Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.1

Question 1:

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer 1:

(i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

(iv) $5005 = 5\times7\times11\times13$

(v) $7429 = 17\times19\times23$

Question 2:

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer 2:

(i) 26 and 91

$26 = 2\times13$

$91 = 7\times13$

$\text{HCF} = 13$

$\text{LCM} = 2\times7\times13 = 182$

$\text{Product of the two numbers} = 26\times91 = 2366$

$\text{HCF}\times\text{LCM} = 13\times182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

$510 = 2\times3\times5\times17$

$92 = 2\times2\times23$

$\text{HCF} = 2$

$\text{LCM} = 2\times2\times3\times5\times17\times23 = 23460$

$\text{Product of the two numbers} = 510\times92 = 46920$

$\text{HCF}\times\text{LCM} = 2\times23460$

$= 46920$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$\text{HCF} = 2 \times 3 = 6$

$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$

Product of the two numbers $= 336 \times 54 = 18144$

$\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

Question 3:

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer 3:

(i) 12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\text{HCF} = 3$

$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17,23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\text{HCF} = 1$

$\text{LCM} = 17 \times 23 \times 29 = 11339$

(iii) $8, 9$ and 25

$$8 = 2 \times 2 \times 2$$
$$9 = 3 \times 3$$
$$25 = 5 \times 5$$
$$\text{HCF} = 1$$
$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$$

Question 4:

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer 4:

$$\text{HCF}(306, 657) = 9$$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 22338$$

Question 5:

Check whether 6^n can end with the digit 0 for any natural number n.

Answer 5:

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Question 6:

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer 6:

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Question 7:

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer 7:

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 × 3 × 3

And, 12 = 2 × 2 × 3

LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.2

Question 1:

Prove that $\sqrt{5}$ is irrational.

Answer 1:

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that $\sqrt{5} = \frac{a}{b}$ Let *a* and *b* have a common factor other than 1. Then we can divide them by the common factor, and assume that *a* and *b* are co-prime.

$$a = \sqrt{5}b$$

$$\Rightarrow a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that *a* is divisible by 5.

Let *a* = 5*k*, where *k* is an integer

$$(5k)^2 = 5b^2$$

$$\Rightarrow 5k^2 = b^2$$

This means that b^2 is divisible by 5 and hence, *b* is divisible by 5.

This implies that *a* and *b* have 5 as a common factor.

And this is a contradiction to the fact that *a* and *b* are co-prime.

Hence, $\sqrt{5}$ *cannot* be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:

Prove that $3 + 2\sqrt{5}$ is irrational.

Answer 2:

Let $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$$

$$\Rightarrow \sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since *a* and *b* are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Question 3:

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ **(ii)** $7\sqrt{5}$ **(iii)** $6+\sqrt{2}$

Answer 3:

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$\frac{1}{\sqrt{2}}=\frac{a}{b}$$

Or

$$\sqrt{2}=\frac{b}{a}$$

$\frac{b}{a}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$7\sqrt{5}=\frac{a}{b}$$

$$\Rightarrow \sqrt{5}=\frac{a}{7b}$$

$\frac{a}{7b}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let $6+\sqrt{2}$ be rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

Since *a* and *b* are integers, $\frac{a}{b}-6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6+\sqrt{2}$ is irrational.