CBSE Class 12 - Mathematics
Sample Paper 04

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If A is square matrix such that $A^2 = I$, then A^{-1} is equal to

   a. O

   b. A + I

   c. I

   d. A

2. Let A be a skew-symmetric matrix of order n then

   a. $|A| = 0$ for all $n \in N$

b. |A| = 0 if n is even

c. None of these

d. |A| = 0 if n is odd

3. $\underset{x\to\infty}{Lt}\left(\frac{2x+3}{2x+1}\right)^x$ is equal to

a. e

b. e^2

c. None of these

d. $e^{1/2}$

4. Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. Find P (A|B)

a. 0.27

b. 0.3

c. 0.2

d. 0.33

5. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that all the five cards are spades?

a. $\frac{5}{1024}$

b. $\frac{3}{1024}$

c. $\frac{7}{1024}$

d. $\frac{1}{1024}$

6. Minimize Z = 5x + 10 y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0

a. Minimum Z = 310 at (60, 0)

b. Minimum Z = 320 at (60, 0)

c. Minimum Z = 330 at (60, 0)

d. Minimum Z = 300 at (60, 0)

7. $\sin^2 25^0 + \sin^2 65^0$ is equal to

a. 1

b. $\frac{1}{2}$

c. 0

d. None of these

8. $\int (\sin(\log x) + \cos(\log x))$ dx is equal to

a. log (sin x - cos x) + c

b. x sin (log x) +C

c. sin (log x) – cos (log x) + C

d. x cos (log x) + C

9. Find the vector and cartesian equations of the planes that passes through the point (1 ,4 ,6) and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$

a. $\left[\vec{r} - \left(\hat{i} + 5\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + 2z + 1 = 0$

b. $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 7\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 5 = 0$

c. $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 1 = 0$

d. $\left[\vec{r} - \left(2\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 3y + z + 1 = 0$

10. If $\vec{a}$ and $\vec{b}$are two collinear vectors, then which of the following are incorrect

a. $\vec{b}$=λ$\vec{a}$ for some scalar λ

b. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

c. $\vec{a} = \pm \vec{b}$

d. the respective components of $\vec{a}\ and\ \vec{b}$ are not proportional

11. Fill in the blanks:

A function f : X $\rightarrow$ Y is said to be a ________ function, if the images of distinct elements of X under f are distinct.

12. Fill in the blanks:

If y = x^3 + tan x, then $\frac{d^2y}{dx^2}$ is ________.

13. Fill in the blanks:

If A and B are symmetric matrices, then AB - BA is a ________ matrix.

14. Fill in the blanks:

The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by ________.

OR

Fill in the blanks:

If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2$ = ________.

15. Fill in the blanks:

If $|\vec{a}|$ = 4 and -3 $\leq \lambda \leq$ 2, then the range of $|\lambda \overline{a}|$ is ________.

OR

Fill in the blanks:

The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is ________.

16. Evaluate $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

17. Evaluate $\int 3^{x+2} dx$

OR

Evaluate $\int_0^3 \frac{dx}{9+x^2}$.

18. Evaluate $\int 3^{x+2} dx$

19. Find the point at which the tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ is equally inclined to the axes.

20. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$

Section B

21. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

OR

Find fog and gof, if f(x) = x + 1, g(x) = 2x + 3.

22. Find the values of x for which the function,

$f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

23. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$ Prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$

24. Find the angle between vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ if $\left|\overrightarrow{a}\right| = \sqrt{3}, \left|\overrightarrow{b}\right| = 2, \overrightarrow{a} . \overrightarrow{b} = \sqrt{6}$

OR

Represent graphically a displacement of 40 km, 30^o East of North.

25. Find the Cartesian equation of the plane $\overrightarrow{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$.

26. If A and B are two events such that P (A) = $\frac{1}{4}$ P (B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ find P(not A and not B).

Section C

27. Let L be the set of all lines in plane and R be the relation in L define if R = {(l_1, L_2): L_1 is $\perp$ to L_2}. Show that R is symmetric but neither reflexive nor transitive.

28. Discuss the continuity of the function f(x) at x = 1/2, when f(x) is defined as follows:

$$f(x) = \begin{cases} 1/2 + x, & 0 \leq x < 1/2 \\ 1, & x = 1/2 \\ 3/2 + x, & 1/2 < x \leq 1 \end{cases}$$

OR

Find the value of a, if the function f(x) defined by f(x) = $\begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$ is continuous at x = 2. Also, discuss the continuity of f(x) at x = 3.

29. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

30. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

31. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

OR

Three cards are drawn at random (without replacement) from a well-shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence, find the mean of the distribution.

32. If a young man rides his motor-cycle at 25 km per hour, he has to spend of Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per h, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to

find the distance to be covered with different speeds. What value is indicated in this question?

Section D

33. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$ and hence find A^4.

OR

Prove: $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

34. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

35. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

36. Find the equation of plane passing through the line of intersection of planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and Parallel to line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

Solution

Section A

1. (d) A

   Explanation:

   If A and B are two square matrices of same order and the product AB= I, the matrix B is called inverse of matrix A.Therefore, if A^2= I, then matrix A is the inverse of itself.

2. (d) |A| = 0 if n is odd

   Explanation:

   Because, the determinant of a skew-symmetric matrix of odd order is always zero and of even order is a non zero perfect square.

3. (b) e^2

   Explanation:

   We know that, $\lim_{x\to\infty}(1+\frac{a}{x})^x = e^a$ ....(1)

   Now, $\lim_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^x = \lim_{x\to\infty}\left(1+\frac{2}{2x+1}\right)^x = e^2$, by eq(1)

4. (b) 0.3

   Explanation:

   Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4 P(A/B) = P(A) = 0.3.

5. (d) $\frac{1}{1024}$

   Explanation:

   Here, probability of getting a spade from a deck of 52 cards = $\frac{13}{52}$ = $\frac{1}{4}$. p = $\frac{1}{4}$, q = $\frac{3}{4}$.
   let , x is the number of spades, then x has the binomial distribution with n = 5 , p = $\frac{1}{4}$,

q = $\frac{3}{4}$.

P(all 5 cards are spades)= P (x = 5) = ${}^5C_5\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$.

6. (d) Minimum Z = 300 at (60, 0)

Explanation:

Objective function is Z = 5x + 10 y ....(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30),(120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300..........................(Min.)
C(40,20)	400

Here , Z = 300 is minimum at (60, 0).

7. (a) 1

Explanation:

$\sin^2 25^0 + \sin^2 65^0$ = $\sin^2(90^o-65^o)+\sin^2 65^o$ = $\cos^2 65^0 + \sin^2 65^0 = 1$

8. (b) x sin (log x) +C

Explanation:

$\int (sin(log\ x) + cos(log\ x))dx$

(Use By Part, Take 1 as II function)

$= \int \sin(log\ x).1dx + \int \cos(\log\ x)dx$

$= (\sin(\log x)).x - \int \cos(\log x)\frac{1}{x}.xdx + \int \cos(log\ x)dx.$

$= x \sin(\log x) + C$

9. (c) $\left[\vec{r} - \left(\hat{i} + 4\hat{j} + 6\hat{k}\right)\right] . \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0; x - 2y + z + 1 = 0$

   Explanation:

   Let $\overrightarrow{a}$

   be the position vector of the point (1, 0, - 2)

   $\therefore \overrightarrow{a} = \hat{i} + 4\hat{j} + 6\hat{k}$, here,

   $\therefore \overrightarrow{n} = \hat{i} - 2\hat{j} + \hat{k}$

   Therefore, the required vector equation of the plane is:

   $\overrightarrow{r} . \overrightarrow{n} = \overrightarrow{a} . \overrightarrow{n}$

   $\Rightarrow \overrightarrow{r}(\hat{i} - 2\hat{j} + \hat{k}) = (\hat{i} + 4\hat{j} + 6\hat{k}).(\hat{i} - 2\hat{j} + \hat{k})$

   $\Rightarrow \overrightarrow{r}(\hat{i} - 2\hat{j} + \hat{k}) = -1$

   On putting $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get:

   $(x\hat{i} + y\hat{j} + z\hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = -1$

   $\Rightarrow x - 2y + z = -1$

10. (b) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

    Explanation:

    If $\vec{a}\ and\ \vec{b}$are two collinear vectors, then, they are parallel to the same line irrespective of their magnitudes and directions.

11. one-one

12. 6x + 2 $\sec^2$x tan x

13. skew symmetric

14. (2, 0, 0)

OR

1

15. [0, 12]

OR

$\frac{2}{3}$

16. Let A = $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

$|A|$ = x (5x + 1) + 7 $\times$ x

= 5x^2 + x + 7x

= 5x^2 + 8x

Hence $|A|$ = 5x^2 + 8x

17. Using $\int kf(x)dx = k\int f(x)dx$, we obtain

$\int 3^{x+2}dx = \int 3^x \cdot 3^2 dx = 9\int 3^x\, dx = 9\left(\frac{3^x}{\log 3}\right) + C$

OR

Let $I = \int_0^3 \frac{dx}{9+x^2} \Rightarrow I = \int_0^3 \frac{dx}{x^2+(3)^2}$

$\Rightarrow I = \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^3 \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$

$\Rightarrow I = \frac{1}{3}\left[\tan^{-1}\left(\frac{3}{3}\right) - \tan^{-1}(0)\right]$

$= \frac{1}{3}\left[\tan^{-1}(1) - 0\right] = \frac{1}{3}\left(\frac{\pi}{4}\right) = \frac{\pi}{12}$

18. Using $\int kf(x)dx = k\int f(x)dx$, we obtain

$\int 3^{x+2}dx = \int 3^x \cdot 3^2 dx = 9\int 3^x\, dx = 9\left(\frac{3^x}{\log 3}\right) + C$

19. we have, $\sqrt{x} + \sqrt{y} = 4$ ....(1)

since the tangent is equally inclined to the axes,

$\frac{dy}{dx} = \tan 45^\circ$ or tan 135^0 i.e 1 or -1. Thus,

$-\frac{\sqrt{y}}{\sqrt{x}} = \pm 1$. This gives y = x

From (1), $\sqrt{x}+\sqrt{y}=4 \Rightarrow x=4$. Also, y = x = 4.

The required point is (4,4).

20. Given $\frac{dy}{dx}=\frac{y}{x} \Rightarrow \frac{dy}{y}=\frac{dx}{x} \Rightarrow \int \frac{dy}{y}=\int \frac{dx}{x}$

$\Rightarrow \log y=\log x+\log c \Rightarrow log\ y=\log\ cx$

$\Rightarrow y=cx$

Section B

21. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$
$\Rightarrow \cos y=\frac{\sqrt{3}}{2}$
$\Rightarrow \cos y=\cos\frac{\pi}{6}$
$\Rightarrow y=\frac{\pi}{6}$
Since, the principal value branch of $\cos^{-1}$ is $[0,\ \pi]$.
Therefore, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

OR

f(x) = x + 1 and g(x) = 2x + 3
Range of f = R $\subseteq$ Domain of g = R $\Rightarrow$ gof exist
Range of g = R $\subseteq$ Domain of R = R $\Rightarrow$ fog exist
Now,
fog(x) = f(g(x)) = f(2x + 3) = (2x + 3) + 1 = 2x + 4
And
gof(x) = g(f(x)) = g(x + 1) = 2 (x + 1) + 3
$\Rightarrow$ gof (x) = 2x + 5

22. we have, f(x)=kx^3-9x^2+9x+3

$$\Rightarrow f'(x)=3kx^2-18x+9$$

Since f(x) is increasing on R, therefore , $f'(x)>0\ \forall\ x\in R$

$\Rightarrow\ 3kx^2 - 18x + 9 > 0, \forall\, x \in\ R$
$\Rightarrow\ kx^2 - 6x + 3 > 0, \forall\, x \in\ R$

$\Rightarrow$ k>0 and 36-12k<0
$[\because\ ax^2 + bx + c > 0, \forall\, x \in\ R \Rightarrow\ a > 0\ and\ discriminant < 0]$

$\Rightarrow\ k > 3$

Hence,f(x) is increasing on R, if k>3.

23. $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$

$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 1 & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ 1 & m & n \\ 0 & 0 & 0 \end{vmatrix}$

$= \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 1 & m & n \\ a & b & c \end{vmatrix} + 0 + 0$

$= \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$

24. $\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

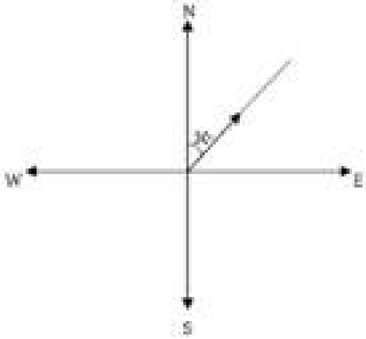
$= \frac{\sqrt{6}}{(\sqrt{3}).(2)} = \frac{\sqrt{2}\times\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{2}.\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}, \theta = \frac{\pi}{4}$

OR

Displacement 40 km, 30^o East of North

$\Rightarrow$ Displacement vector $\overrightarrow{OA}$ (say) such that $\left|\overrightarrow{OA}\right| = 40$ km (given) and vector $\overrightarrow{OA}$ makes an angle 30^o with North in East-North quadrant.

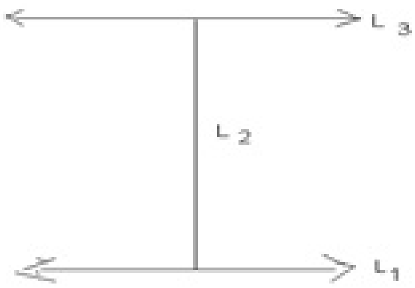


25. Let
$\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$\overrightarrow{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$
$(x\hat{i} + y\hat{i} + z\hat{k}).(\hat{i} + \hat{j} - \hat{k}) = 2$
x + y - z = 2
which is the required equation of the plane.

26. P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$
P (not A) = 1 – P (A) = $1 - \frac{1}{4} = \frac{3}{4}$
P (not B) = 1 – P (B) = $1 - \frac{1}{2} = \frac{1}{2}$
Now, P (A). P (B) $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
$\therefore P(A \cap B)$ = P (A).P (B)
Thus, A and B are independent events.
Therefore, ‘not A’ and ‘not B’ are independent events.
Hence, P (not A and not B) = P (not A). P (not B)
$= \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

Section C

27. R is not reflexive, as a line L_1 cannot be $\perp$ to itself i.e $(L_1, L_1) \notin$ R



Let $(L_1, L_2) \in R$

$\Rightarrow L_1 \perp L_2$

$\Rightarrow L_2 \perp L_1$

$\Rightarrow (L_2,L_1) \in R$

$\Rightarrow$ R is symmetric

Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$,then

$L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be $\perp$ to L_3 in fact $L_1 \mid\mid L_3$

i.e $(L_1,L_2) \in R$, $(L_2,L_3) \in R$.

But $(L_1, L_3) \notin R$

R is not transitive.

28. According to the question, if f(x) = $\begin{cases} 1/2+x, & 0 \le x < 1/2 \\ 1, & x = 1/2 \\ 3/2+x, & 1/2 < x \le 1 \end{cases}$

Then, we have to check continuity of f(x) at $x = \frac{1}{2}$

We shall make use of the continuity of function to check whether f(x) is continuos at $x = \frac{1}{2}$.

Now, LHL = $\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} \left(\frac{1}{2} + x\right)$

$= \lim_{h \to 0} \left(\frac{1}{2} + \frac{1}{2} - h\right) = \frac{1}{2} + \frac{1}{2} = 1$

and RHL = $\lim_{x \to \frac{1}{2}} f(x) = \lim_{x \to \frac{1}{2}^+} \left(\frac{3}{2} + x\right)$

$= \lim_{h \to 0} \left(\frac{3}{2} + \frac{1}{2} + h\right)$

$= \lim_{h \to 0} (2 + h)$ = 2

$\because$LHL $\neq$ RHL at x = 1/2.

$\therefore f(x)$ is discontinuous at x = $\frac{1}{2}$

OR

Giv(en that f(x) = $\begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1, & x > 2 \end{cases}$ is continuous at x = 2

$\therefore$(LHL)$_{x=2}$ = (RHL)$_{x=2}$ = f(2).........(i)

Now, f(2) = a

and LHL = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x - 1)$

$= \lim_{h \to 0}[2(2-h) - 1]$ = 3

From Eq(i), we have

LHL = f(2) $\Rightarrow$ a = 3

Now, let us check the continuity at x = 3.

Consider, $\lim_{x \to 3} f(x) = \lim_{x \to 3}(x+1)$

= 4 = f(3)

$\therefore$ f(x) is continuous at x = 3.

29. According to the question, (x^2 + y^2)$^2 = xy$

On differentiating both sides w.r.t x, we get

$$2(x^2 + y^2)\left[2x + 2y\frac{dy}{dx}\right] = \left[x\frac{dy}{dx} + y\right]$$
$$\Rightarrow 4(x^2 + y^2)\left[x + y\frac{dy}{dx}\right] = \left[y + x\frac{dy}{dx}\right]$$
$$\Rightarrow 4(x^2 + y^2)x + 4(x^2 + y^2)y\frac{dy}{dx} = y + x\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}[4(x^2 + y^2)y - x] = y - 4x(x^2 + y^2)$$
$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x(x^2+y^2)}{4(x^2+y^2)y - x}$$

30. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \text{...(1)}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$I=\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ ...(2)

(1) + (2)

$2I=\int_{\pi/6}^{\pi/3} 1 dx$

$=[x]_{\pi/6}^{\pi/3}$

$=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$

$I=\frac{\pi}{12}$

31. Here, n(S) = 6 $\times$ 6 = 36

Let E = Event of getting a total 10

= {(4, 6) , (5, 5) , (6, 4) }

$\therefore$ n(E) = 3

$\therefore$ P(getting a total of 10) = P(E) = $\frac{n(E)}{n(S)}=\frac{3}{36}=\frac{1}{12}$

and P(not getting a total of 10) = P(E)

$=1-P(E)=1-\frac{1}{12}=\frac{11}{12}$

Thus, P(A getting 10) = P(B getting 10) = $\frac{1}{12}$

and P(A is not getting 10) = P(B is not getting 10)

$=\frac{11}{12}$

Now, P(A winning) =$P(A)+P(\overline{A}\cap\overline{B}\cap A)$

$+P(\overline{A}\cap\overline{B}\cap\overline{A}\cap\overline{B}\cap A)+\ldots$

$=P(A)+P(\overline{A})P(\overline{B})P(A)+P(\overline{A})P(\overline{B})P(\overline{A})$

$P(\overline{B})P(A)+\ldots$

$=\frac{1}{12}+\frac{11}{12}\times\frac{11}{12}\times\frac{1}{12}+\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{1}{12}+\ldots$

$=\frac{1}{12}\left[1+\left(\frac{11}{12}\right)^2+\left(\frac{11}{12}\right)^4+\ldots\right]=\frac{1}{12}\left[\frac{1}{1-\left(\frac{11}{12}\right)^2}\right]$

$\left[\because \text{ the sum of an infinite GP is } S_\infty=\frac{a}{1-r}\right]$

$=\frac{1}{12}\left[\frac{1}{\frac{144-121}{144}}\right]=\frac{12}{23}$

Now, P(B winning)= 1 - P(A winning)

$=1-\frac{12}{23}=\frac{11}{23}$

Hence, the probabilities of winning A and Bare respectively $\frac{12}{23}$ and $\frac{11}{23}$

OR

Let X be a random variable that denotes number of red cards in thee draws.

Here, X can take values 0, 1, 2, 3.

Now, P(X = 0) = P (getting all black cards)

$= \frac{^{26}C_3}{^{52}C_3} = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$

P(X = 1) = P (getting one red card and two black cards)

$= \frac{^{26}C_1 \times ^{26}C_2}{^{52}C_3} = 3 \times \frac{26}{52} \times \frac{26}{51} \times \frac{25}{50} = \frac{13}{34}$

P(X = 2) = P (getting two red cards and one black card)

$= \frac{^{26}C_2 \times ^{26}C_1}{^{52}C_3} = 3 \times \frac{26}{52} \times \frac{25}{51} \times \frac{26}{50} = \frac{13}{34}$

P(X = 3) = P (getting all red cards)

$= \frac{^{26}C_3}{^{52}C_3} = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$

So, the probability distribution of X is as follows

X	0	1	2	3
P(X)	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

Now mean

$E(X) = \sum X_i \cdot P_i = 0 \times \frac{2}{17} + \frac{13}{34} \times 1 + \frac{13}{34} \times 2 + 3 \times \frac{2}{17}$

$= 0 + \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = \frac{51}{34} = 1.5$

32. Let the young man covers x km at the speed of 25 km/h and y km at the speed of 40 km/h. The total distance travelled is $x + y$,

Here, objective function is max $(Z) = x + y$

Cost constraints

According to the question, the cost of 1 km at the speed of 25 km/h = Rs 2

The cost of x km at the speed of 25 km/h = 2x

Also, the cost of 1 km at the speed of 40 km =Rs 5

∴ The cost of y km at the speed of 40 km/h = 5y

So, the total cost of travel (x + y) km= $2x + 5y$

Given, the driver has Rs 100 to spend.

Hence, cost constraints is $2x + 5y \leq 100$

Time constraints

According to the question, Total available time = 1 h

Time to travel a distance of 25 km = 1 h

$\therefore$Time to travel a distance of x km = $\frac{x}{25}$ h

Also, time to travel a distance of 40 km = 1 h

Time to travel a distance of y km = $\frac{y}{40}$ h

$\therefore$The inequation representing time constraint is

$\frac{x}{25} + \frac{y}{40} \leq 1$

$\Rightarrow \quad 8x + 5y \leq 200$

The linear programming problem

is max $(Z) = x + y$

Subject to constraints

2x + 5y$\leq$ 100

8x + 5y$\leq$ 200

x, y $\geq$0

Consider the inequalities as equations,

$2x + 5y = 100$............(i)

$8x + 5y = 200$.....(ii)

$x, y = 0$..............(iii)

Table for line $2x + 5y = 100$ is

x	0	50
y	20	0

So, it passes through the points $(0, 20)$ and $(50, 0)$.

Putting (0,0) in the inequality 2x + 5y$\leq$100, we get

2(0) + 5(0)$\leq$100 $\Rightarrow$ 0$\leq$ 100 [which is true]

The half plane is towards the origin.

Table for line $8x + 5y = 200$ is

x	0	25

y	40	0

So, it passes through the points $(0,40)\ and\ (25,0)$

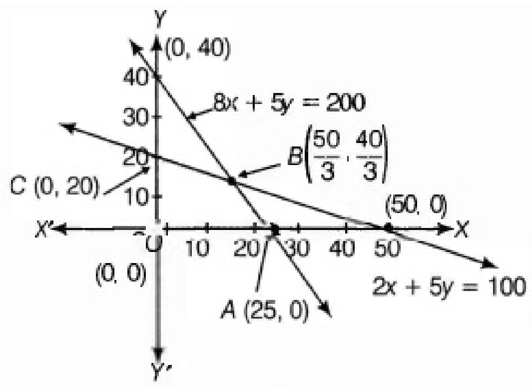
On putting (0,0) in the inequality 8x + 5y$\leq$ 200, we get

8(0 + 5(0)$\leq$ 200 = 0 $\leq$ 200 [which is true]

The half plane is towards the origin

Since, x, y $\geq$ 0, so the feasible region lies in the first quadrant.

The point of intersection of Equations (i) and (ii) is B$\left(\frac{50}{3},\frac{4}{3}\right)$



The corner points of the feasible region $OABC$ are

$O(0,0), A(25,0), B\left(\frac{50}{3},\frac{4}{3}\right)\ and\ C(0,20)$.

Corner Points	Value of Z = x + y
$O(0,0)$	$Z=0+0=0$
$A(25,0)$	$Z=25+0=25$
B$\left(\frac{50}{3},\frac{4}{3}\right)$	Z = $\frac{50}{3}+\frac{40}{3}$= 30(maximum)
$C(0,20)$	$Z=0+20=20$

The maximum value of Z is 30 at point B

Hence, young man cover $\frac{50}{3}$km at the speed of 25 km/h and $\frac{40}{3}$km at the speed of 210 km/h.

He can travel maximum 30 km in 1 hour.

Here, On increasing the speed of motor-cycle, air pollution and expenditure on petrol also increases.

Section D

33. It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\therefore\ A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore$ L.H.S. = A^2 - 5A + 7I

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

= 0 = R.H.S.

$\therefore$ A^2 - 5A + 7I = 0

A^2 = 5A – 7I

A^3 = A^2. A

= 5A^2 – 7AI

= 5A^2 – 7A (Since AI = A)

= 5(5A – 7I) – 7A

= 25A – 35I – 7A

A^3= 18A – 35I

A^4 = A^3.A

= (18A – 35I).A

=18A^2 – 35IA

= 18(5A – 7I) – 35A

= 90A – 126I – 35A

= 55A – 126I

$= 55 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

OR

LHS = $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$

Apply: $C_2 \to C_2 - 2C_1 - 2C_3$

= $\begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2-2b^2-2ca & ca \\ c^2 & c^2-(a-b)^2-2c^2-2ab & ab \end{vmatrix}$

= $\begin{vmatrix} a^2 & -\left(b^2+c^2+a^2\right) & bc \\ b^2 & -\left(b^2+c^2+a^2\right) & ca \\ c^2 & -\left(b^2+c^2+a^2\right) & ab \end{vmatrix}$

Taking $-(a^2 + b^2 + c^2)$ common from C_2,

= $-(b^2 + c^2 + a^2)$ $\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$

= $-(b^2 + c^2 + a^2)$ $\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$ $(R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$

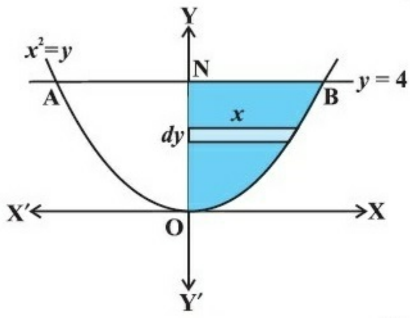
= $-(b^2 + c^2 + a^2)$ (a - b) (c - a) $\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$

= $-(b^2 + c^2 + a^2)$ (a - b) (c - a) [(−(b + a))(−b) − (c)(c + a)]

= (a − b) (b − c) (c − a) (a + b + c) $(a^2 + b^2 + c^2)$

= RHS

34. The required area is shown in fig below by shaded region;



The points of intersection of two curves can be calculated and are (-1,1) and (2,4) as shown in fig.

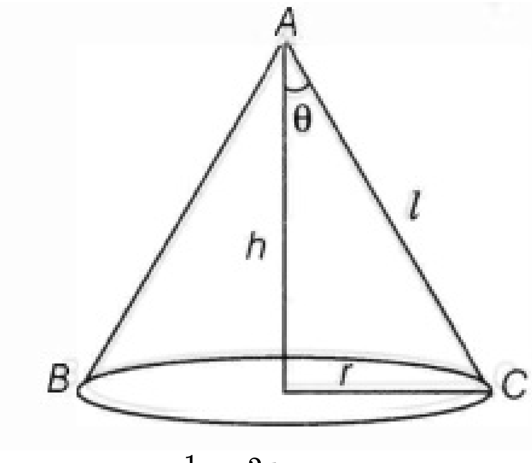
The required area is given as;

$2\int_0^4 xdy = 2$ (area of the region BONB bounded by curve, y - axis and the lines y = 0 and y = 4)

$= 2\int_0^4 \sqrt{y}dy = 2 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{2}$

which is the required area.

35. Let r be the radius of the base, h be the height, V be the volume, S be the surface area of the cone,slant height= AC = l and θ be the semi-vertical angle.



Then, $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad 3V = \pi r^2 h$

$\Rightarrow \quad 9V^2 = \pi^2 r^4 h^2 \quad$ [on squaring both sides]

$\Rightarrow \quad h^2 = \frac{9V^2}{\pi^2 r^4}$ .....(i)

and curved surface area, $S = \pi rl$

$\Rightarrow \quad S = \pi r\sqrt{r^2 + h^2} \quad \left[\because l = \sqrt{h^2 + r^2}\right]$

$\Rightarrow \quad S^2 = \pi^2 r^2 (r^2 + h^2) [\text{ on squaring both sides }]$

$\Rightarrow \quad S^2 = \pi^2 r^2 \left(\frac{9V^2}{\pi^2 r^4} + r^2 \right)$ [from Eq. (i)]

$\Rightarrow \quad S^2 = \frac{9V^2}{r^2} + \pi^2 r^4$ ....(ii)

When S is least, then S^2 is also least.

Now, $\frac{d}{dr}\left(S^2\right) = -\frac{18V^2}{r^3} + 4\pi^2 r^3$....(iii)

For maxima or minima, put $\frac{d}{dr}\left(S^2\right) = 0$

$\Rightarrow \quad -\frac{18V^2}{r^3} + 4\pi^2 r^3 = 0$

$\Rightarrow \quad 18V^2 = 4\pi^2 r^6$

$\Rightarrow \quad 9V^2 = 2\pi^2 r^6$....(iv)

Again, on differentiating Eq. (iii) w.r.t.r, we get

$\frac{d^2}{dr^2}\left(S^2\right) = \frac{54V^2}{r^4} + 12\pi^2 r^2 > 0$

At $r = \left(\frac{9V^2}{2\pi^2}\right)^{1/6}, \frac{d^2}{dr^2}\left(S^2\right) > 0$

So, S^2 or S is minimum, when

$V^2 = 2\pi^2 r^6/9$

On putting $V^2 = 2\pi^2 r^6/9$ in Eq. (i) we get

$2\pi^2 r^6 = \pi^2 r^4 h^2$

$\Rightarrow \quad 2r^2 = h^2$

$\Rightarrow \quad h = \sqrt{2} r$

$\Rightarrow \quad \frac{h}{r} = \sqrt{2}$

$\Rightarrow \quad \cot\theta = \sqrt{2} \quad \left[\text{from the figure, } \cot\theta = \frac{h}{r}\right]$

$\therefore \quad \theta = \cot^{-1}\sqrt{2}$

Hence, the semi-vertical angle of the right circular cone of

given volume and least cured surface area is $\cot^{-1}\sqrt{2}$.

OR

Let the dimensions of the box be x and y. Also, let V denotes its volume and S denotes its total surface area.

Now, S = x^2 + 4xy

Given, x^2 + 4xy = C^2

$\Rightarrow \quad y = \frac{C^2 - x^2}{4x}$.........(i)

Also, volume of the box is given by

V = x^2y

$\Rightarrow \quad V = x^2\left(\frac{C^2-x^2}{4x}\right)$ [From Eq.(i)]

$\Rightarrow \quad V = \frac{xC^2-x^3}{4}$

On differentiating both sides w.r.t. x, we get

$\frac{dV}{dx} = \frac{C^2-3x^2}{4}$

For maxima or minima, put $\frac{dV}{dx}$ = 0

$\Rightarrow \frac{C^2-3x^2}{4} = 0$

$\Rightarrow C^2 = 3x^2$

$\therefore \quad x = C/\sqrt{3}$

Also, $\frac{d^2V}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = \frac{d}{dx}\left(\frac{C^2-3x^2}{4}\right)$

$= \frac{-6x}{4} = \frac{-3x}{2}$

$\therefore \quad \left.\frac{d^2V}{dx^2}\right|_{at x=C/\sqrt{3}} < 0$

This implies V is maximum.

Now, maximum volume at x = $\frac{C}{\sqrt{3}}$ is

$V = \frac{xC^2-x^3}{4}$

$= \frac{1}{4}\left[\frac{C}{\sqrt{3}} \cdot C^2 - \left(\frac{C}{\sqrt{3}}\right)^3\right]\left[\text{put } x = \frac{C}{\sqrt{3}}\right]$

$= \frac{1}{4}\left[\frac{C^3}{\sqrt{3}} - \frac{C^3}{3\sqrt{3}}\right] = \frac{1}{4}\left[\frac{3C^3-C^3}{3\sqrt{3}}\right]$

$= \frac{1}{4} \times \frac{2C^3}{3\sqrt{3}} = \frac{C^3}{6\sqrt{3}}$

Hence, the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

36. Given equations of planes are

2x + y - z - 3 = 0 ....(i)

and 5x - 3y + 4z + 9 = 0 ....(ii)

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) be

$(2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$.......................(iii)

$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + (-3 + 9\lambda) = 0$ ....(iv)

Here, DR's of plane are

$(2+5\lambda, 1-3\lambda, -1+4\lambda)$ Also, given that the plane (iv) is parallel to the line, whose equation is

$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$

DR's of the line are (2, 4, 5).

Since, the plane is parallel to the line.

Hence, normal to the plane is perpendicular to the line,

i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, $a_1=2+5\lambda, b_1=1-3\lambda, c_1=-1+4\lambda$

and $a_2 = 2$, $b_2 = 4$, $c_2 = 5$

$\therefore \quad (2+5\lambda)2+(1-3\lambda)4+(-1+4\lambda)5=0$

$\Rightarrow \quad 4+10\lambda+4-12\lambda-5+20\lambda=0$

$\Rightarrow \quad 18\lambda+3=0 \Rightarrow \lambda=-\frac{3}{18}=-\frac{1}{6}$

On putting $\lambda=-\frac{1}{6}$ in Eq. (iii), we get the required equation of plane as

$(2x+y-z-3)-\frac{1}{6}(5x-3y+4z+9)=0$

12 + 6y - 6z - 1 8 - 5x + 3y - 4z- 9 = 0

7x + 9y - 10z - 27 = 0