SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)#	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	1(5)*	**3(7)**
4.	Determinants	1(1)	1(2)	–	–	**2(3)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)*	–	**3(9)**
7.	Integrals	2(2)*	1(2)*	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)*	1(3)	–	**3(6)**
10.	Vector Algebra	1(1)	–	–	–	**1(1)**
11.	Three Dimensional Geometry	2(2)# + 1(4)	1(2)	–	1(5)*	**5(13)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)#	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Show that the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : a = b\}$ is both symmetric and transitive.

OR

Give an example of a relation, which is transitive but neither reflexive nor symmetric.

2. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, then find the matrix $A^2 - B$.

3. Evaluate : $\int \frac{x-4}{(x-2)^3} \cdot e^x dx$

OR

Evaluate : $\int_0^1 x e^{x^2} dx$

4. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

5. Find the equation of a line passing through (1, 2, –3) and parallel to the line $\frac{x-2}{1}=\frac{y+1}{3}=\frac{z-1}{4}$.

OR

Find the vector equation of the plane passing through a point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$.

6. Prove that the function $f: R \to R$ defined by $f(x) = 3 - 4x$ is onto.

7. Find the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}+5-2\frac{d^2y}{dx^2}=0$.

OR

Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$.

8. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

9. Prove that if E and F are independent events, then the events E' and F' are also independent.

OR

Given two independent events A and B, such that $P(A) = 0.39$ and $P(B) = 0.6$. Find $P(A' \cap B')$.

10. The cartesian equation of a line is $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$. Find its vector equation.

11. Let R be a relation on N defined by $R = \{(1 + x, 1 + x^2) : x \leq 5, x \in N\}$. Then, verify the following :

(a) R is reflexive

(b) Domain of $R = \{2, 3, 4, 5, 6\}$

12. Evaluate : $\int\left(\frac{1-x}{1+x^2}\right)^2 e^x dx$

13. If A and B are two events such that $P(A) = 0.53$, $P(B) = 0.24$ and $P(A \cap B) = 0.42$, then find $P(B' \cap A)$.

14. If $\vec{u}=\hat{i}+2\hat{j}$, $\vec{v}=-2\hat{i}+\hat{j}$ and $\vec{w}=4\hat{i}+3\hat{j}$. Find scalars x and y respectively such that $\vec{w}=x\vec{u}+y\vec{v}$.

15. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Find the probability that they are of the different colours.

16. Find the additive inverse of $A + B$, where A and B are given as $A=\begin{bmatrix}2 & 5\\ 9 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2\\ 3 & -9\end{bmatrix}$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A cricket match is organised between students of Class XI and Class XII for which a team from each class is chosen. Remaining students of Class XI and XII are respectively sitting on the plane's represented by the equation $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=8$, to cheers the team of their own class.

Based on the above answer the following :

(i) The cartesian equation of the plane on which student of class XI are seated is

(a) $2x - y + z = 8$ (b) $2x + y + z = 8$ (c) $x + y + 2z = 5$ (d) $x + y + z = 5$

(ii) The magnitude of the normal to the plane on which student of Class XII are seated, is

(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

(iii) The intercept form of the equation of the plane on which student of Class XII are seated, is

(a) $\frac{x}{5}+\frac{y}{5}+\frac{z}{5}=1$ (b) $\frac{x}{4}+\frac{y}{(-8)}+\frac{z}{8}=1$ (c) $\frac{x}{4}+\frac{y}{8}+\frac{z}{8}=1$ (d) $\frac{x}{5}+\frac{y}{5}+\frac{z}{5/2}=1$

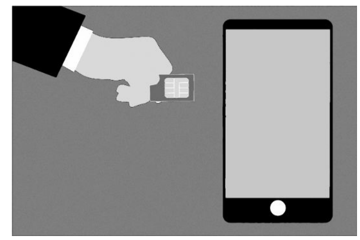
(iv) Which of the following is a student of Class XII?

(a) A sitting at (1, 2, 1) (b) B sitting at (0, 1, 2) (c) C sitting at (4, 1, 1) (d) none of these

(v) The distance of the plane, on which student of Class XII are seated, from the origin is

(a) 8 units (b) $\frac{8}{\sqrt{6}}$ units (c) $\frac{5}{\sqrt{6}}$ units (d) none of these

18. A mobile company in a town has 500 subscribers on its list and collects fix charges of ₹ 300 per year from each subscriber. The company proposes to increase the annual charges and it is believed that for every increase of ₹ 1, one subscriber will discontinue service.



Based on the above information answer the following questions:

(i) If x denote the amount of increase in annual charges of each subscriber, then revenue, R, as a function of x can be represent as

(a) $R(x) = 300 \times 500 \times x$ (b) $R(x) = (300 - 2x)(500 + 2x)$

(c) $R(x) = (500 + x)(300 - x)$ (d) $R(x) = (300 + x)(500 - x)$

(ii) If mobile company increases ₹ 50 as annual charges, then R is equal to

(a) ₹ 157500 (b) ₹ 167500 (c) ₹ 17500 (d) ₹ 15000

(iii) If revenue collected by the mobile company is ₹ 156,400, then value of amount increased as annual charges for each subscriber, is

(a) 40 (b) 160 (c) Both (a) and (b) (d) None of these

(iv) What amount of increase in annual charges will bring maximum revenue?

(a) 100 (b) 200 (c) 300 (d) 400

(v) Maximum revenue is equal to

(a) ₹ 15000 (b) ₹ 160000 (c) ₹ 20500 (d) ₹ 25000

PART - B

Section - III

19. For what value(s) of 'a' the matrix $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ will not be invertible.

20. Evaluate : $\int \frac{x}{1-\sin 2x}\, dx$

OR

Evaluate : $\int \frac{dx}{1+\tan x}$

21. Find the equation of the tangent to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3).

22. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$.

23. Let A and B be two events such that $P(A)=\frac{3}{8}$, $P(B)=\frac{5}{8}$ and $P(A\cup B)=\frac{3}{4}$. Find the value of $P(A|\,B)\cdot P(A'|B)$.

OR

Two events E and F are independent. If $P(E) = 0.3$, $P(E \cup F) = 0.5$, then find $P(E \mid F) - P(F \mid E)$.

24. Find the value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

25. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

26. Find $\frac{dy}{dx}$ for the equation $x^3 + y^3 = \sin(x + y)$.

27. Find the distance between the lines given by $\vec{r}=\hat{i}+\hat{j}+\lambda(\hat{i}-2\hat{j}+3\hat{k})$ and $\vec{r}=(2\hat{i}-3\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$.

28. Solve $\log\left(\frac{dy}{dx}\right)=ax+by$.

OR

Solve the differential equation $(x+y)^2\frac{dy}{dx}=1$.

Section - IV

29. Evaluate : $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx$

30. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is bijective function.

31. Find the local maxima or local minima of $f(x) = x^3 - 6x^2 + 9x + 15$. Also, find the local maximum or local minimum values as the case may be.

OR

Find the values of x for which the function $f(x) = x^x$, $x > 0$ is (a) increasing (b) decreasing.

32. Find a particular solution of the differential equation $\cos\left(\frac{dy}{dx}\right)=a(a\in R);\ y=1$ when $x=0$.

33. If $y = x^{x^x}$, then find $\frac{dy}{dx}$.

34. If α, β are the roots of $ax^2 + bx + c = 0$ and $f(x)$ is continuous at $x = \alpha$, where $f(x)=\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$, for $x \neq \alpha$, then prove that $f(\alpha) = \frac{b^2-4ac}{2}$.

OR

If $f(x)=\frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$, $x \neq 0$ then for f to be continuous everywhere, what should be the value of $f(0)$.

35. Find the area of the region bounded by the parabola $y^2 = 4ax$, its axis and two ordinates $x = a$ and $x = 2a$.

Section - V

36. Solve the following LPP graphically :

Maximize $Z = x + y$

Subject to the constraints,

$2x + 5y \leq 100$

$\frac{x}{25} + \frac{y}{40} \leq 1$

$x \geq 0, y \geq 0$

OR

Find the maximum value of $Z = 5x + 2y$ subject to constraints $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

37. Find the equation of the plane passing through the point $A(1, 2, 1)$ and perpendicular to the line joining the points $P(1, 4, 2)$ and $Q(2, 3, 5)$. Also, find the distance of this plane from the line $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1}$.

OR

Find the coordinates of the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{6}$, which are at a distance of 2 units from the point $(-2, -1, 3)$.

38. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ and $f(x) = x^2 - 5x - 14$, find $f(A)$. Hence obtain A^3.

OR

Solve the following system of equations by matrix method :

$2x + 5y = 1$, $3x + 2y = 7$

1. The set $A = \{x \in Z : 0 \le x \le 12\} = \{0, 1, 2, ..., 12\}$
$R = \{(a, b) : a = b\} = \{(0, 0), (1, 1), (2, 2),..., (12, 12)\}$
(i) Let $(a, b) \in R \Rightarrow a = b \Rightarrow b = a$
$\Rightarrow (b, a) \in R$.
So, R is symmetric.
(ii) Let $(a, b) \in R$ and $(b, c) \in R \Rightarrow a = b = c$
$\Rightarrow a = c \Rightarrow (a, c) \in R$.
So, R is transitive.

OR

Let $A = \{1, 2, 3\}$ and defined a relation R on A as $R = \{(1, 2), (2, 2)\}$.
Then, R is transitive, as $(1, 2), (2, 2) \in R \Rightarrow (1, 2) \in R$
But R is not reflexive, as $1 \in A$ but $(1, 1) \notin R$.
and also R is not symmetric, as $(1, 2) \in R$ but $(2, 1) \notin R$.

2. $A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$\therefore A^2 - B = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1-0 & -4-4 \\ 12+1 & 1-7 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$

3. Let $I = \int \frac{x-4}{(x-2)^3} \cdot e^x dx$

$= \int \left[\frac{x-2}{(x-2)^3} - \frac{2}{(x-2)^3} \right] e^x dx$

$= \int \left[\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right] e^x dx = \frac{e^x}{(x-2)^2} + C$

$\left[\because \int [f(x) + f'(x)] e^x dx = e^x f(x) + C \right]$

OR

Let $I = \int_0^1 x e^{x^2} dx = \int_0^1 e^t \frac{dt}{2}$ [Putting $x^2 = t \Rightarrow 2xdx = dt$]

$= \frac{1}{2}[e^t]_0^1 = \frac{1}{2}(e-1)$

4. We have, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$\Rightarrow 3 - x^2 = 3 - 8 \Rightarrow x^2 = 8$. Hence, $x = \pm 2\sqrt{2}$.

5. Since, the line is parallel to the line
$\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$
$\therefore$ D.R.'s of the required line are <1, 3, 4>
Hence, equation of the line passing through (1, 2, –3) with d.r.'s <1, 3, 4> is $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$

OR

Vector equation of plane passing through a point having position vector $\vec{a}$ and perpendicular to $\vec{n}$ is given by $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$
Here $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$
$\therefore$ Required equation is $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 4 + 3 - 8 = -1$

6. Let $y \in R$ be any real number, such that $f(x) = y$.
$\therefore y = 3 - 4x$
$\Rightarrow 4x = 3 - y \Rightarrow x = \frac{3-y}{4}$
Since, for any real number $y \in R$, there exists $\frac{3-y}{4} \in R$
such that $f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = 3 - 3 + y = y$
Hence, f is onto.

7. We have, $\left(\frac{d^3y}{dx^3}\right)^{2/3} = 2\frac{d^2y}{dx^2} - 5$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left(2\frac{d^2y}{dx^2} - 5\right)^3$ (On cubing both sides)

Clearly, degree is 2.
[$\because$ Power of highest order derivative is 2]

OR

We have, $\frac{dy}{dx} + \frac{1}{\sqrt{x}} y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$
This is a linear differential equation of the form
$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\sqrt{x}}, Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$
$\therefore$ I.F. $= e^{\int P dx} \Rightarrow$ I.F. $= e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$

8. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$.
Now, $P(E) = \frac{10}{15}$, $P(F | E) = \frac{9}{14}$
By multiplication rule of probability, we have
$P(E \cap F) = P(E) \cdot P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

9. Since, E and F are independent events.
$\therefore P(E \cap F) = P(E)\, P(F)$ ...(i)

Now, $P(E' \cap F') = 1 - P(E \cup F)$

$[\because P(E' \cap F') = P((E \cup F)')]$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - P(E) - P(F) + P(E)\,P(F)$ [Using (i)]

$= (1 - P(E))\,(1 - P(F)) = P(E')\,P(F')$

Hence, E' and F' are also independent events.

OR

Since A and B are independent events, therefore A' and B' will also be independent.

So, $P(A' \cap B') = P(A') \cdot P(B') = (1 - P(A))\,(1 - P(B))$

$= (1 - 0.39)\,(1 - 0.6)$

$= 0.61 \times 0.4$

$= 0.244$

10. The given cartesian equation is

$$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$$

The line passes through the point $(-3, 5, -6)$ and is parallel to vector $2\hat{i} + 4\hat{j} + 2\hat{k}$.

Hence, the vector equation of the line is

$\vec{r} = -3\hat{i} + 5\hat{j} - 6\hat{k} + \lambda\,(2\hat{i} + 4\hat{j} + 2\hat{k})$.

11. Clearly, $R = \{(2, 2), (3, 5), (4, 10), (5, 17), (6, 26)\}$

Domain of $R = \{x : (x, y) \in R\} = \{2, 3, 4, 5, 6\}$

and R is not reflexive, as $1 \in N$ but $(1, 1) \notin R$.

12. Let $I = \int\left(\frac{1+x^2-2x}{(1+x^2)^2}\right)e^x dx$

$$= \int\left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right)e^x dx = \frac{1}{1+x^2}e^x + C$$

$$\left[\because \int e^x[f(x) + f'(x)]dx = e^x f(x) + C\right]$$

13. $P(B' \cap A) = P(A - B) = P(A) - P(A \cap B)$

$= 0.53 - 0.42 = 0.11$

14. We have, $\vec{w} = x\vec{u} + y\vec{v}$

$\Rightarrow\ 4\hat{i} + 3\hat{j} = x(\hat{i} + 2\hat{j}) + y(-2\hat{i} + \hat{j})$

$\Rightarrow\ (x - 2y - 4)\hat{i} + (2x + y - 3)\hat{j} = \vec{0}$

$\Rightarrow\ x - 2y - 4 = 0$ and $2x + y - 3 = 0$

$\Rightarrow\ x = 2$ and $y = -1$

15. Total number of possible outcomes $= {}^6C_2 = 15$

Number of favourable outcomes $= {}^2C_1\,{}^4C_1 = 2 \times 4 = 8$

$\therefore$ Required probability $= \dfrac{8}{15}$

16. Let $C = A + B = \begin{bmatrix} 1 & 7 \\ 12 & -6 \end{bmatrix}$

Now, $(-C) = \begin{bmatrix} -1 & -7 \\ -12 & 6 \end{bmatrix}$, which is the additive inverse of $A + B$.

17. (i) (c) : Clearly, the plane for Class XI students is $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$, which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$

$\Rightarrow\ x + y + 2z = 5$, which is the required cartesian equation.

(ii) (b) : Clearly, the plane for Class XII students is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 8$, which is of the form $\vec{r} \cdot \vec{n} = d$

$\therefore$ Normal vector to the plane is $\vec{n} = 2\hat{i} - \hat{j} + \hat{k}$ and its magnitude is $|\vec{n}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

(iii) (b) : The cartesian form is $2x - y + z = 8$, which can be rewritten as

$$\frac{2x}{8} - \frac{y}{8} + \frac{z}{8} = 1$$

$$\Rightarrow\ \frac{x}{4} + \frac{y}{-8} + \frac{z}{8} = 1$$

(iv) (c) : Since, only the point $(4, 1, 1)$ satisfy the equation of plane representing Class XII, therefore C is the student of XII.

(v) (b) : Equation of plane representing Class XII is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 8$, which is not in normal form, as $|\vec{r}| \neq 1$

On dividing both sides by $\sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$, we get

$$\vec{r} \cdot \left(\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}\right) = \frac{8}{\sqrt{6}},$$

which is of the form $\vec{r} \cdot \hat{n} = d$

Thus, the required distance is $\dfrac{8}{\sqrt{6}}$ units.

18. (i) (d) : If x be the amount of increase in annual charges of each subscriber, then number of subscriber reduces to $500 - x$

$\therefore$ Revenue, $R(x) = (300 + x)\,(500 - x)$

$= 150000 + 200x - x^2,\ 0 < x < 500$

(ii) (a) : Clearly, at $x = 50$

$R(50) = 150000 + 200(50) - (50)^2$

$= 150000 + 10000 - 2500 = ₹\ 157500$

(iii) (c) : Since, $150000 + 200x - x^2 = 156400$ (Given)

$\Rightarrow\ x^2 - 200x + 6400 = 0 \Rightarrow x^2 - 160x - 40x + 6400 = 0$

$\Rightarrow\ x(x - 160) - 40(x - 160) = 0 \Rightarrow x = 40, 160$

(iv) (a) : $\dfrac{dR}{dx} = 200 - 2x$ and $\dfrac{d^2R}{dx^2} = -2 < 0$

For maximum revenue, $\dfrac{dR}{dx} = 0$

$\Rightarrow\ x = 100$

$\therefore$ Required amount $= 100$

(v) (b) : Maximum revenue $= R(100)$

$= (300 + 100)\,(500 - 100) = 400 \times 400 = ₹\ 160000$

19. The matrix will not be invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $1(2-5) - a(1-10) + 2(1-4) = 0$

$\Rightarrow$ $-3 + 9a - 6 = 0 \Rightarrow a = 1$

20. Let $I = \int \frac{x}{1-\sin 2x}\,dx = \int \frac{x(1+\sin 2x)}{\cos^2 2x}\,dx$

$$= \int \underset{\text{I}}{x}\left(\underset{\text{II}}{\sec^2 2x + \sec 2x \tan 2x}\right)dx$$

$$= x\left(\frac{\tan 2x}{2} + \frac{\sec 2x}{2}\right)$$

$$-\left(\frac{\log|\sec 2x|}{4} + \frac{\log|\sec 2x + \tan 2x|}{4}\right) + C$$

$$\therefore\ I = \frac{x}{2}(\tan 2x + \sec 2x) - \frac{1}{4}\log|\sec^2 2x + \sec 2x \tan 2x| + C$$

OR

Let $I = \int \frac{dx}{1+\tan x} = \int \frac{\cos x}{\cos x + \sin x}\,dx$

$$= \frac{1}{2}\int \frac{2\cos x}{\cos x + \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{(\cos x + \sin x)}\,dx$$

$$= \left[\frac{1}{2}\int dx + \frac{1}{2}\int\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)dx\right]$$

$$= \frac{x}{2} + \frac{1}{2}\log|\cos x + \sin x| + C$$

21. Here, $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ ...(i)

Differentiating (i) w.r.t. x, we get

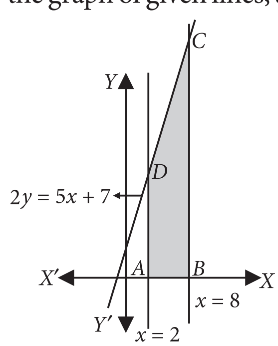
$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\therefore\ \left(\frac{dy}{dx}\right)_{(1,\,3)} = 4 - 18 + 26 - 10 = 2$$

Hence the equation of the tangent to (i) at (1, 3) is

$y - 3 = 2(x - 1) \Rightarrow y = 2x + 1$

22. Let us draw the graph of given lines, as shown below:



$\therefore$ Required area (shown in shaded region)

$$= \int_2^8 y\,dx = \int_2^8 \left(\frac{5x+7}{2}\right)dx$$

$$= \frac{1}{2}\left[\frac{5x^2}{2} + 7x\right]_2^8 = \frac{1}{2}\left[\left\{\frac{5(64)}{2} + 56\right\} - \left\{\frac{5(4)}{2} + 14\right\}\right]$$

$$= \frac{1}{2}[(160+56) - (10+14)] = \frac{1}{2}(216 - 24) = \frac{192}{2}$$

$= 96$ sq. units.

23. Given, $P(A) = \frac{3}{8}, P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$.

Clearly, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = \frac{3+5-6}{8} = \frac{1}{4}$$

Also, we know that $P(A' \cap B) + P(A \cap B) = P(B)$

[As $A' \cap B$ and $A \cap B$ are mutually exclusive events]

$\therefore$ $P(A' \cap B) = P(B) - P(A \cap B) = \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

Now, $P(A\,|\,B) \cdot P(A'\,|\,B) = \frac{P(A \cap B)}{P(B)} \cdot \frac{P(A' \cap B)}{P(B)}$

$$= \frac{1/4}{5/8} \cdot \frac{3/8}{5/8} = \frac{3}{32} \times \frac{64}{25} = \frac{6}{25}$$

OR

Since, E and F are independent events.

$\therefore$ $P(E \cap F) = P(E)\,P(F)$

$\Rightarrow$ $P(E|F) = P(E)$ and $P(F|E) = P(F)$

Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow$ $0.5 = 0.3 + P(F) - 0.3\,P(F)$

$\Rightarrow$ $P(F)(1 - 0.3) = 0.5 - 0.3 \Rightarrow P(F) = \frac{0.2}{0.7} = \frac{2}{7}$

$\therefore$ $P(E\,|\,F) - P(F\,|\,E) = P(E) - P(F)$

$$= 0.3 - \frac{2}{7} = \frac{3}{10} - \frac{2}{7} = \frac{1}{70}$$

24. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{4} - 3\left(\frac{\pi}{3}\right)$

$$= \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$$

$\left[\because\ \text{Principal value of } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \text{ and that of } \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}\right]$

25. Let E = the event of A speaking the truth

and F = the event of B speaking the truth

Then, $P(E) = \frac{60}{100} = \frac{3}{5}$ and $P(F) = \frac{90}{100} = \frac{9}{10}$

Required probability = P (A and B contradicting each other)

$= P(E\bar{F} \text{ or } \bar{E}F) = P(E\bar{F}) + P(\bar{E}F)$

$= P(E)\cdot P(\bar{F}) + P(\bar{E})\cdot P(F)$

[$\because$ E and F are independent events]

$= P(E)\cdot[1 - P(F)] + [1 - P(E)]\cdot P(F)$

$= \frac{3}{5}\left(1-\frac{9}{10}\right)+\left(1-\frac{3}{5}\right)\cdot\frac{9}{10}=\frac{21}{50}=\frac{42}{100}$

Thus, A and B are likely to contradict each other in 42% cases.

26. We have, $x^3 + y^3 = \sin(x + y)$

On differentiating both sides w.r.t. x, we get

$$3x^2+3y^2\frac{dy}{dx}=\cos(x+y)\frac{d}{dx}(x+y)$$
$$=\cos(x+y)\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\ 3x^2+3y^2\cdot\frac{dy}{dx}=\cos(x+y)+\cos(x+y)\frac{dy}{dx}$$

$$\Rightarrow\ 3y^2\frac{dy}{dx}-\cos(x+y)\frac{dy}{dx}=\cos(x+y)-3x^2$$

$$\Rightarrow\ \frac{dy}{dx}\left[3y^2-\cos(x+y)\right]=\cos(x+y)-3x^2$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{\cos(x+y)-3x^2}{3y^2-\cos(x+y)}$$

27. The given lines are parallel.

Here, $\vec{a}_1=\hat{i}+\hat{j}, \vec{a}_2=2\hat{i}-3\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=(2\hat{i}-3\hat{k})-(\hat{i}+\hat{j})\ =\hat{i}-\hat{j}-3\hat{k}$

$$\vec{b}\times(\vec{a}_2-\vec{a}_1)=\hat{i}(6+3)-\hat{j}(-3-3)+\hat{k}(-1+2)$$
$$=9\hat{i}+6\hat{j}+\hat{k}$$

$$|\vec{b}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$$

Shortest distance $=\dfrac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}=\dfrac{|9\hat{i}+6\hat{j}+\hat{k}|}{\sqrt{14}}$

$$=\frac{1}{\sqrt{14}}\sqrt{(9)^2+(6)^2+(1)^2}=\sqrt{\frac{118}{14}}=\sqrt{\frac{59}{7}}\text{ units}$$

28. We have $\log\left(\dfrac{dy}{dx}\right)=ax+by$

$$\Rightarrow\ \frac{dy}{dx}=e^{ax+by}\Rightarrow dy=e^{ax}e^{by}\,dx\Rightarrow e^{-by}\,dy=e^{ax}\,dx$$

$$\Rightarrow\ \frac{e^{-by}}{-b}=\frac{e^{ax}}{a}+C,\qquad\text{[Integrating both sides]}$$

which is the required solution.

OR

We have $(x + y)^2\dfrac{dy}{dx} = 1$ ...(i)

Let $x + y = u \Rightarrow 1 + \dfrac{dy}{dx}=\dfrac{du}{dx}$

(i) becomes, $\dfrac{du}{dx}-1=\dfrac{1}{u^2}\ \Rightarrow\ \dfrac{du}{dx}=\dfrac{1}{u^2}+1=\dfrac{1+u^2}{u^2}$

$$\Rightarrow\ \int\frac{u^2}{u^2+1}du=\int dx+C\ \Rightarrow\ \int\frac{u^2+1-1}{u^2+1}du=x+C$$

$\Rightarrow\ u - \tan^{-1}(u) = x + C$

$\Rightarrow\ (x + y) - \tan^{-1}(x + y) = x + C$

$\Rightarrow\ y - \tan^{-1}(x + y) = C$ is the required solution

29. Let $I=\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

$$\Rightarrow\ I=\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx \qquad\text{...(i)}$$

$$\Rightarrow\ I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\sin x}dx \qquad\text{...(ii)}$$

Adding (i) and (ii), we get

$$2I=\int_0^{\pi}\frac{\pi\sin x}{1+\sin x}dx\ \Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x-\sin^2 x}{\cos^2 x}dx\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\left(\frac{\sin x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\right)dx$$

$$=\frac{\pi}{2}\int_0^{\pi}(\tan x\sec x-\tan^2 x)\,dx$$

$$=\frac{\pi}{2}\int_0^{\pi}[\tan x\sec x-(\sec^2 x-1)]\,dx=\frac{\pi}{2}[\sec x-\tan x+x]_0^{\pi}$$

$$=\frac{\pi}{2}[\sec\pi-\tan\pi+\pi]-\frac{\pi}{2}[\sec 0-\tan 0+0]$$

$$=\frac{\pi}{2}[-1-0+\pi]-\frac{\pi}{2}[1-0+0]=-\frac{\pi}{2}+\frac{\pi^2}{2}-\frac{\pi}{2}$$

$$=\frac{\pi^2}{2}-\pi=\frac{\pi}{2}(\pi-2)$$

30. We have, $f: R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$.

(i) Let $x_1, x_2 \in R_+$ s.t. $f(x_1) = f(x_2)$

$\Rightarrow\ x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow\ x_1 = x_2$ ($\because\ x_1, x_2 \in R_+$)

$\Rightarrow\ f$ is one-one

(ii) $y = f(x)$, where $y \in [4, \infty)$, *i.e.*, $y \ge 4$

$\Rightarrow\ x^2 + 4 = y \Rightarrow x = \sqrt{y-4}$

Now, x is defined if, $y-4\ge 0$ and $\sqrt{y-4}\in R_+$

Thus, for each $y \in [4, \infty)$, we have $x=\sqrt{y-4}\in R_+$

such that $f\left(\sqrt{y-4}\right)=\left(\sqrt{y-4}\right)^2+4=(y-4)+4=y$

$\Rightarrow\ f$ is onto.

$\therefore\ f$ is one-one and onto.

$\Rightarrow\ f$ is bijective function

31. Given that, $f(x) = x^3 - 6x^2 + 9x + 15$

$\Rightarrow f'(x) = 3x^2 - 12x + 9.$

For local maxima or minima, we must have $f'(x) = 0$.

Now, $f'(x) = 0 \Rightarrow 3(x^2 - 4x + 3) = 0$

$\Rightarrow 3(x - 3)(x - 1) = 0$

$\Rightarrow x = 3$ or $x = 1$

Case I : When $x = 3$

In this case, when x is slightly less than 3 then $f'(x)$ is negative and when x is slightly more than 3 then $f'(x)$ is positive.

Thus, $f'(x)$ changes sign from negative to positive as x increases through 3.

So, $x = 3$ is a point of local minima.

$\therefore$ Local minimum value $= f(3) = 15$

Case II : When $x = 1$

In this case, when x is slightly less than 1 then $f'(x)$ is positive and when x is slightly more than 1 then $f'(x)$ is negative.

Thus, $f'(x)$ changes sign from positive to negative as x increases through 1.

So, $x = 1$ is a point of local maxima.

$\therefore$ Local maximum value $= f(1) = 19$.

OR

Given, $f(x) = x^x$

$\Rightarrow f(x) = e^{x \log x}$

$\Rightarrow f'(x) = e^{x\log x} \cdot \dfrac{d}{dx}(x \log x) = x^x(1 + \log_e x)$ ...(i)

(a) $f(x)$ is increasing

$\Rightarrow f'(x) \ge 0 \Rightarrow x^x(1 + \log_e x) \ge 0$ [From (i)]

$\Rightarrow (1 + \log_e x) \ge 0$ [$\because x^x > 0$ when $x > 0$]

$\Rightarrow \log_e x \ge -1 \Rightarrow x \ge e^{-1} \Rightarrow x \in \left[\dfrac{1}{e}, \infty\right)$

$\therefore f(x)$ is increasing on $\left[\dfrac{1}{e}, \infty\right)$.

(b) $f(x)$ is decreasing

$\Rightarrow f'(x) \le 0 \Rightarrow x^x(1 + \log_e x) \le 0$

$\Rightarrow (1 + \log_e x) \le 0$ [$\because x^x > 0$]

$\Rightarrow \log_e x \le -1 \Rightarrow x \le e^{-1} \Rightarrow 0 < x \le \dfrac{1}{e} \Rightarrow x \in \left(0, \dfrac{1}{e}\right]$

$\therefore f(x)$ is decreasing on $\left(0, \dfrac{1}{e}\right]$.

Hence, $f(x)$ is increasing on $\left[\dfrac{1}{e}, \infty\right)$ and decreasing on $\left(0, \dfrac{1}{e}\right]$.

32. We have, $\cos\left(\dfrac{dy}{dx}\right) = a$

$\Rightarrow \dfrac{dy}{dx} = \cos^{-1} a \Rightarrow dy = \cos^{-1} a\, dx$ ...(i)

Integrating (i) both sides, we get

$\int dy = \cos^{-1} a \int dx \Rightarrow y = x\cos^{-1} a + C$

When $x = 0, y = 1 \Rightarrow 1 = C$

Thus, particular solution is $y = x \cos^{-1} a + 1$

$\Rightarrow (y - 1) = x\cos^{-1} a \Rightarrow \cos^{-1} a = \left(\dfrac{y-1}{x}\right)$

$\Rightarrow a = \cos\left(\dfrac{y-1}{x}\right)$

33. Given, $y = x^{x^x}$

Taking log on both sides, we get $\log y = x^x \log x$

Again, taking log on both sides, we get

$\log(\log y) = \log(x^x \log x)$

$\Rightarrow \log(\log y) = \log x^x + \log(\log x)$

$\Rightarrow \log(\log y) = x(\log x) + \log(\log x)$ ...(i)

On differentiating (i) both sides w.r.t. x, we get

$$\frac{1}{\log y}\cdot\frac{1}{y}\cdot\frac{dy}{dx} = \left[x\cdot\frac{1}{x} + \log x \cdot 1\right] + \left[\frac{1}{\log x}\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{1}{y\log y}\cdot\frac{dy}{dx} = 1 + \log x + \frac{1}{x\log x}$$

$$\Rightarrow \frac{dy}{dx} = y\log y\left[1 + \log x + \frac{1}{x\log x}\right]$$

$$= x^{(x^x)} \log x^{(x^x)}\left[1 + \log x + \frac{1}{x\log x}\right]$$

34. Given, α, β are the roots of $ax^2 + bx + c = 0$, therefore $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

Since $f(\alpha)$ is continuous at $x = \alpha$, therefore

$f(\alpha) = \lim\limits_{x\to\alpha} f(x)$

$$\Rightarrow f(\alpha) = \lim_{x\to\alpha}\left(\frac{1-\cos(a(x-\alpha)(x-\beta))}{(x-\alpha)^2}\right)$$

$$= \lim_{x\to\alpha}\left(\frac{1-\cos(a(x-\alpha)(x-\beta))}{a^2(x-\alpha)^2(x-\beta)^2}\cdot a^2(x-\beta)^2\right)$$

$$= \frac{a^2}{2}(\alpha-\beta)^2 \quad \text{...(i)}$$

Now $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = \left(\dfrac{-b}{a}\right)^2 - 4\left(\dfrac{c}{a}\right)$

$= \dfrac{b^2 - 4ac}{a^2}$

From (i), we get $f(\alpha) = \dfrac{b^2 - 4ac}{2}$.

OR

Consider, $\lim_{x\to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0} f(x) = -\lim_{x\to 0} \frac{(256-7x)^{1/8} - 256^{1/8}}{(5x+32)^{1/5} - 32^{1/5}}$$

$$= -\lim_{x\to 0} \frac{\dfrac{(256-7x)^{1/8} - 256^{1/8}}{(256-7x)-(256)} \times (-7x)}{\dfrac{(5x+32)^{1/5} - 32^{1/5}}{(5x+32)-(32)} \times 5x}$$

$$= \frac{\frac{7}{5} \cdot \frac{1}{8} \cdot 256^{\frac{1}{8}-1}}{\frac{1}{5} \cdot 32^{\frac{1}{5}-1}} = \frac{7}{64}$$

35. Equation of parabola is $y^2 = 4ax$

Its axis is $y = 0$ and vertex is $(0, 0)$

$\therefore$ Required area $ABCDA = \int_a^{2a} y\,dx$

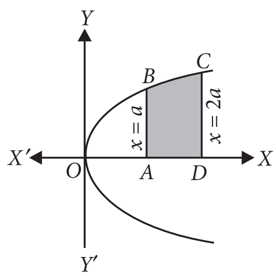
$= 2\sqrt{a} \int_a^{2a} \sqrt{x}\,dx \quad [\because\ y > 0]$

$= 2\sqrt{a} \cdot \frac{2}{3}\left[x^{3/2}\right]_a^{2a}$

$= 2\sqrt{a} \cdot \frac{2}{3}\left[(2a)^{3/2} - (a)^{3/2}\right]$

$= \frac{4}{3}\sqrt{a}\left[a^{3/2}(2^{3/2} - 1)\right]$

$= \frac{4}{3} a^2 [2\sqrt{2} - 1]$ sq. units



36. Given problem is

Maximize $Z = x + y$

Subject to the constraints, $x \ge 0, y \ge 0, 2x + 5y \le 100$,

$\frac{x}{25} + \frac{y}{40} \le 1 \Rightarrow 8x + 5y \le 200$

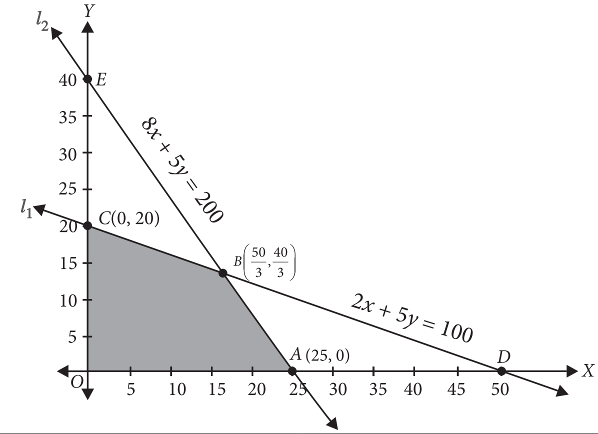
Let us convert the system of the inequations into equations.

$l_1 : 2x + 5y = 100$ and $l_2 : 8x + 5y = 200$

Both the lines intersect at $B\left(\frac{50}{3}, \frac{40}{3}\right)$.

The solution set of the given system is the shaded region $OABC$.

The coordinates of corner points O, A, B, C are $(0, 0)$, $(25, 0)$, $\left(\frac{50}{3}, \frac{40}{3}\right)$ and $(0, 20)$ respectively.



Corner Points	Value of $Z = x + y$
$O(0, 0)$	0
$A(25, 0)$	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Maximum)
$C(0, 20)$	20

So, $Z = x + y$ is maximum when $x = \frac{50}{3}$ and $y = \frac{40}{3}$.

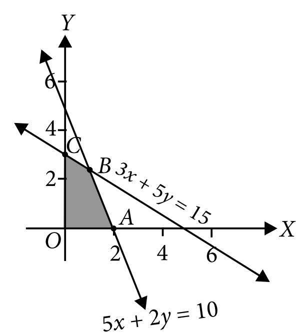
OR

Let us convert the given inequations into equations and draw the corresponding lines.

We have, $3x + 5y = 15$ and $5x + 2y = 10$

i.e., $\frac{x}{5} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{5} = 1$

As $x \ge 0, y \ge 0$, solution lies in first quadrant.



Here, B is the point of intersection of the lines

$3x + 5y = 15$ and $5x + 2y = 10$ *i.e.*, $B = \left(\frac{20}{19}, \frac{45}{19}\right)$

We have points $A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$.

Now, value of $Z = 5x + 2y$ at these points are given below:

$Z(O) = 5(0) + 2(0) = 0$

$Z(A) = 5(2) + 2(0) = 10$

$Z(B) = 5\left(\frac{20}{19}\right) + 2\left(\frac{45}{19}\right) = 10$

$Z(C) = 5(0) + 2(3) = 6$

Thus, Z has maximum value 10 at two points $A(2, 0)$ and $B\left(\frac{20}{19}, \frac{45}{19}\right)$.

37. The line joining the given points $P(1, 4, 2)$ and $Q(2, 3, 5)$ has direction ratios $<1 - 2, 4 - 3, 2 - 5>$ *i.e.*, $<-1, 1, -3>$

The plane through $(1, 2, 1)$ and perpendicular to the line PQ is $-1(x - 1) + 1(y - 2) - 3(z - 1) = 0$

$\Rightarrow \ x - y + 3z - 2 = 0$

Now, direction ratios of line $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1}$ are $2, -1, -1$.

Since $2(1) + (-1)(-1) + (3)(-1) = 2 + 1 - 3 = 0$

$\therefore$ Line is parallel to the plane.

Since, $(-3, 5, 7)$ lies on the given line.

$\therefore$ Distance of the point $(-3, 5, 7)$ from plane is

$$d = \left|\frac{-3-5+3(7)-2}{\sqrt{1+1+9}}\right|$$

$$\Rightarrow \ d = \frac{11}{\sqrt{11}} = \sqrt{11} \text{ units.}$$

OR

$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{6}$ is the given line ...(i)

Let $P(-2, -1, 3)$ lies on the line.

The direction ratios of line (i) are 3, 2, 6

$\therefore$ The direction cosines of line are $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$

Equation (i) may be written as

$$\frac{x+2}{\frac{3}{7}} = \frac{y+1}{\frac{2}{7}} = \frac{z-3}{\frac{6}{7}} \quad \text{..(ii)}$$

Coordinates of any point on the line (ii) may be taken as

$$\left(\frac{3}{7}r-2, \frac{2}{7}r-1, \frac{6}{7}r+3\right)$$

Let $Q \equiv \left(\frac{3}{7}r-2, \frac{2}{7}r-1, \frac{6}{7}r+3\right)$

Given $|r| = 2, \quad \therefore \ r = \pm 2$

Putting the value r, we have

$$Q \equiv \left(\frac{-8}{7}, \frac{-3}{7}, \frac{33}{7}\right)$$

or $\quad Q \equiv \left(\frac{-20}{7}, \frac{-11}{7}, \frac{9}{7}\right)$

38. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$\therefore \ A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot3+(-5)\cdot(-4) & 3\cdot(-5)+(-5)\cdot2 \\ -4\cdot3+2\cdot(-4) & -4\cdot(-5)+2\cdot2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

Given $f(x) = x^2 - 5x - 14$

$\Rightarrow \ f(A) = A^2 - 5A - 14I_2$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25-(-25)-0 \\ -20-(-20)-0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\Rightarrow \ A^2 = 5A + 14I_2$ ...(i)

$\therefore \ A^3 = AA^2 = A(5A + 14I_2)$ (Using (i))

$= A(5A) + A(14I_2) = 5AA + 14(AI_2)$

$= 5A^2 + 14A = 5(5A + 14I_2) + 14A$ (Using (i))

$$= 39A + 70I_2 = 39\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} + 70\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 117 & -195 \\ -156 & 78 \end{bmatrix} + \begin{bmatrix} 70 & 0 \\ 0 & 70 \end{bmatrix} = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

OR

The given equations can be written as

$$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

or $AX = B$, where $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

Now, on premultiplying the above matrix equation by A^{-1}, we get

$(A^{-1}A)X = A^{-1}B$

$\Rightarrow \ X = A^{-1}B$ ...(i)

Now as $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}$, $|A| = -11$ and adj $A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

Now, $X = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ [Using (i)]

$$\Rightarrow \ \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 33 \\ -11 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Hence $x = 3$ and $y = -1$.

☺☺☺