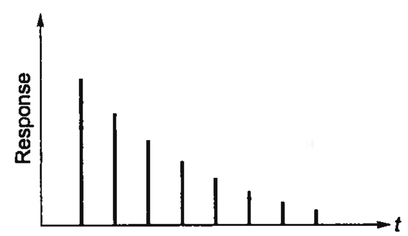
6 Z - Transform

Multiple Choice Questions

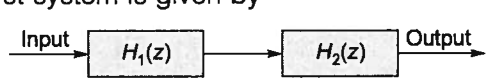
Q.1 The impulse response of a discrete system with a simple pole shown in the below figure.



The pole of the system must be located on the
(a) real axis at $z = -1$
(b) real axis between $z = 0$ and $z = 1$
(c) imaginary axis at $z = j$
(d) imaginary axis between $z = 0$ and $z = j$

[ESE-2000]

Q.2 Consider the compound system shown in the below figure. Its output is equal to input with a delay of two units. If the transfer function of the first system is given by



$$H_1(z) = \frac{z-0.5}{z-0.8},$$

then the transfer function of the second system would be

(a) $H_2(z) = \dfrac{z^{-2}-0.2z^{-3}}{1-0.4z^{-1}}$

(b) $H_2(z) = \dfrac{z^{-2}-0.8z^{-3}}{1-0.5z^{-1}}$

(c) $H_2(z) = \dfrac{z^{-1}-0.2z^{-3}}{1-0.4z^{-1}}$

(d) $H_2(z) = \dfrac{z^{-2}+0.8z^{-3}}{1+0.5z^{-1}}$

[ESE-2000]

Q.3 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I
A. $\alpha^n u(n)$
B. $-\alpha^n u(-n-1)$
C. $-n\alpha^n u(-n-1)$
D. $n\alpha^n u(n)$

List-II
1. $\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$ ROC: $|z| > |\alpha|$
2. $\dfrac{1}{(1-\alpha z^{-1})}$ ROC: $|z| > |\alpha|$
3. $\dfrac{1}{(1-\alpha z^{-1})}$ ROC: $|z| < |\alpha|$
4. $\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})}$ ROC: $|z| < |\alpha|$

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	1	3	4	2
(c)	1	4	3	2
(d)	2	3	4	1

[ESE-2000]

Q.4 **Assertion (A):** The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the Z-plane.
Reason (R): For a causal stable system all the poles should be outside the unit circle in the Z-plane.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

[ESE-2002]

Q.5 **Assertion (A):** The signals $a^n u(n)$ and $a^n u(-n-1)$ have the same Z-transform, $z/(z-a)$.
Reason (R): The Region of Convergence (ROC) for $a^n u(n)$ is $|z| > |a|$, whereas the ROC for $a^n u(-n-1)$ is $|z| < |a|$.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

[ESE-2002]

Q.6 If $X(z) = \dfrac{z+z^{-3}}{z+z^{-1}}$, then $x(n)$ series has
(a) alternate 0s (b) alternate 1s
(c) alternate 2s (d) alternate −1s

[ESE-2002]

Q.7 If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$, then the system function $H(z)$ is equal to
(a) $\dfrac{-z}{z-5}$ and the system is stable
(b) $\dfrac{z}{z-5}$ and the system is stable
(c) $\dfrac{-z}{z-5}$ and the system is unstable
(d) $\dfrac{z}{z-5}$ and the system is unstable

[GATE-2002]

Q.8 The z-transform of the time function $\sum_{k=0}^{\infty} \delta(n-k)$ is
(a) $\dfrac{(z-1)}{z}$ (b) $\dfrac{z}{(z-1)^2}$
(c) $\dfrac{z}{(z-1)}$ (d) $\dfrac{(z-1)^2}{z}$

Q.9 A stable and causal system is described by the difference equation

$$y(n) + \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = -2x(n) + \frac{5}{4}x(n-1)$$

The impulses response of the system is
(a) $h(n) = \left(\frac{1}{4}\right)^n u(n) - 3\left(-\frac{1}{2}\right)^n u(n)$
(b) $h(n) = \left(\frac{1}{4}\right)^n u(n) - 3u(n)$
(c) $h(n) = \left(\frac{1}{4}\right)^{n-1} u(n-1) - 3u(n-1)$
(d) $h(n) = \left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$

Q.10 Consider a system with system function

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}}$$

(a) system is non-causal
(b) system is causal
(c) data is not sufficient
(d) none of these

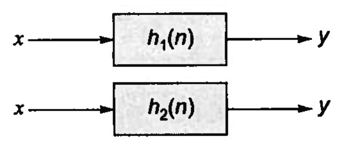
Q.11 $x[n] * u[n-n_0]$ is equal to
(a) $\sum_{k=-\infty}^{n} x[k]$ (b) $\sum_{k=-\infty}^{n_0} x[k]$
(c) $\sum_{k=-\infty}^{n-n_0} x[k]$ (d) $\sum_{k=n_0}^{\infty} x[k]$

Q.12 Find $x[n]$ for the case, when $X(z) = \dfrac{z^2+z}{(z-1)^2}$
(a) $n^2 u(n)$ (b) $2n\,u(n)$
(c) $(2n+1)\,u(n)$ (d) $(n+1)^2 u(n)$

Q.13 If, $h_1(n) = \left(\frac{1}{2}\right)^n$; $n \geq 0$

$h_2(n) = \left(\frac{1}{3}\right)^n$; $n \geq 0$

represented impulse response of two systems. Find *I.R.* of the series cascaded network.



(a) $3(2)^n u(n) + 2(3)^n u(n)$
(b) $3(2)^n u(n) - 2(3)^n u(n)$
(c) $3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n)$
(d) $3\left(\frac{1}{2}\right)^n u(n) + 2\left(\frac{1}{3}\right)^n u(n)$

Q.14 Consider a discrete time LTI system with impulse response $h[n]$ is given by

$$h[n] = \left(-\frac{1}{2}\right)^n u[n-1]$$

Statement 1: The system is causal.
Statement 2: The system is also stable
(a) Both statements are true.
(b) Statement 1 true, but statement 2 is false
(c) Statement 1 is false, but statement 2 is true
(d) Both statements are false

Q.15 A causal LTI system is described by the difference equation, $2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$. The system is stable only if,
(a) $|\alpha| = 2$, $|\beta| < 2$
(b) $|\alpha| > 2$, $|\beta| > 2$
(c) $|\alpha| < 2$, any value of β
(d) $|\beta| < 2$, any value of α

Q.16 A discrete time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n-1]$$

The system is
(a) Causal and stable
(b) Non-Causal and stable
(c) Causal and unstable
(d) Non-causal and unstable

Q.17 If $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be
(a) $\frac{1}{3} < |z| < 3$ (b) $\frac{1}{3} < |z| < \frac{1}{2}$
(c) $\frac{1}{2} < |z| < 3$ (d) $\frac{1}{3} < |z|$

[GATE-2012]

Q.18 The Z-transform corresponding to the Laplace transform function $G(s) = \frac{10}{s(s+5)}$ is
(a) $\frac{2Ze^{-5Z}}{(Z-1)(Z-e^{-T})}$ (b) $\frac{2(1-e^{-5T})Z}{(Z-1)(Z-e^{-5T})}$
(c) $\frac{e^{-5T}}{(Z-1)^2}$ (d) $\frac{e^{-T}}{Z(Z-e^{-3T})}$

[ESE-2012]

Q.19 The Z-transform of a signal $x(n)$ is

$$X(z) = \frac{z}{z-0.4}, \text{ROC } |z| > 0.4$$

The ROC of *ZT* of even, part of $x(n)$ will be
(a) $|z| > 0.4$ (b) $0.4 < |z| < 2.5$
(c) $|z| > 0.2$ (d) $|z| > 0.8$

Q.20 $x(n) = X(z) = \frac{1}{1-z^{-3}}$. If $x(n)$ is a causal signal then, $x(2) = ?$, $x(3) = ?$
(a) 0, 0 (b) 0, 1
(c) 1, 0 (d) 1, 1

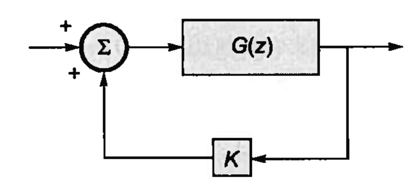
Q.21 The discrete-time signal

$$x[n] \leftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$$

where $\leftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal
(a) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$
(b) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$
(c) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$
(d) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

[GATE-2006]

Q.22 Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0$, $g(1) = g(2) = 1$, $g(3) = g(4) = \ldots = 0$



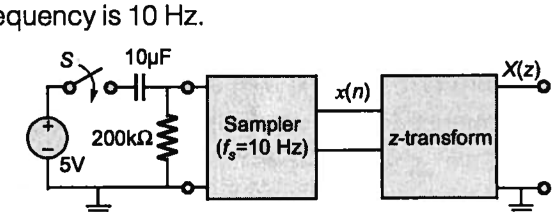
This system is stable for range of values of K
(a) [−1, 1/2] (b) [−1, 1]
(c) [−1/2, 1] (d) [−1/2, 2]

[GATE-2007]

Q.23 $X(z) = 1 - 3z^{-1}$, $Y(z) = 1 + 2z^{-2}$ are Z-transforms of two signals $x[n]$, $y[n]$ respectively. A linear time invariant system has the impulse response h[n] defined by these two signals as $h[n] = x[n-1] * y[n]$ where * denotes discrete time convolution. Then the output of the system for the input $\delta[n-1]$
(a) has Z-transform $z^{-1} X(z) Y(z)$
(b) equals $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$
(c) has Z-transform $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$
(d) does not satisfy any of the above three

[GATE-2007]

Linked Answer Questions (24 and 25):
In the following network, the switch is closed at $t = 0^-$ and the sampling starts from $t = 0$. The sampling frequency is 10 Hz.



Q.24 The sample $x(n)$ ($n = 0, 1, 2, \ldots\ldots$) are given by
(a) $5(1-e^{-0.05n})$ (b) $5e^{-0.05n}$
(c) $5(1-e^{-5n})$ (d) $5e^{-5n}$

[GATE-2008]

Q.25 The expression and the region of convergence of the z-transform of the sampled signal are
(a) $\frac{5z}{z-e^5}, |z| < e^{-5}$
(b) $\frac{5z}{z-e^{-0.05}}, |z| < e^{-0.05}$
(c) $\frac{5z}{z-e^{-0.05}}, |z| > e^{-0.05}$
(d) $\frac{5z}{z-e^{-5}}, |z| > e^{-5}$

[GATE-2008]

Q.26 Let $x[n] = x[-n]$. Let $X(z)$ be the z-transform of $x[n]$. If $0.5 + j0.25$ is a zero of $X(z)$, which one of the following must also be a zero of $X(z)$.
(a) $0.5 - j0.25$ (b) $\frac{1}{(0.5 + j0.25)}$
(c) $\frac{1}{(0.5 - j0.25)}$ (d) $2 + j4$

[GATE-2014]

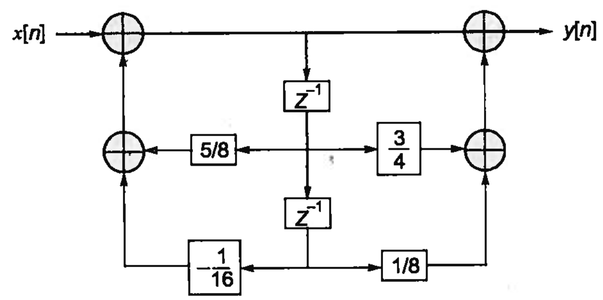
Q.27 The input-output relationship of a causal stable LTI system is given as

$$y[n] = \alpha y[n-1] + \beta x[n]$$

If the impulse response $h[n]$ of this system satisfies the condition $\sum_{n=0}^{\infty} h[n] = 2$, the relationship between α and β is
(a) $\alpha = 1 - \beta/2$ (b) $\alpha = 1 + \beta/2$
(c) $\alpha = 2\beta$ (d) $\alpha = -2\beta$

[GATE-2014]

Q.28 The direct form-II realisation of IIR filter is shown below:



The transfer function is

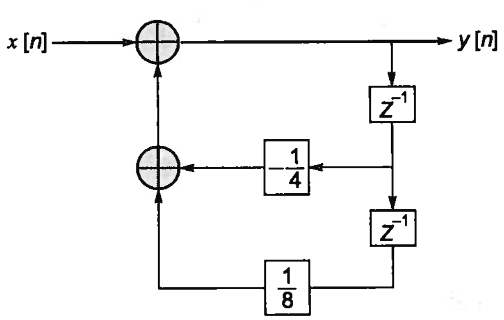
(a) $H(z) = \dfrac{1+\frac{5}{8}z^{-1}-\frac{1}{16}z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$

(b) $H(z) = \dfrac{1-\frac{5}{8}z^{-1}+\frac{1}{16}z^{-2}}{1-\frac{3}{4}z^{-1}-\frac{1}{8}z^{-2}}$

(c) $H(z) = \dfrac{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}{1-\frac{5}{8}z^{-1}+\frac{1}{16}z^{-2}}$

(d) $H(z) = \dfrac{1-\frac{5}{8}z^{-1}+\frac{1}{16}z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$

Q.29 The discrete time LTI system the output $y[n]$ and input $x[n]$ is $u[n]$. given below fig.



The impulse response of the system is

(a) $2\left(-\frac{1}{2}\right)^n u(t) - \left(\frac{1}{4}\right)^n u(t)$

(b) $\left[\frac{1}{2}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{2}\right)^n\right] u(t)$

(c) $\frac{1}{4}\left(\frac{1}{2}\right)^n u(t) - \frac{1}{2}\left(\frac{1}{4}\right)^n u(t)$

(d) $\left(\frac{1}{2}\right)^n u(t) - 2\left(\frac{1}{4}\right)^n u(t)$

Numerical Data Type Questions

Q.30 A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at $n = 4$ is ______.

[GATE-2003]

Q.31 The sequence $x[n]$ corresponding to $X(z)$ defined as

$$X(z) = \frac{z}{2z^2 - 3z + 1}\ ;\ |z| < \frac{1}{2}$$

Will have the value at $n = -2$ is ______.

Q.32 Consider a discrete time signal $x(n)$, whouse z-transform $X(z)$ is given by,

$$X(z) = \frac{z(z-2)}{(z-1)(z-0.5)}$$

value of $x(n)$ for $n = 0$ is ______.

Q.33 The z-transform $X[z]$ of a sequence $x[n]$ is given by $X[z] = \dfrac{0.5}{1-2z^{-1}}$. It is given that the ROC of $X[z]$ includes the unit circle. The value of $x[0]$ is ______.

Q.34 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals ______.

[GATE-2012]

Q.35 The z-transform of a signal is given by $C(z) = \dfrac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$ (a) in. Its final value is ____.

[GATE-1999]

Q.36 $x(n) = X(z) = \dfrac{z^{-2}}{z-3}$, the value of $x(5)$ is ____.

Q.37 $x(n) = 0.5^n u(n)$

$y(n) = x(n) * x(n)$ than

$\sum_{n=-\infty}^{\infty} y(n) =$ ______

Q.38 The sequence $x[n] = 0.5^n u[n]$ is the unit step sequence, is convolved with itself to obtain $y[n]$. Then $\sum_{n=-\infty}^{+\infty} y[n]$ is ______ .

[GATE-2014]

Q.39 An input signal $x(t) = 2 + 5\sin(100\pi t)$ is sampled with a sampling frequency of 400 Hz and applied to the system whose transfer function is represented by

$$\frac{Y(z)}{X(z)} = \frac{1}{N}\left(\frac{1-z^{-N}}{1-z^{-1}}\right)$$

where, N represents the number of samples per cycle. The output $y(n)$ of the system under steady state is ____.

[GATE-2014]

Try Yourself

T1. Let $x[n]$ be a discrete time signal whose z-transform is represented by $X(z)$. The z-transform of the signal

$y[n] = \left(\frac{1}{2}\right)^{n+1} x[n]$ will be

(a) $\frac{1}{2}X\left(\frac{z}{2}\right)$ (b) $\frac{1}{2}X(2z)$

(c) $X(2z)$ (d) $2X(2z)$

[Ans: (b)]

T2. The region of convergence of a signal $x[n]$ whose z-transform is represented as $X(z)$, where

$$x[n] = \begin{cases} 1; & -10 \le n \le 10 \\ 0; & \text{otherwise} \end{cases}$$

(a) $|z| > \frac{1}{10}$

(b) $|z| > 10$

(c) $\frac{1}{10} < |z| < 10$

(d) entire z-plane except $z = 0$ and $z = \infty$

[Ans: (d)]

T3. The Region of convergence of z-transform of a signal $x[n] = (2)^n u[n] - (4)^n u[-n-1]$ is

(a) $|z| > 2$ (b) $|z| < 4$

(c) $2 < |z| < 1/4$ (d) $2 < |z| < 4$

[Ans: (d)]

T4. A causal sequence $x[n]$ is represented by its z-transform as

$$X(z) = \frac{18}{(1-2z^{-1})^2(1+z^{-1})}$$

then $x[n]$ can be represented

(a) $[4(2)^n + 12n(2)^n + 2(-1)^n]\, u[n]$

(b) $[16(2)^n - 12n(2)^n + 2(-1)^n]\, u[n]$

(c) $[4(2)^n + 12n(2^n) - 2(-1)^n]\, u[n]$

(d) $[16(2)^n + 12n(2)^n + 2(-1)^n]\, u[n]$

[Ans: (a)]

T5. Consider a discrete LTI system, not necessarily stable but unit sample response of system is zero for $n < 0$. Which of the following can be transfer function of system.

(i) $\dfrac{(1-z^{-1})^2}{\left(1-\frac{1}{2}z^{-1}\right)}$ (ii) $\dfrac{(z-1)^2}{\left(z-\frac{1}{2}\right)}$

(iii) $\dfrac{\left(z-\frac{1}{4}\right)}{\left(z-\frac{1}{2}\right)^6}$ (iv) $\dfrac{\left(z-\frac{1}{4}\right)^6}{\left(z-\frac{1}{2}\right)^5}$

(a) (*i*) and (*iii*) (b) (*i*) and (*iv*)
(c) (*i*), (*ii*) and (*iii*) (d) (*i*) and (*ii*)

[Ans: (a)]

T6. The z-transform $X(z)$ of a right-sided real sequence $x[n]$ has exactly two poles and one of them is $e^{j\pi/2}$ and there are two zeroes at origin. If $X(1) = 1$, then which one of the following is true

(a) $X(z) = \dfrac{2z^2}{(z-1)^2+2}$ ROC is $\dfrac{1}{2} < |z| < 1$

(b) $X(z) = \dfrac{2z^2}{z^2+1}$ ROC is $|z| > \dfrac{1}{2}$

(c) $X(z) = \dfrac{2z^2}{(z-1)^2+2}$ ROC is $|z| > 1$

(d) $X(z) = \dfrac{2z^2}{(z^2+1)}$ ROC is $|z| > 1$

[Ans: (d)]

T7. The z-transform $F(z)$ of the function $f[nT] = a^{nT}$ $u[n]$ is

(a) $\dfrac{z}{z-a^T}$ (b) $\dfrac{z}{z+a^T}$

(c) $\dfrac{z}{z-a^{-T}}$ (d) $\dfrac{z}{z+a^{-T}}$

[Ans: (a)]

T8. Consider the difference equation

$y[n] - \dfrac{1}{3}y[n-1] = x[n]$ and suppose that

$x[n] = \left(\dfrac{1}{2}\right)^n u[n]$. Assuming the condition of initial rest, the solution for $y[n]$, $n \geq 0$ is

(a) $\left(3\left(\dfrac{1}{3}\right)^n - 2\left(\dfrac{1}{2}\right)^n\right)u[n]$

(b) $\left(-2\left(\dfrac{1}{3}\right)^n + 3\left(\dfrac{1}{2}\right)^n\right)u[n]$

(c) $\left(\dfrac{2}{3}\left(\dfrac{1}{3}\right)^n + \dfrac{1}{3}\left(\dfrac{1}{2}\right)^n\right)u[n]$

(d) $\left(\dfrac{1}{3}\left(\dfrac{1}{3}\right)^n + \dfrac{2}{3}\left(\dfrac{1}{2}\right)^n\right)u[n]$

[Ans: (b)]

■■■■