Relations & Functions

1. A function f : $R_+ \rightarrow R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is : (2024)

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Ans. (A) one-one but not onto

2. A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive. (2024)

Ans.

(a) Reflexive:

$\because |x^2 - x^2| < 8 \ \forall \ x \in A \Rightarrow (x, x) \in R \therefore R$ is reflexive.

(b) Symmetric:

Let $(x,y) \in R$ for some $x,y \in A$

$\therefore |x^2 - y^2| < 8 \Rightarrow |y^2 - x^2| < 8 \Rightarrow (y, x) \in R$

Hence R is symmetric.

(c) Transitive:

$(1,2)$, $(2,3) \in R$ as $|1^2 - 2^2| < 8$, $|2^2 - 3^2| < 8$ respectively

But $|1^2 - 3^2| \nless 8 \Rightarrow (1, 3) \notin R$

Hence R is not transitive

3. A function f is defined from $R \rightarrow R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not. (2024)

Ans. $f(x) = ax + b$

Solving $a + b = 1$ and $2a + b = 3$ to get $a = 2$, $b = -1$

$f(x) = 2x - 1$

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$

Hence f is one – one.

Let y = 2x -1, y ∈ ***R*** (Codomain)

⇒ x = ***y***+1/2 ∈ ***R*** (domain)

Also, f(x) = f(***y***+1/2) = y

∴ f is onto.

4.

If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

(2024)

Ans.

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Put $x = \sin\theta$, $y = \sin\phi$

$$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$$

$$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$$

$$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$$

$$\Rightarrow \theta - \phi = 2\cot^{-1} a$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

5. If y = $(\tan x)^x$, then find dy/dx.

Ans. $y = (\tan x)^x$

log y = x log (tan x)

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$$

$$\frac{dy}{dx} = (\tan x)^x\left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$$

Previous Years' CBSE Board Questions

1.2 Types of Relations

MCQ

1. Let $A = \{3, 5\}$. Then number of reflexive relations on A is
   (a) 2 (b) 4
   (c) 0 (d) 8 *(2023)*

2. Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Then
   (a) $(8, 7) \in R$ (b) $(6, 8) \in R$
   (c) $(3, 8) \in R$ (d) $(2, 4) \in R$ *(2023)*

3. A relation R is defined on N. Which of the following is the reflexive relation?
   (a) $R = \{(x, y): x > y, x, y \in N\}$
   (b) $R = \{(x, y): x + y = 10, x, y \in N\}$
   (c) $R = \{(x, y): xy$ is the square number, $x, y \in N\}$
   (d) $R = \{(x, y): x + 4y = 10, x, y \in N\}$
   (Term I, 2021-22) An

4. The number of equivalence relations in the set $\{1, 2, 3\}$ containing the elements $(1, 2)$ and $(2, 1)$ is
   (a) 0 (b) 1
   (c) 2 (d) 3 *(Term I, 2021-22)*

5. A relation R is defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is
   (a) reflexive and symmetric
   (b) symmetric but not reflexive
   (c) transitive but not reflexive
   (d) reflexive but not symmetric
   (Term I, 2021-22) Ap

6. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing $(1, 3)$ is
   (a) 1 (b) 2
   (c) 3 (d) 4 *(2020)*

7. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is
   (a) symmetric and transitive, but not reflexive
   (b) reflexive and symmetric, but not transitive
   (c) symmetric, but neither reflexive nor transitive
   (d) an equivalence relation *(2020)*

VSA (1 mark)

8. Write the smallest reflexive relation on set $A = \{a, b, c\}$. *(2021 C)*

9. A relation R in a set A is called ________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$. *(2020)* R

10. A relation in a set A is called ________ relation, if each element of A is related to itself. *(2020)* R

11. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. *(AI 2014)*

12. Let $R = \{(a, a^3) : a$ is a prime number less than $5\}$ be a relation. Find the range of R. *(Foreign 2014)*

13. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2$ divides $(a - b)\}$. Write the equivalence class $[0]$. *(Delhi 2014 C)*

SA I (2 marks)

14. Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive. *(2020)*

15. Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.
    Show that this relation R is reflexive and symmetric, but not transitive. *(2020)*

LA I (4 marks)

16. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b|$ is divisible by $2\}$ is an equivalence relation. *(2020)*

17. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. *(2019)*

18. Show that the relation R on the set Z of all integers, given by $R = \{(a, b) : 2$ divides $(a - b)\}$ is an equivalence relation. *(2019)*

19. Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b): a \le b\}$, is reflexive and transitive but not symmetric. *(NCERT, Delhi 2019)*

20. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $3\}$ is an equivalence relation. *(AI 2019)* Ap

21. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$. *(Delhi 2014)*

22. Let R be a relation defined on the set of natural numbers N as follow :
    $R = \{(x, y) \mid x \in N, y \in N$ and $2x + y = 24\}$
    Find the domain and range of the relation R.
    Also, find if R is an equivalence relation or not. *(Delhi 2014 C)* An

LA II (5/6 marks)

23. If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. *(2023, Delhi 2015)*

24. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$, is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. *(2018)*

25. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is divisible by 2$\}$ is an equivalence relation. Write all the equivalence classes of R. *(AI 2015 C)*

1.3 Types of Functions

MCQ

26. The function $f : R \to R$ defined by $f(x) = 4 + 3 \cos x$ is
(a) bijective (b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto *(Term I, 2021-22)* An

27. The number of functions defined from $\{1, 2, 3, 4, 5\} \to \{a, b\}$ which are one-one is
(a) 5 (b) 3
(c) 2 (d) 0 *(Term I, 2021-22)*

28. Let $f : R \to R$ be defined by $f(x) = 1/x$, for all $x \in R$, Then, f is
(a) one-one (b) onto
(c) bijective (d) not defined
(Term I, 2021-22)

29. The function $f : N \to N$ is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

The function f is
(a) bijective
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto
(Term I, 2021-22) Ev

VSA (1 mark)

30. If $f = \{(1, 2), (2, 4), (3, 1), (4, k)\}$ is a one-one function from set A to A, where $A = \{1, 2, 3, 4\}$, then find the value of k. *(2021 C)*

LA I (4 marks)

31. **Case Study :** An organization conducted bike race under two different categories - Boys and girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions.

(i) How many relations are possible from B to G?
(ii) Among all the possible relations from B to G, how many functions can be formed from B to G?
(iii) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, justify your answer.
(2023) Ap

32. Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not. *(2023)*

33. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto. *(2020)*

CBSE Sample Questions

1.2 Types of Relations

MCQ

1. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?
(a) (1, 1) (b) (1, 2) (c) (2, 2) (d) (3, 3)
(Term I, 2021-22) An

2. Let the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4.$\}$ Then [1], the equivalence class containing 1, is
(a) {1, 5, 9} (b) {0, 1, 2, 5}
(c) ϕ (d) A
(Term I, 2021-22) Ev

VSA (1 mark)

3. How many reflexive relations are possible in a set A whose $n(A) = 3$? *(2020-21)* Ap

4. A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation? *(2020-21)*

5. An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$. *(2020-21)*

SA I (2 marks)

6. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R is transitive? Write the equivalence class of 0. *(2020-21)* Ap

SA II (3 marks)

7. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b \text{ is "divisible by 2"}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0, *i.e.*, [0]. *(2020-21)*

LA II (5/6 marks)

8. Given a non-empty set X, define the relation R on $P(X)$ as:
For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive, and not symmetric. *(2022-23)*

9. Define the relation R in the set $N \times N$ as follows:
For $(a, b), (c, d) \in N \times N$, $(a, b)\, R\, (c, d)$ iff $ad = bc$. Prove that R is an equivalence relation in $N \times N$. *(2022-23)*

1.3 Types of Functions

MCQ

10. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as
(a) Surjective function (b) Injective function
(c) Bijective function (d) Function
(Term I, 2021-22) Ev

11. The function $f : R \to R$ defined as $f(x) = x^3$ is
(a) One-one but not onto
(b) Not one-one but onto
(c) Neither one-one nor onto
(d) One-one and onto *(Term I, 2021-22)*

VSA (1 mark)

12. Check whether the function $f : R \to R$ defined as $f(x) = x^3$ is one-one or not. *(2020-21)*

13. A relation R in the set of real numbers **R** defined as $R = \{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify *(2020-21)*

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. **(b):** Total number of reflexive relations on a set having n number of elements $= 2^{n^2 - n}$
Here, $n = 2$
$\therefore$ Required number of reflexive relations $= 2^{2^2 - 2}$
$= 2^{4-2} = 2^2 = 4$

2. **(b):** Given, $R = \{(a, b) : a = b - 2, b > 6\}$
Since, $b > 6$, so $(2, 4) \notin R$
Also, $(3, 8) \notin R$ as $3 \neq 8 - 2$
and $(8, 7) \notin R$ as $8 \neq 7 - 2$
Now, for (6, 8), we have
$8 > 6$ and $6 = 8 - 2$, which is true
$\therefore$ $(6, 8) \in R$

3. **(c):** Consider, $R = \{(x, y) : xy \text{ is the square number}, x, y \in N\}$
As, $xx = x^2$, which is the square of natural number x.
$\Rightarrow$ $(x, x) \in R$. So, R is reflexive.

Concept Applied
- A relation R in a set A is called reflexive, if $(a, a) \in R$, for all $a \in A$.

4. **(c):** Equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) are
$R_1 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)\}$
$R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1) (2, 3), (3, 2), (1, 1), (2, 2), (3, 3)\}$
$\therefore$ Number of equivalence relations is 2.

Concept Applied
- A relation R in a set A is called an equivalence relation, if R is reflexive, symmetric and transitive.

5. **(d):** Given, aRb, $a, b \in Z$
Reflexive : For $a \in Z$, we have
$a^2 - 7a{\cdot}a + 6a^2 = a^2 - 7a^2 + 6a^2 = 0 \Rightarrow (a, a) \in R$
$\therefore$ Relation is reflexive.
Symmetric : Since, $(6, 1) \in R$
As, $6^2 - 7 \times 6 \times 1 + 6 \times 1^2 = 36 - 42 + 6 = 0$
But $(1, 6) \notin R$. $\therefore$ Relation is not symmetric.

6. **(b):** Equivalence relations in the set containing the element (1, 3) are
$R_1 = \{(1, 1), (3, 3), (1, 3), (3, 1), (5, 5)\}$
$R_2 = \{(1, 1), (3, 3), (5, 5), (1, 5), (5, 1), (3, 5), (5, 3), (1, 3), (3, 1)\}$
$\therefore$ There are 2 possible equivalence relations.

7. **(c):** Given $R = \{(1, 2), (2, 1), (1, 1)\}$ is a relation on set {1, 2, 3}
Reflexive : Clearly $(2, 2), (3, 3) \notin R$
$\therefore$ R is not a reflexive relation.
Symmetric : Now, $(1, 2) \in R$ and $(2, 1) \in R$ $\therefore$ R is symmetric.
Transitive : Now, $(2, 1) \in R$ and $(1, 2) \in R$ but $(2, 2) \notin R$
$\therefore$ R is not transitive relation.
R is symmetric, but neither reflexive nor transitive.

8. We have, $A = \{a, b, c\}$
A relation R on the set A is said to be reflexive if $(a, a) \in R$, $\forall\, a \in A$
$\therefore$ $R = \{(a, a), (b, b), (c, c)\}$ is the required smallest reflexive relation on A.

9. A relation R in a set A is called symmetric, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

10. A relation in a set A is called reflexive relation, if each element of A is related to itself.

11. Here, $R = \{(x, y) : x + 2y = 8x, y \in N\}$.
For $x = 1, 3, 5, ...$
$x + 2y = 8$ has no solution in N.
For $x = 2$, we have $2 + 2y = 8 \Rightarrow y = 3$
For $x = 4$, we have $4 + 2y = 8 \Rightarrow y = 2$
For $x = 6$, we have $6 + 2y = 8 \Rightarrow y = 1$
For $x = 8, 10, ...$
$x + 2y = 8$ has no solution in N.
$\therefore$ Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$

12. Given relation is
$R = \{(a, a^3) : a$ is a prime number less than $5\}$.
$\therefore$ $R = \{(2, 8), (3, 27)\}$. So, the range of R is $\{8, 27\}$.

13. Here, $R = \{(a, b) : 2$ divides $(a - b)\}$
$\therefore$ Equivalence class of $[0] = \{a \in A : (a, 0) \in R\}$.
$\Rightarrow$ $(a - 0)$ is divisible by 2 and $a \in A \Rightarrow a = 0, 2, 4$
Thus $[0] = \{0, 2, 4\}$.

14. We have, $R = \{(a, b) : a < b\}$, where $a, b \in \mathbb{R}$
(i) Symmetric : Let $(x, y) \in R$, *i.e.*, $x R y \Rightarrow x < y$
But $y \not< x$, so $(x, y) \in R \Rightarrow (y, x) \notin R$
Thus, R is not symmetric.
(ii) Transitive : Let $(x, y), (y, z) \in R$
$\Rightarrow$ $x < y$ and $y < z \Rightarrow x < z$
$\Rightarrow$ $(x, z) \in R$. Thus, R is transitive.

15. We have, $R = \{(x, y) \in W \times W : x$ and y have at least one letter in common$\}$
Reflexive : Clearly $(x, x) \in R$, because same words will contains all common letters.
$\Rightarrow$ R is reflexive.
Symmetric : Let $(x, y) \in R$ *i.e.*, x and y have at least one letter in common.
$\Rightarrow$ y and x will also have at least one letter in common.
$\Rightarrow$ $(y, x) \in R$
$\Rightarrow$ R is symmetric.
Transitive : Let, x = LAND, y = NOT and z = HOT
Clearly $(x, y) \in R$ as x and y have a common letter and $(y, z) \in R$ as y and z have 2 common letters.
but $(x, z) \notin R$ as x and z have no letter in common.
Hence, R is not transitive.

Concept Applied

- A relation R in a set A is not transitive if for $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$

16. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(a, b) : |a - b|$ is divisible by $2\}$
(i) Reflexive : For any $a \in A$
$|a - a| = 0$, which is divisible by 2.
Thus, $(a, a) \in R$. So, R is reflexive.
(ii) Symmetric : For any $a, b \in A$
Let $(a, b) \in R$
$\Rightarrow$ $|a - b|$ is divisible by $2 \Rightarrow |b - a|$ is divisible by 2
$\Rightarrow$ $(b, a) \in R$ $\therefore$ $(a, b) \in R \Rightarrow (b, a) \in R$ $\therefore$ R is symmetric.
(iii) Transitive : For any $a, b, c \in A$
Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2.
$\Rightarrow$ $a - b = \pm 2k_1$ and $b - c = \pm 2k_2$ $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $a - b + b - c = \pm 2(k_1 + k_2) \Rightarrow a - c = \pm 2k_3$ $\forall\, k_3 \in N$
$\Rightarrow$ $|a - c|$ is divisible by $2 \Rightarrow (a, c) \in R$ $\therefore$ R is transitive.
Hence, R is an equivalence relation.

17. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and a relation R on A defined as $R = \{(a, b) : b = a + 1\}$
Reflexive : Let $(a, a) \in R$
$\Rightarrow$ $a = a + 1 \Rightarrow a - a = 1 \Rightarrow 0 = 1$, which is not possible.
$\therefore$ $(a, a) \notin R \Rightarrow R$ is not reflexive.
Symmetric : Let $(a, b) \in R \Rightarrow b = a + 1$ ...(i)
Now, if $(b, a) \in R$
$\Rightarrow$ $a = b + 1 \Rightarrow b = b + 1 + 1$ (using (i))
$\Rightarrow$ $b = b + 2 \Rightarrow b - b = 2 \Rightarrow 0 = 2$, which is not possible
$\Rightarrow$ $(b, a) \notin R \Rightarrow R$ is not symmetric.
Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $b = a + 1$ and $c = b + 1 \Rightarrow c = a + 1 + 1$
$\Rightarrow$ $c = a + 2 \neq a + 1 \Rightarrow (a, c) \notin R \Rightarrow R$ is not transitive.

18. We have, $R = \{(a, b) : 2$ divides $(a - b)\}$
Reflexive : For any $a \in Z$, $a - a = 0$ and 2 divides 0.
$\Rightarrow$ $(a, a) \in R$ for every $a \in Z$ $\therefore$ R is a reflexive.
Symmetric : Let $(a, b) \in R$
$\Rightarrow$ 2 divides $(a - b)$
$\Rightarrow$ $a - b = 2m$, for some $m \in Z$
$\Rightarrow$ $b - a = 2m$
$\Rightarrow$ 2 divides $b - a$
$\Rightarrow$ $(b, a) \in R$
$\therefore$ R is symmetric.
Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ 2 divides $(a - b)$ and 2 divides $(b - c)$
$\Rightarrow$ $a - b = 2m$ and $b - c = 2n$ for some $m, n \in Z$
$\Rightarrow$ $a - b + b - c = 2m + 2n$
$\Rightarrow$ $a - c = 2(m + n)$
$\Rightarrow$ 2 divides $a - c$
$\Rightarrow$ $(a, c) \in R$
$\therefore$ R is transitive.
Hence, R is an equivalence relation.

19. We have, $R = \{(a, b) : a \leq b,\ a, b \in \mathbb{R}\}$
(i) Reflexive : Since $a \leq a$ $\therefore$ $aRa\ \forall\, a \in R$
Hence, R is reflexive.
(ii) Symmetric : $(a, b) \in R$ such that $aRb \Rightarrow a \leq b \not\Rightarrow b \leq a$
So, $(b, a) \notin R$.
Hence, R is not symmetric.
(iii) Transitive : Let $a, b, c \in R$ such that aRb and bRc
Now, $aRb \Rightarrow a \leq b$ ... (i) and $bRc \Rightarrow b \leq c$ ... (ii)
From (i) and (ii), we have $a \leq b \leq c \Rightarrow a \leq c$ $\therefore$ aRc
Hence, relation R is transitive.

20. We have, $A = \{x \in Z : 0 \leq x \leq 12\}$
$\therefore$ $A = \{0, 1, 2, 3, ..., 12\}$
Also, $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $3\}$

(i) Reflexive : For any $a \in A$,
$|a - a| = 0$, which is divisible by 3
Thus, $(a, a) \in S$ $\therefore$ S is reflexive.
(ii) Symmetric : Let $(a, b) \in S$
$\Rightarrow$ $|a - b|$ is divisible by 3.
$\Rightarrow$ $|b - a|$ is divisible by 3 $\Rightarrow$ $(b, a) \in S$ i.e. $(a, b) \in S \Rightarrow (b, a) \in S$
$\therefore$ S is symmetric.
(iii) Transitive :
Let $(a, b) \in S$ and $(b, c) \in S$
$\Rightarrow$ $|a - b|$ is divisible by 3 and $|b - c|$ is divisible by 3.
$\Rightarrow$ $(a - b) = \pm 3k_1$ and $(b - c) = \pm 3k_2$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $(a - b) + (b - c) = \pm 3(k_1 + k_2)$
$\Rightarrow$ $(a - c) = \pm 3(k_1 + k_2)$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $|a - c|$ is divisible by 3 $\Rightarrow$ $(a, c) \in S$ $\therefore$ S is Transitive.
Hence, S is an equivalence relation.

Concept Applied

- A relation R in a set A is called
  (i) reflexive, if $(a, a) \in R$, for all $a \in A$
  (ii) symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$
  (iii) transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$

21. Given $A = \{1, 2, 3, 4, ..., 9\}$
To show : R is an equivalence relation.
(i) Reflexive : Let (a, b) be an arbitrary element of $A \times A$. Then, we have $(a, b) \in A \times A \Rightarrow a, b \in A$
$\Rightarrow$ $a + b = b + a$ (by commutativity of addition on $A \subset N$)
$\Rightarrow$ $(a, b)\, R\, (a, b)$
Thus, $(a, b)\, R\, (a, b)$ for all $(a, b) \in A \times A$. So, R is reflexive.
(ii) Symmetric : Let $(a, b), (c, d) \in A \times A$ such that $(a, b)\, R\, (c, d)$
$\Rightarrow$ $a + d = b + c \Rightarrow b + c = a + d$
$\Rightarrow$ $c + b = d + a$ (by commutativity of addition on $A \subset N$)
$\Rightarrow$ $(c, d)\, R\, (a, b)$.
Thus, $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in A \times A$.
So, R is symmetric.
(iii) Transitive : Let $(a, b), (c, d), (e, f) \in A \times A$ such that $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$
Now, $(a, b)\, R\, (c, d) \Rightarrow a + d = b + c$ ...(i)
and $(c, d)\, R\, (e, f) \Rightarrow c + f = d + e$ ...(ii)
Adding (i) and (ii), we get $(a + d) + (c + f) = (b + c) + (d + e)$
$\Rightarrow$ $a + f = b + e \Rightarrow (a, b)\, R\, (e, f)$
Thus, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Rightarrow (a, b)\, R\, (e, f)$.
So, R is transitive. $\therefore$ R is an equivalence relation.
Equivalence class of $[(2, 5)] = \{(x, y) \in N \times N : (x, y)\, R (2, 5)\}$
$= \{(x, y) \in N \times N : x + 5 = y + 2\}$
$= \{(x, y) \in N \times N : y = x + 3\} = \{(x, x + 3) : x \in A\}$
$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$.

Answer Tips

- First, prove the given relation is an equivalence relation and then find the equivalence class by using the given relation.

22. Here, $R = \{(x, y) \mid x \in N, y \in N$ and $2x + y = 24\}$
$R = \{(1, 22), (2, 20), (3, 18), ..., (11, 2)\}$
Domain of $R = \{1, 2, 3, 4, ..., 11\}$
Range of $R = \{2, 4, 6, 8, 10, 12, ..., 22\}$
R is not reflexive as if $(2, 2) \in R \Rightarrow 2 \times 2 + 2 = 6 \neq 24$
In fact R is neither symmetric nor transitive.
$\Rightarrow$ R is not an equivalence relation.

23. (i) Reflexive : Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N$
$\Rightarrow$ $ab(b + a) = ba(a + b)$
[by commutativity of addition and multiplication on N]
$\Rightarrow$ $(a, b)\, R\, (a, b)$
So, R is reflexive on $N \times N$.
(ii) Symmetric : Let $(a, b), (c, d) \in N \times N$ such that $(a, b)\, R\, (c, d)$.
$\Rightarrow$ $ad(b + c) = bc(a + d) \Rightarrow cb(d + a) = da(c + b)$
[by commutativity of addition and multiplication on N]
Thus, $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in N \times N$.
So, R is symmetric on $N \times N$.
(iii) Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$. Then,
$(a, b)\, R\, (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \quad ...(i)$$

and $(c, d)\, R\, (e, f) \Rightarrow cf(d + e) = de(c + f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \quad ...(ii)$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$$

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$$

$\Rightarrow$ $af(b + e) = be(a + f) \Rightarrow (a, b)\, R\, (e, f)$
So, R is transitive on $N \times N$.
Hence, R is an equivalence relation.

24. We have, $A = \{x \in Z : 0 \le x \le 12\}$
$\therefore$ $A = \{0, 1, 2, 3, ..., 12\}$
and $S = \{(a, b) : |a - b|$ is divisible by $4\}$
(i) Reflexive : For any $a \in A$, $|a - a| = 0$, which is divisible by 4. Thus, $(a, a) \in R$ $\therefore$ R is reflexive.
(ii) Symmetric : Let $(a, b) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 4
$\Rightarrow$ $|b - a|$ is divisible by 4 $\Rightarrow$ $(b, a) \in R$
i.e., $(a, b) \in R \Rightarrow (b, a) \in R$ $\therefore$ R is symmetric.
(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4
$\Rightarrow$ $a - b = \pm 4k_1$ and $b - c = \pm 4k_2$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $(a - b) + (b - c) = \pm 4(k_1 + k_2)$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $a - c = \pm 4(k_1 + k_2)$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $|a - c|$ is divisible by $4 \Rightarrow (a, c) \in R$ $\therefore$ R is transitive.
Hence, R is an equivalence relation.
The set of elements related to 1 is $\{1, 5, 9\}$.
Equivalence class for $[2]$ is $\{2, 6, 10\}$.

Concept Applied

- In a relation R in a set A, the set of all elements related to any element $a \in A$ is denoted by $[a]$
  i.e., $[a] = \{x \in A : (x, a) \in R\}$
  Here, $[a]$ is called an equivalence class of $a \in A$.

25. We have, $A = \{1, 2, 3, 4, 5\}$
and $R = \{(a, b) : |a - b|$ is divisible by 2$\}$
(i) Reflexive : For any $a \in A$,
$|a - a| = 0$, which is divisible by 2
Thus, $(a, a) \in R$ $\therefore$ R is reflexive.
(ii) Symmetric : Let $(a, b) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2
$\Rightarrow$ $|b - a|$ is divisible by 2 $\Rightarrow$ $(b, a) \in R$
i.e., $(a, b) \in R \Rightarrow (b, a) \in R$ $\therefore$ R is symmetric.
(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2
$\Rightarrow$ $a - b = \pm 2k_1$ and $b - c = \pm 2k_2$; $\forall k_1, k_2 \in N$
$\Rightarrow$ $(a - b) + (b - c) = \pm 2(k_1 + k_2)$; $\forall k_1, k_2 \in N$
$\Rightarrow$ $(a - c) = \pm 2(k_1 + k_2)$; $\forall k_1, k_2 \in N$
$\Rightarrow$ $|a - c|$ is divisible by 2 $\Rightarrow (a, c) \in R$ $\therefore$ R is transitive.
Hence, R is an equivalence relation.
Further R has only two equivalence classes, namely $[1] = [3] = [5] = \{1, 3, 5\}$ and $[2] = [4] = \{2, 4\}$.

26. (d): We have, $f(x) = 4 + 3\cos x$, $\forall x \in R$

At $x = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = 4 + 3\cos\frac{\pi}{2} = 4 \Rightarrow f\left(-\frac{\pi}{2}\right) = 4 + 3\cos\left(-\frac{\pi}{2}\right) = 4$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$, But $\frac{\pi}{2} \neq -\frac{\pi}{2}$

Therefore, f is not one-one.
As $-1 \le \cos x \le 1, \forall x \in R \Rightarrow 1 \le 4 + 3\cos x \le 7, \forall x \in R$
$\Rightarrow$ $f(x) \in [1, 7]$, where $[1, 7]$ is subset of R. $\therefore$ f is not onto.

Concept Applied

- Range of $\cos x$ is $[-1, 1]$.

27. (d): $\because f : X \to Y$ is one-one, if different element of X have different image in Y under f. But here, no such situation is possible.

28. (d): Given $f(x) = \frac{1}{x}$, for all $x \in R$

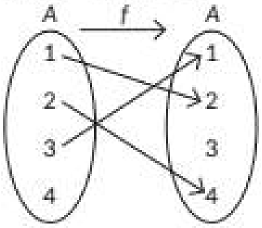
At $x = 0 \in R$, $f(x)$ is not defined.

29. (c): Given, $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Now, $f(1) = \frac{1+1}{2} = 1, f(2) = \frac{2}{2} = 1$

$\Rightarrow$ $f(1) = f(2)$ but $1 \neq 2$ $\therefore$ f is not one-one.
But f is onto ($\because$ range of f is N.)

30. We have, $A = \{1, 2, 3, 4\}$ function $f : A \to A$ is one-one and $f(1) = 2, f(2) = 4, f(3) = 1, f(4) = k$



As f is one-one, so no two element of A has same image in A.
$\therefore$ $f(4) = 3 \Rightarrow k = 3$

Concept Applied

- For a function to be one-one, no two elements should have the same image in A.

31. (i) Here $n(B) = 3$ and $n(G) = 2$
$\therefore$ Number of relation from B to $G = 2^{3 \times 2} = 2^6$
(ii) Number of functions formed from B to $G = 2^3 = 8$
(iii) We have, $R = \{(x, y) = x$ and y are students of the same sex$\}$
$\therefore$ R is reflexive as $(x, x) \in R$.
R is symmetric as $(x, y) \in R \Rightarrow (y, x) \in R$.
Since, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$
Hence, R is an equivalence relations.

OR

We have $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$
Since, elements b_1 and b_3 have the same image, therefore, the functions is not one-one but it is many one functions.
Since, every element in G has its pre-image in B, so the functions is onto.
For bijection, function should be one-one and onto both.
Hence, the function is surjective but not injective.

32. The function $f : R - \left\{-\frac{4}{3}\right\} \to R$ is given by $f(x) = \frac{4x}{3x+4}$.

One-one : Let $x, y \in R - \left\{-\frac{4}{3}\right\}$ such that $f(x) = f(y)$

$\Rightarrow$ $\frac{4x}{3x+4} = \frac{4y}{3y+4}$

$\Rightarrow$ $4x(3y + 4) = 4y(3x + 4) \Rightarrow 12xy + 16x = 12xy + 16y$
$\Rightarrow$ $16x = 16y \Rightarrow x = y$
$\therefore$ f is one-one.
Onto : Let y be an arbitrary element of R. Then $f(x) = y$

$\Rightarrow$ $\frac{4x}{3x+4} = y \Rightarrow 4x = 3xy + 4y \Rightarrow 4x - 3xy = 4y \Rightarrow x = \frac{4y}{4-3y}$

As $y \in R - \left\{\frac{4}{3}\right\}, \frac{4y}{4-3y} \in R$

Also, $\frac{4y}{4-3y} \neq \frac{-4}{3}$ as if

$\frac{4y}{4-3y} = -\frac{4}{3} \Rightarrow 12y = 12y - 16$, which is not possible.

Thus, $x = \frac{4y}{4-3y} \in R - \left\{-\frac{4}{3}\right\}$ such that

$$f(x) = f\left(\frac{4x}{3x+4}\right) = \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right) + 4} = \frac{16y}{12y + 16 - 12y} = \frac{16y}{16} = y$$

So, every element in $R - \left\{\frac{4}{3}\right\}$ has pre-image in $R - \left\{-\frac{4}{3}\right\}$

$\therefore$ f is not onto.

33. Given, $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$

$= \frac{x}{1-x}$ $\quad (\because x \in (-\infty, 0), |x| = -x)$

For one-one : Let $f(x_1) = f(x_2), x_1, x_2 \in (-\infty, 0)$

$\Rightarrow \quad \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1(1-x_2) = x_2(1-x_1)$

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2), \Rightarrow x_1 = x_2$

$\therefore$ f is one-one

For onto : Let $f(x) = y$

$\Rightarrow \quad y = \frac{x}{1-x} \Rightarrow y(1-x) = x \Rightarrow y - xy = x$

$\Rightarrow \quad x + xy = y \Rightarrow x(1+y) = y \Rightarrow x = \frac{y}{1+y}$

Here, $y \in (-1, 0)$

So, x is defined for all values of y in codomain. $\therefore$ f is onto.

Concept Applied

- A function $f : A \to B$ is called
  - (i) one-one or injective function, if distinct elements of A have distinct images in B. *i.e.*, for $a, b \in A, f(a) = f(b) \Rightarrow a = b$
  - (ii) onto or surjective function, if for every element $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

CBSE Sample Questions

1. (b): We have, $(1, 2) \in R$ but $(2, 1) \notin R$
So, (1, 2) should be removed from R to make it an equivalence relation. **(1)**

2. (a): We have, $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$
$\therefore$ The set of elements related to 1 is $\{1, 5, 9\}$.
So, equivalence class for [1] is $\{1, 5, 9\}$ **(1)**

3. Number of reflexive relations on a set having n elements $= 2^{n(n-1)}$
So, required number of reflexive relations $= 2^{3(3-1)} = 2^6$ **(1)**

4. We have, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$
which is reflexive and transitive.
For R to be symmetric (1, 2) should be removed from R. **(1)**

5. As we know that, union of all equivalence classes of a set is the set itself.
$\therefore \quad A_1 \cup A_2 \cup A_3 = A$
Also, $A_1 \cap A_2 \cap A_3 = \phi$
[$\because$ Equivalence classes are either equal or disjoint] **(1)**

6. Let $(a, b) \in R$ and $(b, c) \in R$. Then, 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$
So, 2 divides $[(a - b) + (b - c)]$
$\Rightarrow$ 2 divides $(a - c) \Rightarrow (a, c) \in R$. So, relation R is transitive. **(1)**
Equivalence class of $0 = \{0, \pm 2, \pm 4, \pm 6, ...\}$ **(1)**

7. (i) Reflexive : Since, $a + a = 2a$ which is even.
$\therefore \quad (a, a) \in R \ \forall \ a \in Z$
Hence, R is reflexive. **(1/2)**
(ii) Symmetric : If $(a, b) \in R$, then $a + b = 2\lambda \Rightarrow b + a = 2\lambda$
$\Rightarrow (b, a) \in R$. Hence, R is symmetric. **(1)**
(iii) Transitive : If $(a, b) \in R$ and $(b, c) \in R$
then $a + b = 2\lambda$ .... (i) and $b + c = 2\mu$ ... (ii)
Adding (i) and (ii), we get
$a + 2b + c = 2(\lambda + \mu) \Rightarrow a + c = 2(\lambda + \mu - b)$
$\Rightarrow \quad a + c = 2k$, where $k = \lambda + \mu - b \Rightarrow (a, c) \in R$
Hence, R is transitive. **(1)**
Equivalence class containing 0 *i.e.*,
$[0] = \{..., -4, -2, 0, 2, 4, ...\}$ **(1/2)**

8. We have, a relation R on X such that, $(A, B) \in R$ iff $A \subset B$ for $A, B \in P(X)$. **(1/2)**
Reflexive : Clearly every set is a subset of itself.
$\Rightarrow \quad (A, A) \in R$
$\therefore \quad R$ is reflexive. **(1)**
Symmetric : Let $(A, B) \in R$
$\Rightarrow \quad A \subset B$
$\Rightarrow \quad B$ is a super set of A. **(1/2)**
$\Rightarrow \quad B \not\subset A \Rightarrow (B, A) \notin R$
$\therefore \quad R$ is not symmetric. **(1)**
Transitive : Let $(A, B) \in R$ and $(B, C) \in R$, for all $A, B, C \in P(X)$
$\Rightarrow \quad A \subset B$ and $B \subset C \Rightarrow A \subset B \subset C$ **(1/2)**
$\Rightarrow \quad A \subset C \Rightarrow (A, C) \in R$
$\therefore \quad R$ is transitive. **(1)**
Hence, R is reflexive and transitive but not symmetric. **(1/2)**

9. Reflexive : Let $(a, b) \in N \times N$. Then $ab = ba$
(By commutativity of multiplication of natural number)
$\Rightarrow \quad (a, b) R (b, a)$
Thus, $(a, b) R (b, a)$ for all $(a, b) \in N \times N$
So, R is reflexive. **(1)**
Symmetric : Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$
$\Rightarrow ad = bc \Rightarrow bc = ad \Rightarrow cb = da$
(By commutativity of multiplication of natural numbers)
$\Rightarrow \quad (c, d) R (a, b)$
Thus, $(a, b) R (c, d) = (c, d) R (a, b)$ for $(a, b), (c, d) \in N \times N$
So, R is symmetric. **(1)**
Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$
Now, $(a, b) R (c, d) \Rightarrow ad = bc$ ...(i)
and $(c, d) R (e, f) \Rightarrow cf = de$ ...(ii)
Multiplying (i) and (ii), we get $ad \cdot cf = bc \cdot de$ **(1)**
$\Rightarrow \quad af = be \Rightarrow (a, b) R (e, f)$
Thus, $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$ **(1)**
So, R is transitive.
$\therefore \quad R$ is an equivalence relation. **(1)**

10. (b): As every pre-image $x \in A$, has a unique image $y \in B$.
$\Rightarrow \quad f$ is injective function. **(1)**

11. (d): Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$
$\Rightarrow \quad x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.
Let $f(x) = x^3 = y$ for some arbitrary element $y \in R \Rightarrow x = y^{1/3}$
$\Rightarrow \quad f(y^{1/3}) = y$
Every image $y \in R$ has a unique pre-image in R.
$\Rightarrow \quad f$ is onto
$\therefore \quad f$ is one-one and onto. **(1)**

12. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$.
$\Rightarrow \quad (x_1)^3 = (x_2)^3$
$\Rightarrow \quad x_1 = x_2$, hence $f(x)$ is one-one. **(1)**

13. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$
$\therefore \quad R = \{(a, b) : \sqrt{a} = b\}$ is not a function. **(1)**