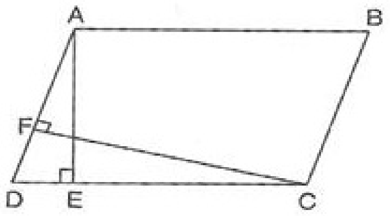
CBSE Test Paper 03

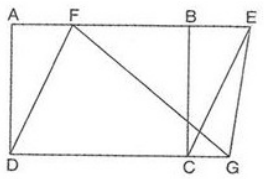
CH-9 Areas of Parallelograms & Triangles

1. ABCD is a parallelogram. If AB = 12 cm, AE 7.5 cm, CF = 15 cm, then AD is equal to



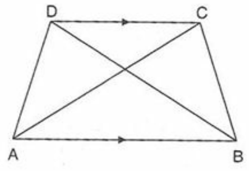
   a. 3 cm.
   b. 6 cm.
   c. 10.5 cm.
   d. 8 cm.

2. ABCD is a rectangle in which AB = 8 units and AD = 3 units. If DCEF is a parallelogram, then the area of $\triangle EFG$ is



   a. 16 sq units.
   b. 24 sq units.
   c. 12 sq units.
   d. 6 sq units.

3. ABCD is a trapezium in which $AB \parallel DC$.If $ar(\triangle ABD) = 24\ cm^2$ and AB = 8 cm, then the height of $\triangle ABC$ is



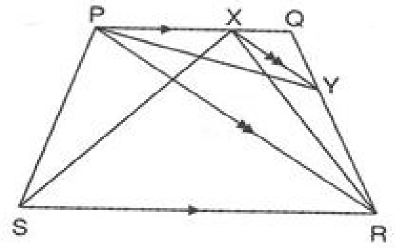
a. 4 cm.

b. 6 cm.

c. 3 cm.

d. 8 cm.

4. PQRS is a trapezium with $PQ \parallel SR.$ A line parallel to PR intersects PQ at X and QR at Y. If $ar(\triangle PYR) = 5\ cm^2$, then $ar\ (\triangle PXS)$ is



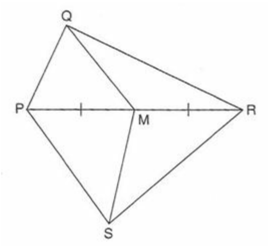
a. $5\ cm^2$.

b. $2.5\ cm^2$.

c. $10\ cm^2$.

d. $7.5\ cm^2$.

5. In quadrilateral PQRS, M is the mid-point of PR. If ar (SMQR) is $18\ cm^2$, then ar (PQMS) is



a. $12\ cm^2$.

b. $36\ cm^2$.

c. $18\ cm^2$.

d. $24\ cm^2$.

6. Fill in the blanks:

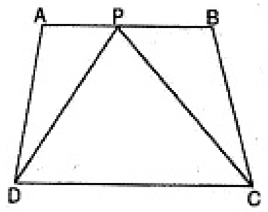
   The median of a triangle divides it into two triangles of ________.

7. Fill in the blanks:

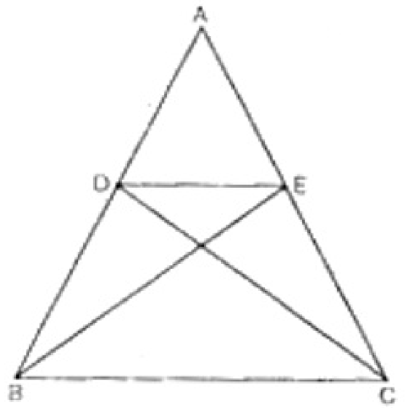
   Two triangles having the same base and equal areas lie between the ________.

8. The diagonal of a square is 10 cm. Find its area.

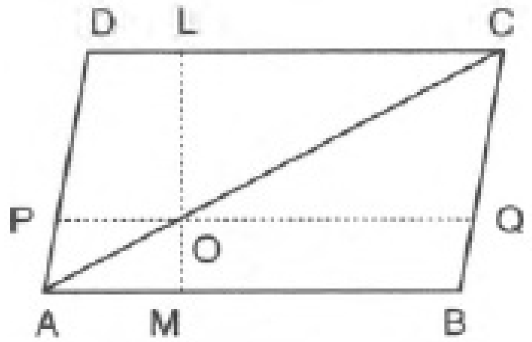
9. Is the given figure lie on the same base and between a same parallels. In such a case, write the common base and the two parallels.



10. D and E are points on sides AB and AC respectively of ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.



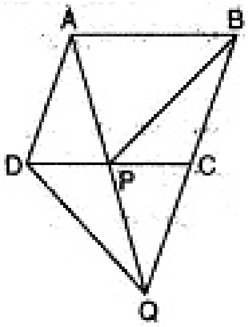
11. In the figure, ABCD is a $||^{gm}$. O is any point on AC. PQ || AB and LM II AD. Prove that ar($||^{gm}$ DLOP) = ar($||^{gm}$ BMOQ).



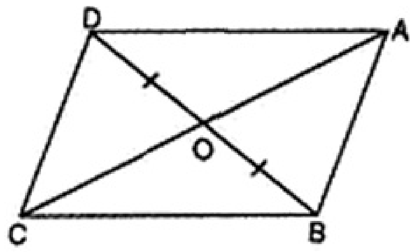
12. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar($\triangle$AOD) = ar($\triangle$BOC).

13. A point O inside a rectangle PQRS is joined to the angular points. Prove that the sum of the areas of two of the triangles so formed is equal to the sum of the other two triangles.

14. In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar(BPC) = ar(DPQ).



15. In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.



If AB = CD, then show that :

i. ar(DOC) = ar(AOB)
ii. ar(DCB) = ar(ACB)
iii. DA || CB or ABCD is a parallelogram.

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Solution

1. (b) 6 cm.

   Explanation:

   Here, Area of parallelogram ABCD with base AB = AB $\times$ AE = 12 $\times$ 7.5 = 90 sq. cm

   Therefore, Area of parallelogram ABCD with base AD = 90 sq. cm

   $\Rightarrow$ AD $\times$ CF = 90

   $\Rightarrow$ AD $\times$ 15 = 90

   $\Rightarrow$ AD = 6 cm
2. (c) 12 sq units.

   Explanation:

   Area of rectangle ABCD = 8 $\times$ 3 = 24 sq. cm

   Since rectangle ABCD and parallelogram DCEF are on the same base CD and between the same parallels, therefore,

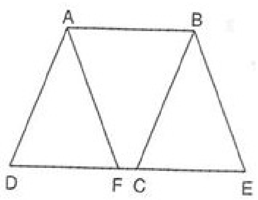
   $\text{area}(\text{ABCD}) = \text{area}(\| gm\text{DCEF}) = 24$ sq. cm

   Also, triangle EFG and parallelogram DCEF are on the same base EF and between the same parallels, therefore,

   $\text{area}(\triangle\text{EFG}) = \frac{1}{2} \times \text{area}(\| gm\text{DCEF}) = \frac{1}{2} \times 24 = 12$ sq. cm
3. (b) 6 cm.

   Explanation:

   In the given figure,

   $\text{area}(\triangle\text{ABD}) = \text{area}(\triangle\text{ABC})$

   $\Rightarrow$ $\text{area}(\triangle\text{ABC}) = 24$ Sq. cm

   $\Rightarrow \frac{1}{2}$ $\times$ AB $\times$ Height of area (ΔABC) = 24

   $\Rightarrow \frac{1}{2} \times 8 \times$ Height of area $(\triangle\text{ABC})$ = 24

$\Rightarrow$ Height of area $(\triangle \mathrm{ABC})$ = 6 cm

4. (a)$5\ cm^2$.

Explanation:

Since triangle PXS and and PXR are on the same base PX and between the same parallels, the
$\text{area}\,(\triangle \mathrm{PXS}) = \text{area}\,(\triangle \mathrm{PXR})$.....(i)
Similarly, $\text{area}\,(\triangle \mathrm{PRY}) = \text{area}\,(\triangle \mathrm{PXR})$.....(ii)
From eq.(i) and (ii), we have
$\text{area}\,(\triangle \mathrm{PXS}) = \text{area}\,(\triangle \mathrm{PRY})$
$\Rightarrow \text{area}\,(\triangle \mathrm{PXS})$ = 5 cm^2

5. (c) $18\ cm^2$.

Explanation:

Since M is mid-point of PR, therefore, QM is the median of triangle PQR.
$\Rightarrow \text{area}\,(\triangle \mathrm{PQM}) = \text{area}\,(\triangle \mathrm{QMR})$ ....(i)
SImilarly, $\text{area}\,(\triangle \mathrm{PSM}) = \text{area}\,(\triangle \mathrm{SMR})$ ....(ii)
Adding eq.(i) and (ii), we have
$\text{area}\,(\triangle \mathrm{PQM}) + \text{area}\,(\triangle \mathrm{PSM}) = \text{area}\,(\triangle \mathrm{QMR}) + \text{area}\,(\triangle \mathrm{SMR})$
$\Rightarrow \text{area}\,(\mathrm{SQRM}) = \text{area}\,(\triangle \mathrm{PQMS})$
$\Rightarrow \text{area}\,(\triangle \mathrm{PQMS}) = 18\ \mathrm{cm}^2$

6. equal area

7. same parallels

8. Diagonal of a square = $\sqrt{2}$ a
$\Rightarrow$ 10 = $\sqrt{2}$ a
$\Rightarrow$ a = $\frac{10}{\sqrt{2}}$

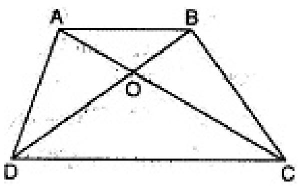
Area of the square = $a^2 = \left(\frac{10}{\sqrt{2}}\right)^2 = \frac{100}{2}$ = 50 cm^2.

9. $\triangle$PDC and quadrilateral ABCD lie on the same base DC and between the same parallels DC and AB.

10. Given: ar ($\triangle$DBC) = ar ($\triangle$EBC)
To prove: DE$\|$BC
Proof: Since two triangles of equal area have common base BC. Therefore DE$\|$BC
[$\because$ Two triangles having same base (or equal bases) and equal areas lie between the same parallel]

11. Since a diagonal of a parallelogram divides it into two triangles of equal area.
$\therefore$ ar ($\triangle$ADC) = ar ($\triangle$ABC)
$\Rightarrow$ ar ($\triangle$APO) + ar ($||^{gm}$ DLOP) + ar ($\triangle$OLC) = ar ($\triangle$AOM) + ar ($||^{gm}$ BMOQ) + ar ($\triangle$OQC) ....(i)
Since, AO and OC are diagonals of parallelograms AMOP and OQCL respectively.
$\therefore$ ar ($\triangle$APO) = ar ($\triangle$AOM) ....(ii)
and, ar ($\triangle$ OLC) = ar ($\triangle$ OQC) ....(iii)
Subtracting (ii) and (iii) from (i), we get
ar ($||^{gm}$ DLOP) = ar ($||^{gm}$ BMOQ)

12. Given: Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

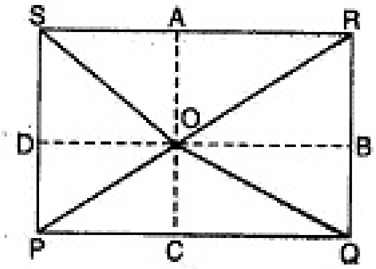


To Prove : ar($\triangle$AOD) = ar($\triangle$BOC)
Proof : $\triangle$ABD and $\triangle$ABC are on the same base AB and between the same parallels AB and DC.
$\therefore$ ar($\triangle$ABD) = ar($\triangle$ABC) . . .[Two triangles on the same base and between the same parallels are equal in area]
$\Rightarrow$ ar($\triangle$ABD) – ar($\triangle$AOB) = ar($\triangle$ABC) – ar($\triangle$AOB) . . .[Subtracting the same areas from both sides]
$\Rightarrow$ ar($\triangle$AOD) = ar($\triangle$BOC)

13. Given: A point O inside a rectangle PQRS is joined to the angular points P, Q, R, S.

To Prove : ar($\triangle$POQ) + ar($\triangle$ROS) = ar($\triangle$POS) + ar($\triangle$QOR)

Construction : Draw OA $\perp$ RS, OB $\perp$ RQ, OC $\perp$ PQ and OD $\perp$ PS.



Proof : ar($\triangle$POQ) = $\frac{(PQ)(OC)}{2}$

ar($\triangle$ROS) = $\frac{(SR)(OA)}{2} = \frac{(PQ)(OA)}{2}$ . . . .[As SP = PQ . . . opp. sides of rectangle]

$\therefore$ ar($\triangle$POQ) + ar($\triangle$ROS) = $\frac{(PQ)(OC)}{2} = \frac{(PQ)(OA)}{2}$

$= \frac{(PQ)(OC+OA)}{2} = \frac{(PQ)(AC)}{2}$ . . .(1)

Similarly,

ar($\triangle$QOR) = $\frac{(RQ)(OB)}{2}$

ar($\triangle$POS) $= \frac{(PS)(OD)}{2} = \frac{(RQ)(OD)}{2}$. . .[As PS = RQ opp. sides of rectangle]

$\therefore$ ar($\triangle$QOR) + ar($\triangle$POS)= $\frac{(RQ)(OB)}{2} + \frac{(RQ)(OD)}{2}$

$= \frac{(RQ)(OB+OD)}{2}$

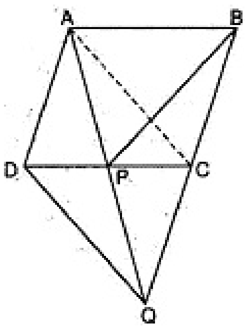
$= \frac{(RQ)(BD)}{2} = \frac{(AC)(PQ)}{2}$ . . .(2)

ar($\triangle$POQ) + ar($\triangle$ROS) = ar($\triangle$QOR) + ar($\triangle$POS) . . . [From (1) and (2)]

14. Given: ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. AQ intersects DC at P.

To Prove : ar($\triangle$BPC) = ar($\triangle$DPQ)

Construction : Join AC.



Proof : $\triangle$QAC and $\triangle$QDC are on the same base QC and between the same parallels AD and QC.

$\therefore$ ar($\triangle$QAC) = ar($\triangle$QDC) . . . [Two triangles on the same base and between the same parallels are equal] . . .(1)

$\Rightarrow$ ar($\triangle$QAC) – ar($\triangle$QPC) = ar($\triangle$QDC) – ar($\triangle$QPC) . . .[Subtracting the same areas from both sides]

$\Rightarrow$ ar($\triangle$PAC) = ar(QDP) . . . (2)

As $\triangle$PAC and $\triangle$PBC are on the same base PC and between the same parallels AB and DC.

$\therefore$ ar($\triangle$PAC) = ar($\triangle$PBC) . . .[Two triangles on the same base and between the same parallels are equal] . . .(3)

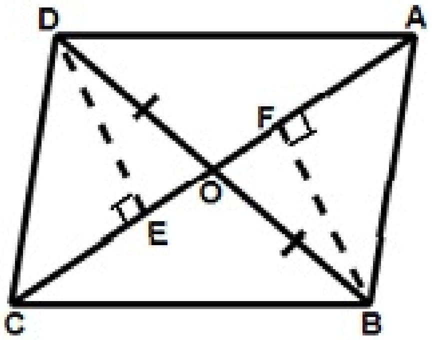
ar($\triangle$PBC) = ar($\triangle$QDP) . . . [From (2) and (3)]

$\Rightarrow$ ar($\triangle$BPC) = ar($\triangle$DPQ).

15. Given: Diagonals AC and BD of a quadrilateral ABCD intersect at O such that OB = OD.

To Prove : IF AB = CD then,

Construction: Draw DE AC and BF AC.



Proof :

$\therefore$ DA | | CB

$\therefore$ ar($\triangle$DOC) = ar($\triangle$AOB)

i. ar($\triangle$DOC) = ar($\triangle$AOB)
ii. ar($\triangle$DCB) = ar($\triangle$ACB)
iii. DA | | CB or ABCD is a parallelogram.
iv. In $\triangle$ADB,

   As AO is a median.

   $\therefore$ ar($\triangle$AOD) = ar($\triangle$AOB) . . .[As median of a triangle divides it into two triangles of equal areas] . . . (1)

   In $\triangle$CBD,

As CO is a median

$\therefore$ ar($\triangle$COD) = ar($\triangle$COB) . . [As median of a triangle divides it into two triangles of equal areas] . . .(2)

ar($\triangle$AOD) + ar($\triangle$COD) = ar($\triangle$AOB) + ar($\triangle$COB) . . .[Adding (1) and (2)]

$\Rightarrow$ ar($\triangle$ACD) = ar(ACB)

$\Rightarrow \frac{(AC)\times(DE)}{2} = \frac{(AC)\times(BF)}{2}$ . . . .[As area of triangle$= \frac{Base \times corresponding\ altitude}{2}$]

DE = BF . . .(3)

In right$\triangle$s DEC and BFA,

DC = BA . . .[Given]

DE = BF . . .[From (3)]

$\therefore$ $\triangle$DEC$\cong$$\triangle$BFA . . .[R.H.S. rule]

$\therefore$ $\angle$DCE = $\angle$BAF . . .[c.p.c.t.]

But these angles form a pair of equal alternate interior angles.

$\therefore$ DC || AB

As CD = AB and DC || AB

$\therefore$ quadrilateral ABCD is a parallelogram . . . .[As quadrilateral is a || gm if a pair of opp. sides is parallel and equal]

v. As ABCD is a parallelogram

   OC = OA . . .[Diagonals of a parallelogram bisect each other] . . . (5)

   ar($\triangle$DOC) = $\frac{OC \times DE}{2}$

   ar($\triangle$AOB) = $\frac{OA \times BF}{2}$

   As DE = BF . . .[From (3)]

   and OC = OA . . [From (5)

vi. From (i)

   ar($\triangle$DOC) = ar($\triangle$AOB)

   $\Rightarrow$ ar($\triangle$DOC) + ar($\triangle$OCB) = ar($\triangle$AOB) + ar(OCB) . . .[Adding same areas on both sides]

   $\Rightarrow$ ar($\triangle$DCB) = ar($\triangle$ACB)