CBSE Test Paper 04
Chapter 11 Three Dimensional Geometry

1. Find the distance of the point (0, 0, 0) from the plane 3x – 4y + 12 z = 3.
   a. $\frac{3}{13}$
   b. $\frac{7}{13}$
   c. $\frac{9}{13}$
   d. $\frac{5}{13}$
2. What is the shortest distance between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ where $\lambda, \mu \in R$?
   a. $S.D. = \left|\frac{\vec{b}\times\left(\overrightarrow{a_2}+\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$
   b. $S.D. = \left|\frac{\vec{b}\times\left(-\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$
   c. $S.D. = \left|\frac{-\vec{b}\times\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$
   d. $S.D. = \left|\frac{\vec{b}\times\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$
3. Cartesian equation of a plane that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1$ and $A_2x + B_2y + C_2z + D_2 = 0$ is
   a. $(A_1x + B_1y - C_1z + D_1) + \lambda (A_2x + B_2y + C_2z + D_2) = 0$
   b. $(-A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0$
   c. $(A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0$
   d. $(A_1x - B_1y + C_1z + D_1) + \lambda (A_2x + B_2y + C_2z + D_2) = 0$
4. Find the equations of the planes that passes through three points (1, 1, 0), (1, 2, 1), (–2, 2, –1).
   a. 2x + 3y – 3z = 5

b. 3x + 3y – 3z = 5
c. 2x + 5y – 3z = 5
d. 2x + 3y – 7z = 5

5. Find the shortest distance between the lines
$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

a. $2\sqrt{31}$
b. $2\sqrt{27}$
c. $2\sqrt{23}$
d. $2\sqrt{29}$

6. The vector equation of a line that passes through two points whose positions vectors are $\vec{a}$ and $\vec{b}$ is ________.
7. The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by ________.
8. A plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4). The equation of plane is ________.

9. If a line makes angles 90°, 60° and θ with X, Y and Z-axes respectively, where θ is acute angle, then find θ.

10. The equation of a line is 5x-3= 15y+7= 3-10z. Write the direction cosines of the line.

11. Write the equation of the straight line through the point (α, β, γ) and parallel to Z-axis.

12. Cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$ write the direction ratios of a line parallel to AB.

13. Find the direction cosines of a line which makes equal angles with the co-ordinate axes.

14. Find the angle between the lines
$$\overrightarrow{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$$
$$\overrightarrow{r} = \left(2\hat{i} - \hat{j} - 56\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$$

15. Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equation of a line passing through a point (3,2,-4) and parallel to line l_1.

16. Find the direction cosines of the unit vector $\perp$ to the plane $\overrightarrow{r} \cdot (6i - 3j - 2k) + 1 = 0$ passing through the origin.

17. If the points (1,1,p) and (-3,0,1)be equidistant from the plane $\vec{r}.\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) + 13 = 0$, then find the value of p.

18. Find x such that four points A (3, 2, 1), B (4, x, 5), C (4, 2, -2) and D (6, 5, -1) are coplanar.

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Solution

1. a. $\frac{3}{13}$

   Explanation: As we know that the length of the perpendicular from point P(x_1, y_1, z_1) from the plane $a_1x + b_1y + c_1z + d_1 = 0$ is given by:
   $\frac{|a_1x+b_1y+c_1z+d_1|}{\sqrt{a^2{}_1+b^2{}_1+c^2{}_1}}$.
   Here, $P(0, 0, 0)$ is the point and equation of plane is 3x – 4y + 12 z – 3.
   Therefore, the perpendicular distance is: $\frac{|0-0+0-3|}{\sqrt{9+16+144}} = \frac{|-3|}{\sqrt{169}} = \frac{3}{13}$ units.

2. d. $S.D. = \left|\frac{\vec{b}\times\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$

   Explanation: In vector form the shortest distance between two parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}\ and\ \vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ where $\lambda, \mu \in R$ is given by:
   $S.D. = \left|\frac{\vec{b}\times\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{|\vec{b}|}\right|$

3. c. (A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0.

   Explanation: In Cartesian coordinate system: Cartesian equation of a plane that passes through the intersection of two given plane
   (A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0 is given by :
   (A_1x + B_1y + C_1z + D_1) + λ(A_2x + B_2y + C_2z + D_2) = 0

4. a. 2x + 3y – 3z = 5

   Explanation: In cartesian co-ordinate system: Equation of plane passing through three non collinear point (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by:
   $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$
   Therefore, the equation of the planes that posses through three points (1, 1, 0),

(1, 2, 1), (-2, 2, -1) is given by:

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(-2)-(y-1)(3)+3z=0$$

$$\Rightarrow 2x+3y-3z=5$$

5. d. $2\sqrt{29}$

Explanation: On comparing the given equations with: in the Cartesian form two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

We get;

$x_1=-1, y_1=-1, z_1=-1; a_1=7, b_1=-6, c_1=1$ and

$x_2=3, y_2=5, z_2=7; a_2=1, b_2=-2, c_2=1$

now, the shortest distance the line is given by:

$$S.D.=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$S.D.=\frac{1}{\sqrt{116}}\begin{vmatrix} 3-(-1) & 5-(-1) & 7-(-1) \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$=\frac{1}{\sqrt{116}}\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$=\frac{1}{\sqrt{116}}[-16-36-64]$$

$$=\frac{1}{\sqrt{116}}|-116|=\sqrt{116}=2\sqrt{29}$$

6. $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$
7. (2, 0, 0)
8. $\frac{x}{2}$ + $\frac{y}{3}$ + $\frac{z}{4}$ = 1
9. We need to find the value of angle θ. Now,let I, m and n be the direction cosines of the given line Therefore, we have ,

   $l=\cos 90^\circ=0$

   $m=\cos 60^\circ=\frac{1}{2}$

   $n=\cos\theta$

$\because \quad l^2+m^2+n^2=1$

$\therefore 0+\left(\frac{1}{2}\right)^2+\cos^2\theta=1$

$\Rightarrow \quad \cos^2\theta=1-\frac{1}{4}=\frac{3}{4}$

$\Rightarrow \quad \cos\theta=\frac{\sqrt{3}}{2}$

[$\because$ cosθ cannot be negative as θ is an acute angle]

$\Rightarrow \quad \cos\theta=\cos 30^\circ$

$\theta=30^\circ$

10. We need to find the direction cosines of the line. Here, we are given equation of a line in the following form.Now, we have,

    5x - 3 = 15y + 7 = 3 - 10z .... (i)

    Let us first convert the equation in standard form

    $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ .... (ii)

    Let us divide Eq. (i) by LCM (coefficients of x, y and z). i.e. LCM (5, 15, -10) = 30

    Now, the Eq. (i) becomes

    $\frac{5x-3}{30}=\frac{15y+7}{30}=\frac{3-10z}{30}$

    $\Rightarrow \quad \frac{x-\frac{3}{5}}{6}=\frac{y+\frac{7}{15}}{2}=\frac{z-\frac{3}{10}}{-3}$

    On comparing the above equation with Eq.(ii), we get 6, 2, -3 are the direction ratios of the given line.

    Now, the direction cosines of given line are

    $\frac{6}{\sqrt{6^2+2^2+(-3)^2}} \frac{2}{\sqrt{6^2+2^2+(-3)^2}}$ and $\frac{-3}{\sqrt{6^2+2^2+(-3)^2}}$ i.e,$\left(\frac{6}{7},\frac{2}{7},\frac{-3}{7}\right)$

11. According to the question, the required line is parallel to z-axis.

    The vector equation of a line parallel to Z-axis is $\vec{m}=0\hat{i}+0\hat{i}+\hat{k}$

    Then, the required line passes through the point $A(\alpha,\beta,\gamma)$ whose position vector is $\vec{r}_1=\alpha\hat{i}+\hat{\beta}\hat{j}+\gamma\hat{j}$ and is parallel to the vector $\vec{m}=(0\hat{i}+0\hat{j}+\hat{k})$

    $\therefore$ The equation is $\vec{r}=\vec{r}_1+\lambda\vec{m}$

    $=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(0\hat{i}+0\hat{j}+\hat{k})$

    $=(\hat{\alpha}\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda\hat{k}$

12. Given equation of a line can be written is

    $\frac{x-\frac{1}{2}}{1}=\frac{y-4}{-7}=\frac{z+1}{2}$

The direction ratios of a line parallel to AB are (1, -7, 2)

13. Let a line make equal angles α, α, α with the co-ordinate axes.
$\therefore$ Direction cosines of the line are $\cos\alpha, \cos\alpha, \cos\alpha$...(i)
$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1 \left[\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1\right]$
$\Rightarrow 3\cos^2\alpha = -1$
$\Rightarrow \cos^2\alpha = \frac{1}{3}$
$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$
Putting $\cos\alpha = \pm\frac{1}{\sqrt{3}}$ in eq. (i), direction cosines of the required line making equal angles with the co-ordinate axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

14. Let θ is the angle between the given lines
$\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$
and
$\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$
$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$
$= \left|\frac{\left(\hat{i}-\hat{j}-2\hat{k}\right)\cdot\left(3\hat{i}-5\hat{j}-4\hat{k}\right)}{\left|\hat{i}-\hat{j}-2\hat{k}\right|\left|3\hat{i}-5\hat{j}-4\hat{k}\right|}\right|$
$= \left|\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right| = \frac{16}{\sqrt{6}\sqrt{50}}$
$= \frac{16}{\sqrt{6}\ 5\sqrt{2}}$
$= \frac{16}{\sqrt{2}\times\sqrt{3}\times 5\times\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{16\sqrt{3}}{2\times3\times5}$
$\cos\theta = \frac{8\sqrt{3}}{15}$
$\theta = \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

15. Firstly, we write the given equations of lines in standard form to determine the direction ratios of both lines by using the condition
$a_1a_2 + b_1b_2 + c_1c_2 = 0$ to get the value of p.
Further, we use the formula $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$
Now, equation of the given lines can be written in standard form as:

$l_1 : \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}$
and $l_2 : \frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$
Direction ratios of these lines are $\left(-3, \frac{p}{7}, 2\right)$ and $\left(-\frac{3p}{7} - 1, -5\right)$ respectively.
We know that, two lines of direction ratios (a_1, b_2, c_2) and (a_2, b_2, c_2) are perpendicular to each other, if
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\therefore \quad (-3)\left(-\frac{3p}{7}\right) + \left(\frac{p}{7}\right)(1) + (2)(-9) = 0$
$\Rightarrow \quad \frac{9p}{7} + \frac{p}{7} - 10 = 0$
$\Rightarrow \quad \frac{10p}{7} = 10$
$\Rightarrow \quad p = 7$
Hence, the value of p is 7.
Aso, we know that, the equation of a line which passes through the point (x_1, y_1, z_1) with direction ratios a, b, c is given by
$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
Since, requied line is parallel to line l_1 .
a = -3, $b = \frac{7}{7}$ and c = 2
Therefore, equation of line passing through the point (3,2,-4) and having direction ratios (-3,1,2) is $\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$
$\therefore \quad \frac{3-x}{3} = \frac{y-2}{1} = \frac{z+4}{2}$

16. $\overrightarrow{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) = -1$
$\overrightarrow{r} \cdot \left(-6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 1$ ..... (1)
$\left|-6\hat{i} + 3\hat{j} + 2\hat{k}\right| = \sqrt{36 + 9 + 4} = 7$
Dividing equation (1) by 7,
$\overrightarrow{r} \cdot \left(\frac{-6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right) = \frac{1}{7}$
$\hat{n} = \frac{-6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \, [\because \overrightarrow{r} . \hat{n} = d]$
Hence direction cosines of $\hat{n}$ is $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$.

17. The given plane is $\vec{r}. (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$
$(x\hat{i} + y\hat{j} + z\hat{k}). (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

$3x+4y-12z+13=0$...(1)

This plane is equidistant from the points (1,1,p) and (-3,0,1)

$\frac{|3(1)+4(1)-12p+13|}{\sqrt{3^2+4^2+(-12)^2}}=\frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{3^2+4^2+(-12)^2}}$

$|20-12p|=|-8|$

$20-12p=\pm8$

p = 1 or $\frac{7}{3}$

18. The equation of plane through (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

∴ Equation of a plane passing through A(3, 2, 1), (C(4, 2, -2), D(6, 5, -1) is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}=0$$

Expanding along R_1, we get,

(x - 3)(9) - (y - 2) (-2 + 9) + (z - 1)3 = 0

⇒9(x - 3) -7(y - 2) +3(z - 1) = 0

⇒9x - 27 - 7y + 14 + 3z - 3 = 0

⇒9x - 7y + 3z - 16 = 0 ...(i)

Since the points A,B,C,D are coplanar, thereforeB(4, x, 5) lies on (i).

∴ from (i), we have,

$9\times4-7x+3\times5-16=0$

⇒36 - 7x + 15 - 16 = 0

⇒7x = 35

⇒x = 5