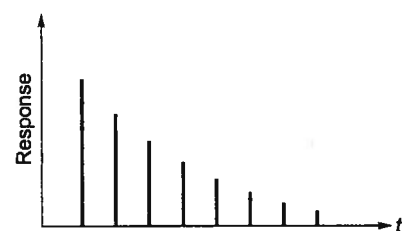




## Multiple Choice Questions

- Q.1 The impulse response of a discrete system with a simple pole shown in the below figure.

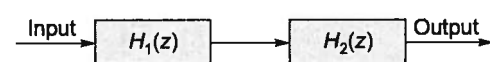


The pole of the system must be located on the

- real axis at  $z = -1$
- real axis between  $z = 0$  and  $z = 1$
- imaginary axis at  $z = j$
- imaginary axis between  $z = 0$  and  $z = j$

[ESE-2000]

- Q.2 Consider the compound system shown in the below figure. Its output is equal to input with a delay of two units. If the transfer function of the first system is given by



$$H_1(z) = \frac{z - 0.5}{z - 0.8}$$

then the transfer function of the second system would be

- $H_2(z) = \frac{z^{-2} - 0.2z^{-3}}{1 - 0.4z^{-1}}$
- $H_2(z) = \frac{z^{-2} - 0.8z^{-3}}{1 - 0.5z^{-1}}$

$$(c) H_2(z) = \frac{z^{-1} - 0.2z^{-3}}{1 - 0.4z^{-1}}$$

$$(d) H_2(z) = \frac{z^{-2} + 0.8z^{-3}}{1 + 0.5z^{-1}}$$

[ESE-2000]

- Q.3 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

- $\alpha^n u(n)$
- $-\alpha^n u(-n-1)$
- $-\alpha^n u(-n-1)$
- $n\alpha^n u(n)$

List-II

- $\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$  ROC:  $|z| > |\alpha|$
- $\frac{1}{(1 - \alpha z^{-1})}$  ROC:  $|z| > |\alpha|$
- $\frac{1}{(1 - \alpha z^{-1})}$  ROC:  $|z| < |\alpha|$
- $\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})}$  ROC:  $|z| < |\alpha|$

Codes:

- |     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 2 | 4 | 3 | 1 |
| (b) | 1 | 3 | 4 | 2 |
| (c) | 1 | 4 | 3 | 2 |
| (d) | 2 | 3 | 4 | 1 |

[ESE-2000]

- Q.4 Assertion (A): The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the Z-plane.

Reason (R): For a causal stable system all the poles should be outside the unit circle in the Z-plane.

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true but R is NOT the correct explanation of A.
- A is true but R is false.
- A is false but R is true.

[ESE-2002]

- Q.5 Assertion (A): The signals  $a^n u(n)$  and  $a^n u(-n-1)$  have the same Z-transform,  $z/(z-a)$ .

Reason (R): The Region of Convergence (ROC) for  $a^n u(n)$  is  $|z| > |a|$ , whereas the ROC for  $a^n u(-n-1)$  is  $|z| < |a|$ .

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true but R is NOT the correct explanation of A.
- A is true but R is false.
- A is false but R is true.

[ESE-2002]

- Q.6 If  $X(z) = \frac{z + z^{-3}}{z + z^{-1}}$ , then  $x(n)$  series has

- alternate 0s
- alternate 1s
- alternate 2s
- alternate -1s

[ESE-2002]

- Q.7 If the impulse response of a discrete-time system is  $h[n] = -5^n u[-n-1]$ , then the system function  $H(z)$  is equal to

- $\frac{-z}{z-5}$  and the system is stable
- $\frac{z}{z-5}$  and the system is stable
- $\frac{-z}{z-5}$  and the system is unstable
- $\frac{z}{z-5}$  and the system is unstable

[GATE-2002]

- Q.8 The z-transform of the time function  $\sum_{k=0}^{\infty} \delta(n-k)$  is

- $\frac{(z-1)}{z}$
- $\frac{z}{(z-1)^2}$
- $\frac{z}{(z-1)}$
- $\frac{(z-1)^2}{z}$

- Q.9 A stable and causal system is described by the difference equation

$$y(n) + \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = -2x(n) + \frac{5}{4}x(n-1)$$

The impulse response of the system is

- $h(n) = \left(\frac{1}{4}\right)^n u(n) - 3\left(-\frac{1}{2}\right)^n u(n)$
- $h(n) = \left(\frac{1}{4}\right)^n u(n) - 3u(n)$
- $h(n) = \left(\frac{1}{4}\right)^{n-1} u(n-1) - 3u(n-1)$
- $h(n) = \left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$

- Q.10 Consider a system with system function

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}}$$

- system is non-causal
- system is causal
- data is not sufficient
- none of these

- Q.11  $x[n] * u[n - n_0]$  is equal to

- $\sum_{k=-\infty}^n x[k]$
- $\sum_{k=-\infty}^{n_0} x[k]$
- $\sum_{k=-\infty}^{n-n_0} x[k]$
- $\sum_{k=n_0}^{\infty} x[k]$

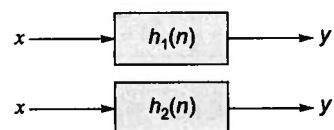
- Q.12 Find  $x[n]$  for the case, when  $X(z) = \frac{z^2 + z}{(z-1)^2}$

- $n^2 u(n)$
- $2n u(n)$
- $(2n+1) u(n)$
- $(n+1)^2 u(n)$

Q.13 If,  $h_1(n) = \left(\frac{1}{2}\right)^n$ ;  $n \geq 0$

$$h_2(n) = \left(\frac{1}{3}\right)^n; n \geq 0$$

represented impulse response of two systems. Find I.R. of the series cascaded network.



- (a)  $3(2)^n u(n) + 2(3)^n u(n)$   
 (b)  $3(2)^n u(n) - 2(3)^n u(n)$   
 (c)  $3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n)$   
 (d)  $3\left(\frac{1}{2}\right)^n u(n) + 2\left(\frac{1}{3}\right)^n u(n)$

Q.14 Consider a discrete time LTI system with impulse response  $h[n]$  is given by

$$h[n] = \left(-\frac{1}{2}\right)^n u[n-1]$$

Statement 1: The system is causal.

Statement 2: The system is also stable

- (a) Both statements are true.  
 (b) Statement 1 true, but statement 2 is false  
 (c) Statement 1 is false, but statement 2 is true  
 (d) Both statements are false

Q.15 A causal LTI system is described by the difference equation,  $2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$ . The system is stable only if,

- (a)  $|\alpha| = 2, |\beta| < 2$   
 (b)  $|\alpha| > 2, |\beta| > 2$   
 (c)  $|\alpha| < 2$ , any value of  $\beta$   
 (d)  $|\beta| < 2$ , any value of  $\alpha$

Q.16 A discrete time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n-1]$$

The system is

- (a) Causal and stable

- (b) Non-Causal and stable  
 (c) Causal and unstable  
 (d) Non-causal and unstable

Q.17 If  $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$ , then the region of convergence (ROC) of its Z-transform in the Z-plane will be

- (a)  $\frac{1}{3} < |z| < 3$  (b)  $\frac{1}{3} < |z| < \frac{1}{2}$   
 (c)  $\frac{1}{2} < |z| < 3$  (d)  $\frac{1}{3} < |z|$

[GATE-2012]

Q.18 The Z-transform corresponding to the Laplace transform function  $G(s) = \frac{10}{s(s+5)}$  is

- (a)  $\frac{2Ze^{-5Z}}{(Z-1)(Z-e^{-T})}$  (b)  $\frac{2(1-e^{-5T})Z}{(Z-1)(Z-e^{-5T})}$   
 (c)  $\frac{e^{-5T}}{(Z-1)^2}$  (d)  $\frac{e^{-T}}{Z(Z-e^{-3T})}$

[ESE-2012]

Q.19 The Z-transform of a signal  $x(n)$  is

$$X(z) = \frac{z}{z-0.4}, \text{ ROC } |z| > 0.4$$

The ROC of ZT of even, part of  $x(n)$  will be

- (a)  $|z| > 0.4$  (b)  $0.4 < |z| < 2.5$   
 (c)  $|z| > 0.2$  (d)  $|z| > 0.8$

Q.20  $x(n) = X(z) = \frac{1}{1-z^{-3}}$ . If  $x(n)$  is a causal signal

then,  $x(2) = ?$ ,  $x(3) = ?$

- (a) 0, 0 (b) 0, 1  
 (c) 1, 0 (d) 1, 1

Q.21 The discrete-time signal

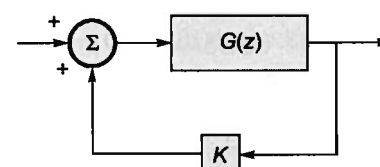
$$x[n] \leftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$$

where  $\leftrightarrow$  denotes a transform-pair relationship, is orthogonal to the signal

- (a)  $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$   
 (b)  $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$   
 (c)  $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$   
 (d)  $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

[GATE-2006]

Q.22 Consider the discrete-time system shown in the figure where the impulse response of  $G(z)$  is  $g(0) = 0$ ,  $g(1) = g(2) = 1$ ,  $g(3) = g(4) = \dots = 0$



This system is stable for range of values of K

- (a)  $[-1, 1/2]$  (b)  $[-1, 1]$   
 (c)  $[-1/2, 1]$  (d)  $[-1/2, 2]$

[GATE-2007]

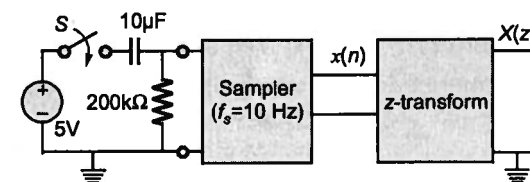
Q.23  $X(z) = 1 - 3z^{-1}$ ,  $Y(z) = 1 + 2z^{-2}$  are Z-transforms of two signals  $x[n]$ ,  $y[n]$  respectively. A linear time invariant system has the impulse response  $h[n]$  defined by these two signals as  $h[n] = x[n-1] * y[n]$  where  $*$  denotes discrete time convolution. Then the output of the system for the input  $\delta[n-1]$

- (a) has Z-transform  $z^{-1} X(z) Y(z)$   
 (b) equals  $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$   
 (c) has Z-transform  $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$   
 (d) does not satisfy any of the above three

[GATE-2007]

Linked Answer Questions (24 and 25):

In the following network, the switch is closed at  $t = 0$  and the sampling starts from  $t = 0$ . The sampling frequency is 10 Hz.



Q.24 The sample  $x(n)$  ( $n = 0, 1, 2, \dots$ ) are given by  
 (a)  $5(1 - e^{-0.05n})$  (b)  $5e^{-0.05n}$   
 (c)  $5(1 - e^{-5n})$  (d)  $5e^{-5n}$

[GATE-2008]

Q.25 The expression and the region of convergence of the z-transform of the sampled signal are

- (a)  $\frac{5z}{z - e^{-5}}, |z| < e^{-5}$   
 (b)  $\frac{5z}{z - e^{-0.05}}, |z| < e^{-0.05}$   
 (c)  $\frac{5z}{z - e^{-0.05}}, |z| > e^{-0.05}$   
 (d)  $\frac{5z}{z - e^{-5}}, |z| > e^{-5}$

[GATE-2008]

Q.26 Let  $x[n] = x[-n]$ . Let  $X(z)$  be the z-transform of  $x[n]$ . If  $0.5 + j0.25$  is a zero of  $X(z)$ , which one of the following must also be a zero of  $X(z)$ .

- (a)  $0.5 - j0.25$  (b)  $\frac{1}{(0.5 + j0.25)}$   
 (c)  $\frac{1}{(0.5 - j0.25)}$  (d)  $2 + j4$

[GATE-2014]

Q.27 The input-output relationship of a causal stable LTI system is given as

$$y[n] = \alpha y[n-1] + \beta x[n]$$

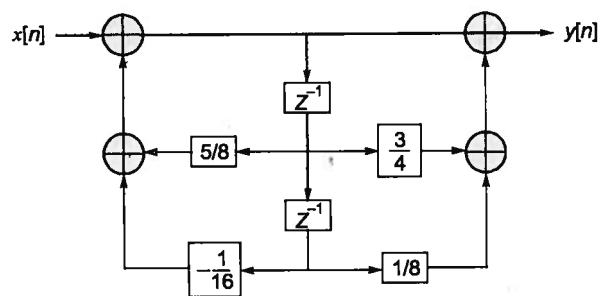
If the impulse response  $h[n]$  of this system satisfies the condition  $\sum_{n=0}^{\infty} h[n] = 2$ , the

relationship between  $\alpha$  and  $\beta$  is

- (a)  $\alpha = 1 - \beta/2$  (b)  $\alpha = 1 + \beta/2$   
 (c)  $\alpha = 2\beta$  (d)  $\alpha = -2\beta$

[GATE-2014]

Q.28 The direct form-II realisation of IIR filter is shown below:



The transfer function is

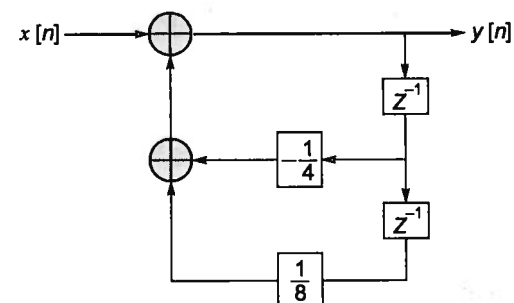
(a)  $H(z) = \frac{1 + \frac{5}{8}z^{-1} - \frac{1}{16}z^{-2}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

(b)  $H(z) = \frac{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}}{1 - \frac{3}{4}z^{-1} - \frac{1}{8}z^{-2}}$

(c)  $H(z) = \frac{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}}$

(d)  $H(z) = \frac{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

Q.29 The discrete time LTI system the output  $y[n]$  and input  $x[n]$  is  $u[n]$ , given below fig.



The impulse response of the system is

(a)  $2\left(-\frac{1}{2}\right)^n u(t) - \left(\frac{1}{4}\right)^n u(t)$

(b)  $\left[\frac{1}{2}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{2}\right)^n\right] u(t)$

(c)  $\frac{1}{4}\left(\frac{1}{2}\right)^n u(t) - \frac{1}{2}\left(\frac{1}{4}\right)^n u(t)$

(d)  $\left(\frac{1}{2}\right)^n u(t) - 2\left(\frac{1}{4}\right)^n u(t)$



### Numerical Data Type Questions

Q.30 A sequence  $x(n)$  with the z-transform  $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$  is applied as an input to a linear, time-invariant system with the impulse response  $h(n) = 2\delta(n-3)$  where

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise} \end{cases}$$

The output at  $n = 4$  is \_\_\_\_\_.

[GATE-2003]

Q.31 The sequence  $x[n]$  corresponding to  $X(z)$  defined as

$$X(z) = \frac{z}{2z^2 - 3z + 1}; |z| < \frac{1}{2}$$

Will have the value at  $n = -2$  is \_\_\_\_\_.

Q.32 Consider a discrete time signal  $x(n)$ , whose z-transform  $X(z)$  is given by,

$$X(z) = \frac{z(z-2)}{(z-1)(z-0.5)}$$

value of  $x(n)$  for  $n = 0$  is \_\_\_\_\_.

Q.33 The z-transform  $X[z]$  of a sequence  $x[n]$  is given

by  $X[z] = \frac{0.5}{1-2z^{-1}}$ . It is given that the ROC of

$X[z]$  includes the unit circle. The value of  $x[0]$  is \_\_\_\_\_.

Q.34 Let  $y[n]$  denote the convolution of  $h[n]$  and  $g[n]$ , where  $h[n] = (1/2)^n u[n]$  and  $g[n]$  is a causal sequence. If  $y[0] = 1$  and  $y[1] = 1/2$ , then  $g[1]$  equals \_\_\_\_\_.

[GATE-2012]

Q.35 The z-transform of a signal is given by

$$C(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2} \quad (a) \text{ in. Its final value is } \_\_\_\_\_\_.$$

[GATE-1999]

Q.36  $x(n) = X(z) = \frac{z^{-2}}{z-3}$ , the value of  $x(5)$  is \_\_\_\_\_.

Q.37  $x(n) = 0.5^n u(n)$

$y(n) = x(n) * x(n)$  then

$$\sum_{n=-\infty}^{\infty} y(n) = \_\_\_\_\_\_.$$

Q.38 The sequence  $x[n] = 0.5^n u[n]$  is the unit step sequence, is convolved with itself to obtain

$$y[n]. \text{ Then } \sum_{n=-\infty}^{\infty} y[n] \text{ is } \_\_\_\_\_\_.$$

[GATE-2014]

Q.39 An input signal  $x(t) = 2 + 5 \sin(100\pi t)$  is sampled with a sampling frequency of 400 Hz and applied to the system whose transfer function is represented by

$$\frac{Y(z)}{X(z)} = \frac{1}{N} \left( \frac{1-z^{-N}}{1-z^{-1}} \right)$$

where,  $N$  represents the number of samples per cycle. The output  $y(n)$  of the system under steady state is \_\_\_\_\_.

[GATE-2014]



### Try Yourself

T1. Let  $x[n]$  be a discrete time signal whose z-transform is represented by  $X(z)$ . The z-transform of the signal

$$y[n] = \left(\frac{1}{2}\right)^{n+1} x[n] \text{ will be}$$

- (a)  $\frac{1}{2}X\left(\frac{z}{2}\right)$  (b)  $\frac{1}{2}X(2z)$   
(c)  $X(2z)$  (d)  $2X(2z)$

[Ans: (b)]

T2. The region of convergence of a signal  $x[n]$  whose z-transform is represented as  $X(z)$ , where

$$x[n] = \begin{cases} 1; & -10 \leq n \leq 10 \\ 0; & \text{otherwise} \end{cases}$$

- (a)  $|z| > \frac{1}{10}$   
(b)  $|z| > 10$   
(c)  $\frac{1}{10} < |z| < 10$   
(d) entire z-plane except  $z = 0$  and  $z = \infty$

[Ans: (d)]

T3. The Region of convergence of z-transform of a signal

$$x[n] = (2)^n u[n] - (4)^n u[-n-1] \text{ is}$$

- (a)  $|z| > 2$  (b)  $|z| < 4$   
(c)  $2 < |z| < 1/4$  (d)  $2 < |z| < 4$

[Ans: (d)]

T4. A causal sequence  $x[n]$  is represented by its z-transform as

$$X(z) = \frac{18}{(1-2z^{-1})^2(1+z^{-1})}$$

then  $x[n]$  can be represented

- (a)  $[4(2)^n + 12n(2)^n + 2(-1)^n] u[n]$   
(b)  $[16(2)^n - 12n(2)^n + 2(-1)^n] u[n]$   
(c)  $[4(2)^n + 12n(2)^n - 2(-1)^n] u[n]$   
(d)  $[16(2)^n + 12n(2)^n + 2(-1)^n] u[n]$

[Ans: (a)]

T5. Consider a discrete LTI system, not necessarily stable but unit sample response of system is zero for  $n < 0$ . Which of the following can be transfer function of system.

(i)  $\frac{(1-z^{-1})^2}{\left(1-\frac{1}{2}z^{-1}\right)}$  (ii)  $\frac{(z-1)^2}{\left(z-\frac{1}{2}\right)}$

(iii)  $\frac{\left(z-\frac{1}{4}\right)}{\left(z-\frac{1}{2}\right)^6}$  (iv)  $\frac{\left(z-\frac{1}{4}\right)^6}{\left(z-\frac{1}{2}\right)^5}$

- (a) (i) and (iii)      (b) (i) and (iv)  
 (c) (i), (ii) and (iii)      (d) (i) and (ii)

[Ans: (a)]

T6. The z-transform  $X(z)$  of a right-sided real sequence  $x[n]$  has exactly two poles and one of them is  $e^{j\pi/2}$  and there are two zeroes at origin. If  $X(1) = 1$ , then which one of the following is true

(a)  $X(z) = \frac{2z^2}{(z-1)^2+2}$  ROC is  $\frac{1}{2} < |z| < 1$

(b)  $X(z) = \frac{2z^2}{z^2+1}$  ROC is  $|z| > \frac{1}{2}$

(c)  $X(z) = \frac{2z^2}{(z-1)^2+2}$  ROC is  $|z| > 1$

(d)  $X(z) = \frac{2z^2}{(z^2+1)}$  ROC is  $|z| > 1$

[Ans: (d)]

T7. The z-transform  $F(z)$  of the function  $f[nT] = a^{nT} u[n]$  is

(a)  $\frac{z}{z-a^T}$

(b)  $\frac{z}{z+a^T}$

(c)  $\frac{z}{z-a^{-T}}$

(d)  $\frac{z}{z+a^{-T}}$

[Ans: (a)]

T8. Consider the difference equation

$$y[n] - \frac{1}{3}y[n-1] = x[n] \text{ and suppose that}$$

$x[n] = \left(\frac{1}{2}\right)^n u[n]$ . Assuming the condition of initial rest, the solution for  $y[n]$ ,  $n \geq 0$  is

(a)  $\left(3\left(\frac{1}{3}\right)^n - 2\left(\frac{1}{2}\right)^n\right)u[n]$

(b)  $\left(-2\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{2}\right)^n\right)u[n]$

(c)  $\left(\frac{2}{3}\left(\frac{1}{3}\right)^n + \frac{1}{3}\left(\frac{1}{2}\right)^n\right)u[n]$

(d)  $\left(\frac{1}{3}\left(\frac{1}{3}\right)^n + \frac{2}{3}\left(\frac{1}{2}\right)^n\right)u[n]$

[Ans: (b)]

