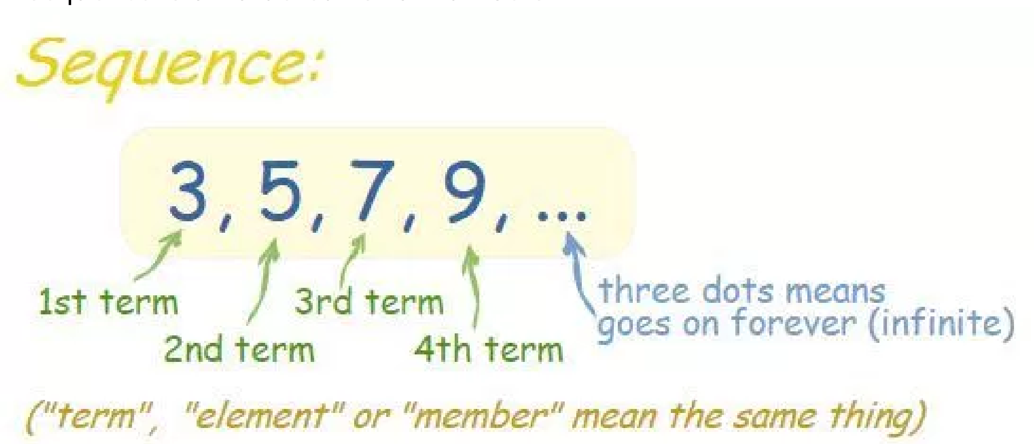
10. Sequences and Series

In the previous lesson, we learned about pattern of numbers. In this lesson we discuss about Sequences.

A sequence is an ordered list of numbers.



The sum of the terms of a sequence is called a **series**.

- Each number of a sequence is called a term (or element) of the sequence.
- A finite sequence contains a finite number of terms (you can count them). 1, 4, 7, 10, 13
- An infinite sequence contains an infinite number of terms (you cannot count them). 1, 4, 7, 10, 13, . . .
- The terms of a sequence are referred to in the subscripted form shown below, where the natural number subscript refers to the location (position) of the term in the sequence.

$$\underset{a_1}{1},\ \underset{a_2}{4},\ \underset{a_3}{7},\ \underset{a_4}{10},\ \underset{a_5}{13},\ \underset{a_6}{16},\ \ldots$$

The general form of a sequence is represented:

- The domain of a sequence consists of the counting numbers 1, 2, 3, 4, … and the range consists of the terms of the sequence.
- The terms in a sequence may, or may not, have a pattern, or a related formula.
- For some sequences, the terms are simply random.

Let’s examine some sequences that have patterns:

Sequences often possess a definite pattern that is used to arrive at the sequence’s terms.

It is often possible to express such patterns as a formula. In the sequence shown at the left, an explicit formula may be:

$$a_n = 12n$$

where n represents the term's position in the sequence.

$$\overset{a_1}{\underset{12(1)}{12}},\ \overset{a_2}{\underset{12(2)}{24}},\ \overset{a_3}{\underset{12(3)}{36}},\ \ldots$$

Examples:

1. Write the first three terms of the sequence whose n^{th} term is given by the explicit formula:

$$a_n = 2n - 1$$

ANSWER: Remember that n is a natural number (starting with $n = 1$).

$$a_1 = 2(1) - 1 = 1$$
$$a_2 = 2(2) - 1 = 3$$
$$a_3 = 2(3) - 1 = 5$$

Notice that n is replaced with the number of the term you are trying to find.

2. Find the 5^{th} and 10^{th} terms of the sequence whose n^{th} term is given by: $a_n = \frac{n}{n+1}$

ANSWER: Remember that n corresponds to the location of the term. Use $n = 5$ and $n = 10$.

$$a_5 = \frac{5}{5+1} = \frac{5}{6}$$

$$a_{10} = \frac{10}{10+1} = \frac{10}{11}$$

3. Write an explicit formula for the n^{th} term of a sequence of negative even integers starting with -2.

ANSWER: Get a visual of the terms. -2, -4, -6, -8, ...
Compare the terms to the numbers associated with their locations and look for a pattern.

Notation	Location	Term
a_1	1	-2
a_2	2	-4
a_3	3	-6
a_4	4	-8

Look for a pattern. In this example, each term can be found by multiplying the location number by -2.
A formula could be:

$$a_n = -2n$$

4. Find the first 4 terms of the sequence $a_n = (-1)^n(n^2 + 3)$

$$a_1 = (-1)^1(1^2 + 3) = -4$$
$$a_2 = (-1)^2(2^2 + 3) = +7$$
$$a_3 = (-1)^3(3^2 + 3) = -12$$
$$a_4 = (-1)^4(4^2 + 3) = +19$$

Notice how the terms are alternating signs between negative and positive.

Keep this pattern in mind (involving powers of -1) when asked to write formulas for sequences.

$(-1)^n(n^2 + 3)$ yields -4, 7, -12, 19, ...
$(-1)^{n+1}(n^2 + 3)$ yields 4, -7, 12, -19, ...

What is the pattern of numbers?

PATTERNS

Patterns arise due to special arrangements of numbers associated with geometrical figures. The numbers are of different kinds like

1. Natural numbers
2. Polygonal numbers
   (a) Triangular numbers
   (b) Square numbers

(c) Pentagonal numbers
(d) Hexagonal numbers, etc.

Examples

1. Patterns in natural numbers

(a)

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$
$$12345 \times 9 + 6 = 111111$$
$$123456 \times 9 + 7 = 1111111$$
$$1234567 \times 9 + 8 = 11111111$$
$$12345678 \times 9 + 9 = 111111111$$

(b)

$$9 \times 9 + 7 = 88$$
$$98 \times 9 + 6 = 888$$
$$987 \times 9 + 5 = 8888$$
$$9876 \times 9 + 4 = 88888$$
$$98765 \times 9 + 3 = 888888$$
$$987654 \times 9 + 2 = 8888888$$
$$9876543 \times 9 + 1 = 88888888$$
$$98765432 \times 9 + 0 = 888888888$$

(c)

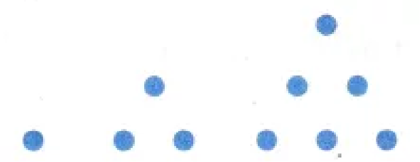
$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

(d)

$$(11)^2 = 121$$
$$(111)^2 = 12321$$
$$(1111)^2 = 1234321$$
$$(11111)^2 = 123454321$$

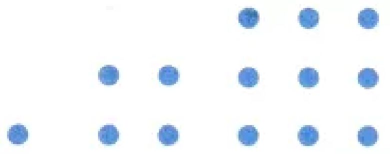
2. Patterns in polygonal numbers

(a) Triangular numbers:



Here the numbers of dots up to any stage are given by the pattern
1, 3, 6, 10, 15, ……
These numbers arise from triangles and so these are called triangular numbers.

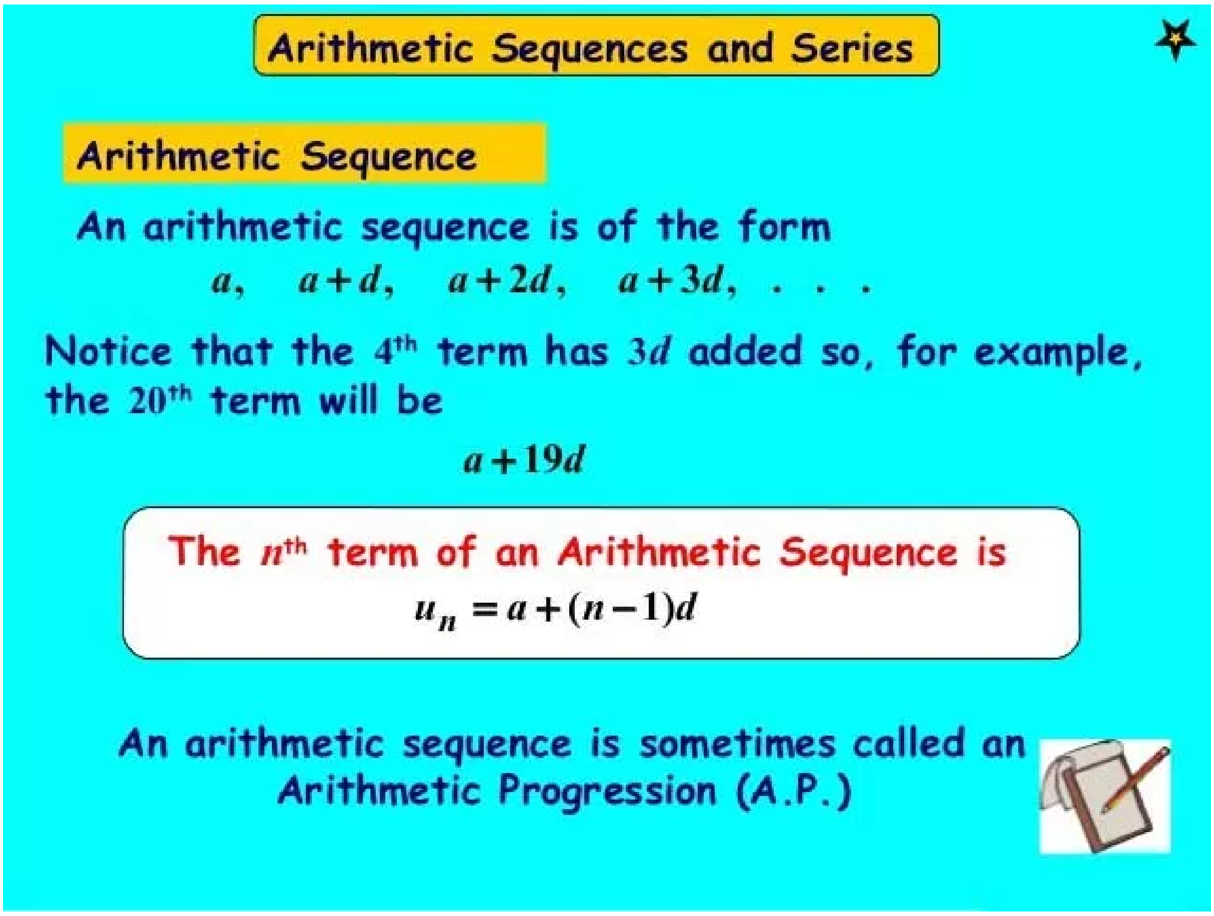
(b) Square numbers:

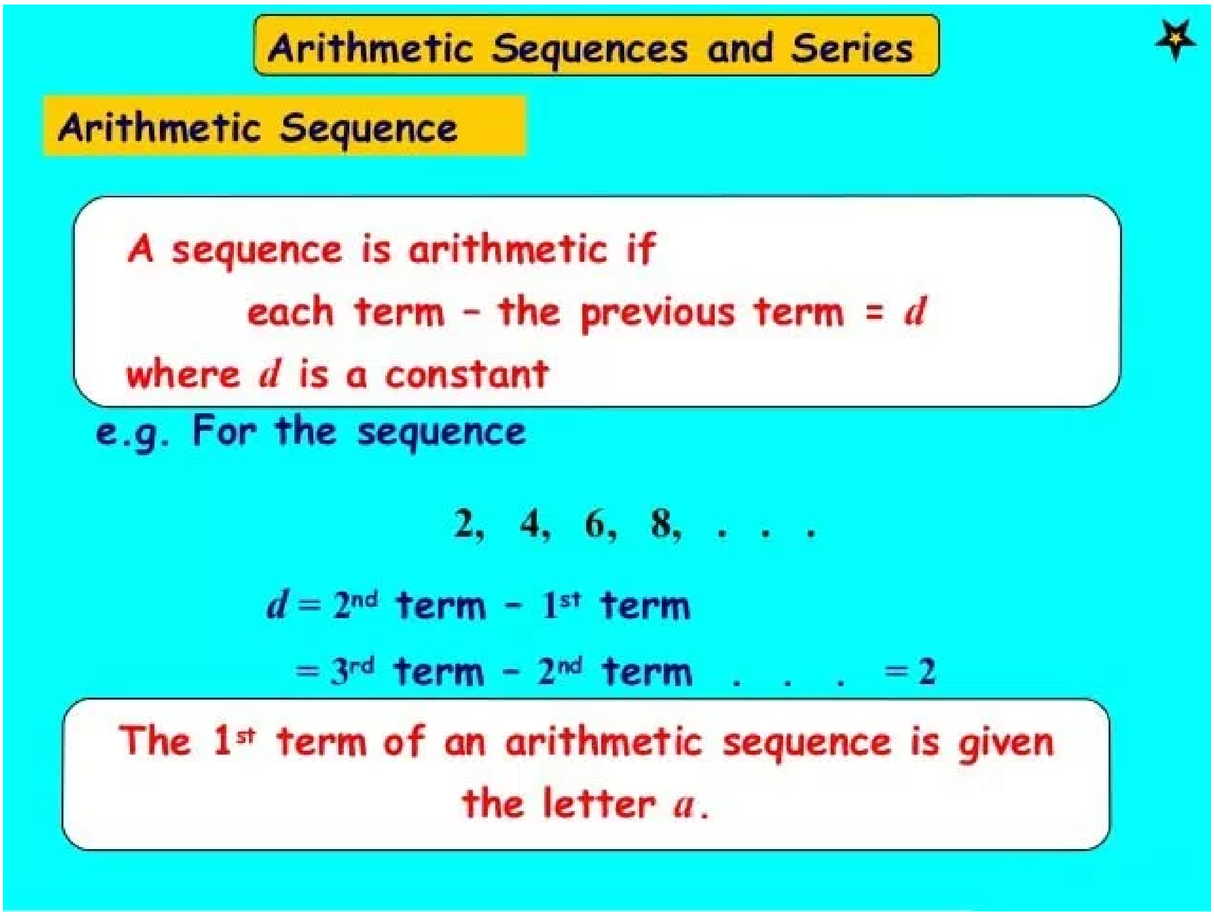


Here the numbers of dots upto any stage are given by the pattern
1, 4, 9, 16, 25, 36, ……
These numbers arise from squares and are called square numbers.

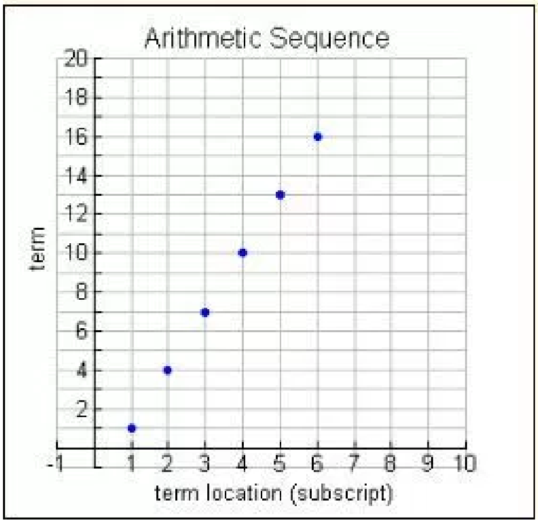
Arithmetic Sequences and Series

A sequence is an ordered list of numbers.
The sum of the terms of a sequence is called a **series**.





While some sequences are simply random values, other sequences have a definite pattern that is used to arrive at the sequence's terms.
Two such sequences are the arithmetic and geometric sequences. Let's investigate the arithmetic sequence.



If a sequence of values follows a pattern of adding a fixed amount from one term to the next, it is referred to as an arithmetic sequence. The number added to each term is constant (always the same).

The fixed amount is called the common difference, d, referring to the fact that the difference between two successive terms yields the constant value that was added. To find the common difference, subtract the first term from the second term.
Notice the linear nature of the scatter plot of the terms of an arithmetic sequence. The domain consists of the counting numbers 1, 2, 3, 4, … and the range consists of the terms of the sequence. While the x value increases by a constant value of one, the y value increases by a constant value of 3 (for this graph).

- An arithmetic sequence is of the form

$$a, \quad a+d, \quad a+2d, \quad a+3d, \quad . \; . \; .$$

- The n^{th} term is $u_n = a+(n-1)d$

- The sum of n terms of an arithmetic series is given by

$$S_n = \frac{n}{2}(a+l) \quad \text{or} \quad S_n = \frac{n}{2}(2a+(n-1)d)$$

Examples:

Arithmetic Sequence	Common Difference, d	
1, 4, 7, 10, 13, 16, ...	$d = 3$	add 3 to each term to arrive at the next term, or...the difference $a_2 - a_1$ is 3.
15, 10, 5, 0, -5, -10, ...	$d = -5$	add -5 to each term to arrive at the next term, or...the difference $a_2 - a_1$ is -5.
$1, \frac{1}{2}, 0, -\frac{1}{2}, \ldots$	$d = -\frac{1}{2}$	add -1/2 to each term to arrive at the next term, or....the difference $a_2 - a_1$ is -1/2.

Formulas used with arithmetic sequences and arithmetic series:

To **find any term** of an **arithmetic sequence**:

$$a_n = a_1 + (n-1)d$$

where a_1 is the first term of the sequence, d is the common difference, n is the number of the term to find.

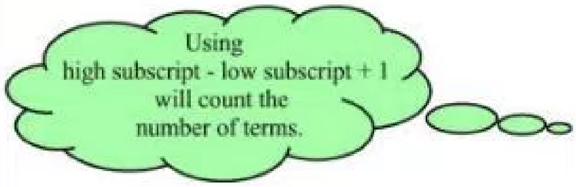
Note: a_1 is often simply referred to as a

To find the **sum of a certain number of terms** of an **arithmetic sequence**:

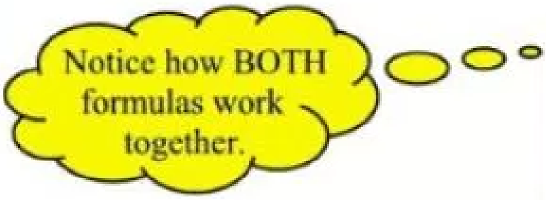
$$S_n = \frac{n(a_1 + a_n)}{2}$$

where S_n is the sum of n terms (n^{th} **partial sum**), a_1 is the first term, a_n is the n^{th} term.

Examples:

Question	Answer
1. Find the common difference for this arithmetic sequence 5, 9, 13, 17 ...	**1.** The common difference, d, can be found by subtracting the first term from the second term, which in this problem yields **4.** Checking shows that 4 is the difference between all of the entries.
2. Find the common difference for the arithmetic sequence whose formula is $a_n = 6n + 3$	**2.** The formula indicates that 6 is the value being added (with increasing multiples) as the terms increase. A listing of the terms will also show what is happening in the sequence (start with $n = 1$). 9, 15, 21, 27, 33, ... The list shows the common difference to be 6.
3. Find the 10th term of the sequence 3, 5, 7, 9, ...	**3.** $n = 10$; $a_1 = 3$, $d = 2$ $a_n = a_1 + (n-1)d$ $a_{10} = 3 + (10-1)2$ $a_{10} = 21$ The tenth term is 21.
4. Find a_7 for an arithmetic sequence where $a_1 = 3x$ and $d = -x$.	**4.** n = 7; $a_1 = 3x$, $d = -x$ $a_n = a_1 + (n-1)d$ $a_7 = 3x + (7-1)(-x)$ $a_7 = 3x + 6(-x) = -3x$
5. Find t_{15} for an arithmetic sequence where $t_3 = -4 + 5i$ and $t_6 = -13 + 11i$ 	**5.** Notice the change of labeling from a to t. The letter used in labeling is of no importance. Get a visual image of this problem $\underbrace{\quad}_{t_1}, \underbrace{\quad}_{t_2}, \underbrace{-4+5i}_{t_3}, \underbrace{\quad}_{t_4}, \underbrace{\quad}_{t_5}, \underbrace{-13+11i}_{t_6}$ Using the third term as the "first" term, find the common difference from these known terms. $a_n = a_1 + (n-1)d$ $t_6 = t_3 + (4-1)d$ $-13 + 11i = -4 + 5i + (4-1)d$ $-13 + 11i = -4 + 5i + 3d$ $-9 + 6i = 3d$ $-3 + 2i = d$ Now, from t_3 to t_{15} is 13 terms. $t_{15} = -4 + 5i + (13-1)(-3 + 2i) = -4 + 5i - 36 + 24i$ $= -40 + 29i$
6. Find a formula for the sequence 1, 3, 5, 7, ...	**6.** A formula will relate the subscript number of each term to the actual value of the term. $a_n = 2n - 1$ Substituting $n = 1$, gives 1. Substituting $n = 2$, gives 3, and so on.
7. Find the 25th term of the sequence -7, -4, -1, 2, ...	**7.** $n = 25$; $a_1 = -7$, $d = 3$ $a_n = a_1 + (n-1)d$ $a_{25} = -7 + (25-1)3$ $a_{25} = 65$

8. Find the sum of the first 12 positive even integers.



8. The word "sum" indicates the need for the sum formula.

positive even integers: 2, 4, 6, 8, ...

$n = 12;\ a_1 = 2, d = 2$

We are missing a_{12}, for the sum formula, so we use the "any term" formula to find it.

$$a_n = a_1 + (n-1)d$$

$$a_{12} = 2 + (12-1)2$$

$$a_{12} = 24$$

Now, let's find the sum:

$$S_{12} = \frac{12(2+24)}{2} = 156$$

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9. Insert 3 arithmetic means between 7 and 23.

> *Note:* An **arithmetic mean** is the term between any two terms of an arithmetic sequence. It is simply the average (mean) of the given terms.

9. While there are several solution methods, we will use our arithmetic sequence formulas.

Draw a picture to better understand the situation.

7, ____, ____, ____, 23

This set of terms will be an arithmetic sequence.

We know the first term, a_1, the last term, a_n, but not the common difference, d. *This question makes NO mention of "sum", so avoid that formula.*

Find the common difference:

$$a_n = a_1 + (n-1)d$$

$$23 = 7 + (5-1)d$$

$$23 = 7 + 4d$$

$$16 = 4d$$

$$4 = d$$

Now, insert the terms using d.

7, <u>11</u>, <u>15</u>, <u>19</u>, 23

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10. Find the number of terms in the sequence 7, 10, 13, ..., 55.

10. $a_1 = 7, a_n = 55,\ d = 3$. We need to find n.

This question makes NO mention of "sum", so avoid that formula.

$$a_n = a_1 + (n-1)d$$

$$55 = 7 + (n-1)3$$

$$55 = 7 + 3n - 3$$

$$55 = 4 + 3n$$

$$51 = 3n$$

$$17 = n$$

When solving for n, be sure your answer is a positive integer. There is no such thing as a fractional number of terms in a sequence!

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11. A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theater has 20 rows of seats, how many seats are in the theater?

11. The seating pattern is forming an arithmetic sequence.

60, 68, 76, ...

We wish to find "the sum" of all of the seats.

$n = 20,\ a_1 = 60,\ d = 8$ and we need a_{20} for the sum.

$$a_n = a_1 + (n-1)d$$

$$a_{20} = 60 + (20-1)8 = 212$$

Now, use the sum formula:

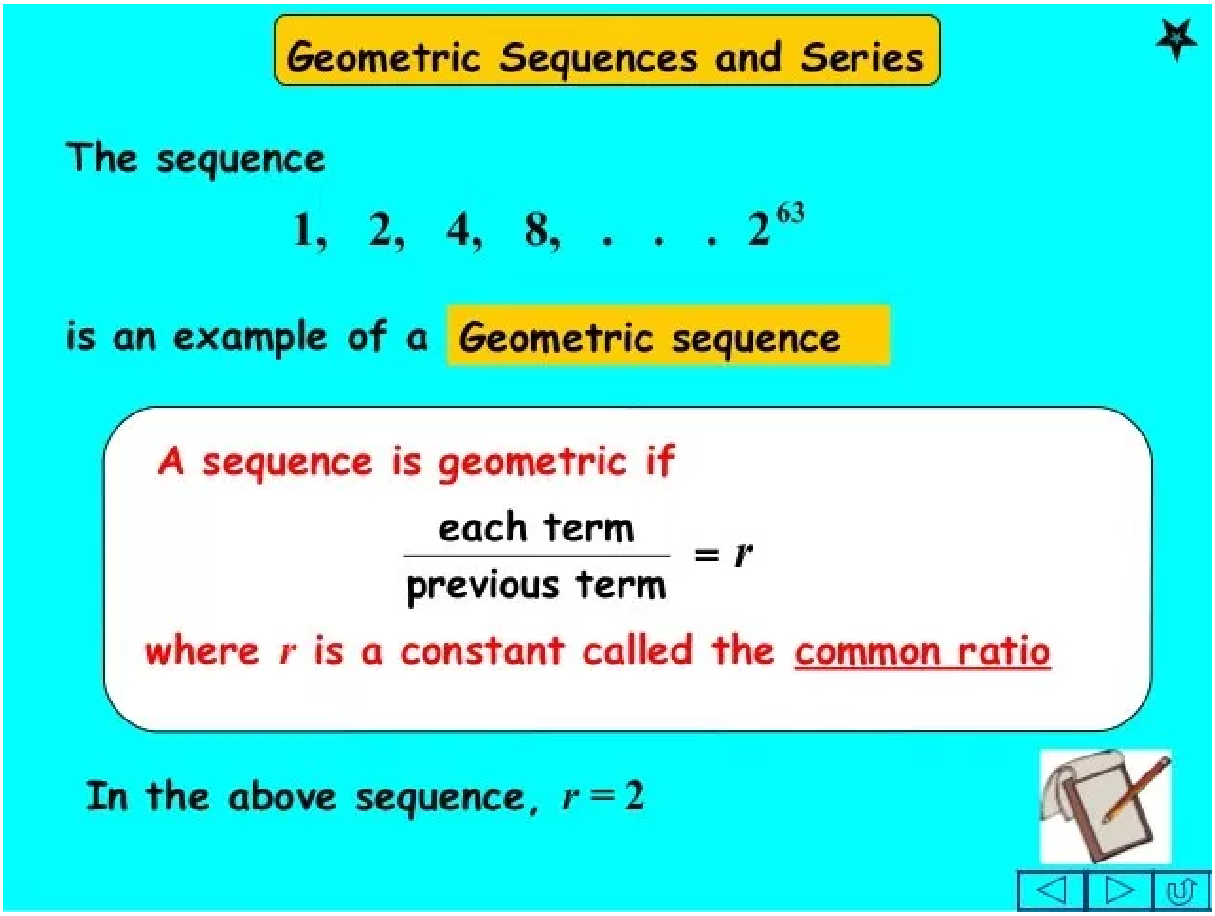
$$S_n = \frac{n(a_1 + a_n)}{2}$$

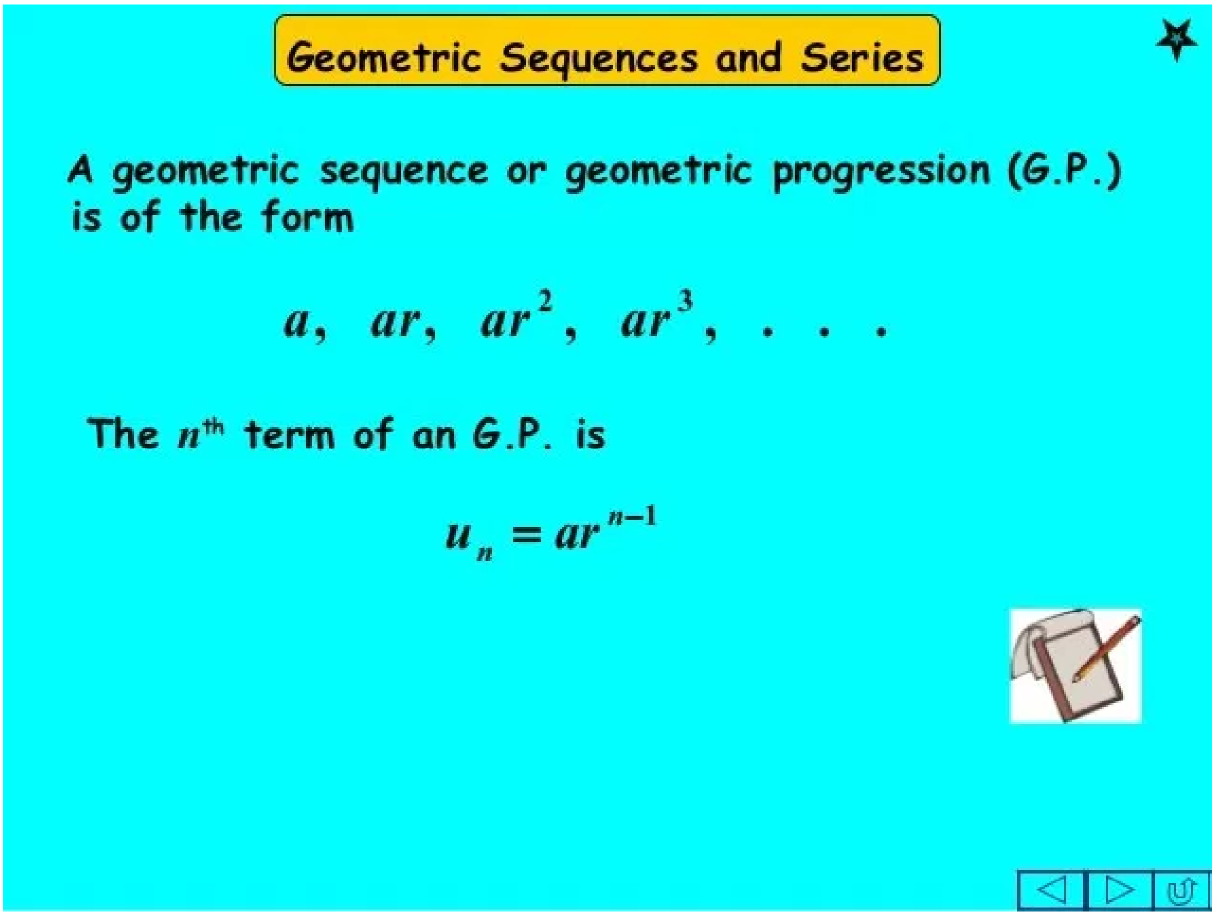
$$S_{20} = \frac{20(60+212)}{2} = 2720$$

There are 2720 seats.

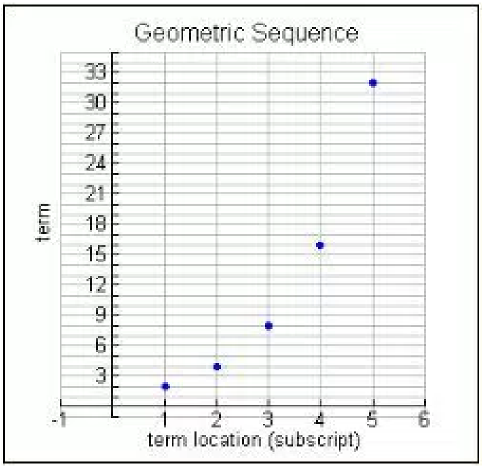
Geometric Sequences and Series

A sequence is an ordered list of numbers.
The sum of the terms of a sequence is called a series.





While some sequences are simply random values, other sequences have a definite pattern that is used to arrive at the sequence's terms.
Two such sequences are the arithmetic and geometric sequences. Let's investigate the geometric sequence.



If a sequence of values follows a pattern of multiplying a fixed amount (not zero) times each term to arrive at the following term, it is referred to as a geometric sequence. The number multiplied each time is constant (always the same).

The fixed amount multiplied is called the common ratio, r, referring to the fact that the ratio (fraction) of the second term to the first term yields this common multiple. To find the common ratio, divide the second term by the first term.

Notice the non-linear nature of the scatter plot of the terms of a geometric sequence. The domain consists of the counting numbers 1, 2, 3, 4, … and the range consists of the terms of the sequence. While the x value increases by a constant value of one, the y value increases by multiples of two (for this graph).
Examples:

Geometric Sequence	Common Ratio, r	
5, 10, 20, 40, …	$r = 2$	multiply each term by 2 to arrive at the next term or...divide a_2 by a_1 to find the common ratio, 2.
-11, 22, -44, 88, …	$r = -2$	multiply each term by -2 to arrive at the next term .or...divide a_2 by a_1 to find the common ratio, -2.
$4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \frac{64}{81}, \ldots$	$r = \frac{2}{3}$	multiply each term by 2/3 to arrive at the next term or...divide a_2 by a_1 to find the common ratio, 2/3.

Formulas used with geometric sequences and geometric series:

To **find any term** of a **geometric sequence**:

$$a_n = a_1 \cdot r^{n-1}$$

where a_1 is the first term of the sequence, r is the common ratio, n is the number of the term to find.

To find the **sum of a certain number of terms** of a **geometric sequence**:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where S_n is the sum of n terms (n^{th} **partial sum**), a_1 is the first term, r is the common ration.

Examples:

Question	Answer
1. Find the common ratio for the sequence $6,\ -3,\ \frac{3}{2},\ -\frac{3}{4},\ \ldots$	**1**. The common ratio, r, can be found by dividing the second term by the first term, which in this problem yields **-1/2.** Checking shows that multiplying each entry by -1/2 yields the next entry.
2. Find the common ratio for the sequence given by the formula $a_n = 5(3)^{n-1}$	**2.** The formula indicates that 3 is the common ratio by its position in the formula. A listing of the terms will also show what is happening in the sequence (start with $n = 1$). 5, 15, 45, 135, ... The list also shows the common ratio to be 3.
3. Find the 7th term of the sequence 2, 6, 18, 54, ...	**3**. $n = 7$; $a_1 = 2, r = 3$ $a_n = a_1 \cdot r^{n-1}$ $a_7 = 2 \cdot 3^{7-1} = 1458$ The seventh term is 1458.
4. Find the 11th term of the sequence $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$	**4.** $n = 11$; $a_1 = 1, r = -1/2$ $a_{11} = 1 \cdot \left(-\frac{1}{2}\right)^{11-1} = \frac{1}{1024}$
5. Find a_8 for the sequence 0.5, 3.5, 24.5, 171.5, ...	**5.** $n = 8$; $a_1 = 0.5,\ r = 7$ $a_n = a_1 \cdot r^{n-1}$ $a_8 = 0.5 \cdot 7^{8-1} = 411{,}771.5$
6. Evaluate using a formula: $\sum_{k=1}^{5} 3^k$	**6.** Examine the summation $\sum_{k=1}^{5} 3^k = 3^1 + 3^2 + 3^3 + 3^4 + 3^5$ This is a geometric series with a common ratio of 3. $n = 5$; $a_1 = 3, r = 3$ $S_5 = \frac{3(1-3^5)}{1-3} = \frac{-726}{-2} = 363$

7. Find the sum of the first 8 terms of the sequence -5, 15, -45, 135, ...	7. The word "sum" indicates a need for the sum formula. $n = 8;\ a_1 = -5,\ r = -3$ $S_8 = \frac{-5(1-(-3)^8)}{1-(-3)}$ $S_8 = \frac{-5(1-6561)}{4} = \frac{32800}{4} = 8200$
8. The third term of a geometric sequence is 3 and the sixth term is 1/9. Find the first term.	**8.** Think of the sequence as "starting with" 3, until you find the common ratio. ___, ___, 3, ___, ___, $\frac{1}{9}$ *For this modified sequence:* $a_1 = 3,\ a_4 = 1/9,\ n = 4$ $a_n = a_1 \cdot r^{n-1}$ $\frac{1}{9} = 3 \cdot r^{4-1}$ $\frac{1}{27} = r^3$ $\frac{1}{3} = r$ Now, work backward multiplying by 3 (or dividing by 1/3) to find the actual first term. $a_1 = 27$
9. A ball is dropped from a height of 8 feet. The ball bounces to 80% of its previous height with each bounce. How high (*to the nearest tenth of a foot*) does the ball bounce on the fifth bounce?	**9.** Set up a model drawing for each "bounce". 6.4, 5.12, ___, ___, ___ The common ratio is 0.8. $a_n = a_1 \cdot r^{n-1}$ $a_n = 6.4 \cdot (.8)^{5-1} = 2.62144$ Answer: 2.6 feet

What is Arithmetico–Geometric Sequence?

Arithmetico-geometric Progression (A.G.P.)

Definition:
The combination of arithmetic and geometric progression is called arithmetico-geometric progression.

n^{th} term of A.G.P.

If $a_1, a_2, a_3, \ldots\ldots a_n, \ldots\ldots$ is an A.P. and $b_1, b_2, b_3, \ldots\ldots b_n, \ldots\ldots$ is a G.P., then the sequence $a_1b_1, a_2b_2, a_3b_3, \ldots\ldots, a_nb_n, \ldots\ldots$ is said to be an arithmetico-geometric sequence.
Thus, the general form of an arithmetico geometric sequence is a, (a + d)r, (a + 2d)r^2, (a + 3d)r^3, ………
From the symmetry we obtain that the n^{th} term of this sequence is $[a + (n - 1)d]r^{n-1}$.
Also, let a, (a + d)r, (a + 2d)r^2, (a + 3d)r^3, ……… be an arithmetico-geometric sequence.
Then, a, (a + d)r, (a + 2d)r^2, (a + 3d)r^3, ……… is an arithmetico-geometric series.

Sum of A.G.P.

(1) **Sum of n terms :** The sum of n terms of an arithmetico-geometric sequence $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, ...$ is given by

$$S_n = \begin{cases} \frac{a}{1-r} + dr\frac{(1-r^{n-1})}{(1-r)^2} - \frac{\{a+(n-1)d\}r^n}{1-r}, \text{ when } r \neq 1 \\ \frac{n}{2}[2a+(n-1)d], \text{ when } r = 1 \end{cases}$$

(2) **Sum of infinite sequence:** Let $|r| < 1$. Then $r^n, r^{n-1} \to 0$ as $n \to \infty$ and it can also be shown that $n.r^n \to 0$ as $n \to \infty$. So, we obtain that

$$S_n \to \frac{a}{1-r} + \frac{dr}{(1-r)^2}, \text{ as } n \to \infty.$$

In other words, when $|r| < 1$ the sum to infinity of an arithmetico-geometric series is $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

Method for finding sum

This method is applicable for both sum of n terms and sum of infinite number of terms.
First suppose that sum of the series is S, then multiply it by common ratio of the G.P. and subtract. In this way, we shall get a G.P., whose sum can be easily obtained.

Method of difference

If the differences of the successive terms of a series are in A.P. or G.P., we can find n^{th} term of the series by the following steps :

Step I: Denote the n^{th} term by T_n and the sum of the series upto n terms by S_n.
Step II: Rewrite the given series with each term shifted by one place to the right.
Step III: By subtracting the later series from the former, find T_n.
Step IV: From T_n, S_n can be found by appropriate summation.

Example : Consider the series 1+ 3 + 6 + 10 + 15 +…..to n terms. Here differences between the successive terms are 3 – 1, 6 – 3, 10 – 6, 15 – 10, …….i.e., 2, 3, 4, 5,…… which are in A.P. This difference could be in G.P. also. Now let us find its sum:

$$S = 1 + 3 + 6 + 10 + 15 + ... + T_{n-1} + T_n$$

$$S = \quad 1 + 3 + 6 + 10 + ... + T_{n-1} + T_n$$

Subtracting, we get

$$0 = 1 + 2 + 3 + 4 + 5 + ... + (T_n - T_{n-1}) - T_n$$

$$\Rightarrow \quad T_n = 1 + 2 + 3 + 4 + ... \text{ to } n \text{ terms.}$$

$$\Rightarrow \quad T_n = \frac{1}{2}n(n+1) \qquad \therefore \; S_n = \Sigma T_n = \frac{1}{2}[\Sigma n^2 + \Sigma n]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right] = \frac{n(n+1)(n+2)}{6}.$$

Miscellaneous series

Special series

(1) Sum of first n natural numbers

$$= 1 + 2 + 3 + ... + n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2} \text{ .}$$

(2) Sum of squares of first n natural numbers

$$= 1^2 + 2^2 + 3^2 + ... + n^2 = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6} \text{ .}$$

(3) Sum of cubes of first n natural numbers

$$= 1^3 + 2^3 + 3^3 + 4^3 + ... + n^3 = \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2 \text{ .}$$

Recursive Sequences

Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term. Recursion requires that you know the value of the term immediately before the term you are trying to find.

A recursive formula always has two parts:
1. the starting value for a1.
2. the recursion equation for an as a function of an-1 (the term before it.)

Recursive formula:	Same recursive formula:
$a_1 = 4$ $a_n = 2a_{n-1}$	$a_1 = 4$ $a_{n+1} = 2a_n$

Be sure you understand that the two formulas at the left say the same thing. Different textbooks write recursive formulas in different ways.
A recursive formula may list the first two (or more) terms as starting values, depending upon the natrue of the sequence. In such caes, the an portion of the formula is dependent upon the previous two (or more) terms..

Examples:

1. Write the first four terms of the sequence: $a_1 = -4$
$a_n = a_{n-1} + 5$

$a_1 = -4$
$n = 2:\ a_2 = a_{2-1} + 5 = 1$
$n = 3:\ a_3 = a_{3-1} + 5 = 6$
$n = 4:\ a_4 = a_{4-1} + 5 = 11$

In recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: -4, 1, 6, 11

2. Consider the sequence 2, 4, 6, 8, 10, ...

Explicit formula:
$a_n = 2n$

Recursive formula:
$a_1 = 2$
$a_n = a_{n-1} + 2$

Certain sequences, such as this arithmetic sequence, can be represented in more than one manner. This sequence can be represented as either an explicit (general) formula or a recursive formula.

3. Consider the sequence 3, 9, 27, 81, ...

Explicit formula:
$a_n = 3^n$

Recursive formula:
$a_1 = 3$
$a_n = 3a_{n-1}$

Certain sequences, such as this geometric sequence, can be represented in more than one manner. This sequence can be represented as either an explicit formula or a recursive formula.

4. Consider the sequence 2, 5, 26, 677, ...

Recursive formula:
$a_1 = 2$
$a_n = (a_{n-1})^2 + 1$

This sequence is neither arithmetic nor geometric. It does, however, have a pattern of development based upon each previous term.

5. Write the first 5 terms of the sequence $a_1 = 3$
$a_n = (-1)^n \cdot 5a_{n-1}$

$a_1 = 3$
$a_2 = (-1)^2 \cdot 5a_{2-1} = 5 \cdot 3 = 15$
$a_3 = (-1)^3 \cdot 5a_{3-1} = (-1) \cdot 5 \cdot 15 = -75$
$a_4 = (-1)^4 \cdot 5a_{4-1} = 5 \cdot (-75) = -375$
$a_5 = (-1)^5 \cdot 5a_{5-1} = (-1) \cdot 5 \cdot (-375) = 1875$

Notice how the value of n is used as the exponent for the value (-1). Also, remember that in recursive formulas, each term is used to produce the next term. Follow the movement of the terms through the set up at the left.

Answer: 3, 15, -75, -375, 1875