + 3 = 12 units
∴ l(PT) = 12 units
Co-ordinate of point Q is 5
Co-ordinate of point R is 2
5 > 2
∴d(Q, R) = 5 – 2= 3 units
∴ l(QR) = 3 units
Co-ordinate of point Q is 5
Co-ordinate of point S is -7
5 > -7
∴d(Q, S) = 5 – (-7) = 5 + 7 = 12 units
∴ l(QS) = 12 units
Co-ordinate of point Q is 5
Co-ordinate of point T is 9
9 > 5
∴d(Q, T) = 9 – (5) = 4 units
∴ l(QT) = 4 units
Co-ordinate of point R is 2
Co-ordinate of point S is -7
2 > -7
∴d(R, S) = 2 – (-7) = 2 + 7 = 9 units
∴ l(RS) = 9 units
Co-ordinate of point R is 2
Co-ordinate of point T is 9
9 > 2
∴d(RT) = 9 – 2 = 7 units
∴ l(RT) = 7 units
Co-ordinate of point S is -7
Co-ordinate of point T is 9
9 > -7
∴d(S, T) = 9 – (-7) = 9 + 7 = 16 units
∴ l(ST) = 16 units

Solution 8:

P is the midpoint of seg AB.

$\therefore\ l(AP) = \frac{1}{2} \times l(AB)$

$\therefore\ l(AP) = \frac{1}{2} \times 7$

$\therefore\ l(AP) = 3.5$ cm

Solution 9:

Q is the midpoint of CD
∴ l(CD) = 2 l(CQ)
∴ l(CD) = 2×4.5
∴ l(CD) = 9 cm

Solution 10:

7 > 5.4 > 4
∴ l(AB) > l(AP) > l(BP)
∴seg(AB) > seg(AP) > seg(BP)
∴ AB > AP >BP

Solution 11:

l(AB) = l(AC) = 5 cm
∴seg AB ≅ seg AC
l(BC) = l(DE) = 5.5 cm
∴seg BC ≅ seg DE
l(CD) = l(CE) = 4 cm
∴seg CD ≅ seg CE

Exercise – 1.3

Solution 1:

1. Yes, two acute angles measuring 30° and 60° have their sum 90°.
2. No, because the sum of the measures of two obtuse angles cannot be 90°.
3. No, because the sum of the measures of two right angles is 180°
4. No, because the sum of the measures of two acute angles is always less than 180°
5. No, because the sum of the measures of two obtuse angles is always greater than 180°.
6. Yes, because the sum of the measures of two right angles is 180°
7. Yes, because a linear pair of angles is adjacent as well as supplementary.
8. Yes, if the sum of the measures of adjacent angles adds to 90°
9. Yes, AOB and COB are obtuse and adjacent angles.

Solution 2(i):

Measure of the given angle = 60°
Measure of its supplementary angle = 180° – 60° = 120°

Solution 2(ii):

Measure of the given angle = 138°
Measure of its supplementary angle = 180° – 138° = 42°

Solution 2(iii):

$$\text{Measure of the given angle} = \frac{3}{5} \times \text{right angle} = \frac{3}{5} \times 90^\circ = 3 \times 18^\circ = 54^\circ$$

$$\text{Measure of its supplementary angle} = 180^\circ - 54^\circ = 126^\circ$$

Solution 2(iv):

Measure of the given angle = (180 – r)°
Measure of its supplementary angle = 180°- (180 – r)°= r°

Solution 2(v):

Measure of the given angle = (90 + r)°
Measure of its supplementary angle = 180° – (90 + r)°
= 180° – 90° – r°
= (90 – r)°

Solution 2(vi):

Measure of the given angle = 87°
Measure of its supplementary angle = 180° – 87° = 93°

Solution 2(vii):

Measure of the given angle = 124°
Measure of its supplementary angle = 180° – 124° = 56°

Solution 2(viii):

Measure of the given angle = 108°
Measure of its supplementary angle = 180° – 108° = 72°

Solution 3(i):

Measure of the given angle = 58°
Measure of its complementary angle = 90° – 58° = 32°

Solution 3(ii):

Measure of the given angle = 16°
Measure of its complementary angle = 90° – 16° = 74°

Solution 3(iii):

$$\text{Measure of the given angle} = \frac{2}{3} \text{ of right angle}$$
$$= \frac{2}{3} \times 90^\circ$$
$$= 2 \times 30^\circ$$
$$= 60^\circ$$
$$\text{Measure of its complementary angle} = 90^\circ - 60^\circ = 30^\circ$$

Solution 3(iv):

Measure of the given angle = $(a + b)^\circ$
Measure of its complementary angle = $90^\circ - (a + b)^\circ = (90 - a - b)^\circ$

Solution 3(v):

Measure of the given angle = $(90 - r)^\circ$
Measure of its complementary angle = $90^\circ - (90 - r)^\circ$
$= 90^\circ - 90^\circ + r^\circ$
$= r^\circ$

Measure of the given angle = 78°
Measure of its complementary angle = 90° – 78° = 12°

Solution 3(vii):

Measure of the given angle = 68°
Measure of its complementary angle = 90° – 68° = 22°

Solution 3(viii):

Measure of the given angle = 56°
Measure of its complementary angle = 90° – 56° = 34°

Solution 4:

$m\angle AOC + m\angle BOC = 180^\circ$ (angles forming a linear pair)

$\therefore (3x + 5)^\circ + (2x - 25)^\circ = 180^\circ$

$\therefore 5x^\circ - 20^\circ = 180^\circ$

$\therefore 5x^\circ = 180^\circ + 20^\circ$

$\therefore 5x^\circ = 200^\circ$

$\therefore x^\circ = \dfrac{200^\circ}{5}$

$\therefore x^\circ = 40^\circ$

$\therefore m\angle AOC = 3(40^\circ) + 5^\circ$

$= 120^\circ + 5^\circ$

$= 125^\circ$

$\therefore \angle BOC = 2(40^\circ) - 25^\circ$

$= 80^\circ - 25^\circ$

$= 55^\circ$

Solution 5:

$\angle AOC = \angle BOE$.... (vertically opposite angle)

But, $m\angle BOE = 72^\circ$

$\therefore m\angle AOC = 72^\circ$ ....(i)

$m\angle AOC + m\angle BOC = 180^\circ$.... (angles in a linear pair)

$\therefore 72^\circ + m\angle BOC = 180^\circ$

$\therefore m\angle BOC = 180^\circ - 72^\circ$

$\therefore m\angle BOC = 108^\circ$ ....(ii)

$\angle AOE = \angle BOC$ .... (vertically opposite angle)

$\therefore m\angle AOE = 108^\circ$.... [from(ii)]

Exercise – 1.4

Solution 1:

1. Alternate angles are congruent. (Converse of alternates angle test)
2. All three lines are parallel to each other. (If two lines are parallel to the same line then they are parallel to each other.)
3. Both angles are congruent.
4. One and only one such line can be drawn.
5. Let the interior angles formed be 2x and 7x.
   The converse of interior angles test:
   If two lines are parallel then the interior angles formed by a transversal are supplementary.
   $\therefore 2x + 7x = 180°$
   $\therefore 9x = 180°$
   $\therefore x = 20°$
   The measure of the greater angle = 7x
   $= 7 \times 20°$
   $= 140°$

Solution 2:

Given that $r = 20°$
$a = r$ (vertically opposite angles)
$\therefore a = 20°$
a and b are interior angles.

Converse of interior angles test:
If two lines are parallel then the interior angles formed by a transversal are supplementary.

$\therefore a + b = 180°$ (converse of interior angles test)
$\therefore 20° + b = 180°$
$\therefore b = 180° - 20°$
$\therefore b = 160°$
$$\therefore \frac{a}{b} = \frac{20°}{160°}$$
$$\therefore \frac{a}{b} = \frac{1}{8}$$
$\therefore a : b = 1 : 8$

Solution 3:

Given that line $l \parallel$ line n

on transversal OC

$\angle EOC = \angle OCD$ (Converse of alternate anlges test)

$\therefore m\angle OCD = 110^\circ$

$\therefore m\angle OCA + m\angle ACD = 110^\circ$

$\therefore 30^\circ + m\angle ACD = 110^\circ$

$\therefore m\angle ACD = 110^\circ - 30^\circ$

$\therefore m\angle ACD = 80^\circ$ .... (i)

Now, line m $\parallel$ line n

$\therefore m\angle BAC + m\angle ACD = 180^\circ$ (converse of interior angles test)

$\therefore m\angle BAC + 80^\circ = 180^\circ$ [from(i)]

$\therefore m\angle BAC = 180^\circ - 80^\circ$

$\therefore m\angle BAC = 100^\circ$

$\therefore m\angle A = 100^\circ$

Solution 4:

Given that $m\angle PEB = 70°$

$\angle AEF = \angle PEB$ (Vertically opposite angles)

$\therefore m\angle AEF = 70°$

$m\angle PEB + m\angle BEF = 180°$ (Angles in linear pair)

$\therefore 70° + m\angle BEF = 180°$

$\therefore m\angle BEF = 180° - 70°$

$\therefore m\angle BEF = 110°$

$m\angle PEA = m\angle BEF$ (Vertically opposite angles)

$\therefore m\angle PEA = 110°$

$m\angle BEF + m\angle EFD = 180°$ (converse of interior angles theorem)

$\therefore 110° + m\angle EFD = 180°$

$\therefore m\angle EFD = 180° - 110°$

$\therefore m\angle EFD = 70°$

$\therefore \angle EFD = \angle CFQ$ (Vertically opposite angles)

$\therefore m\angle CFQ = 70°$

$\angle EFD + \angle DFQ = 180°$ (Angles in linear pair)

$70° + m\angle DFQ = 180°$

$\therefore m\angle DFQ = 180° - 70°$

$\therefore m\angle DFQ = 110°$

$\angle DFQ = \angle CFE$ (Vertically opposite angles)

$\therefore m\angle CFE = 110°$