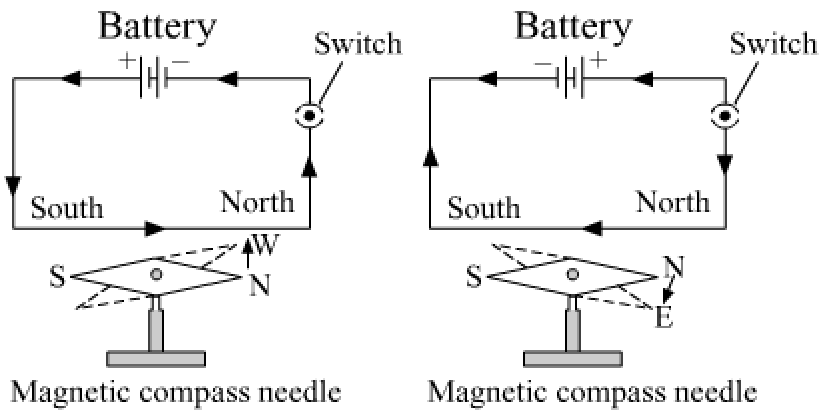
Moving Charges And Magnetism

Magnetic Field

The magnetic effect of current was first discovered by H.C. Oersted. He observed that the flow of charges in a conductor produces magnetic effect around it. He demostrated this phenomenon using the experimental set-up shown below.



When the current in the circuit was anticlockwise, the magnetic needle was found to deflect towards West. The needle deflected towards East when the direction of current was reversed. This experiment proved that current has magnetic effect associated with it.

The Biot–Savart Law

Let:

XY = Current-carrying conductor

I = Current in the conductor

dl = Infinitesimal element of the conductor

*d*B = Magnetic field at point P

r = Distance of point *P* from the element

According to the Biot–Savart law, the magnetic field is proportional to the current and element length and inversely proportional to the square of the distance.

That is,

$$dB \propto \frac{I\, dl \times r}{r^3}$$

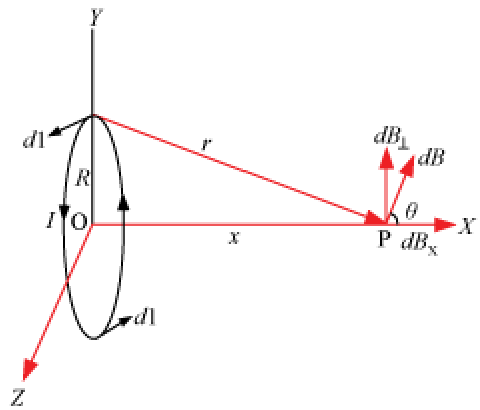
$= \frac{\mu_0}{4\pi} \frac{I\, dl \times r}{r^3}$

Here,

$\frac{\mu_0}{4\pi}$ = Constant of proportionality = 10^{-7} Tm/A

μ_0 = Permeability in free space

Magnetic field on the axis of a circular current loop



I = Current in the loop

R = Radius of the loop

X-axis = Axis of the loop

X = Distance between O and P

dl = Conducting element of the loop

- According to the Biot–Savart law, the magnetic field at P is

$dB = \frac{\mu_0}{4\pi} \frac{I\,|dl \times r|}{r^3}$

$r^2 = x^2 + R^2$

$|dl \times r| = rdl$ (Because they are perpendicular)

$\therefore dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{(x^2 + R^2)}$

- dB has two components: dB_x and $dB_\perp$. $dB_\perp$ is cancelled out and only the x-component remains.

  $\therefore dB_x = dB\cos\theta$

  $$\cos\theta = \frac{R}{\left(x^2+R^2\right)^{1/2}}$$

  $$\therefore dB_x = \frac{\mu_0 I\, dl}{4\pi}\,\frac{R}{\left(x^2+R^2\right)^{3/2}}$$

- Summation of dl over the loop is given by $2\pi R$.

  $$\therefore B = B = B_x\,\hat{i} = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}\,\hat{i}$$

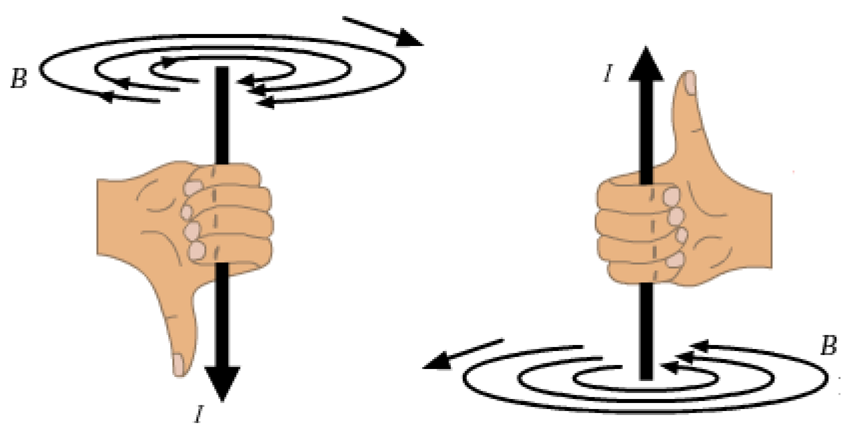
- For the magnetic field at the centre of the loop, $x = 0$.

  $$\therefore B = \frac{\mu_0 I}{2R}\,\hat{i}$$

Right-Hand Thumb Rule

Maxwell's right-hand thumb rule indicates the direction of magnetic field if the direction of current is known.

According to this rule, if we grasp the current-carrying wire in our right hand such that our thumb points in the direction of the current, then the direction in which our fingers encircle the wire will tell the direction of the magnetic field lines around the wire.

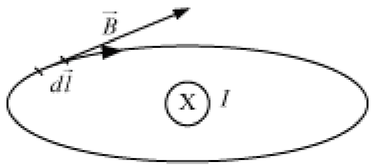


When the thumb points upwards, the curled fingers are anticlockwise. So, the direction of the magnetic field is anticlockwise.

When the thumb points downwards, the curled fingers are clockwise. So, the direction of the magnetic field is clockwise.

Ampere's Circuital Law

- Consider a closed plane curve, with current *I* passing through the curve.



- The boundary is made up of a number of small elements, each of length *dl.*
- The tangential component of the magnetic field (B_T) is multiplied to the element *dl* (i.e., $B_T\, dl$).
- All such products are added together to form an integral $\oint B \cdot dl$.
- Ampere's law states that this integral is equal to μ_0 times the total current (*I*) passing through the surface.
i.e., $\oint B \cdot dl = \mu_0 I$
- When the boundary of the loop is a circle and the magnetic field is tangential to its circumference, we get:

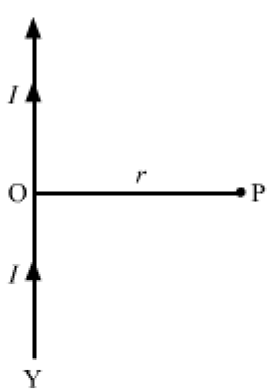
$$B \times 2\pi r = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

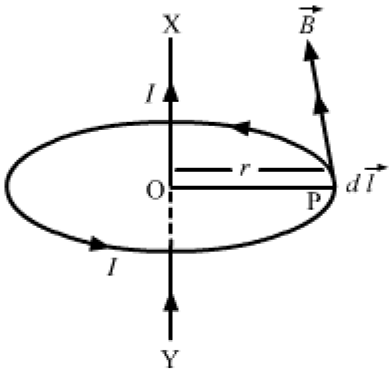
- Ampere's circuital law holds true for steady currents, which do not fluctuate with time.

Applications of Ampere's Law

(1) Magnetic field due to a long straight conductor carrying current:



Let XY be an infinitely long straight conductor.
I = current through the conductor
r = distance of point P from XY
B→B→ = magnetic field at point P due to current I
As the Amperian loop, we choose an imaginary circle with radius r perpendicular to the conductor.
Let dl→dl→ be the length of a small element of the circle around the wire.



According to Ampere's law, we have:

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 I \quad ...(1)$$

But $\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \oint Bdl \cos\theta$

Here, angle θ is zero.

$\therefore \cos\theta = \cos 0 = 1$

Further, due to symmetry, B is the same at all points along the closed path.

$$\therefore \oint \overrightarrow{B} \cdot \overrightarrow{dl} = B \oint dl = B\,(2\pi r) \quad ...(2)$$

From equations (1) and (2), we get:

$$B\,(2\pi r) = \mu_0 I$$

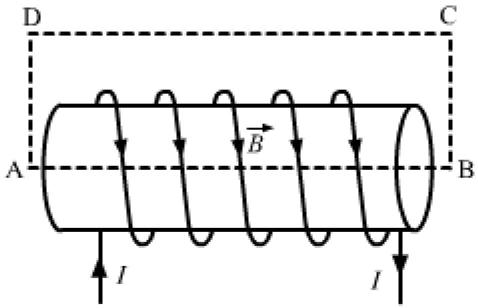
$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right)$$

(2) Magnetic induction along the axis of a long straight solenoid:

A solenoid consists of an insulating long wire closely wound in the form of helix. Its length is large as compared to its diameter.
Consider a long straight solenoid.
Suppose that

n = number of turns per unit length
F = cylindrical former
I = current flowing through the solenoid
B→B→ = magnetic induction at a point well inside the solenoid
ABCD is the rectangular path.
AB = L (length of the rectangular path ABCD)



The number of turns enclosed by ABCD is nL. Since current I flows through the rectangular path nL times, the total current flowing through the rectangular path is nLI.

According to Ampere's circuital law, we have:

$$\oint \vec{B} . \vec{dl} = \mu_0 (nLI) \quad ...(1)$$

For closed loop ABCDA, we have:

$$\oint \vec{B} . \vec{dl} = \int_A^B \vec{B} . \vec{dl} + \int_B^C \vec{B} . \vec{dl} + \int_C^D \vec{B} . \vec{dl} + \int_D^A \vec{B} . \vec{dl}$$

The direction of $\vec{B}$ is perpendicular to BC and AD.

$$\int_B^C \vec{B} . \vec{dl} = \int_D^A \vec{B} . \vec{dl} = 0$$

The magnetic field lines are widely spaced outside the solenoid, hence it is a very weak field (practically zero).

$$\therefore \int_C^D \vec{B} . \vec{dl} = 0$$

$$\text{Thus, } \oint \vec{B} . \vec{dl} = \int_A^B \vec{B} . \vec{dl} + 0 + 0 + 0$$

$$\Rightarrow \oint \vec{B} . \vec{dl} = \int_A^B B . dl$$

$$\Rightarrow \oint \vec{B} . \vec{dl} = \int_A^B Bdl \cos\theta = \int_A^B B . dl$$

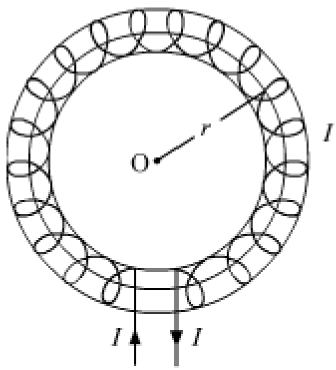
$$\Rightarrow \oint \vec{B} . \vec{dl} = B \int_A^B dl$$

$$\Rightarrow \oint \vec{B} . \vec{dl} = B . L \ (\because \ \int dl = L) \quad \ldots \left(2\right)$$

Comparing equations (1) and (2), we get:
$\mu_0 nLI = BL$
$\therefore B = \mu_0 nI$

At a point near the end, the magnetic induction is given by $B = 1/2\ \mu_0 nI$

(3) Magnetic induction along the axis of a toroid:



A Toroid is a hollow circular ring on which a large number of turns of a wire are closely wound.

Consider a toroidal solenoid with centre O as shown.
Suppose that
r = average radius of the toroid
I = current through the solenoid
To determine the magnetic field produced at the centre along the axis of the toroid due to current *I*, we imagine a Amperian loop of radius *r* and traverse it in clockwise direction.
According to Ampere's circuital law, we have:

$$\oint \overrightarrow{B} \,.\, \overrightarrow{dl} = \mu_0 I$$

The total current flowing through the toroid is NI, where N is the total number of turns.

$$\oint \overrightarrow{B} \,.\, \overrightarrow{dl} = \mu_0 \left(NI\right) \quad ...(1)$$

Now, $\overrightarrow{B}$ and $\overrightarrow{dl}$ are in the same direction

$$\oint \overrightarrow{B} \,.\, \overrightarrow{dl} = B. \oint dl$$

$$\Rightarrow \oint \overrightarrow{B} \,.\, \overrightarrow{dl} = B\left(2\pi r\right) \quad \ldots \left(2\right)$$

Comparing equations (1) and (2), we get:

$$B\left(2\pi r\right) = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

If n is the number of turns per unit length, then

$$n = \frac{N}{2\pi r}$$

$$\therefore B = \mu_0 nI$$

This is the expression for magnetic induction along the axis of a toroid.

Magnetic Force

- Static charges produce an electric field while current or moving charges produce magnetic field (B).
- Magnetic field of several sources is the vector addition of magnetic field of each individual source.

Lorentz Force

- Consider a point charge q moving in the presence of both electric and magnetic fields.

Let

q – Point charge

v – Velocity of point charge

t – Time

r – Distance

B (r) – Magnetic field

E (r) – Electric field

∴ Force on the charge, $\vec{F} = q\left[\vec{E}(r) + v \times B(r)\right] = F_{electric} + F_{magnetic}$

This force is called Lorentz force.

- Force due to magnetic field depends on q, v, B. Force on negative charge is opposite to that of positive charge.
- Magnetic force is a vector product of velocity (v) and magnetic field (B). It vanishes, if v and B are parallel or anti-parallel.
- Magnetic force is zero, if charge is not moving.
- Unit of magnetic field (B) is tesla (T).

Magnetic Force on a Current Carrying Conductor Placed in Magnetic Field:

A straight rod carrying current is considered.

Let

A – Cross-sectional area of the rod

l – Length of the rod

n – Number density of mobile charge carriers

I – Current in the rod

v_d – Average drift velocity of mobile charge carrier

B – External magnetic field

Force on the carriers,

$F = (nAl)\, qv_d \times B$

Since current density, $j = nqv_d$

$\therefore F = [(nqv_d)Al] \times B$

$F = [jAl] \times \mathrm{B}$

$F = I\, l \times B$

Where,

l is the vector magnitude of length of the rod

- For a wire of arbitrary shape, $F = \sum Idl \times B$

Motion in Electic & Magnetic Fields

- When a charged particle having charge q moves inside a magnetic field $\vec{B}$ with velocity v, it experiences a force $\vec{F} = q(\vec{v} \times \vec{B})$.
- When $\vec{v}$ is perpendicular to $\vec{B}$, the force $\vec{F}$ on the charged particle acts as the centripetal force and makes it move along a circular path.
- Let m be the mass of charged particle and r be the radius of the circular path.

Then $q\left(\vec{v}\times\vec{B}\right)=\frac{mv^2}{r}$

$\because v$ and B are at right angles

$$\therefore qvB=\frac{mv^2}{r}$$
$$\Rightarrow r=\frac{mv}{Bq}$$

Time period of the circular motion of a charged particle is given by

$$T=\frac{2\pi r}{v}$$
$$=\frac{2\pi}{v}\frac{mv}{Bq}$$

$$\Rightarrow T=\frac{2\pi m}{Bq}$$

- Angular frequency, $\omega=\frac{2\pi}{T}$

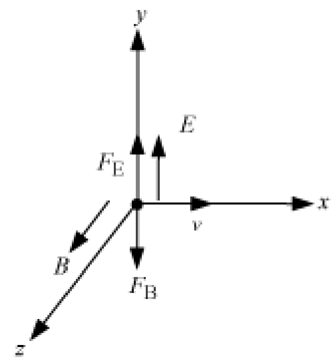
$$\boxed{\therefore \omega=\frac{Bq}{m}}$$

This is often called cyclotron frequency.

Velocity Selector

- Force in the presence of magnetic and electric field, $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$
- Consider that the electric and the magnetic field are perpendicular to each other and, also, perpendicular to the velocity of the particle.
  Then we have:

$$\vec{E}=E\,\hat{j},\vec{B}=B\,\hat{k},\vec{B},\vec{v}=v\hat{i}$$
$$\vec{F}_E=q\,\vec{E}=qE\,\hat{j},\vec{F}_B=q\vec{v}\times\vec{B},=q(v\hat{i}\times B\hat{k})=-qB\,\hat{j}$$
$$\therefore \vec{F}=q(E-v\,B)\,\hat{j}$$



- If we adjust the values of $\vec{E}$ and $\vec{B}$ such that magnitudes of the two forces are equal, then the total force on the charge will be zero and it will move in the fields undeflected. This happens when
  $qE = qvB$

  $$\therefore v = \frac{E}{B}$$

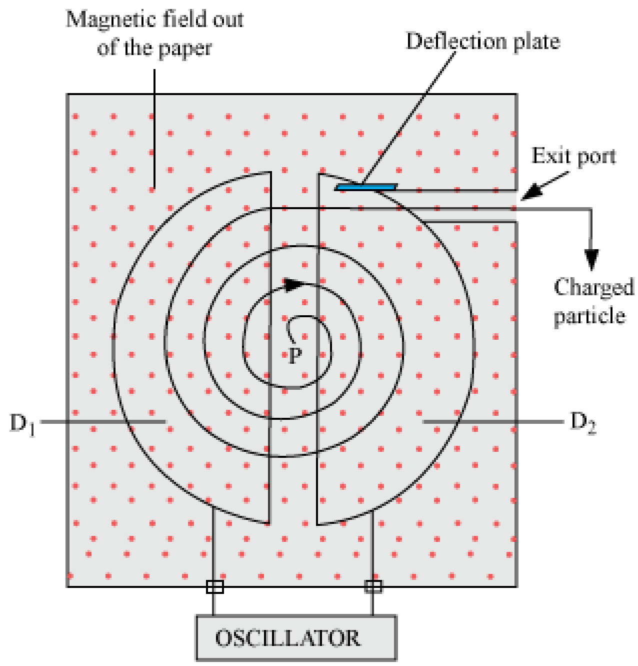
- The above condition can be used to select a charged particle of particular velocity from the charges moving with different speeds. Therefore, it is called velocity selector.

  Cyclotron

- It is used to accelerate charged particles or ions to high energy.

- Electric and magnetic fields are used in combination to increase the energy.

- The working of a cyclotron is based on the fact that the frequency of revolution of a charged particle is not dependent on the energy.



- The particle moves inside the two semicircular disc-like metal containers D_1 and D_2, called dees.
- When the particle moves from one dee to another, it is acted upon by the electric field.
- The electric field changes sign alternately. Therefore, the particle is accelerated by the electric field, which increases the energy of the particle.
- The increase in energy increases the radius of the circular path. Hence, the path is a spiral one.
- Time period of revolution is given by

$$T = \frac{1}{\nu_C} = \frac{2\pi m}{qB}$$

$$\nu_C = \frac{qB}{2\pi m}$$

This frequency is called the cyclotron frequency.

- When the frequency of an applied voltage is equal to the cyclotron frequency, it is called resonance condition.
- The radius of trajectory is given by

$$r = \frac{vm}{qB}$$

$$\therefore v = \frac{rqB}{m}$$

Now, kinetic energy of the ion = $\frac{1}{2}mv^2$

$$= \frac{1}{2}m\frac{r^2q^2B^2}{m^2}$$

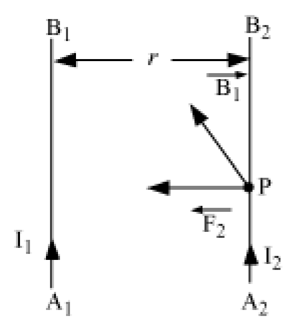
$$\boxed{= \frac{1}{2}\frac{r^2q^2B^2}{m}}$$

Limitations of a Cyclotron

- It cannot accelerate uncharged particles like neutrons.
- There is a limit of speed beyond which a charged particle cannot be accelerated by a cyclotron.
- It cannot accelerate electrons as their charge to mass ratio is very high due to which they cannot remain in phase with the field.
- It cannot produce highly energetic particles with energy of the order of 500 MeV.

Force Between Two Parallel Current Carrying Conductors

Force between Two Parallel Currents



- A_1B_1 and A_2B_2 are two infinity long straight conductors.

- I_1 and I_2 are the current flowing through them and these are r distance apart.

- Magnetic field induction at a point P on conductor A_2B_2 due to current I_1 passing through A_1B_1 is

$$B_1 = \frac{\mu_0 2I_1}{4\pi r}$$

- Unit length of A_2B_2 will experience a force as

$$F_2 = B_1 I_2 \times 1 = B_1 I_2$$
$$\therefore F_2 = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$$

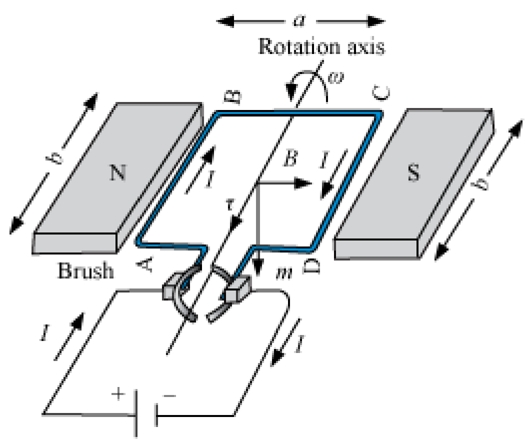
- Conductor A_1B_1 also experiences the same amount of force, directed towards A_2B_2. Hence, A_1B_1 and A_2B_2 attract each other.

- Two linear parallel conductors carrying currents in the same direction attract each other while in opposite direction repel each other.

Ampere

- One ampere is the current which flows through each of the two parallel uniform long linear conductors, which are placed in free space at a distance of 1 m from each other and which attract or repel each other with a force of 2×10^{-7} N/m of their lengths.

Torque on Current Loop



Case I - The rectangular loop is placed such that the uniform magnetic field B is in the plane of loop.

- No force is exerted by the magnetic field on the arms AD and BC.

- Magnetic field exerts a force F_1 on arm AB.

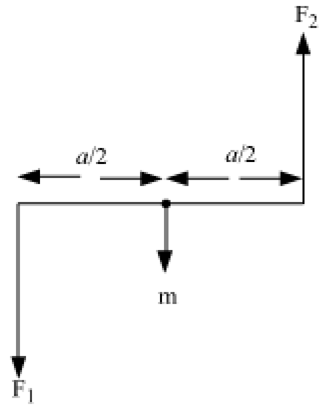
$\therefore F_1 = IbB$

- Magnetic field exerts a force F_2 on arm CD.

$\therefore F_2 = IbB = F_1$

Net force on the loop is zero.

- The torque on the loop rotates the loop in anti-clockwise direction.



- Torque, τ = $F_1\frac{a}{2}+F_2\frac{a}{2}$

$= IbB\frac{a}{2}+IbB\frac{a}{2}$

= $I(ab)$B

τ = IAB

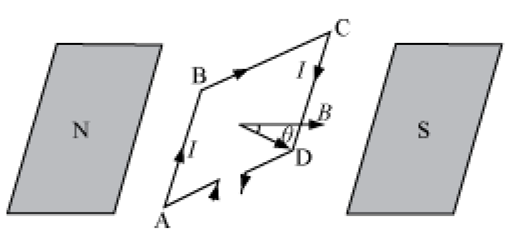
If there are 'n' such turns the torque will be nIAB

Where, b → Breadth of the rectangular coil

a → Length of the rectangular coil

A = ab → Area of the coil

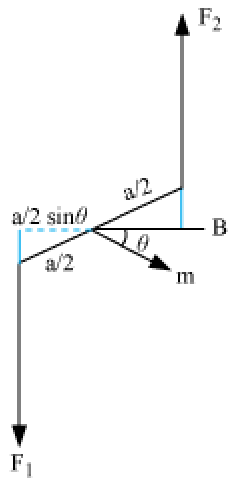
Case II - Plane of the loop is not along the magnetic field, but makes angle with it.



- Angle between the field and the normal is θ.
- Forces on BC and DA are equal and opposite and they cancel each other as they are collinear.
- Force on AB is F_1 and force on CD is F_2.

$F_1 = F_2 = IbB$

- Magnitude of torque on the loop as in the figure:



$\therefore \tau = F_1 \dfrac{a}{2} \sin\theta + F_2 \dfrac{a}{2} \sin\theta$

$= Iab\ B\sin\theta$

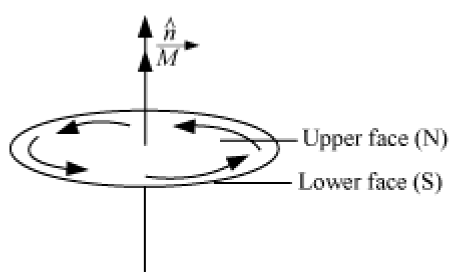
$\tau = IAB \sin\theta$

If there are 'n' such turns the torque will be $nIAB \sin\theta$

- Magnetic moment of the current, $m = IA$

$\boxed{\therefore\ \vec{\tau} = \vec{m} \times \vec{B}}$

Circular Current Loop as a Magnetic Dipole



- A current carrying wire can be considered as a magnetic dipole.
- Its upper face has current flowing in anti-clockwise direction.

∴ It has North polarity.

- Its lower face has current flowing in clockwise direction.

  ∴ It has South polarity.

- Magnetic dipole moment of current loop (M) is
- directly proportional to strength of current (I)

  ∴ $M \propto I$

- directly proportional to the area of enclosed loop (A)

  ∴ $M \propto A$

  $\Rightarrow$ $M = kIA$

  Where, k is constant of proportionality

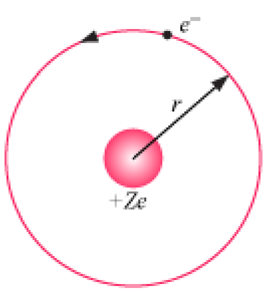
  For $k = 1$,

  $M = AI$

- For N such turns,

  $$\boxed{M = NIA}$$

Magnetic dipole moment of a revolving electron



- An electron of charge e revolves in a path of radius r having a uniform angular velocity ω.
- Current, $i = \dfrac{\text{Charge}}{\text{Time}} = \dfrac{e}{t}$

  Where, t is period of revolution of electron

$$\therefore t = \frac{2\pi}{\omega}$$

$$\therefore i = \frac{e\omega}{2\pi}$$

- Let r be the radius of the orbit.

  $\therefore$ Area of the orbit, $A = \pi r^2$

- Magnetic moment of the atom, $M = iA$

$$M = \frac{\omega e}{2\pi}\pi r^2$$

$$\therefore M = \frac{1}{2}e\omega r^2 \qquad \text{........ (1)}$$

- According to Bohr's theory, an electron in an atom can revolve only in a certain stationary orbit.

- Therefore, the angular momentum of electron (mvr) is an integral multiple of $\left(\frac{h}{2\pi}\right)$.

$$\therefore mvr = \frac{nh}{2\pi}$$

$$\because v = r\omega$$

$$\therefore m(r\omega)r = \frac{nh}{2\pi}$$

$$\omega r^2 = \frac{nh}{2\pi m}$$

From equation (1),

$$M = \frac{1}{2}e \cdot \frac{nh}{2\pi m}$$

$$= n\frac{eh}{4\pi m} = n\mu_B$$

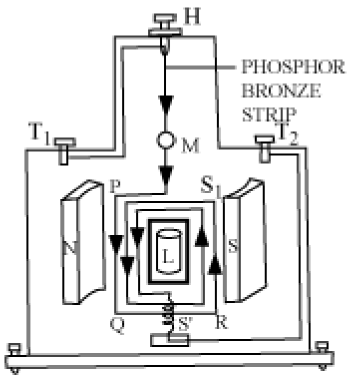
$$\mu_B = \frac{eh}{4\pi m}$$

Where, μ_B is the least value of atomic dipole moment known as Bohr's magneton.

Moving Coil Galvanometer

Principle:

The working of a moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



Working:

Suppose that a coil *PQRS* is suspended freely in a magnetic field.

Let l = length *PQ* or *RS* of the coil;

b = breadth *QR* or *SP* of the coil and

n = number of turns in the coil

Area of each turn of the coil, $A = l \times b$

Let B = strength of the magnetic field in which the coil is suspended.

and I = current passing through the coil in the direction *PQRS*

Let α be the angle, at any instant, which the normal drawn on the plane of the coil makes with the direction of the magnetic field.

The rectangular coil carrying current, when placed in the magnetic field, experiences a torque whose magnitude is given by
$\tau = nIBA \sin\alpha$

Due to the deflecting torque, the coil rotates and the suspension wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze stripe due to the rotation of the coil and let K be the restoring torque per unit twist of the phosphor bronze stripe.
Then total restoring torque produced = $k\theta$

In equilibrium position of the coil, the deflecting torque is equal to the restoring torque.

$\therefore NIBA = k\theta$

$\Rightarrow I = \frac{k}{NBA}\theta \text{ or } G\theta$

Here, $\frac{k}{NBA} = G = a$ [constant for a galvanometer]

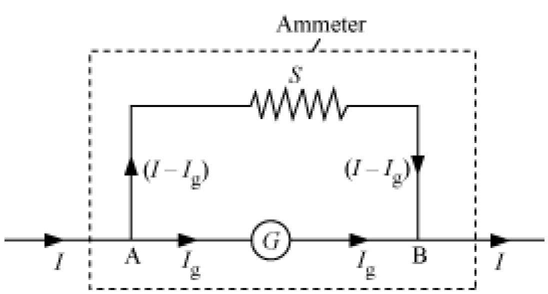
It is known as the galvanometer constant.

- Current sensitivity of the galvanometer is the deflection per unit current.
  $\therefore \frac{\phi}{I} = \frac{NAB}{k}$

- Voltage sensitivity is the deflection per unit voltage.

$$\therefore \frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V}$$
$$= \frac{NAB}{k}\frac{1}{R} \qquad (\because V = IR)$$

Conversion of a galvanometer into an ammeter



- A shunt (low resistance) is connected in parallel with a galvanometer.

- $S = \left(\frac{I_g}{I - I_g}\right)G$

Here,

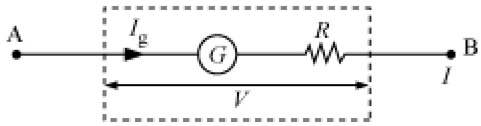
I = total current in circuit

G = resistance of the galvanometer

S = resistance of the shunt

I_g = current through the galvanometer

Conversion of a galvanometer into a voltmeter



- A high resistance is connected in series with the galvanometer.
- $$R = \frac{V}{I_g} - G$$

Here,
V = potential difference across the terminals A and B

I_g = current through the galvanometer

R = high resistance

G = resistance of the galvanometer

Advantages of a Moving Coil Galvanometer

- It is not affected by the Earth's magnetic field. Also, it can work efficiently in any significant magnetic disturbance.
- It has a high value of torque/weight ratio.
- It is highly accurate and reliable.
- Its scales are uniform.

Disadvantages of Moving Coil Galvanometer

- It cannot be used to measure alternating currents.
- Its restoring torque is produced by a spring connected to it, therefore it cannot be easily changed.
- Its restoring torque is affected by the change in temperature.
- The phosphor bronze fibre suspension and hair helical springs cannot bear severe stress. Thus, they can get easily damaged on application of severe stress.

Sensitivity of a Moving Coil Galvanometer

- The sensitivity of the moving coil galvanometer depends on the amount of deflection for any change in the current.
- Its sensitivity increases with increasing deflection for the small change in the current.

- The sensitivity of a moving coil galvanometer is the ratio of the deflection to the change in current. It is given by $S = d\theta/dI$

  The current flowing through a moving coil galvanometer is given by

  $$I = \left(\frac{C}{NBA}\right)\theta$$

  On differentiation, we get:

  $$dI = \left(\frac{C}{NBA}\right)d\theta$$

  $$\Rightarrow \frac{d\theta}{dI} = \left(\frac{NBA}{C}\right)$$

  $$\therefore S = \left(\frac{NBA}{C}\right) \qquad \ldots (1)$$

  Here, N = number of turns in the coil
  B = magnetic induction of the magnetic field
  A = area of the rectangular coil
  C = twist constant of the suspension wire

  From equation (1), it can be concluded that the sensitivity of a moving coil galvanometer can be increased by:

- increasing the number of turns(N) of the coil
- increasing the magnetic induction (B)
- increasing the area of the coil (A)
- decreasing the restoring torque per unit angular displacement

Accuracy of a Moving Coil Galvanometer

- The accuracy of the moving coil galvanometer is more if the relative error in the measurement of current is less.
- The current flowing through the coil of the galvanometer is given by

$$I = \left(\frac{C}{NBA}\right)\theta \text{ ....(2)}$$

On differentiation, we get:

$$dI = \left(\frac{C}{NBA}\right)d\theta \qquad \ldots (3)$$

Dividing (3) by (2), we get:

$$\frac{dI}{I} = \frac{d\theta}{\theta}$$

The ratio dI/I known as the fractional error in current. If the value of the fractional error is small for a moving coil galvanometer, it is said to be more accurate.