Probability

Ex. 4.1

Answer 1-i.

Three coins are tossed.

∴ The sample space is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ n(S) = 8

P is the event of getting at least two heads.

[At least two heads means two heads or more than two heads but not less than two.]

Less than two heads = 1 head

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ P = {HHH, HHT, HTH, THH} ... (1)

∴ n(P) = 4.

Q is the event of getting no heads.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ Q = {TTT} ... (2) ∴ n(Q) = 1.

R is the event of getting head on the second coin.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ R = {HHH, HHT, THH, THT} ... (3)

∴ n(R) = 4.

From results (1) and (2), P ∩ Q = Ø

From results (2) and (3), Q ∩ R = Ø

∴ P and Q are mutually exclusive events. Also, Q and R are mutually exclusive events.

Answer 1-ii.

A die is thrown.

∴ The sample space S = {1, 2, 3, 4, 5, 6}.

∴ n(S) = 6.

P is the event of getting an odd number.

S = {1, 2, 3, 4, 5, 6}

∴ P = {1, 3, 5} ... (1)

∴ n(P) = 3.

Q is the event of getting an even number.

Q is the event of getting an even number.

S = {1, 2, 3, 4, 5, 6}

∴ Q = {2, 4, 6} ... (2)

∴ n(Q) =3.

R is the event of getting a prime number.

S = {1, 2, 3, 4, 5, 6}

∴ R = {2, 3, 5} ...(3)

∴ n(R) = 3.

From results (1) and (2),

Q = P' and P ∪ Q = S.

∴ Events P and Q are complementary as well as exhaustive events.

Also, P ∩ Q =Ø

∴ Events P and Q are mutually exclusive events.

Answer 1-iii.

Two dice are thrown.

∴The sample space is S

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1,6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3),(4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4),(5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}

∴ n(S) = 36.

P is the event that the sum of the scores on the upper faces is a multiple of 6.

S = {(1, 1), (1, 2), (1, 3), (1, 4), <u>(1, 5)</u>, (1,6),

(2, 1), (2, 2), (2, 3), <u>(2, 4)</u>, (2, 5), (2, 6),

(3, 1), (3, 2), <u>(3, 3)</u>, (3, 4), (3, 5), (3, 6),

(4, 1), <u>(4, 2)</u>, (4, 3),(4, 4), (4, 5), (4, 6),

<u>(5, 1)</u>, (5, 2), (5, 3), (5, 4),(5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), <u>(6, 6)</u>}

∴ P = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)}

∴ n(P) = 6.

Q is the event that the sum of the scores appearing on the upper faces is at least 10.

At least 10 means, 10 or more than 10

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1,6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3),(4, 4), (4, 5), <u>(4, 6)</u>,

(5, 1), (5, 2), (5, 3), (5, 4), <u>(5, 5)</u>, <u>(5, 6)</u>,

(6, 1), (6, 2), (6, 3), <u>(6, 4)</u>, <u>(6, 5)</u>, <u>(6, 6)</u>}

∴ Q = { (4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

∴ n(Q) = 6.

R is the event that the same score appears on both the dice.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1,6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

∴ n(R) = 6.

Answer 1-iv.

Here there are three red balls R_1, R_2, R_3;

Let the three white balls be W_1, W_2, W_3;

Let the three green balls be G_1, G_2, G_3.

∴ The sample space is

S = {$R_1, R_2, R_3, W_1, W_2, W_3, G_1, G_2, G_3$}

∴ n(S) = 9

P is the event that the ball is red.

∴ P = {R_1, R_2, R_3}

∴ n(P) = 3. ...(1)

Q is the event that the ball is not green.

∴ Q = {$R_1, R_2, R_3, W_1, W_2, W_3$}

∴ n(Q) = 6. ... (2)

R is the event that the ball is red or white.

∴R = {$R_1, R_2, R_3, W_1, W_2, W_3$}

∴ n(R) = 6. ... (3)

The events Q and R are same events.

Answer 1-v.

While forming two-digit numbers using the given digits, 0 cannot be used at the tens place.

Hence, the sample space is

S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54}

∴ n(S) = 25.

P is the event that the number is even.

S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54}

∴ P = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52,
54}

∴ n(P) = 13. .... (1)

Q is the event that the number is divisible by 3.

S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54}

∴ Q = {12, 15, 21, 24, 30, 42, 45, 51, 54}

∴ n(Q) = 9. .... (2)

R is the event that the number is greater than 5O.

S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54}

∴ R = {51, 52, 53, 54}

∴ n(R) = 4.

Answer 1-vi.

The sample space (for a coin)

S = (H, T)

∴ n(S) = 2

The sample space (for a die)

S = (1, 2, 3, 4, 5, 6)

∴ n(S) = 6

Hence, if a coin is tossed and a die is thrown simultaneously, the sample space

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}
∴ n(S) = 12

P is the event of getting a head and an odd number. S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

∴ P = {(H, 1), (H, 3), (H, 5)}

∴ n(P) = 3. ... (1)

Q is the event of getting H or T and an even number.

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

∴ Q = {(H, 2), (H, 4), (H, 6), (T, 2), (T, 4), (T, 6)}

∴ n(Q) = 6. ... (2)

(R is the event of getting a number on die greater than 7 and a tail.

There is no number greater than 6 on a die. ∴ R= Ø
∴ n(R) = 0. ... (3)

Here, P ∩ Q = Ø

P ∩ R = Ø

and Q ∩ R = Ø.

Hence the events P and Q; P and R; Q and R are mutually exclusive events.

Answer 1-vii.

Let the 3 men be M_1, M_2, M_3 and 2 women be W_1, W_2.
A committee of two is to be formed.
∴ The sample space is
$S = \{M_1M_2, M_1M_3, M_2M_3, M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$ ... (1)
∴ $n(S) = 10$

P is the event that the committee should contain at least one woman.
At least one woman means, 1 woman or more than one, i.e. two women in this case.

$S = \{M_1M_2, M_1M_3, M_2M_3, \underline{M_1W_1}, \underline{M_1W_2}, \underline{M_2W_1}, \underline{M_2W_2}, \underline{M_3W_1}, \underline{M_3W_2}, \underline{W_1W_2}\}$
∴ $P = \{M_1W_1, M_2W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$ ...(2)
∴ $n(P) = 7$.

Q is the event that the committee should contain one man and one woman.
$S = \{M_1M_2, M_1M_3, M_2M_3, \underline{M_1W_1}, \underline{M_1W_2}, \underline{M_2W_1}, \underline{M_2W_2}, \underline{M_3W_1}, \underline{M_3W_2}, \underline{W_1W_2}\}$
∴ $Q = \{M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2\}$ ... (3)
∴ $n(Q) = 6$.

R is the event that there is no woman in the committee.
$S = \{M_1M_2, M_1M_3, M_2M_3, M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$
∴ $R = \{M_1M_2, M_1M_3, M_2M_3\}$ ... (4)
∴ $n(R) = 3$.
From (1), (2) and (4), $R = P'$
∴ The events P and R are complementary events.
Also, $P \cup R = S$
∴ Events P and R are mutually exhaustive events.
From (3) and (4),
∴ $Q \cap R = \emptyset$
∴ The events Q and R are mutually exclusive events.

Ex. 4.2

Answer 1.

Let S be the sample space.

Then S = {HH, HT, TH, TT}∴ n(S) = 4.

i. Let A be the event where at least one tails turns up.

At least one tail means 1 tails or more than 1 tails.
If S = {HH, HT, TH, TT}
∴ A = {HT, TH, TT}
∴ n(A) = 3.

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{3}{4}$$

ii. Let B be the event where no heads turns up.

If S = {HH, HT, TH, TT}
Then B = {TT}
∴ n(B) = 1

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{1}{4}$$

iii. Let C be the event that at the most one tails turns up.

At the most one tails means not more than one tails or less than or equal to one tails.
S = {HH, HT, TH, TT}
Then C= {HH, HT, TH}
∴ n(C) = 3.

$$P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{3}{4}$$

Answer 2.

Let S be the sample space.
Then S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
∴ n(S) = 8

(i) Let A be the event of getting heads on the middle coin.
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
∴ A = {HHH, HHT, THH, THT}∴ n(A)= 4

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{4}{8} = \frac{1}{2}$$

(ii) Let B be the event of getting exactly one tails.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Then B = {HHT, HTH, THH}
n(B) = 3.

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{3}{8}$$

(iii) Let C be the event of getting no tails.
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Then C = {HHH} n(C) = 1

$$\text{Now, } P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{1}{8}$$

Answer 3.

Let S be the sample space.
Then S = {1, 2, 3, 4, 5, 6}∴ n(S) = 6

(i) Let A be the event of getting an odd number.
S = {1, 2, 3, 4, 5, 6}
Then A = {1, 3, 5} ∴ n(A)=3.

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$$

(ii) Let B be the event of getting a perfect square.
S = {1, 2, 3, 4, 5, 6} Then B = {1, 4} ∴ n(B) = 2.

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{2}{6} = \frac{1}{3}$$

(iii) Let C be the event of getting a number greater than 3.
S = {1, 2, 3, 4, 5, 6}
Then C = {4, 5, 6}
∴ n(C) = 3

$$\text{Now, } P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{3}{6} = \frac{1}{2}$$

Answer 4.

The sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4,5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
This contains 36 sample points. ∴ n(S) = 36

(i) Let A be the event that the sum of the numbers on the upper faces is divisible by 9.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4,5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Then A = {(3, 6), (4, 5), (5, 4), (6, 3)} ∴ n(A) = 4

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{4}{36} = \frac{1}{9}$$

(ii) Let B be the event that the sum of the numbers on their upper faces is at the most 3.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4,5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Then B = {(1, 1), (1, 2), (2, 1)} ∴ n(B) = 3

Now, $P(B) = \frac{n(B)}{n(S)}$

$\therefore P(B) = \frac{3}{36} = \frac{1}{12}$

(iii) Let C be the event that the number on the upper face of the first die is less than the number on the upper face of the second die.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4,5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Then C = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2,4), (2,5), (2, 6), (3, 4), (3, 5),
(3, 6), (4, 5), (4, 6), (5, 6)}
∴ n(C) = 15

Now, $P(C) = \frac{n(C)}{n(S)}$

$\therefore P(C) = \frac{15}{36} = \frac{5}{12}$

Answer 5.

The sample space is

S = {1, 2, 3, 4, 5, 6,.... , 18, 19, 20}

∴ n(S) = 20

(i) Let A be the event that the number on the card is a

prime number.

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Then A = {2, 3, 5, 7, 11, 13, 17, 19}∴ n(A) = 8.

$$\text{Now, } P(A)=\frac{n(A)}{n(S)}$$

$$\therefore P(A)=\frac{8}{20}=\frac{2}{5}$$

(ii) Let B be the event that the number on the card is a

perfect square.

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 , 17, 18, 19, 20}

Then B = {1, 4, 9, 16}

∴ n(B) = 4.

$$\text{Now, } P(B)=\frac{n(B)}{n(S)}$$

$$\therefore P(B)=\frac{4}{20}=\frac{1}{5}$$

(iii) Let C be the event that the number on the card is a multiple of 5.

S = {I, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Then C = {5, 10, 15, 20}

∴ n(C) = 4.

$$\text{Now, } P(C)=\frac{n(C)}{n(S)}$$

$$\therefore P(C)=\frac{4}{20}=\frac{1}{5}$$

Answer 6.

The sample space is

S = {10, 12, 13, 14, 20, 21, 23, 24, 30, 31, 32, 34, 40, 41, 42, 43}∴ n(S) = 16.

(i) Let A be the event that the number formed is an even number.

S = {10, 12, 13, 14, 20,21,23,24,30,31,32,34,40, 41, 42, 43}

Then A = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42}∴ n(A) = 10

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{10}{16} = \frac{5}{8}$$

(ii) Let B be the event that the number formed is greater than 40.

S = {10, 12, 13, 14, 20,21,23,24,30,31,32,34,40, 41, 42, 43} Then B = {41, 42, 43}

∴ n(B) = 3

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{3}{16}$$

(iii) Let C be the event that the number formed is a prime number.

S = {10, 12, 13, 14, 20,21,23,24,30,31,32,34,40, 41, 42, 43}

Then C = {13, 23, 31, 41, 43} ∴ n(C) = 5

$$\text{Now, } P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{5}{16}$$

Answer 7.

Here, there are three boys B_1, B_2, B_3 and two girls G_1, G_2.

A committee of two is to be formed.∴ The sample space is

S = {B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1,

B_3G_2, G_1G_2}

∴ n(S) = 10

(i) Let A be the event that the committee contains at least one girl.

S = {B_1B_2, B_1B_3, B_2B_3, $\underline{B_1G_1}$, $\underline{B_1G_2}$, $\underline{B_2G_1}$, $\underline{B_2G_2}$, $\underline{B_3G_1}$,

$\underline{B_3G_2}$, $\underline{G_1G_2}$}

Then A = {B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2}

∴ n(A) = 7

$$\text{Now, } P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{7}{10}$$

(ii) Let B be the event that the committee contains one boy and one girl.

S = {B_1B_2, B_1B_3, B_2B_3, $\underline{B_1G_1}$, $\underline{B_1G_2}$, $\underline{B_2G_1}$, $\underline{B_2G_2}$, $\underline{B_3G_1}$,

$\underline{B_3G_2}$, G_1G_2}

Then B = {B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2}

∴ n(B) = 6.

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{6}{10} = \frac{3}{5}$$

(iii) Let C be the event that the committee contains only boys.

S = {B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1,

B_3G_2, G_1G_2}

Then C= {B_1B_2, B_1B_3, B_2B_3}∴ n(C) =3.

$$\text{Now, } P(C)=\frac{n(C)}{n(S)}$$

$$\therefore P(C)=\frac{3}{10}$$

Answer 8.

One card can be drawn out of 52 cards in 52 ways.

∴ The sample space S contains 52 sample points. ∴ n(S) = 52.

(i) A is the event of getting a black card.

Number of black cards in a pack of 52 cards = 26

(13 spades and 13 clubs).

One card can be drawn out of 26 black cards in 26

ways.

∴ n(A) = 26

$$\text{Now, } P(A)=\frac{n(A)}{n(S)}$$

$$\therefore P(A)=\frac{26}{52}=\frac{1}{2}$$

(ii) B is the event of not getting a black card.

Out of 52 cards,

Number of black cards = 26 Number of red cards = 26

Not getting a black card means getting a red card.

One red card can be drawn from 26 red cards.

This can be done in 26 ways.

$\therefore$ n(B) = 26 ... [Not getting a black card means getting a red card.]

$$\text{Now, } P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{26}{52} = \frac{1}{2}$$

(iii) C is the event of getting a card bearing number between 2 to 5 including 2 and 5.
There are 13 cards in each of the four suits, Spades, Hearts, Diamonds and Clubs.

There are 4 cards bearing numbers 2, 3, 4, and 5 in each of the four suits.

$\therefore$ There are in all 4 x 4 = 16 cards.

One card can be drawn out of 16 cards in 16 ways.$\therefore$ n(C) = 16.

$$\text{Now, } P(C) = \frac{n(C)}{n(S)}$$

$$\therefore P(C) = \frac{16}{52} = \frac{4}{13}$$