CBSE Test Paper 04

CH-06 Linear Inequalities

1. The marks scored by Rohit in two tests were 65 and 70. Find the minimum marks he should score in the third test to have an average of atleast 65 marks.
   a. 60
   b. 65
   c. 75
   d. 70
2. Solve the system of inequalities $4\text{x} + 3 \geq 2\text{x} + 17 , 3\text{x} - 5 < - 2$
   a. no solution
   b. $(-\frac{3}{2}, \frac{2}{5})$
   c. $(-4, 12)$
   d. (-2 , 2)
3. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.
   a. (11 , 13) , (13 , 15) , (15 , 17) , (17 , 19)
   b. (9 , 11) , (13 , 15) , (15 , 17) , (17 , 19)
   c. (11 , 13) , (13 , 15) , (15 , 17) , (17 , 21)
   d. (11 , 13) , (13 , 15) , (17 , 19) , (19 , 21)
4. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then.
   a. length $\leq$ 20
   b. breadth $>$ 20
   c. length $<$ 20
   d. breadth $\geq$ 20
5. Solve the system of inequalities : $\text{x} - 5 > 0, \frac{2x-4}{x+2} < 2$
   a. $\text{x} > 5$
   b. none of these
   c. $\text{x} > 2$
   d. $\text{x} < - 2$
6. Fill in the blanks:

The region represented by x > 0 and y < 0 lies in ________ quadrant.

7. Fill in the blanks:

   The graph of a linear inequality involving sign < or > is always an ________ half plane.

8. Solve: x + 5 > 4x -10

9. Solve: 2 (3 - x) $\geq \frac{x}{5}$ + 4

10. Solve the inequalities: $-12 < 4 - \frac{3x}{-5} \leqslant 2$

11. Find all pairs of consecutive odd positive integers, both of which are smaller than 18, such that their sum is more than 20.

12. The cost and revenue functions of a product are given by C(x) = 20x + 4000 and R(x) = 60x + 2000 respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

13. Solve the following system of inequalities graphically: 2x - y > 1, x - 2y < -1

14. Solve the following system of inequalities graphically: $2x + y \geqslant 8, x + 2y \geqslant 10$

15. Solve the system of inequalities graphically.
    x + y $\leq$ 5
    4x + y $\geq$ 4
    x + 5y $\geq$ 5
    x $\leq$ 4
    y $\leq$ 3

Solution

1. (a) 60

   Explanation: Let x be the mark obtained by Rohit in the third test.

   Then
   $\frac{65+70+x}{3} \geq 65$
   $\Rightarrow 135 + x \geq 195 \Rightarrow x \geq 60$

   Hence Rohit should get a minimum of 60 marks to get an average of atleast 65 marks.

2. (a) no solution

   Explanation:

   $4\mathrm{x} + 3 \geq 2\mathrm{x} + 17$

   $\Rightarrow 4\mathrm{x} + 3 - 3 \geq 2\mathrm{x} + 17 - 3$

   $\Rightarrow 4\mathrm{x} \geq 2\mathrm{x} + 14$
   $\Rightarrow 4\mathrm{x} - 2\mathrm{x} \geq 2\mathrm{x} + 14 - 2\mathrm{x}$
   $\Rightarrow 2x \geq 14$
   $\Rightarrow x \geq 7$
   $\Rightarrow x \in [7, \infty)$

   $3\mathrm{x} - 5 < -2$
   $\Rightarrow 3x < -2 + 5$
   $\Rightarrow 3x < 3$
   $\Rightarrow x < 1$
   $\Rightarrow x \epsilon (-\infty, 1)$

   Hence solution set is $[7, \infty) \bigcap (-\infty, 1) = \phi$

   which implies no solution exists.

3. (a) (11 , 13) , (13 , 15) , (15 , 17) , (17 , 19)

Explanation: Let the consecutive odd natural numbers be x and x+2.
According to the question , x>10 and

Now $x+(x+2)<40 \Rightarrow 2x<38 \Rightarrow x<19$

Hence the maximum value of x is 17 and minimum value is 11.

So the possible pairs of odd natural numbers are (11 , 13) , (13 , 15) , (15 , 17) , (17 , 19)

4. (d) breadth $\geq$ 20

   Explanation:

   For the rectangle let length=L and breadth=B , then we have the perimeter= $2(L+B)$

   According to the question L=3B and perimeter ≥ 160

   Now perimeter ≥ 160

   $\Rightarrow 2(L+B) \geq 160$

   $\Rightarrow 2(3B+B) \geq 160$ [Using L=3B]

   $\Rightarrow 2(4B) \geq 160$

   $\Rightarrow 8B \geq 160$

   $\Rightarrow B \geq 20$

5. (a) x $>$ 5

   Explanation:

   x $-$ 5 $>$ 0

   $\Rightarrow x>5$

   $\Rightarrow x \epsilon(5, \infty)$

   Now $\frac{2x-4}{x+2}<2$

$\frac{2x-4}{x+2}-2<0$
$\Rightarrow \frac{2x-4-2(x+2)}{x+2}<0$
$\Rightarrow \frac{2x-4-2x-4}{x+2}<0$
$\Rightarrow \frac{-8}{x+2}<0$
$\Rightarrow x+2>0 \left[\because \frac{a}{b}<0, a<0 \quad \Rightarrow \quad b>0\right]$
$\Rightarrow x>-2$
$\Rightarrow x \epsilon(-2, \infty)$
Hence the solution set is $(5, \infty) \cap(-2, \infty)=(5, \infty)$

which means x $>\ 5$

6. 4th

7. open

8. $\Rightarrow$ x - 4x > -10 - 5
$\Rightarrow$ -3x > -15
$\Rightarrow$ 3x < 15
$\Rightarrow$ x < $\frac{15}{3}$
$\Rightarrow$ x < 5
$\therefore$ (-∞, 5) is the solution set.

9. $\Rightarrow 6-2x \geq \frac{x}{5}+4$
$\Rightarrow$ -2x - $\frac{x}{5}$ $\geq$ 4 - 6
$\Rightarrow \frac{-11x}{5} \geq -2$
$\Rightarrow \frac{11x}{5} \leq 2$
$\Rightarrow$ x $\leq \frac{10}{11}$
Therefore, $\left(-\infty, \frac{10}{11}\right]$ is the solution set.

10. We have $-12<4-\frac{3x}{-5} \leqslant 2$
$\Rightarrow -16<\frac{-3x}{-5} \leqslant -2$
$\Rightarrow -16<\frac{3x}{5} \leqslant -2 \Rightarrow -80<3x \leqslant -10$
$\Rightarrow \frac{-80}{3}<x \leqslant \frac{-10}{3}$

11. Let x be the smaller of the two consecutive odd positive integers. Then, the other odd

integers is x + 2.

It is given that both the integers are smaller than 18 and their sum is more than 20.

Therefore,

x + 2 < 18 and, x + (x + 2) > 20

$\Rightarrow$ x < 16 and 2x + 2 > 20

$\Rightarrow$ x < 16 and x > 9 $\Rightarrow$ 9 < x < 16 $\Rightarrow$ x = 11, 13, 15 [$\because$ x is an odd integers]

Hence, the requiered pairs odd odd integers are (11, 13), (13, 15) and (15, 17).

12. We know that, Profit = Revenue - Cost

= (60x + 2000) - (20x + 4000)

= 40x - 2000

To earn some profit, 40x - 2000 > 0

$\Rightarrow$ 40x - 2000 + 2000 > 2000 [adding 2000 on both sides]

$\Rightarrow$ 40x > 2000

$\Rightarrow \quad \frac{40x}{40} > \frac{2000}{40}$ [dividing both sides by 40]

$\therefore$ x > 50

Hence, the manufacturer must sell more than 50 items to realise some profit.

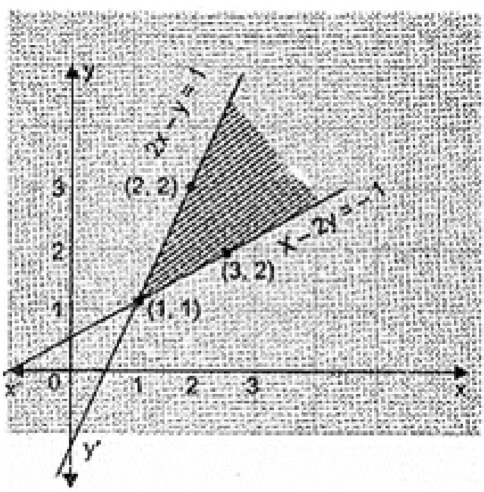
13. The given inequality is 2x - y > 1

Draw the graph of the line 2x - y = 1

Table of values satisfying the equation

2x - y = 1

X	1	2
Y	1	3



Putting (0, 0) in the given inequation, we have

$2 \times 0 - 0 > 1 \Rightarrow 0 > 1$, which is false.

$\therefore$ Half plane of 2x - y > 1 is away from origin.

Also the given inequality is x - 2y < -1

Draw the graph of the line x - 2y = -1

Table of values satisfying the equation x - 2y = -1

X	1	2
Y	1	2

Putting (0, 0) in the given inequation, we have

$0 - 2 \times 0 < -1 \Rightarrow 0 < -1$ which is false

$\therefore$ Half plane of x - 2y < - 1 is away from origin.

14. The given inequality is $2x + y \geqslant 8$.

Draw the graph of the line 2x + y = 8.

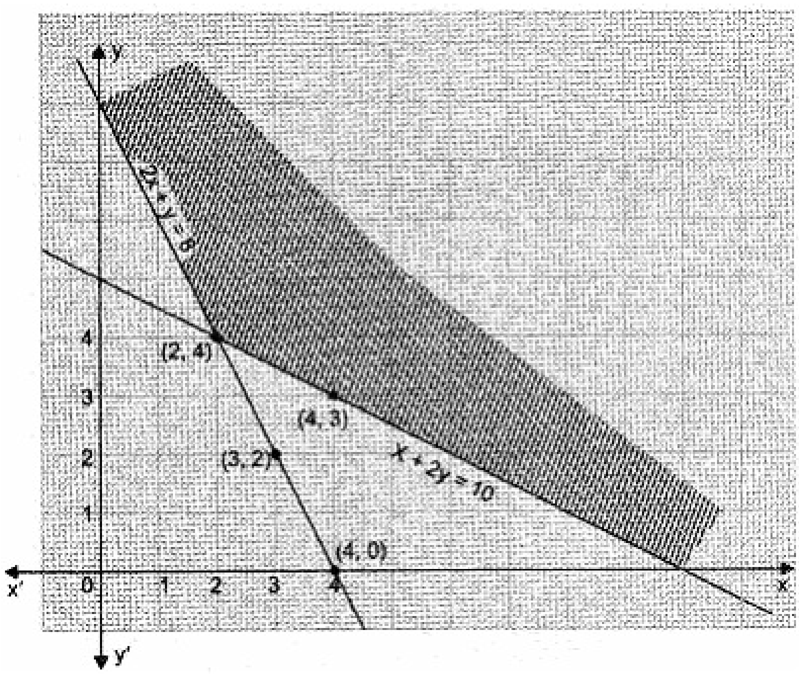


Table of values satisfying the equation 2x + y = 8

X	3	4
Y	2	0

Putting (0, 0) in the given inequation, we have

$2 \times 0 + 0 \geqslant 8 \Rightarrow 0 \geqslant 8$, which is false

$\therefore$ Half plane of $2x + y \geqslant 8$ is away from origin.

Also the given inequality is $x + 2y \geqslant 10$

Draw the graph of the line x + 2 y = 10

Table of the values satisfying the equation x + 2y = 10

X	2	4
Y	4	3

Putting (0, 0) in the given inequation, we have

$0 + 2 \times 0 \geqslant 10 \Rightarrow 0 \leqslant 10$, which is false.

$\therefore$ Half plane of $x + 2y \geqslant 10$ is away from origin.

15. We have,

$x + y \leq 5$ ... (i)

$4x + y \geq 4$ ... (ii)

$x + 5y \geq 5$ ... (iii)

$x \leq 4$ ... (iv)

$y \leq 3$ ... (v)

Take inequality (i)

$x + y \leq 5$

Convert it into linear equation i.e.,

x + y = 5

x	0	5
y	5	0

Thus, the line x + y = 5 passes through points (0,5) and (5.0)

Now, putting x = 0 and y = 0 in inequality (i), we get $0 \leq 5$, which is true.

$\therefore$ For inequality $x + y \leq 5$ shade the region which contains the origin.

Take inequality (ii)

$4x + y \geq 4$

Convert it into linear equation i.e.,

4x + y = 4

x	0	1
y	4	0

Thus, the line $4x + y = 4$ passes through points (0, 4) and (1, 0)

Now, on putting $x = 0$ and $y = 0$ in inequality (ii), we get

$4(0) + 0 \geq 4$

$\Rightarrow 0 \geq 4$, which is false.

$\therefore$ For inequality $4x + y \geq 4$, shade the region which does not contain origin.

Take inequality (iii)

$x + 5y \geq 5$

Convert it into linear equation i.e.,

$x + 5y = 5$

x	0	5
y	1	0

Thus, the line $x + 5y = 5$ passes through points (0, 1) and (5,0).

Now, on putting $x = 0$ and $y = 0$ in inequality (iii), we get

$0 \geq 5$, which is false.

$\therefore$ For inequality $x + 5y \geq 5$, shade the region which does not contain the origin.

Take inequality (iv)

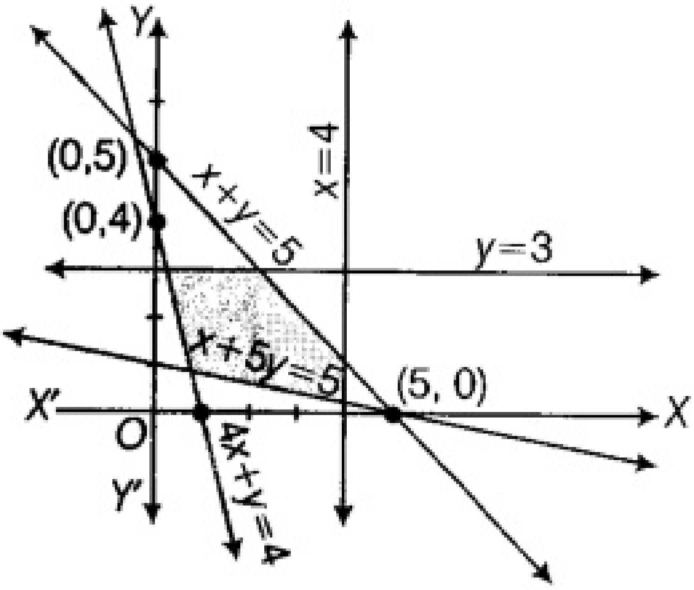
$x \leq 4$

Linear equation corresponding to inequality (iv) is $x = 4$. This is a line parallel to Y-axis at a distance 4 units to the right of Y-axis and for this inequality shaded region contains the origin.

Take inequality (v)

$y \leq 3$

Linear equation corresponding to inequality (v) is $y = 3$. This is a line parallel to X-axis at a distance of 3 units above the X-axis and for this inequality shaded region contains the origin.



Hence, the common region is shaded. Any point in this region represents a solution of given inequalities.