# Chapter 12. Probability

## Conditional Probability and Independent Events

### 4 Marks Questions

**1.** A couple has 2 children. Find the probability that both are boys, if it is known that

(i) one of them is a boy.

(ii) the older child is a boy.

**Delhi 2014C, 2008C; All India 2014, 2010**

> Firstly, write the sample space of given data. Then, use concept of conditional probability $P(A/B) = \frac{P(A \cap B)}{P(B)}$ to get the desired result.

Let $B$ represents older child which is a boy and $b$ represents younger child which is also a boy. Also, let $G$ represents older child which is a girl and $g$ represents younger child which is also a girl. The sample space of the given question is

$$S = \{Bb, Bg, Gg, Gb\}$$

$\therefore \quad n(S) = 4$

Let $A$ be the event that both children are boys.

Then, $\quad A = \{Bb\}$

$\therefore \quad n(A) = 1$ **(1)**

(i) Let $B$ : Atleast one of the children is a boy

$\therefore \quad B = \{Bb, Bg, Gb\}$ and $n(B) = 3$

and $\quad P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$ ...(i)

Here, $A \cap B = \{Bb\}$, then $n(A \cap B) = 1$

$\therefore \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$ ...(ii)

We have to find $P(A/B)$, we know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0 \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

Hence, required probability is $\frac{1}{3}$. **(1½)**

(ii) Let $B$ : The older child is a boy.

Then, $B = \{Bb, Bg\}$

$\therefore \quad n(B) = 2$

and $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$ ...(iv)

Here, $A \cap B = \{Bb\}$, then $n(A \cap B) = 1$

and $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{4}$ ...(v)

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/4}{1/2} = \dfrac{1}{2}$

Hence, required probability is $\dfrac{1}{2}$. **(1½)**

**2.** Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls? Given that

(i) the youngest is a girl?

(ii) atleast one is a girl? **Delhi 2014**

Let $B$ represents elder child which is a boy and $b$ represents younger child which is also a boy and $G$ represents elder child which is a girl and $g$ represents younger child which is also a girl.

If a family has two children, then all possible cases are

$$S = \{Bb, Bg, Gg, Gb\}$$

$\therefore \quad n(S) = 4$

Let us define event $A$ : Both children are girls, then

$$A = \{Gg\} \Rightarrow n(A) = 1$$

(i) Let $E_1$ : The event that youngest child is a girl.

Then,

$$E_1 = \{Bg, Gg\}, \text{ so } P(E_1) = \frac{2}{4} = \frac{1}{2}$$

and $A \cap E_1 = \{Gg\}$, so $P(A \cap E_1) = \frac{1}{4}$

Now, $P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \frac{1/4}{1/2} = \frac{1}{2}$

$\therefore$ Required probability $= \frac{1}{2}$

(ii) Let $E_2$ : The event that atleast one is girl.

Then, $E_2 = \{Bg, Gg, Gb\}$

$\Rightarrow$ $n(E_2) = 3$, so $P(E_2) = \frac{3}{4}$

and $(A \cap E_2) = \{Gg\}$

so $P(A \cap E_2) = \frac{1}{4}$

Now, $P\left(\frac{A}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3}$

$\therefore$ Required probability $= \frac{1}{3}$

**3.** $A$ speaks truth in 75% of the cases, while $B$ in 90% of the cases. In what per cent of cases, are they likely to contradict each other in stating the same fact? Do you think that statement of $B$ is true? **All India 2013**

Let $A_T$ : Event that $A$ speaks truth.

and $B_T$ : Event that $B$ speaks truth.

Given, $P(A_T) = \frac{75}{100}$, then $P(\bar{A}_T) = 1 - \frac{75}{100}$

$[\because P(\bar{A}) = 1 - P(A)]$

$$= \frac{25}{100}$$

and $P(B_T) = \frac{90}{100}$, then

$$P(\bar{B}_T) = 1 - \frac{90}{100} = \frac{10}{100} \quad \textbf{(1)}$$

Now, $P$ ($A$ and $B$ are contradict to each other)

$$= P(A_T \cap \bar{B}_T) + P(\bar{A}_T \cap B_T)$$

$$= P(A_T) \cdot P(\bar{B}_T) + P(\bar{A}_T) \cdot P(B_T) \quad \textbf{(1)}$$

[$\because$ events $A_T$ and $B_T$ are independent events]

$$= \frac{75}{100} \times \frac{10}{100} + \frac{25}{100} \times \frac{90}{100}$$

$$= \frac{750 + 2250}{10000} = \frac{3000}{10000} = \frac{3}{10}$$

$\therefore$ Percentage of $P$ ($A$ and $B$ are contradict to each other) $= \frac{3}{10} \times 100 = 30\%$ **(1)**

We think that statement of $B$ may be false. **(1)**

**4.** $P$ speaks truth in 70% of the cases and $Q$ in 80% of the cases. In what per cent of cases are they likely to agree in stating the same fact?

Do you think, when they agree, means both are speaking truth? **All India 2013**

Let $P_T$ : Event that $P$ speaks truth.

and $Q_T$ : Event that $Q$ speaks truth.

Given,

$$P(P_T) = \frac{70}{100}, \text{ then } P(\bar{P}_T) = 1 - \frac{70}{100} = \frac{30}{100}$$

and $P(Q_T) = \frac{80}{100}$,

Then $P(\bar{Q}_T) = 1 - \frac{80}{100} = \frac{20}{100}$ **(1)**

$P$ ($A$ and $B$ are agree to each other)

$$= P(P_T \cap Q_T) + P(\bar{P}_T \cap \bar{Q}_T)$$

$$= P(P_T) \cdot P(Q_T) + P(\bar{P}_T) \cdot P(\bar{Q}_T)$$ **(1)**

[$\because$ events $P_T$ and $Q_T$ are independent events]

$$= \frac{70}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{20}{100}$$

$$= \frac{5600}{10000} + \frac{600}{10000} = \frac{6200}{10000} = \frac{62}{100}$$

$\therefore$ Percentage of $P$($A$ and $B$ are agree to each other)

$$= \frac{62}{100} \times 100 = 62\%$$ **(1)**

No, agree does not mean that they are speaking truth. **(1)**

**5.** $A$ speaks truth in 60% of the cases, while $B$ in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of $B$ will carry more weight as he speaks truth in more number of cases than $A$? **Delhi 2013**

Let $A_T$ : Event that $A$ speaks truth.

and $B_T$ : Event that $B$ speaks truth.

Given, $P(A_T) = \frac{60}{100}$,

then $P(\bar{A}_T) = 1 - \frac{60}{100} = \frac{40}{100}$

and $P(B_T) = \frac{90}{100}$,

then $P(\bar{B}_T) = 1 - \frac{90}{100} = \frac{10}{100}$ **(1)**

Now, $P$ ($A$ and $B$ are contradict to each other)

$= P(A_T \cap \bar{B}_T) + P(\bar{A} \cap B_T)$

$= P(A_T) \cdot P(\bar{B}_T) + P(\bar{A}_T) \cdot P(B_T)$

[$\because$ events $A_T$ and $B_T$ are independent events]

$= \frac{60}{100} \times \frac{10}{100} + \frac{40}{100} \times \frac{90}{100}$

$= \frac{600 + 3600}{10000} = \frac{4200}{10000} = \frac{42}{100}$ **(1½)**

$\therefore$ Percentage of $P$ ($A$ and $B$ are contradict to each other) $= \frac{42}{100} \times 100 = 42\%$ **(1/2)**

Yes, the statement of $B$ will carry more weights as he speaks truth in more number of cases than $A$. **(1)**

**6.** The probabilities of two students $A$ and $B$ coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$, respectively. Assuming that the events, '$A$ coming in time' and '$B$ coming in time' are independent, find the probability of only one of them coming to the school in time.

Write atleast one advantage of coming to school in time. **Value Based Question; Delhi 2013**

Given, probability of student $A$ coming in time,

$$P(A) = \frac{3}{7}$$

then probability of student $A$ not coming in time,

$$P(\overline{A}) = 1 - \frac{3}{7} = \frac{4}{7} \quad \textbf{(1)}$$

and probability of student $B$ coming in time,

$$P(B) = \frac{5}{7}$$

then probability of student $B$ not coming in time,

$$P(\overline{B}) = 1 - \frac{5}{7} = \frac{2}{7} \quad \textbf{(1)}$$

Now, required probability

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$$

[∵ events $A$ and $B$ are independent events]

$$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49} \quad \textbf{(1)}$$

Regular attendance and punctuality are important, if children want to take advantage of the learning opportunities offered by the school. Also, it is vital to the educational process and encourages for a good pattern of work. **(1)**

**7.** In a hockey match, both teams $A$ and $B$ scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team $A$ was asked to start, then find their respective probabilities of winning the match and state whether the decision of the referee was fair or not. **Value Based Question; Delhi 2013C**

Let $E_1$ : Event of $A$ getting six.

and $E_2$: Event of $B$ getting six.

In throwing a die,

Total number of elements in sample space, $n(S) = 6$

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{6}$$

and probability of not getting a six

$$P(\bar{E_1}) = P(\bar{E_2}) = 1 - \frac{1}{6} = \frac{5}{6} \qquad \textbf{(1)}$$

Since, the referee gives first chance to captain $A$ for throwing a die.

$\therefore$ Probability of $A$ winning

$$= [P(E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) + \ldots]$$

$$= P(E_1) + P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(E_1) + \ldots$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \ldots\right]$$

$$= \frac{1}{6}\left(\frac{1}{1 - \frac{25}{36}}\right)$$

$\left[\because \text{ sum of an infinite GP, } S_\infty = \frac{a}{1-r} \text{ Here, } a = 1 \text{ and } r = 5/6\right]$

$$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11} \qquad \textbf{(1)}$$

Probability of $B$ winning

$$= P(\bar{E}_1 \cap E_2) + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap E_2) + \ldots$$

$$= P(\bar{E}_1) \cdot P(E_2) + P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(\bar{E}_1) \cdot P(E_2) + \ldots$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots$$

$$= \frac{5}{[\ldots]}\left[1 + \left(\frac{5}{[\ldots]}\right)^2 + \ldots\right]$$

$= \frac{5}{36}\left[1 + \left(\frac{25}{36}\right) + \ldots\right]$

[∵ here, series is an infinite GP]

$$= \frac{5}{36}\left(\frac{1}{1-\frac{25}{36}}\right) = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11} \quad \textbf{(1)}$$

Here, we see that $P(A) > P(B)$.

Hence, team $A$ has more chance of winning the match.

As the referee first give a chance to team $A$, so it is not a fair decision. **(1)**

**8.** Probabilities of solving a specific problem independently by $A$ and $B$ are $\frac{1}{2}$ and $\frac{1}{3}$, respectively. If both try to solve problem independently, then find the probability that

(i) problem is solved.

(ii) exactly one of them solves the problem.

**Delhi 2011**

> The problem is solved means atleast one of them solve it. Suppose $A$ and $B$ are independent events, then their complements are independent.

Let $P(A)$ = Probability that $A$ solves the problem.

$P(B)$ = Probability that $B$ solves the problem.

$P(\overline{A})$ = Probability that $A$ does not solve the problem.

and $P(\overline{B})$ = Probability that $B$ does not solve problem. **(1)**

According to the question, we have

$$P(A) = \frac{1}{2}$$

then $$P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

[∵ $P(A) + P(\overline{A}) = 1$] **(1/2)** and $\quad P(B) = \frac{1}{3}$

then $\quad P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$ **(1)**

(i) $P$ (problem is solved)

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B) + P(A) \cdot P(B)$$

[$\because$ $A$ and $B$ are independent events]

$$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$$

$$= \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Hence, probability that the problem is solved, is $\frac{2}{3}$. **(1½)**

***Alternate Method***

$P$ (problem is solved)

$= 1 - P$ (none of them solve the problem)

$$= 1 - P(\bar{A} \cap \bar{B})$$

$$= 1 - P(\bar{A}) \cdot P(\bar{B}) = 1 - \left(\frac{1}{2} \times \frac{2}{3}\right)$$ **(1)**

$$\left[\because P(\bar{A}) = \frac{1}{2} \text{ and } P(\bar{B}) = \frac{2}{3}\right]$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$ **(1½)**

(ii) $P$ (exactly one of them solve the problem)

$$= (P \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$$

$$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$ **(1½)**

**9.** 12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number. **All India 2008**

Let us define the events as

$A$ : Number on the drawn card is more than 3.

$B$ : Number on the drawn card is an even. number **(1)**

Here, total elements in sample space, $n(S) = 12$

$\therefore \quad A = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$

and $\quad B = \{2, 4, 6, 8, 10, 12\}$

So, $\quad A \cap B = \{4, 6, 8, 10, 12\}$ **(1)**

Then, $\quad n(A) = 9, \quad n(B) = 6$ and

$n(A \cap B) = 5 \quad P(A) = \frac{n(A)}{n(S)} = \frac{9}{12} = \frac{3}{4}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{5}{12}$ **(1)**

Now, we have to find $P(B/A)$.

By using conditional probability, we have

$$P(B/A) = \frac{P(B \cap A)}{P(A)}, P(A) \neq 0 = \frac{5/12}{3/4}$$

$$[\because P(B \cap A) = P(A \cap B) = 5/12]$$

$$= \frac{5}{12} \times \frac{4}{3} = \frac{5}{9}$$

Hence, required probability is 5/9. **(1)**

## 6 Marks Questions

**10.** Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4', given that 'there is atleast one tail'. **Delhi 2014C**

Let $S$ = Sample space of the experiment

$= \{HT, HH, T1, T2, T3, T4, T5, T6\}$ **(1)**

$A$ = Event that die shows a number greater than 4 $= \{T5, T6\}$ **(1)**

$B$ = Event that there is atleast one tail

$= (H\,T)\,(T\,1)\,(T\,2)\,(T\,3)\,(T\,4)\,(T\,5)\,(T\,6)$

$$A \cap B = (T\,5), (T\,6)$$ **(1)**

Then, $P(A \cap B) = P(T5) + P(T6)$

$$= \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{6}$$ **(1)**

and $P(B) = P(HT) + P(T1) + P(T2) + P(T3) + P(T4) + P(T5) + P(T6)$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} = \frac{3}{4}$$ **(1)**

Hence required probability =

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/4} = \frac{2}{9}$$ **(1)**

**11.** In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses. **All India 2014C**

In a throw of a die, the probability of getting a six is 1/6 and the probability of not getting a 6 is 5/6. **(1)**

**Case I** If he gets a six in the first throw, then the required probability is 1/6. **(1)**

Amount he will receive = ₹ 5 **(1)**

**Case II** If he does not get a six in the first throw and gets a six in second throw, then probability

$$= \left(\frac{5}{6} \times \frac{1}{6}\right) = \frac{5}{36}$$

Amount he will receive = − 1 + 5 = ₹ 4 **(1)**

**Case III** If he does not get a six in the first two throws and gets a six in the third, then probability

$$= \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) = \frac{25}{216}$$

Amount he will receive = − 1 − 1 + 5 = ₹ 3 **(1)**

**Case IV** If he does not get any six, then probability

$$= \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) = \frac{125}{216}$$

Amount he will receive = − 1 − 1 − 1 = ₹ − 3 **(1)**

Now, expected value he can win

$$= \left(\frac{1}{6}\right)5 + \left(\frac{5}{36}\right)4 + \left(\frac{25}{216}\right)3 + \left(\frac{125}{216}\right)(-3) = 0$$

**(1)**

## Baye's Theorem and Probability Distribution

### 4 Marks Questions

**1.** An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be atleast 3 successes.

**All India 2014**

Let $p$ denote the probability of getting succees in the experiment and $q$ denote the probability of getting failure in the experiment.

According to question,

$$p = 3q$$

$$\Rightarrow \quad p = 3(1-p) \qquad [\because p+q=1]$$

$$\Rightarrow \quad p = 3-3p$$

$$\Rightarrow \quad p+3p=3 \quad \Rightarrow \quad 4p=3$$

$$\Rightarrow \quad p=\frac{3}{4} \text{ and then, } q=1-\frac{3}{4}=\frac{1}{4} \qquad \textbf{(1)}$$

Let $X$ denotes the number of successes in 5 trials. Then, $X$ follows binomial distribution with $n=5$, $p=\frac{3}{4}$ and $q=\frac{1}{4}$ such that

$$P(X=r) = {}^5C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{5-r}; r=0,1,2,\ldots,5 \; \textbf{(1)}$$

Hence, required probability = $P(X \geq 3)$

$= P(X = 3) + P(X = 4) + P(X = 5)$

$$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2 + {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right)^1 + {}^5C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0 \quad \textbf{(1)}$$

$$= \frac{5 \times 4}{2} (3)^3 \left(\frac{1}{4}\right)^5 + 5(3)^4 \left(\frac{1}{4}\right)^5 + 1(3)^5 \left(\frac{1}{4}\right)^5 \quad \textbf{(1)}$$

$$= (3)^3 \left(\frac{1}{4}\right)^5 [10 + 15 + 9]$$

$$= \frac{27 \times 34}{1024}$$

$$= \frac{918}{1024} = \frac{459}{512} \quad \textbf{(1)}$$

**2.** Three cards are drawn at random (without replacement) from a well-shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence, find the mean of the distribution. **Foreign 2014**

Let the random variable $x$ = Number of red cards

Here, $x$ can take values 0, 1, 2, 3.

Now, $P(X=0) = P(\text{all black cards})$

$$= P(B) \times P(BB / B) \times P(BBB / BB)$$

$$= \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$$

$$P(X=1) = P(RBB) + P(BRB) + P(BBR)$$

$$= 3 \times P(R) \times P(B/R) \times P(B/RB)$$

$$= 3 \times \frac{26}{52} \times \frac{26}{51} \times \frac{25}{50} = \frac{13}{34}$$ **(1)**

$$P(X=2) = P(RRB) + P(RBR) + P(BRR)$$

$$= 3 \times P(R) \times P(RR/R) \times P(B/RR)$$

$$= 3 \times \frac{26}{52} \times \frac{25}{51} \times \frac{26}{50} = \frac{13}{34}$$

$$P(X=3) = P(\text{ All red cards})$$

$$= P(R) \times P(RR/R) \times P(RRR/R)$$

$$= \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$$ **(1)**

So, the probability distribution of $X$ is as follows:

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X)$ | $\frac{2}{17}$ | $\frac{13}{34}$ | $\frac{13}{34}$ | $\frac{2}{17}$ |

**(1)**

Now, mean

$E(X) = \Sigma X_i \cdot P_i$

$$= 0 \times \frac{2}{17} + \frac{13}{34} \times 1 + \frac{13}{34} \times 2 + 3 \times \frac{2}{17}$$

$$= 0 + \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = 1.5$$ **(1)**

**3.** A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age $X$ of the selected student is recorded. What is the probability distribution of the random variable $X$? Find the mean of $X$. **All India 2014C**

> Firstly, find the probability of respective ages and make a probability distribution table then using the formula
>
> $$\text{Mean}(X) = \Sigma x P(X), \text{ calculate mean.}$$

Here, total students $= 15$

The ages of students in ascending order are 14, 14, 15, 16, 16, 17, 17, 17, 18, 19, 19, 20, 20, 20 and 21.

Now, $P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$,

$P(X = 16) = \frac{2}{15}$, $P(X = 17) = \frac{3}{15}$,

$$P(X=18)=\frac{1}{15}, P(X=19)=\frac{2}{15},$$

$$P(X=20)=\frac{3}{15}, P(X=21)=\frac{1}{15} \quad \textbf{(1)}$$

Therefore, the probability distribution of random variable $X$ is as follows:

| $X$ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|
| **Number of students** | 2 | 1 | 2 | 3 | 1 | 2 | 3 | 1 |
| $P(X)$ | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ |

**(1)**

The third row gives the probability distribution of $X$.

$\therefore$ Mean $(X) = \Sigma X \cdot P(x)$ **(1)**

$$=\left[\frac{14\times 2+15\times 1+16\times 2+17\times 3+18\times 1+19\times 2+20\times 3+21\times 1}{15}\right]$$

$$=\left[\frac{28+15+32+51+18+38+60+21}{15}\right]$$

$$=\frac{263}{15}=17.53 \quad \textbf{(1)}$$

**4.** A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red? **Delhi 2014C**

Let $E_1$ : First ball is red

$E_2$ : First ball is black

$A$ : Second ball is red **(1)**

Then, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{7}{10}$

$\therefore \quad P\left(\frac{A}{E_1}\right) = \frac{2}{9}$ and $P\left(\frac{A}{E_2}\right) = \frac{3}{9}$ **(1)**

Then, by Baye's theorem, probability of second selected ball is red when first selected ball is also red is given by

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$ **(1)**

$$= \frac{\frac{3}{10} \times \frac{2}{9}}{\frac{3}{10} \times \frac{2}{9} + \frac{7}{10} \times \frac{3}{9}} = \frac{6}{27} = \frac{2}{9}$$ **(1)**

**5.** Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also, find the mean of the distribution. What values are described in this question?

**Value Based Question; Delhi 2013C**

Total number of honest people, $n(S) = 30$

Number of people speaking truth $= 20$

Number of people not speaking truth

$$= 30 - 20 = 10$$

$P(X=0) = P$ (none speak truth)

$$= \frac{{}^{10}C_2}{{}^{30}C_2} = \frac{\frac{10 \times 9}{2 \times 1}}{\frac{30 \times 29}{2 \times 1}} = \frac{9}{87} \qquad \textbf{(1/2)}$$

$P(X=1) = P$ (only one of them speak truth)

$$= \frac{{}^{20}C_1 \times {}^{10}C_1}{{}^{30}C_2} = \frac{20 \times 10}{\frac{30 \times 29}{2 \times 1}} = \frac{40}{87} \quad \textbf{(1/2)}$$

$P(X=2) = P$ (both of them speak truth)

$$= \frac{{}^{20}C_2}{{}^{30}C_2} = \frac{\frac{20 \times 19}{2 \times 1}}{\frac{30 \times 29}{2 \times 1}} = \frac{38}{87} \qquad \textbf{(1/2)}$$

Then, probability distribution is given below:

| ***X*** | 0 | 1 | 2 |
|---|---|---|---|
| ***P* (*X*)** | $\frac{9}{87}$ | $\frac{40}{87}$ | $\frac{38}{87}$ |

**(1)**

$\therefore$ Mean $= \sum X_i \cdot P_i = X_1 \cdot P_1 + X_2 \cdot P_2 + X_3 \cdot P_3$

$$= 0 \times \frac{9}{87} + 1 \times \frac{40}{87} + 2 \times \frac{38}{87}$$

$$= 0 + \frac{40}{87} + \frac{76}{87} = \frac{116}{87} \qquad \textbf{(1/2)}$$

In this question, value of speaking truth is highlighted as out of the group of 30 honest people, 20 always speak the truth. It clearly depicted the value of truthfulness and morality. **(1)**

**6.** Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time, a patient can choose anyone of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods, is more beneficial for the patient? **Delhi 2013**

Let $E_1$ : The patient follows meditation and yoga.

$E_2$ : The patient uses drug, then $E_1$ and $E_2$ are mutually exclusive

and $\quad P(E_1) = P(E_2) = 1/2$

Also, at $E$ : The selected patient suffers a heart attack. **(1)**

Then, $\quad P(E/E_1) = \dfrac{40}{100}\left(1-\dfrac{30}{100}\right) = \dfrac{28}{100}$ **(1/2)**

and $\quad P(E/E_2) = \dfrac{40}{100}\left(1-\dfrac{25}{100}\right) = \dfrac{30}{100}$ **(1/2)**

$\therefore$ $P$ (patient who suffers heart attack follows meditation and yoga) $= P(E_1/E)$

$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}$$ **(1/2)**

[using Baye's theorem]

$$= \frac{\frac{28}{100} \times \frac{1}{2}}{\frac{28}{100} \times \frac{1}{2} + \frac{30}{100} \times \frac{1}{2}} = \frac{28}{58} = \frac{14}{29} \quad \textbf{(1)}$$

Yoga course and meditation are more beneficial for the heart patient. **(1/2)**

**7.** How many times must a man toss a fair coin, so that the probability of having atleast one head is more than 80%? **Delhi 2012**

Let man tosses the coin $n$ times. Now, given that, $P$(having atleast one head) > 80%

i.e. $$P(X \geq 1) > \frac{80}{100} \quad \textbf{(1)}$$

where, $X$ is the number of heads.

$$\Rightarrow \quad 1 - P(X = 0) > \frac{80}{100}$$

$$\Rightarrow \quad 1 - {}^nC_0 p^0 q^n > \frac{80}{100} \quad \textbf{(1)}$$

[using $P(X) = {}^nC_r p^r q^{n-r}$]

$$\Rightarrow \quad 1 - {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{8}{10}$$

$$\left[\because p = \text{probability of getting a head once} = \frac{1}{2} \text{ and } q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}\right]$$

$$\Rightarrow \quad 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow \frac{1}{2^n} < 1 - \frac{4}{5}$$

$$\Rightarrow \quad \frac{1}{2^n} < \frac{1}{5} \Rightarrow 2^n > 5 \quad \ldots(i)$$

Inequality (i) is satisfied for $n \geq 3$.

Hence, coin must be tossed 3 or more times. **(2)**

**8.** Two cards are drawn simultaneously (without replacement) from a well-shuffled deck of 52 cards. Find the mean and variance of number of red cards. **All India 2012**

> Firstly, find the probability distribution table of number of red cards. Then using this table, find the mean and variance.

Let $X$ be the number of red cards. Then, $X$ can take values 0, 1 and 2. **(1/2)**

Now, $P(X=0) = P$ (having no red card)

$$= \frac{^{26}C_2}{^{52}C_2} = \frac{26 \times 25/(2 \times 1)}{52 \times 51/(2 \times 1)}$$

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

$P(X=1) = P$(having one red card)

= Probability of getting one red card and other black card

$$= \frac{^{26}C_1 \times {}^{26}C_1}{^{52}C_2} = \frac{26 \times 26 \times 2}{52 \times 51} = \frac{26}{51}$$

$P(X = 2) = P(\text{having two red cards})$

$$= \frac{^{26}C_2}{^{52}C_2} = \frac{26 \times 25/2 \times 1}{52 \times 51/2 \times 1} = \frac{25}{102}$$

$\therefore$ The probability distribution of number of red cards is given below:

| **X** | 0 | 1 | 2 |
|---|---|---|---|
| **P (X)** | $\frac{25}{102}$ | $\frac{26}{51}$ | $\frac{25}{102}$ |

**(1½)**

Now, we know that, mean $= \sum X \cdot P(X)$

and variance $= \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2$

| **X** | **P(X)** | **X · P(X)** | **$X^2$ · P(X)** |
|---|---|---|---|
| 0 | $\frac{25}{102}$ | 0 | 0 |
| 1 | $\frac{26}{51}$ | $\frac{26}{51}$ | $\frac{26}{51}$ |
| 2 | $\frac{25}{102}$ | $\frac{25}{51}$ | $\frac{50}{51}$ |

**(1/2)**

$$\therefore \text{Mean} = \sum X \cdot P(X) = 0 + \frac{26}{51} + \frac{25}{51} = \frac{51}{51} = 1$$

$$\text{Variance} = \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2$$

$$= \frac{76}{51} - (1)^2 \quad \left[\because \sum X^2 \cdot P(X) = \frac{76}{51}\right]$$

$$= \frac{76}{51} - 1 = \frac{76 - 51}{51} = \frac{25}{51}$$ **(1½)**

**9.** Find the mean number of heads in three tosses of a coin. **HOTS; Foreign 2011**

Firstly, we write the probability distribution table for the given experiment. Then, we find mean by using formula

Mean $= \sum x_i p_i$

Let $X$ = Number of heads when a coin is tossed three times.

Sample space of given experiment is

$S = \{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT\}$ **(1)**

$X$ can take values 0, 1, 2 and 3.

Now, $P(X = 0) = P$ (no head occur) $= \frac{1}{8}$

$P(X = 1) = P$ (one head occur) $= \frac{3}{8}$

$P(X = 2) = P$ (two head occur) $= \frac{3}{8}$

$P(X = 3) = P$ (three head occur) $= \frac{1}{8}$ **(1½)**

$\therefore$ The probability distribution is as follows:

| **X** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **P (X)** | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

Now, for finding mean

| **X** | **P (X)** | $X_iP_i$ |
|---|---|---|
| 0 | 1/8 | 0 |
| 1 | 3/8 | 3/8 |
| 2 | -3/8 | 6/8 |
| 3 | 1/8 | 3/8 |
| **Total** | | 12/8 |

$\therefore$ Mean $= \sum X_i \cdot P_i = \frac{12}{8} = \frac{3}{2}$ **(1½)**

**10.** A random variable $X$ has following probability distributions:

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2 + k$ |

Find (i) $k$ (ii) $P(X < 3)$ (iii) $P(X > 6)$

(iv) $P(0 < X < 3)$ **HOTS; All India 2011**

Firstly use the result that sum of all the probabilities of an experiment is one, to find $k$ and then find other values by using this value of $k$.

(i) We know that, total probability of an experiment = 1

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow \quad 10k^2 + 9k - 1 = 0$$

$$\Rightarrow \quad 10k^2 + 10k - k - 1 = 0$$

$$\Rightarrow \quad 10k(k + 1) - 1(k + 1) = 0$$

$$\Rightarrow \quad (10k - 1)(k + 1) = 0$$

$$\Rightarrow \quad k = \frac{1}{10} \text{ or } -1$$

But $k = -1$ is rejected because probability cannot be negative.

$$\therefore \quad k = \frac{1}{10} \qquad \textbf{(1)}$$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0 + k + 2k = 3k$$

$$= 3\left(\frac{1}{10}\right) \qquad \left[\because k = \frac{1}{10}\right]$$

$$= \frac{3}{10} \qquad \textbf{(1)}$$

(iii) $P(X > 6) = P(X = 7) = 7k^2 + k$

$$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10} \qquad \left[\because k = \frac{1}{10}\right]$$

$$= \frac{7}{100} + \frac{1}{10} = \frac{7 + 10}{100} = \frac{17}{100} \qquad \textbf{(1)}$$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$$= k + 2k = 3k$$

$$= 3\left(\frac{1}{10}\right) \qquad \left[\because k = \frac{1}{10}\right]$$

$$= \frac{3}{10} \qquad \textbf{(1)}$$

**11.** Find the probability distribution of number of doublets in three tosses of a pair of dice.
**All India 2011C; Delhi 2010C**

We know that, when a pair of dice is thrown, then total number of outcomes = 36

Also, probability of getting a doublet in one throw $= \frac{6}{36} = \frac{1}{6}$

[$\because$ doublets in pair of dice are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)]

$\therefore$ Probability of not getting a doublet

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

Let $X$ = Number of doublets in three tosses of pair of dice

So, $X$ can take values 0,1, 2 and 3. **(1)**

Now, $P(X = 0) = P$ (not getting a doublet)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$P(X = 1) = P$ (getting a doublet once only)

$= P$ (getting a doublet in 1st throw)

$+ P$ (getting a doublet in 2nd throw)

$+ P$ (getting a doublet in 3rd throw)

$$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216} \qquad \textbf{(1)}$$

$P(X = 2) = P$ (getting a doublet two times)

$= P$ (doublet in 1st and 2nd throw)
$+ P$ (doublet in 2nd and 3rd throw)
$+ P$ (doublet in 1st and 3rd throw)

$$= \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{5}{216} + \frac{5}{216} + \frac{5}{216} = \frac{15}{216}$$ **(1)**

$P(X = 3) = P$ (getting a doublet in all the three throws)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

∴ The probability distribution is as follows:

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X)$ | $\frac{125}{216}$ | $\frac{75}{216}$ | $\frac{15}{216}$ | $\frac{1}{216}$ |

**(1)**

**12.** Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of number of aces. **Delhi 2011C; All India 2008C**

Let $X$ = Number of aces

Since, two cards are drawn, so $X$ can take values 0, 1 and 2.

Now, probability of getting an ace $= \frac{4}{52} = \frac{1}{13}$

And probability of not getting an ace **(1)**

$$= 1 - \frac{1}{13} = \frac{12}{13}$$

$P(X = 0) = P$ (not getting an ace card)

$$= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$$

$P(X = 1) = P$ (getting one ace card)

$$= \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169}$$ **(1)**

$P(X = 2) = P$ (getting two ace cards)

$$= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$ **(1)**

$\therefore$ The probability distribution is as follows:

| ***X*** | 0 | 1 | 2 |
|---|---|---|---|
| ***P*(*X*)** | $\frac{144}{169}$ | $\frac{24}{169}$ | $\frac{1}{169}$ |

**(1)**

**13.** Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the probability distribution of number of aces. Also, find the mean of distribution. **All India 2010C**

Let $X$ denotes the number of ace cards. Since, two cards are drawn, so $X$ can take values 0, 1 and 2. We know that, probability of getting an ace card

$$= \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$ **(1/2)**

Now, $P(X = 0) = P$ (getting no ace card)

$$= \frac{^4C_0 \times {^{48}C_2}}{^{52}C_2} = \frac{\frac{1 \times 48 \times 47}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}}$$

$$= \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$ **(1)**

$P(X = 1) = P$ (getting one ace card)

$$= \frac{^4C_1 \times {^{48}C_1}}{^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2 \times 1}}$$

$$= \frac{4 \times 48 \times 2 \times 1}{52 \times 51} = \frac{32}{221}$$ **(1)**

$P(X = 2) = P$ (getting both ace cards)

$$= \frac{^4C_2}{^{52}C_2} = \frac{\frac{4 \times 3}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$ **(1)**

∴ The probability distribution is as follows:

| ***X*** | 0 | 1 | 2 |
|---|---|---|---|
| ***P(X)*** | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$ |

Now, required mean $= \sum X_i \cdot P_i$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$

$$= 0 + \frac{32}{221} + \frac{2}{221} = \frac{34}{221}$$ **(1/2)**

**14.** In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing? **Delhi 2010; All India 2009**

We know that, probability of getting correct answer in a multiple choice exam with three possible answers $= \frac{1}{3}$

$\therefore \quad p = \frac{1}{3}$ and $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

But total number of questions = 5

$\therefore \qquad n = 5$

So, we have $n = 5, p = \frac{1}{3}$ and $q = \frac{2}{3}$ **(1)**

$\therefore$ Required probability

$= P$ (getting four or more correct answers)

$= P(4) + P(5)$ ...(i)

Using binomial distribution, we know that

$P(X) = {}^nC_r \, p^r \, q^{n-r}$ **(1)**

$\therefore \quad P(4) = {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 = 5 \times \frac{1}{81} \times \frac{2}{3} = \frac{10}{243}$

and $P(5) = {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 = 1 \times \frac{1}{243} \times 1 = \frac{1}{243}$

**(1)**

Hence, required probability $= P(4) + P(5)$ [from Eq.(i)]

$$= \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$$ **(1)**

**15.** A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of die. **HOTS; Delhi 2009**

In the given question, we have to find probability of getting third six in the sixth throw of a die, i.e. the two sixes are obtained in first five throws. Probability of this event is obtained by using binomial distribution and then required probability $= P$(2 sixes in first five throws) $\times P$ (third six in sixth throw)

Firstly, we find the probability of 2 sixes in first five throws by using binomial distribution. For this, we have $n = 5$.

$p$ = Probability of getting a six $= \frac{1}{6}$,

$q = 1 - \frac{1}{6} = \frac{5}{6}$ and $r = 2$. **(1)**

Using binomial distribution, we have

$$P(X) = {}^nC_r\, p^r\, q^{n-r}$$

$$\therefore\ P(2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

$$= \frac{5\,!}{2\,!\,3\,!} \times \frac{1}{36} \times \frac{125}{216} \quad \left[\because {}^nC_r = \frac{n!}{(n-r)!\ r!}\right]$$

$$= \frac{5 \times 4}{2} \times \frac{1}{36} \times \frac{125}{216}$$

$$= \frac{10 \times 125}{36 \times 216} = \frac{625}{3888}$$ **(1½)**

Now, $P$ (obtaining third six in the sixth throw) $= P$ (2 sixes in first five throws) $\times P$ (third six in sixth throw)

$$= \frac{625}{3888} \times \frac{1}{6} \quad [\because P \text{ (getting six on a die)} = 1/6]$$

Hence,required probability is $\frac{625}{23328}$. **(1½)**

**16.** Three cards are drawn successively with replacement from a well-shuffled deck of 52 cards. If getting a card of spade is a success, then find the probability distribution of number of successes. **Delhi 2009C**

Let $X$ denotes number of spade cards. Since, three cards are drawn, so $X$ can take values 0, 1, 2 and 3. Further, use the binomial distribution and solve it.

We know that, probability of getting a spade card $= \frac{13}{52} = \frac{1}{4}$, i.e. $p = \frac{1}{4}$ **(1/2)**

and probability of not getting a spade card

$= 1 - \frac{1}{4} = \frac{3}{4}$, i.e. $q = \frac{3}{4}$ **(1/2)**

We know that, probability of $X$ success by using the binomial distribution is given by

$$P(X) = {}^nC_r\, p^r q^{n-r}$$

Here, we have $n = 3, p = \frac{1}{4}, q = \frac{3}{4}$

$$\therefore P(X=0) = {}^3C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$P(X=1) = {}^3C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^2 = 3 \times \frac{1}{4} \times \frac{9}{16} = \frac{27}{64}$$

$$P(X=2) = {}^3C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1 = 3 \times \frac{1}{16} \times \frac{3}{4} = \frac{9}{64}$$

$$P(X=3) = {}^3C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 = 1 \times \frac{1}{64} \times 1 = \frac{1}{64}$$

**(2)**

Hence, the required probability distribution is as follows

| ***X*** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| ***P* (*X*)** | $\frac{27}{64}$ | $\frac{27}{64}$ | $\frac{9}{64}$ | $\frac{1}{64}$ |

**(1)**

**17.** A pair of dice is thrown 4 times. If getting a doublet is a success, then find the probability distribution of number of successes. **Delhi 2008**

We know that, the probability of getting doublet $= \frac{6}{36} = \frac{1}{6}$

[$\because$ doublets in a pair of dice are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)]

i.e. $p = \frac{1}{6}$

Probability of not getting a doublet

$$= 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

i.e. $q = \frac{5}{6}$

Here, we have $n = 4, p = \frac{1}{6}, q = \frac{5}{6}$ **(1)**

Let $X$ denotes the number of doublets, then $X$ can take values 0, 1, 2, 3 and 4.

We know that, by binomial distribution

$$P(X) = {}^nC_r\, p^r\, q^{n-r}$$

Using above formula, we find

$P(X = 0)$, $P(X = 1)$, $P(X = 2)$, $P(X = 3)$ and $P(X = 4)$.

$$\therefore \quad P(X = 0) = {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4$$

$$= 1 \times 1 \times \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X = 1) = {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 \qquad \textbf{(1)}$$

$$= 4 \times \frac{1}{6} \times \frac{125}{216} = \frac{500}{1296}$$

$$P(X = 2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{4!}{2!2!} \times \frac{1}{36} \times \frac{25}{36}$$

$$= \frac{6 \times 25}{36 \times 36} = \frac{150}{1296}$$

$$P(X = 3) = {}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 \qquad \textbf{(1)}$$

$$= 4 \times \frac{1}{216} \times \frac{5}{6} = \frac{20}{1296}$$

$$P(X = 4) = {}^4C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0$$

$$= 1 \times \left(\frac{1}{6}\right)^4 \times 1 = \frac{1}{1296}$$

∴ The required probability distribution is as follows:

| **X** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **P(X)** | $\frac{625}{1296}$ | $\frac{500}{1296}$ | $\frac{150}{1296}$ | $\frac{20}{1296}$ | $\frac{1}{1296}$ |

*Or*

| **X** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **P(X)** | $\frac{625}{1296}$ | $\frac{125}{324}$ | $\frac{75}{648}$ | $\frac{5}{324}$ | $\frac{1}{1296}$ |

**(1)**

**18.** A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the four lost cards being a spade. **Delhi 2014**

Let $E_1$ : Event that lost card is spade

$\Rightarrow \quad n(E_1) = 13$

$E_2$ : Event that lost card is not spade

$\Rightarrow n(E_2) = 52 - 13 = 39$ and total cards =52

$\Rightarrow n(S) = 52$ **(1)**

Since, $E_1$ and $E_2$ are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = \frac{13}{52} = \frac{1}{4}$ and $P(E_2) = \frac{39}{52} = \frac{3}{4}$ **(1)**

Let $E$ : Event that three cards, drawn from the remaining pack are spades

When one spade is lost, then there are 12 spade cards out of 51 cards.

The three cards can be drawn out of spade in ${}^{12}C_3$ ways.

Similarly, 3 spade cards can be drawn out of 51 cards in ${}^{51}C_3$ ways.

The probability of getting three cards when one spade card is lost, is given by $P\left(\frac{E}{E_1}\right)$.

$\therefore P\left(\frac{E}{E_1}\right) = P$ (getting three spade cards, when a lost card was spade)

$$= \frac{{}^{12}C_3}{{}^{51}C_3} = \frac{\frac{12 \times 11 \times 10}{3 \times 2 \times 1}}{\frac{51 \times 50 \times 49}{3 \times 2 \times 1}}$$

$$= \frac{12 \times 11 \times 10}{51 \times 50 \times 49}$$

and $P\left(\frac{E}{E_2}\right) = P$ (getting three spade cards, when a lost card was not spade)

$$= \frac{^{13}C_3}{^{51}C_3} = \frac{\frac{13 \times 12 \times 11}{3 \times 2 \times 1}}{\frac{51 \times 50 \times 49}{3 \times 2 \times 1}} = \frac{13 \times 12 \times 11}{51 \times 50 \times 49} \quad \textbf{(1)}$$

By Baye's theorem, $P$ (the four lost cards being a spade)

$$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right) \cdot P(E_1)}{P\left(\frac{E}{E_1}\right) \cdot P(E_1) + P\left(\frac{E}{E_2}\right) \cdot P(E_2)}$$

$$= \frac{\frac{12 \times 11 \times 10}{51 \times 50 \times 49} \times \frac{1}{4}}{\frac{12 \times 11 \times 10}{51 \times 50 \times 49} \times \frac{1}{4} + \frac{13 \times 12 \times 11}{51 \times 50 \times 49} \times \frac{3}{4}}$$

$$= \frac{10}{10 + 13 \times 3} = \frac{10}{49} \quad \textbf{(1)}$$

### 6 Marks Questions

**19.** Two numbers are selected at random (without replacement) from the first six positive integers. Let $X$ denotes the larger of the two numbers obtained. Find the probability distribution of the random variable $X$ and hence, find the mean of the distribution. **All India 2014**

The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways.

Since, $X$ denotes the larger of the two numbers obtained. So, the random variable $X$ may have values 2, 3, 4, 5 or 6.

$P(X = 2) = P$ [2 and a number less than 2 are selected, i.e. (1, 2), (2, 1)]

$$= \frac{2}{30} = \frac{1}{15} \quad \textbf{(1)}$$

$P(X = 3) = P$ [3 and a number less than 3 are selected, i.e. (1, 3), (2, 3), (3, 1) and (3, 2)]

$$= \frac{4}{30} = \frac{2}{15} \quad \textbf{(1)}$$

$P(X = 4) = P$ [4 and a number less than 4 are selected, i.e. (1, 4), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3)]

$$= \frac{6}{30} = \frac{3}{15} \quad \textbf{(1)}$$

$P(X = 5) = P$ [5 and a number less than 5 are selected, i.e. (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)]

$$= \frac{8}{30} = \frac{4}{15} \quad \textbf{(1)}$$

and $P(X = 6) = P$ [6 and a number less than 6 are selected i.e. (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)]

$$= \frac{10}{30} = \frac{1}{3} \quad \textbf{(1)}$$

Therefore, the required probability distribution is as follows:

| **X** | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| **P(X)** | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{4}{15}$ | $\frac{1}{3}$ |

Mean of the distribution $= \sum X \cdot P(X)$

$$= 2 \times \frac{1}{15} + 3 \times \frac{2}{15} + 4 \times \frac{3}{15} + 5 \times \frac{4}{15} + 6 \times \frac{1}{3}$$

$$= \frac{2 + 6 + 12 + 20 + 30}{15} = \frac{70}{15} = \frac{14}{3} \quad \textbf{(1)}$$

**20.** There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed and it shows head. What is the probability that it was the two headed coin? **All India 2014**

Let $E_1$ : The event that the coin chosen is two headed having head on both faces.

$E_2$ : The event choosing a biased coin that comes up heads 75% of the times.

and $E_3$ : The event choosing a biased coin that comes up tails 40% of the times

= the event choosing a biased coin that comes up heads 60% of the times.

Since, $E_1$, $E_2$ and $E_3$ are mutually exclusive and exhaustive events. **(1)**

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3} \quad \textbf{(1)}$$

Let $E$ : Tossed coin shows up a head

Then, $P\left(\frac{E}{E_1}\right) = P$ (coin showing head, when coin is two headed) $= 1$

$P\left(\frac{E}{E_2}\right) = P$ (coin showing heads, when coin is

biased having heads 75% of the times)

$$= \frac{75}{100} = \frac{3}{4} \quad \textbf{(1)}$$

and $P\left(\frac{E}{E_3}\right) = P$ (coin showing heads,when coin is biased having heads 60% of the times)

$$= \frac{60}{100} = \frac{3}{5} \quad \textbf{(1)}$$

The probability that the coin is two headed, given that it shows head, is given by $P\left(\frac{E_1}{E}\right)$.

Using Baye's theorem, we get

$$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right) \cdot P(E_1)}{P\left(\frac{E}{E_1}\right) \cdot P(E_1) + P\left(\frac{E}{E_2}\right) \cdot P(E_2) + P\left(\frac{E}{E_3}\right) \cdot P(E_3)}$$

**(1)**

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{1}{3}} = \frac{1}{1 + \frac{3}{4} + \frac{3}{5}}$$

$$= \frac{20}{20 + 15 + 12} = \frac{20}{47} \quad \textbf{(1)}$$

**21.** An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the balls in the urn are white? **All India 2014C**

Let $A$ : Two balls drawn are white

$E_1$ : All the balls are white

$E_2$ : Three balls are white

$E_3$ : Two balls are white **(1)**

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ **(1)**

and $P\left(\frac{A}{E_1}\right) = \frac{^4C_2}{^4C_2} = 1,\ P\left(\frac{A}{E_2}\right) = \frac{^3C_2}{^4C_2} = \frac{1}{2}$

$$P\left(\frac{A}{E_3}\right) = \frac{^2C_2}{^4C_2} = \frac{1}{6}$$ **(1)**

$\therefore$ Probability that all balls in the urn are white

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$ **(1)**

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{1}{6}} = \frac{6}{10}$$ **(1)**

$= 0.6$ **(1)**

**22.** Five cards are drawn one by one, with replacement, from a well-shuffled deck of 52 cards. Find the probability that

(i) all the five cards are diamonds.

(ii) only 3 cards are diamonds.

(iii) none is a diamond. **Foreign 2014**

Let $X$ represents the number of diamond cards among the five cards drawn. Since, the drawing card is with replacement, the trials are Bernoulli trials. In a well-shuffled deck of 52 cards, there are 13 diamond cards.

$p = P(\text{success}) = P(\text{a diamond card is drawn})$

$$= \frac{13}{52} = \frac{1}{4}$$

and $q = P(\text{failure}) = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$ **(1)**

Thus, $X$ has a binomial distribution with $n = 5, P = \frac{1}{4}$ and $q = \frac{3}{4}$ **(1)**

Therefore, $P(X = r) = {}^nC_r\, p^2 q^{n-r}$, where

$$r = 0, 1, 2, 3, 4, 5$$

$$P(X = r) = {}^5C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{5-r}$$ **(1)**

Now, (i) $P$(all the five cards are diamonds)

$$P(X = 5) = {}^5C_5\, p^5 q^0 = 1p^5 = \left(\frac{1}{4}\right)^5$$

$$= \frac{1}{1024}$$ **(1)**

(ii) $P$(only three cards are diamonds) $= P(X = 3)$

$$= {}^5C_3\, p^3 q^2 = \frac{5 \cdot 4 \cdot 3}{3!} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$$

$$= \frac{60}{1 \times 2 \times 3} \times \frac{3^2}{4^2} = \frac{90}{1024} = \frac{45}{512}$$ **(1)**

(iii) $P$(None is a diamond) $= P(X = 0)$

$$= {}^5C_0 p^0 q^5 = (q)^5 = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$ **(1)**

**23.** An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively.

One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

**Foreign 2014; All India 2009C; Delhi 2008**

Let us define the events as

$E_1$ : Insured person is a scooter driver

$E_2$ : Insured person is a car driver

$E_3$ : Insured person is a truck driver

$A$ : Insured person meets with an accident

Total insurance $= 12000$ **(1)**

Now, $P(E_1)$ = Probability that the insured person is a scooter driver $= \frac{2000}{12000} = \frac{1}{6}$

$P(E_2)$ = Probability that the insured person is a car driver $= \frac{4000}{12000} = \frac{1}{3}$

and $P(E_3)$ = Probability that the insured person is a truck driver $= \frac{600}{12000} = \frac{1}{2}$

Also, $P(A/E_1)$ = Probability that scooter driver meets with an accident = 0.01 [given]

$P(A/E_2)$ = Probability that car driver meets with an accident = 0.03 [given]

and $P(A/E_3)$ = Probability that truck driver meets with an accident = 0.15 [given] **(2)**

The probability that the person meets with an accident was a scooter driver,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{\left[\begin{array}{r}P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) \\ + P(E_3)\cdot P(A/E_3)\end{array}\right]}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{1}{6}\times 0.01}{\left[\begin{array}{r}\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right) \\ +\left(\frac{1}{2}\times 0.15\right)\end{array}\right]}$$ **(1)**

$$= \frac{0.00167}{0.00167 + 0.01 + 0.075}$$

$$= \frac{0.00167}{0.08667} = 0.019$$

Hence, required probability is 0.019. **(1)**

**24.** A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is '1'. Find the probability that it is actually 1.

Delhi 2014C

Let $E_1$ = Event that 1 occurs

$E_2$ = Event that 1 does not occur

$A$ = Event that the man reports that 1 occur

Then, $$P(E_1) = \frac{1}{6}$$

and $$P(E_2) = \frac{5}{6}$$ **(1)**

$\therefore$ $P$ (man reports that 1 occurs when 1occur)

$$= P\left(\frac{A}{E_1}\right) = \frac{3}{5}$$ **(1)**

and $P$(man reports that 1 occur but 1 does not occur)

$$= P\left(\frac{A}{E_2}\right) = \frac{2}{5}$$ **(1)**

Thus, by Baye's theorem, we get

$P$(get actually 1 when he reports that 1 occur)

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$ **(1)**

$$\Rightarrow \quad = \frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5}+\frac{5}{6}\times\frac{2}{5}}$$ **(1)**

$$\Rightarrow \quad = \frac{3}{13}$$ **(1)**

**25.** From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence, find the mean of the distribution. **Delhi 2014**

It is given that out of 15 bulbs, 5 are defective.

$\therefore$ Number of non-defective bulbs

$$= 15 - 5 = 10 \quad \textbf{(1)}$$

4 bulbs are drawn from a lot with replacement.

Let $\quad p = P$ (drawn defective bulb)

$$= \frac{5}{15} = \frac{1}{3} \quad \textbf{(1)}$$

and $\quad q = P$ (drawn non-defective bulb)

$$= \frac{10}{15} = \frac{2}{3} \quad \textbf{(1)}$$

Let $X$ be the random variable which denotes the defective bulbs. So, $X$ may have values 0, 1, 2, 3 or 4.

Using binomial distribution, we have

$$P(X = r) = {}^nC_r \, (p)^r \, (q)^{n-r}$$

Therefore,

$$P(X = 0) = {}^4C_0 p^0 q^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

$$P(X = 1) = {}^4C_1 p^1 q^3 = 4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{32}{81}$$

$$P(X = 2) = {}^4C_2 p^2 q^2 = 6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81}$$

$$P(X = 3) = {}^4C_3 p^3 q^1 = 4\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right) = \frac{8}{81}$$

and $P(X=4) = {}^4C_4 p^4 q^0 = 1\left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 = \frac{1}{81}$

**(1)**

Therefore, the required probability distribution is as follows:

| $X$ | $P(X)$ | $X \cdot P(X)$ |
|---|---|---|
| 0 | $\frac{16}{81}$ | 0 |
| 1 | $\frac{32}{81}$ | $\frac{32}{81}$ |
| 2 | $\frac{24}{81}$ | $\frac{48}{81}$ |
| 3 | $\frac{8}{81}$ | $\frac{24}{81}$ |
| 4 | $\frac{1}{81}$ | $\frac{4}{81}$ |
| | | $\sum X \cdot P(X) = \frac{108}{81}$ |

**(1)**

Hence, mean $= \sum X \cdot P(X)$

$$= \frac{108}{81} = \frac{12}{9} = \frac{4}{3} = 1.33$$ **(1)**

**26.** In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10%, respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian? What value is reflected in the question? **Value Based Question; Delhi 2013C**

Total number of people in a group, $n(S) = 400$

Let $E_1$ : Event of getting a person who is smoker and non-vegetarian

$E_2$ : Event of getting a person who is smoker and vegetarian

$E_3$ : Event of getting a person who is non-smoker and vegetarian

and $A$ = Event of getting a chest disease

$$\therefore \quad n(E_1) = 160$$

$$n(E_2) = 100$$

and $\quad n(E_3) = 400 - (160 + 100) = 140$ **(1/2)**

$\therefore \; P(E_1) = P$(a person who is smoker and non-vegetarian)

$$= \frac{n(E_1)}{n(S)} = \frac{160}{400} = \frac{2}{5}$$ **(1/2)**

$P(E_2) = P$

(a person who is smoker and vegetarian)

$$= \frac{n(E_2)}{n(S)} = \frac{100}{400} = \frac{2}{8}$$ **(1/2)**

and $P(E_3) = P$ (a person who is non-smoker and vegetarian)

$$= \frac{n(E_3)}{n(S)} = \frac{140}{400} = \frac{7}{20}$$ **(1/2)**

Also, $P\left(\frac{A}{E_1}\right) = P$ (a person having a chest disease,who is smoker and non-vegetarian)

$$= 35\% = \frac{35}{100} = \frac{7}{20}$$ **(1/2)**

$P\left(\frac{A}{E_2}\right) = P$ (a person having a chest disease, who is smoker and vegetarian)

$$= 20\% = \frac{20}{100} = \frac{1}{5} \qquad \textbf{(1/2)}$$

and $P\left(\frac{A}{E_3}\right) = P$ (a person having a chest disease, who is non-smoker and vegetarian)

$$= 10\% = \frac{10}{100} = \frac{1}{10} \qquad \textbf{(1/2)}$$

∴ The probability that the selected person is smoker and non-vegetarian, who is suffer from chest disease,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$$

[by Baye's theorem]

$$= \frac{\frac{2}{5} \times \frac{7}{20}}{\frac{2}{5} \times \frac{7}{20} + \frac{2}{8} \times \frac{1}{5} + \frac{7}{20} \times \frac{1}{10}} \qquad \textbf{(1/2)}$$

$$= \frac{\frac{7}{50}}{\frac{7}{50} + \frac{1}{20} + \frac{7}{200}} = \frac{\frac{7}{50}}{\frac{28 + 10 + 7}{200}} = \frac{\frac{7}{50}}{\frac{45}{200}}$$

$$= \frac{7}{45} \times \frac{200}{50} = \frac{28}{45} \qquad \textbf{(1)}$$

The value reflected in this question is 'smoking is injurious for health'. **(1)**

**27.** Among the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain $A$ grade and 20% of day scholars attain A grade in their annual exams. At the end of year, one student is chosen at random from the college and he has an $A$ grade, what is the probability that the student is a hostelier? **Delhi 2012, 2011C**

> Let $E_1, E_2, \ldots, E_n$ be $n$ mutually exclusive and exhaustive events and $E$ be the any event, then by Baye's theorem $P\left(\frac{E_i}{E}\right) = \frac{P(E_i) \cdot P(E/E_i)}{\sum P(E_i) \cdot P(E/E_i)}$;
>
> where, $i = 1, 2, \ldots, n$.

Let us define the events as

$E_1$ : Students reside in a hostel

$E_2$ : Students are day scholars

$A$ : Students get $A$ grade

Now, given that

$P(E_1) =$ Probability that student reside in a hostel

$= 60\% = \frac{60}{100}$ **(1)**

and $P(E_2) =$ Probability that students are day scholars $= 1 - \frac{60}{100} = \frac{40}{100}$ **(1)**

Also, $P(A/E_1) =$ Probability that students who are hostelier get $A$ grade $= 30\% = \frac{30}{100}$

and $P(A/E_2) =$ Probability that students who are day scholars get $A$ grade $= 20\% = \frac{20}{100}$ **(1)**

$\therefore$ The probability that the selecting student is a hosteler having $A$ grade,

$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{\ldots}$

$$P(E_1/A) = \frac{\quad}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\left(\frac{60}{100}\times\frac{30}{100}\right)+\left(\frac{40}{100}\times\frac{20}{100}\right)} \quad \textbf{(1)}$$

$$= \frac{1800}{1800+800} = \frac{1800}{2600} = \frac{18}{26} = \frac{9}{13}$$

**(1)**

**28.** Suppose a girl throws a die. If she gets 5 or 6, then, she tosses a coin 3 times and note the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and note whether a head or tail is obtained. If she obtained exactly one head, then what is the probability that she throw 1, 2, 3 or 4 with the die?

**All India 2012**

Let us define the events as.

$E_1$ : Girl gets 5 or 6 on a die

$E_2$ : Girl gets 1, 2, 3 or 4 on a die

$A$ : She gets exactly one head

Now, $P(E_1)$ = Probability of getting 5 or 6 on a die $= \frac{2}{6} = \frac{1}{3}$

and $P(E_2)$ = Probability of getting 1, 2, 3 or 4 on a die $= \frac{4}{6} = \frac{2}{3}$ **(1)**

Also, $P(A/E_1)$ = Probability that girl get exactly one head when she throws coin thrice

$$= \frac{3}{8}$$ **(1)**

[$\because$ {$HHH, TTT, HHT, HTH, THH, TTH, THT, TTH$}]

$P(A/E_2)$ = Probability that girl get exactly one head when she throws coin once

$$= \frac{1}{2}$$ **(1)**

The probability that she throws 1, 2, 3 or 4 with the die for getting exactly one head,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\left(\frac{1}{3} \times \frac{3}{8}\right) + \left(\frac{2}{3} \times \frac{1}{2}\right)}$$ **(1)**

$$= \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{3+8}{24}} = \frac{\frac{1}{3}}{\frac{11}{24}} = \frac{1}{3} \times \frac{24}{11} = \frac{8}{11}$$ **(1)**

**29.** Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females. **HOTS; Delhi 2011**

Let us define the events as

$E_1$ : Person selected is a male

$E_2$ : Person selected is a female

$A$ : Person selected has grey hair **(1)**

Now, $P(E_1) = P$ (person selected is a male) $= 1/2$

and $P(E_2) = P$ (person selected is a female) $= \frac{1}{2}$

[$\because$ there are equal number of males and females, given]

Now, $P(A/E_1) =$ Probability of selecting a person having grey hair, who is male $= \frac{5}{100}$

[given]

and $P(A/E_2) =$ Probability of selecting a person having grey hair, who is female $= \frac{0.25}{100}$

[given] **(2)**

The probability of selecting person is a male having grey hair,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$\Rightarrow \quad P(E_1/A) = \frac{\frac{1}{2} \times \frac{5}{100}}{\left(\frac{1}{2} \times \frac{5}{100}\right) + \left(\frac{1}{2} \times \frac{0.25}{100}\right)} \quad \textbf{(1)}$$

$$= \frac{\frac{5}{200}}{\frac{5}{200} + \frac{0.25}{200}} = \frac{5}{5 + 0.25}$$

$$= \frac{5}{5.25} = \frac{500}{525} = \frac{100}{105} = \frac{20}{21}$$

Hence, required probability is $\frac{20}{21}$. **(1)**

**30.** Bag I contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. **Delhi 2011, 2010**

Let us define the events as

$E_1$ : Bag I is selected

$E_2$ : Bag II is selected

$A$ : Ball drawn is found to be red

Now, $$P(E_1) = P(E_2) = \frac{1}{2} \quad \textbf{(1)}$$

[∵ there are two bags, I and II and probability of their selections is 50%, i.e. 1/2]

Also, $P(A/E_1)$ = Probability of drawing a red ball from bag I $= \frac{3}{3+4}$

[∵ bag I contains 3 red and 4 black balls]

$$\Rightarrow \quad P(A/E_1) = \frac{3}{7} \quad \textbf{(1)}$$

and $P(A/E_2)$ = Probability of drawing a red ball from bag II $= \frac{5}{5+6}$

[∵ bag II contains 5 red and 6 black balls]

$$\Rightarrow \quad P(A/E_2) = \frac{5}{5+6} = \frac{5}{11} \quad \textbf{(1)}$$

The probability that the red ball is drawn from bag II is,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

[by Baye's theorem ]**(1)**

$$\Rightarrow P(E_2/A) = \frac{\frac{1}{2} \times \frac{5}{11}}{\left(\frac{1}{2} \times \frac{3}{7}\right) + \left(\frac{1}{2} \times \frac{5}{11}\right)} \quad \textbf{(1)}$$

$$= \frac{\frac{5}{22}}{\frac{3}{14} + \frac{5}{22}} = \frac{\frac{5}{22}}{\frac{33 + 35}{154}} = \frac{5}{22} \times \frac{154}{68}$$

$$\Rightarrow \quad P(E_2/A) = \frac{35}{68}$$

Hence, required probability is $\frac{35}{68}$. **(1)**

**31.** A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. **HOTS; Delhi 2011; All India 2008C**

Let us define the events as

$E_1$ : Six appears on a die

$E_2$ : Six does not appear on the die

$A$ : Man reports that it is a six

Now, $P(E_1) = P$ (six appears on a die) $= \frac{1}{6}$

and $P(E_2) = P$ (six does not appear on a die)

$$= 1 - \frac{1}{6} = \frac{5}{6} \qquad \textbf{(1)}$$

$P(A/E_1)$ = Probability that six appears on a die and man also reports that it is a six, i.e. he is speaking the truth $= \frac{3}{4}$ **(1)**

[$\because$ man speaks truth 3 out of 4 times, so the probability of speaking truth is 3/4, given]

$P(A/E_2)$ = Probability that six does not appears on the die but man reports it is a six, i.e. he is not speaking the truth $= 1 - \frac{3}{4} = \frac{1}{4}$ **(1)**

The probability of getting a six, when man reports that it was a six,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$P(E_1/A) = \frac{\left(\frac{1}{6} \times \frac{3}{4}\right)}{\left(\frac{1}{6} \times \frac{3}{4}\right) + \left(\frac{5}{6} \times \frac{1}{4}\right)} \qquad \textbf{(1)}$$

$$= \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} = \frac{\frac{3}{24}}{\frac{8}{24}} = \frac{3}{24} \times \frac{24}{8} = \frac{3}{8}$$

Hence, required probability is $\frac{3}{8}$. **(1)**

**32.** A factory has two machines $A$ and $B$. Past record shows that machine $A$ produced 60% of items of output and machine $B$ produced 40% of items. Further, 2% of items, produced by machine $A$ and 1% produced by machine $B$ were defective. All the items are put into a stockpile and then one item is chosen at random from this and this is found to be defective. What is the probability that it was produced by machine $B$? **Foreign 2011**

Let us define the events as

$E_1$ : Items produced by machine $A$

$E_2$ : Items produced by machine $B$

$A$ : Item is defective

Now,

$P(E_1) =$ Probability that item is produced by machine $A = 60\% = \frac{60}{100}$

$P(E_2) =$ Probability that item is produced by machine $B = 40\% = \frac{40}{100}$ **(1)**

$P(A/E_1) =$ Probability that defective item is produced by machine $A = 2\% = \frac{2}{100}$ **(1)**

$P(A/E_2) =$ Probability that defective item is produced by machine $B = 1\% = \frac{1}{100}$ **(1)**

The probability that the machine $B$ produced a defective item is

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$ **(1)**

[by Baye's theorem]

$$= \frac{\frac{40}{100} \times \frac{1}{100}}{\left(\frac{60}{100} \times \frac{2}{100}\right) + \left(\frac{40}{100} \times \frac{1}{100}\right)}$$ **(1)**

$$= \frac{40}{120 + 40} = \frac{40}{160} = \frac{1}{4}$$

Hence, required probability is 1/4. **(1)**

**33.** There are three coins. One is a two headed coin (having head on both faces), other is a biased coin that comes up heads 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed and it shows head. What is the probability that it was the two headed coin?

**Foreign 2011; Delhi 2009**

Let us define the events as

$E_1$ : A two headed coin is selected

$E_2$ : A biased coin is selected

$E_3$ : Unbiased coin is selected

$A$ : Head comes up

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ **(1)**

Also, $P(A/E_1) =$ Probability that head comes up on a two headed coin $= 1$

$P(A/E_2) =$ Probability that head comes up on a biased coin $= 75\% = \frac{75}{100}$

and $P(A/E_3)$ = Probability that head comes up on an unbiased coin $= \frac{1}{2}$ **(2)**

The probability that the two headed coin shows head, is

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{\left[\begin{array}{r} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ + P(E_3) \cdot P(A/E_3) \end{array}\right]}$$

[by Baye's theorem] **(1)**

$$\Rightarrow P(E_1/A) = \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{75}{100} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

**(1)**

$$= \frac{1}{1 + \frac{75}{100} + \frac{1}{2}}$$

$$= \frac{1}{\frac{100 + 75 + 50}{100}}$$

$$= \frac{100}{225} = \frac{4}{9}$$

Hence, required probability is $\frac{4}{9}$. **(1)**

**34.** There are three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of gold, then what is the probability that the other coin in box is also of gold? **All India 2011**

Let us define the events as

$E_1$ : Box I is selected

$E_2$ : Box II is selected

$E_3$ : Box III is selected

$A$ : The drawn coin is a gold coin

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ **(1)**

Also, $P(A/E_1)$ = Probability that a gold coin is drawn from box I

$= \frac{2}{2} = 1$ [$\because$ box I contain both gold coins]

$P(A/E_2)$ = Probability that a gold coin is drawn from box II = 0 [$\because$ box II has both silver coins]

and $P(A/E_3)$ = Probability that a gold coin is drawn from box III $= \frac{1}{2}$

[$\because$ box III contains 1 gold and 1 silver coin] **(2)**

The probability that other coin in box is also of gold = The probability that the drawing gold coin from bag I,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{\left[\begin{array}{l} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ \qquad + P(E_3) \cdot P(A/E_3) \end{array}\right]}$$

[By Baye's theorem] **(1)**

$$= \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$ **(1)**

$$= \frac{1}{1 + 0 + \frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

Hence, required probability is $\frac{2}{3}$. **(1)**

**35.** There are three coins. One is a two tailed coin (having tail on both faces), another is a biased coin that comes up heads 60% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed and it shows tail. What is the probability that it is a two tailed coin?

**All India 2011C**

Do same as Que 33. [**Ans.** $\frac{10}{19}$]

**36.** In a class, 5% of boys and 10% of girls have an IQ of more than 150. In the class, 60% are boys and rest are girls. If a student is selected at random and found to have an IQ of more than 150, then find the probability that the student is a boy. **All India 2010C**

Let us define the events as

$E_1$ : Boys are selected

$E_2$ : Girls are selected

$A$ : The student has an IQ of more than 150 students.

Now, $P(E_1) = 60\% = \frac{60}{100}$

and $P(E_2) = 40\% = \frac{40}{100}$ **(1)**

[$\because$ in the class 60% students are boys, so 40% are girls, given]

and $P(A/E_1)$ = Probability that boys has an IQ of more than 150

$= 5\% = \frac{5}{100}$ **(1)**

and $P(A/E_2)$ = Probability that girls has an IQ of more than 150

$= 10\% = \frac{10}{100}$ **(1)**

The probability that the selected boy having IQ more than 150 is

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$ **(1)**

[by Baye's theorem]

$$= \frac{\frac{60}{100} \times \frac{5}{100}}{\left(\frac{60}{100} \times \frac{5}{100}\right) + \left(\frac{40}{100} \times \frac{10}{100}\right)}$$ **(1)**

$$= \frac{300}{300 + 400} = \frac{300}{700} = \frac{3}{7}$$

Hence, required probability is $\frac{3}{7}$. **(1)**

**37.** In a bolt factory, machines $A$, $B$ and $C$ manufacture 25%, 35% and 40% of total production, respectively. Out of their total output, 5%, 4% and 2% are defective bolts. A bolt is drawn at random and is found to be defective. What is the probability that it is manufactured by machine $B$? **Delhi 2010C**

Let us define the events as

$E_1$ : Bolt is manufactured by machine $A$

$E_2$ : Bolt is manufactured by machine $B$

$E_3$ : Bolt is manufactured by machine $C$

$A$ : Bolt drawn is defective

Now, $P(E_1)$ = Probability that bolt is manufactured by machine $A = 25\% = \frac{25}{100}$

$P(E_2)$ = Probability that bolt is manufactured by machine $B = 35\% = \frac{35}{100}$

and $P(E_3)$ = Probability that bolt is manufactured by machine $C = 40\% = \frac{40}{100}$ **(1)**

Also, $P(A/E_1)$ = Probability that machine $A$ produces defective bolt

$= 5\% = \frac{5}{100}$

$P(A/E_2)$ = Probability that machine $B$ produces defective bolts $= 4\% = \frac{4}{100}$

$P(A/E_3)$ = Probability that machine $C$ produces defective bolt $= 2\% = \frac{2}{100}$ **(2)**

The probability that the defective bolt is manufactured by machine $B$,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{\left[\begin{array}{r} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ + P(E_3) \cdot P(A/E_3) \end{array}\right]}$$ **(1)**

[by Baye's theorem]

$$= \frac{\frac{35}{100} \times \frac{4}{100}}{\left[\left(\frac{25}{100} \times \frac{5}{100}\right) + \left(\frac{35}{100} \times \frac{4}{100}\right) + \left(\frac{40}{100} \times \frac{2}{100}\right)\right]} \quad \textbf{(1)}$$

$$= \frac{140}{125 + 140 + 80} = \frac{140}{345} = \frac{28}{69}$$

Hence, required probability is $\frac{28}{69}$. **(1)**

**38.** A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both of clubs. Find the probability of the lost card being of clubs. **Delhi 2010**

Let us define the events as

$E_1$ : The lost card is a diamond

$E_2$ : The lost card is a spade

$E_3$ : The lost card is a club

$E_4$ : The lost card is a heart

$A$ : The two drawn cards are clubs

Now, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$ **(1)**

Also, $P(A/E_1) =$ Probability that two drawn cards are clubs when the lost card is a diamond

$$= \frac{{}^{13}C_2}{{}^{51}C_2}$$

Similarly, $P(A/E_2) = P(A/E_4) = \frac{{}^{13}C_2}{{}^{51}C_2}$

and $P(A/ E_3) =$ Probability that two drawn cards are clubs when the lost card is of also a club

$$= \frac{{}^{12}C_2}{{}^{51}C_2} \quad \textbf{(2)}$$

The probability that the lost card is a club, when two club cards are drawn,

$$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{\begin{bmatrix} P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) \\ + P(E_3)\cdot P(A/E_3) + P(E_4)\cdot P(A/E_4) \end{bmatrix}}$$

[By Baye's theorem] **(1)**

$$= \frac{\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}}{\begin{bmatrix} \left(\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}\right) + \left(\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}\right) \\ + \left(\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}\right) + \frac{1}{4}\times\left(\frac{^{13}C_2}{^{51}C_2}\right) \end{bmatrix}}$$ **(1)**

$$= \frac{\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}}{\frac{3}{4}\cdot\frac{^{13}C_2}{^{51}C_2} + \frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}} = \frac{^{12}C_2}{3\cdot{}^{13}C_2 + {}^{12}C_2}$$

$$\begin{bmatrix} \because\ ^{12}C_2 = \frac{12!}{2!10!} = \frac{12\times 11}{2} = 66 \\ \text{and } ^{13}C_2 = \frac{13!}{2!11!} = \frac{13\times 12}{2} = 78 \end{bmatrix}$$

$$= \frac{66}{3\times 78 + 66} = \frac{66}{234+66} = \frac{66}{300} = \frac{11}{50}$$

Hence, required probability is 11/50. **(1)**

**39.** From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of number of defective bulbs? **Delhi 2010**

Firstly, let $X$ be the random variable which gives the number of defective bulbs. Here, $X = 0, 1$ and 2 because 2 bulbs are drawn from a lot of 10 bulbs. Now, find respective probabilities at $X = 0, 1$ and 2 to get the required probability distribution.

Given, total number of bulbs = 10

Number of defective bulbs = 3

Number of bulbs which are not defective = 7

Now, let $X$ be the number of defective bulbs drawn. We take $X = 0, 1$ and 2, as bulbs are drawn from a lot of 10 bulbs. **(2)**

$\therefore P(X = 0)$ = Probability that no defective bulb is drawn

$$= \frac{{}^3C_0 \times {}^7C_2}{{}^{10}C_2} = \frac{1 \times \frac{7!}{2!5!}}{\frac{10!}{2!8!}} = \frac{7!}{2!5!} \times \frac{2!8!}{10!}$$

$$= \frac{7 \times 6}{10 \times 9} = \frac{42}{90} = \frac{14}{30} = \frac{7}{15}$$

$$\Rightarrow \quad P(X = 0) = \frac{7}{15}$$ **(1)**

$P(X = 1)$ = Probability that one defective bulb is drawn, i.e. the other drawn bulb is non-defective

$$= \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2} = \frac{3 \times 7}{\frac{10!}{2!8!}} \qquad [\because {}^nC_1 = n]$$

$$= \frac{3 \times 7 \times 8! \times 2!}{10!} = \frac{3 \times 7 \times 2}{10 \times 9} = \frac{14}{30} = \frac{7}{15}$$ **(1)**

$P(X = 2)$ = Probability that both the bulbs drawn are defective

$$= \frac{{}^3C_2}{{}^{10}C_2} = \frac{3}{\frac{10!}{2 \times 8!}} \qquad [\because {}^nC_{n-1} = n]$$

$$= \frac{3 \times 2! \times 8!}{10!} = \frac{3 \times 2}{10 \times 9} = \frac{6}{90}$$

$$= \frac{2}{30} = \frac{1}{15}$$ **(1)**

∴ The required probability distribution is as

∴ The required probability distribution is as follows:

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | 7/15 | 7/15 | 1/15 |

**(1)**

**40.** Two cards are drawn simultaneously (or successively without replacement) from a well-shuffled deck of 52 cards. Find the mean, variance and standard deviation of number of aces. **All India 2009C**

> Firstly, find probability distribution, then use following formulae:
> Mean $= \sum X_i P_i$
> and variance $= \sum X_i^2 \cdot P_i - (\sum X_i \cdot P_i)^2$

Given, total number of cards = 52

Also, total ace cards = 4

and other cards = 52 − 4 = 48

Let $X$ denotes the number of ace cards drawn. We take values of $X$ as $X = 0, 1, 2$.

Now, $P(X = 0)$ = Probability that no ace card is drawn, i.e. both the cards drawn are non-aces

$$= \frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{\frac{48 \times 47}{2}}{\frac{52 \times 51}{2}} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$P(X = 1)$ = Probability that one ace card is drawn, i.e. the other drawn card will be non-ace card

$$= \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2}} = \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}$$

**(1½)**

and $P(X = 2)$ = Probability that both drawn cards are aces

4!

$$= \frac{^4C_2}{^{52}C_2} = \frac{\frac{\overline{2!2!}}{52!}}{2!50!} = 4 \times 3 \times \frac{50!}{52!} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

∴ The probability distribution is given below:

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$ |

Now, mean $= \sum X_i \cdot P_i$ ...(i)

and variance $= \sum X_i^2 \cdot P_i - (\sum X_i \cdot P_i)^2$ ...(ii)

∴ We have,

| $X$ | $P(X)$ | $X_i \cdot P_i$ | $X_i^2 \cdot P_i$ |
|---|---|---|---|
| 0 | $\frac{188}{221}$ | 0 | 0 |
| 1 | $\frac{32}{221}$ | $\frac{32}{221}$ | $\frac{32}{221}$ |
| 2 | $\frac{1}{221}$ | $\frac{2}{221}$ | $\frac{4}{221}$ |
| **Total** | | $\frac{34}{221}$ | $\frac{36}{221}$ |

**(1)**

Here $\sum X_i \cdot P_i = \frac{34}{221}$ and $\sum X_i^2 \cdot P_i = \frac{36}{221}$

On putting above values in Eqs. (i) and (ii), we get

$$\text{Mean} = \frac{34}{221} = 0.154 \qquad \textbf{(1/2)}$$

$$\text{Variance} = \frac{36}{221} - \left(\frac{34}{221}\right)^2 \quad \text{[from Eq. (ii)]}$$

$$= \frac{36}{221} - \frac{1156}{48841} = \frac{7956 - 1156}{48841}$$

$$= \frac{6800}{48841} = 0.139 \qquad \textbf{(1/2)}$$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}}$

$$= \sqrt{\frac{6800}{48841}} = \frac{82.46}{221} = 0.37 \quad \textbf{(1/2)}$$

Hence, mean $= 0.154$, variance $= 0.139$ and standard deviation $= 0.37$ **(1/2)**

**41.** A bag contains 4 red and 4 black balls. Other bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that ball is drawn from first bag? **Delhi 2009C**

Do same as Que 30. [**Ans.** –2/3]

**42.** A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4.

Let us define the events as

$E_1$ : A number greater than 4 appears on the die

$E_2$ : A number greater than 4 does not appear on the die

$A$ : Man reports that it is a number greater than 4

Now, $P(E_1)$ = Probability of getting a number greater than $4 = \frac{2}{6} = \frac{1}{3}$

[$\because$ there are only two numbers greater than 4, i.e. 5 and 6]

and $P(E_2)$ = Probability that the number greater than 4 does not appear

$$= 1 - \frac{1}{3} = \frac{2}{3} \quad \textbf{(1)}$$

Also, $P(A/E_1)$ = Probability that the man reports it is a number greater than 4 on the die and it is so, i.e. he is speaking truth $= \frac{3}{5}$ **(1)**

[$\because$ given that he speaks truth 3 out of 5 times]

and $P(A/E_2)$ = Probability that man reports it is a number greater than 4 on the die but it is not so, i.e. he is not speaking truth $= 1 - \frac{3}{5} = \frac{2}{5}$ **(1)**

The probability of getting a number greater than 4 when man reports that it was greater than 4,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad \textbf{(1)}$$

[by Baye's theorem]

$$= \frac{\frac{1}{3} \times \frac{3}{5}}{\left(\frac{1}{3} \times \frac{3}{5}\right) + \left(\frac{2}{3} \times \frac{2}{5}\right)} \quad \textbf{(1)}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5} + \frac{4}{5}} = \frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$$

Hence, the required probability is $\frac{3}{7}$. **(1)**

**43.** Coloured balls are distributed in three bags as shown in the following table:

| Bag | Colour of Balls | | |
|---|---|---|---|
| | **Black** | **White** | **Red** |
| I | 1 | 2 | 3 |
| II | 2 | 4 | 1 |
| III | 4 | 5 | 3 |

A bag is selected at random and then two balls are randomly drawn from selected bag. They happen to be black and red. What is the probability that they come from bag I?

**All India 2009**

Let us define the events as

$E_1$ : Bag I is selected

$E_2$ : Bag II is selected

$E_3$ : Bag III is selected

$A$ : Two drawn balls are found to be black and red

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ **(1)**

Also, $P(A/E_1)$ = Probability that two drawn balls red and black are drawn from bag I

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

[∵ bag I contains total 6 balls including 1 black and 3 red balls]

$$= \frac{1 \times 3}{\frac{6\,!}{2\,!\,4\,!}} = \frac{1 \times 3 \times 2 \times 4\,!}{6\,!} = \frac{1 \times 3 \times 2}{6 \times 5}$$

$$\Rightarrow \qquad P(A/E_1) = \frac{1}{5}$$

and $P(A/E_2)$ = Probability that red and black balls are drawn from bag II

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^7C_2}$$

[∵ bag II contains total 7 balls including 2 black balls and 1 red ball]

$$= \frac{2 \times 1}{\frac{7\,!}{2\,!\,5\,!}} = \frac{2 \times 1 \times 2\,! \times 5\,!}{7\,!} = \frac{2 \times 2}{7 \times 6} = \frac{2}{21}$$ **(1)**

$P(A/E_3)$ = Probability that red and black balls are drawn from bag III

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

[$\because$ bag III contains total12 balls including 4 black balls and 3 red balls]

$$= \frac{4 \times 3}{\frac{12\,!}{2\,!10\,!}}$$

$$= \frac{4 \times 3 \times 2\,! \times 10\,!}{12\,!} = \frac{4 \times 3 \times 2}{12 \times 11} = \frac{2}{11} \quad \textbf{(1)}$$

The probability that the two drawing balls which are black and red, are come from bag I,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{\begin{bmatrix} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ + P(E_3) \cdot P(A/E_3) \end{bmatrix}}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\left(\frac{1}{3} \times \frac{1}{5}\right) + \left(\frac{1}{3} \times \frac{2}{21}\right) + \left(\frac{1}{3} \times \frac{2}{11}\right)} \quad \textbf{(1)}$$

$$= \frac{\frac{1}{5}}{\left(\frac{1}{5} + \frac{2}{21} + \frac{2}{11}\right)} = \frac{\frac{1}{5}}{\frac{231 + 110 + 210}{1155}}$$

$$= \frac{1}{5} \times \frac{1155}{551} = \frac{231}{551}$$

Hence, required probability is $\frac{231}{551}$. **(1)**

**44.** Two groups are competing for the post of board of directors of a corporation. The probabilities that the first and second group wins are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product was introduced by second group. **Delhi 2009**

Let us define the events as

$E_1$ : First group wins

$E_2$ : Second group wins

$A$ : The new product is introduced

Now, $P(E_1)$ = Probability that first group wins $= 0.6$

and $P(E_2)$ = Probability that second group wins $= 0.4$

and $P(A/E_1)$ = Probability that first group wins and it introduce the new product $= 0.7$

$P(A/E_2)$ = Probability that second group wins and it introduce the new product $= 0.3$ **(2)**

The probability that product is introduced and it was introduced by second group,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$\Rightarrow P(E_2/A) = \frac{0.4 \times 0.3}{(0.6 \times 0.7) + (0.4 \times 0.3)}$$ **(1)**

$$= \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54} = \frac{12}{54} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$. **(1)**

**45.** Three bags contain balls as shown in the table.

| Bag | White Balls | Black Balls | Red Balls |
|---|---|---|---|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 3 | 2 |

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bag III? **Delhi 2009**

Do same as Que 43. $\left[\textbf{Ans.} - \frac{5}{17}\right]$

**46.** A company has two plants to manufacture motorcycles. 70% motorcycles are manufactured at the first plant, while 30% are manufactured at the second plant. At first plant, 80% motorcycles are rated of the standard quality, while at the second plant, 90% are rated of standard quality.

A motorcycle, randomly picked up and is found to be of standard quality. Find the probability that it has come out from the second plant. **Delhi 2008**

Let us define the events as

$E_1$ : Motorcycle manufactured in plant I

$E_2$ : Motorcycle manufactured in plant II

$A$ : Motorcycle of standard quality **(1)**

Now, $P(E_1) = \frac{70}{100} = \frac{7}{10}$, $P(A/E_1) = \frac{80}{100} = \frac{8}{10}$

$P(E_2) = \frac{30}{100} = \frac{3}{10}$, $P(A/E_2) = \frac{90}{100} = \frac{9}{10}$ **(2)**

The probability that the plant II manufactured a standard quality of motorcycle,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad \textbf{(1)}$$

$$= \frac{\frac{3}{10} \times \frac{9}{10}}{\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{9}{10}} \quad \textbf{(1)}$$

$$= \frac{27}{56 + 27} = \frac{27}{83} \quad \textbf{(1)}$$

**47.** A doctor is to visit a patient. From the past experience, it is known that the probabilities of doctor coming by train, bus, scooter and taxi are $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$ and $\frac{2}{5}$, respectively. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively but by taxi, he will not be late. When he arrives he is late; what is the probability that he came by bus? **Delhi 2008C**

Let us define the events as

$E_1$ : Doctor comes by train

$E_2$ : Doctor comes by bus

$E_3$ : Doctor comes by scooter

$E_4$ : Doctor comes by a taxi

$A$ : Doctor arrives late

Now, given $P(E_1) = \frac{1}{10}$, $P(E_2) = \frac{1}{5}$,

$$P(E_3) = \frac{3}{10} \text{ and } P(E_4) = \frac{2}{5} \qquad \textbf{(1)}$$

Also, $P(A/E_1)$ = Probability that doctor arrives late when he comes by train $= \frac{1}{4}$ [given]

$P(A/E_2)$ = Probability that doctor arrives late when he comes by bus $= \frac{1}{3}$ [given] **(1)**

$P(A/E_3)$ = Probability that doctor arrives late when he comes by scooter $= \frac{1}{12}$ [given]

and $P(A/E_4)$ = Probability that doctor arrives late when he comes by taxi $= 0$ **(1)**

[∵ given when the doctor comes by taxi, he will not late]

The probability that the doctor coming by bus arrives late,

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{\left[\begin{array}{l} P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) \\ + P(E_3) \cdot P(A/E_3) + P(E_4) \cdot P(A/E_4) \end{array}\right]}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{1}{5} \times \frac{1}{3}}{\left[\left(\frac{1}{10} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{1}{3}\right) + \left(\frac{3}{10} \times \frac{1}{12}\right)\right]}$$

$$+ \left(\frac{2}{5} \times 0\right)\Bigg]$$

$$= \frac{\frac{1}{15}}{\frac{1}{40} + \frac{1}{15} + \frac{3}{120}} = \frac{\frac{1}{15}}{\frac{3+8+3}{120}} = \frac{\frac{1}{15}}{\frac{14}{120}} \quad \textbf{(1)}$$

$$= \frac{1}{15} \times \frac{120}{14} = \frac{4}{7}$$

Hence, required probability is $\frac{4}{7}$. **(1)**

**48.** In a bulb factory, machines $A$, $B$ and $C$ manufacture 60%, 30% and 10% bulbs, respectively. 1%, 2% and 3% of bulbs produced respectively by $A$, $B$ and $C$ are found to be defective. A bulb is picked up at random from total production and found to be defective. Find the probability that this bulb was produced by machine $A$.

**All India 2008**

Do same as Que 37. $\left[\textbf{Ans.} - \frac{5}{17}\right]$