Short Answer Type Questions – II

[3 marks]

Que 1. If the perimeter of a semicircular protractor is 66 cm, find the diameter of the protractor (Take $\pi = 22/7$).

Sol. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

$\Rightarrow \quad \pi r + 2r = 66$ [$\therefore$ Perimeter of a semicircle $= \pi r + 2r$]

$\Rightarrow \quad r\left(\frac{22}{7} + 2\right) = 66 \qquad \Rightarrow \quad \frac{36}{7} r = 66$

$\Rightarrow \quad r = \frac{66 \times 7}{36} = \frac{77}{6} = cm$

$\therefore$ Diameter of the protractor $= 2r = 2 \times \frac{77}{6} = \frac{77}{3} = 25\frac{2}{3\,cm}$

Que 2. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then

Diameter = 2r cm and circumference = 2πr cm

According to question,

$\therefore$ Circumference = Diameter + 16.8

$\Rightarrow \quad 2\pi r = 2r + 16.8$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 2r + 16.8 \qquad \Rightarrow \qquad 44r = 14r + 16.8 \times 7$

$\Rightarrow \qquad 44r = 14r = 117.6 \qquad$ or $\qquad 30r = 117.6$

$\Rightarrow \qquad r = \frac{117.6}{30} = 3.92$

Hence, radius = 3.92 cm.

Que 3. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

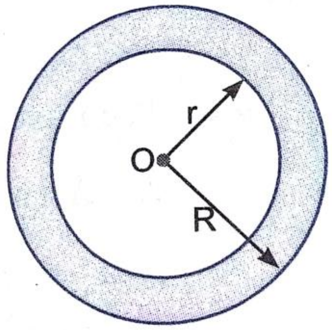

Fig. 12.9

Sol. Let the outer and inner radii of the ring be R m and r m respectively. Then,

$2\pi R = 396$ and $2\pi r = 352$

$\Rightarrow \quad 2 \times \frac{22}{7} \times R = 396$ and $\Rightarrow \quad 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow \quad R = 396 \times \frac{7}{22} \times \frac{1}{2}$ and $r = 352 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow \quad R = 63\, m$ and $r = 56\, m$

Hence, width of the track = (R – r) m = (63 – 56) m = 7 m

Que 4. The inner circumference of a circular track [Fig. 12.10] is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹ 2 per metre.

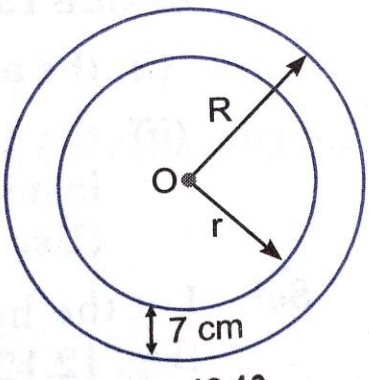

Fig. 12.10

Sol. Let the inner and outer radii of the circular track be r m and R m respectively. Then,

Inner circumference = $2\pi r = 220$ m

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220 \quad \Rightarrow \quad r = \frac{220 \times 7}{2 \times 22} = 35\, m$

Since the track is 7 m wide everywhere. Therefore,

R = Outer radius = r + 7 = (35 + 7) m = 42 m

∴ Outer circumference = $2\pi R = 2 \times \frac{22}{7} \times 42 \text{ m} = 264 \text{ m}$

Rate of fencing = ₹ 2 per metre

∴ Total cost of fencing = (Circumference × Rate) = ₹ (264 × 2) = ₹ 528

Que 5. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Sol. The diameter of a wheel = 80 cm.

So, radius of the wheel = 40 cm.

Now, distance travelled in one complete revolution of wheel = 2π × 40 = 80π

Since, speed of the car is 66 Km/h

So, distance travelled in 10 minutes = $\frac{66 \times 100000 \times 10}{60}$

$= 11 \times 100000\ cm = 1100000\ cm.$

So, Number of complete revolutions in 10 minutes

$$= \frac{1100000}{80\pi} = \frac{110000}{8 \times \frac{22}{7}}$$

$$= \frac{110000 \times 7}{8 \times 22} = \frac{70000}{16} = 4375$$

Que 6. An umbrella has 8 ribs which are equally spaced (Fig. 12.11). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

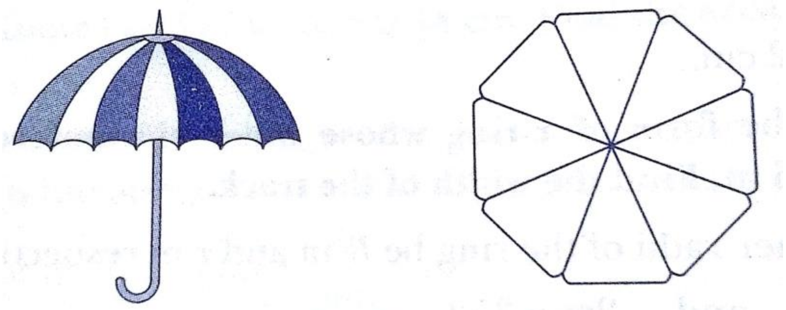

Fig. 12.11

Sol. We have, r = 45 cm

∴ Area between two consecutive ribs $= \frac{1}{8} \times \pi r^2$

$$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 = \frac{11 \times 45 \times 45}{4 \times 7}$$

$$= \frac{22275}{28} = 795.54\ cm^2$$

Que 7. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (Fig. 12.12). Find
(i) the area of that part of the field in which the horse can graze;
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m.
(Use π = 3.14)

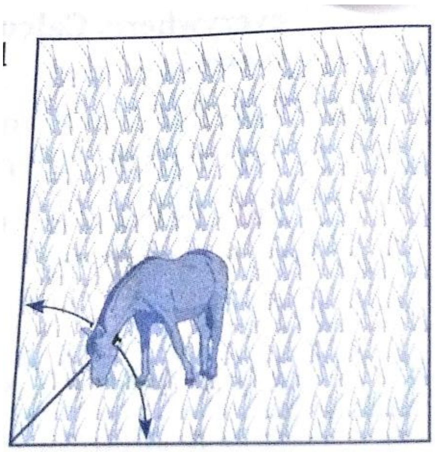

Fig. 12.12

Sol. Let the horse be tied at point O and the length of the rope is OH (Fig. 12.12).
Thus, (i) the area of the part of the field in which the horse can graze
= Area of the quadrant of a circle (OAHB)

$$= \frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times 5 \times 5 = \frac{78.5}{4} = 19.625\ m^2$$

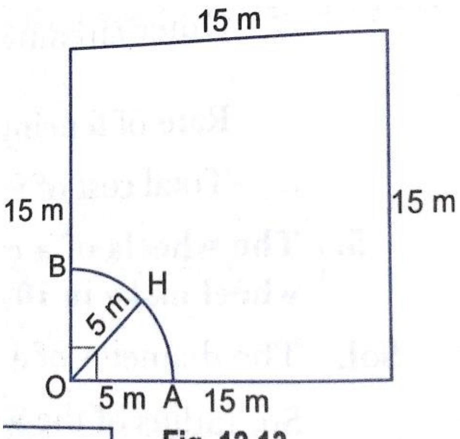

Fig. 12.13

(ii) Now, r = 10 m and

$$\therefore \text{Required area} = \frac{\pi r^2}{4}$$

$$= \frac{3.14 \times (10)^2}{4} = \frac{3.14 \times 100}{4}$$

$$= \frac{314}{4} = 78.5\ m^2$$

Increase in the grazing area

$$= (78.5 - 19.625)\ m^2$$

$$= 58.875\ m^2$$

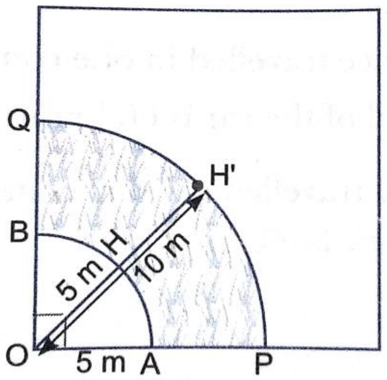

Fig. 12.14

Que 8. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115 °. Find the total area cleaned at each sweep of the blades.

Sol. We have, r = 25 cm and $\theta = 115°$

∴ Total area cleaned at each sweep of the blades

= 2 × (Area of the sector having radius 25 cm and angle$\theta = 115°$).

$$= 2 \times \frac{\theta}{360°} \times \pi r^2 = 2 \times \frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25$$

$$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} = \frac{158125}{126} = 1254.96\ cm^2$$

Que 9. In Fig. 12.15, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown. Find the area of the shaded region.

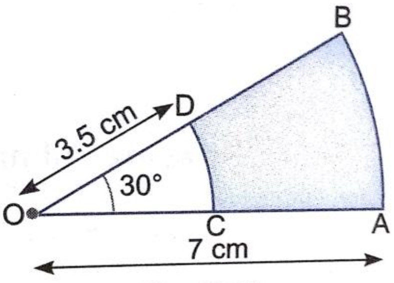

Fig. 12.15

Sol. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30° in a circle of radius 7 cm.

$$\Rightarrow \quad A_1 = \left\{\frac{30}{360} \times \frac{22}{7} \times (7)^2\right\}\ cm^2 \qquad \left[Using: A = \frac{\theta}{360} \times \pi r^2\right]$$

$\Rightarrow \quad A_1 = \left\{\frac{1}{12} \times \frac{22}{7} \times 7 \times 7\right\} \; cm^2 \quad \Rightarrow \; A_1 = \frac{77}{6} \; cm^2$

A_2 = Area of a sector of angle 30° in a circle of radius 3.5 cm.

$$\Rightarrow \; A_2 = \left\{\frac{30}{360} \times \frac{22}{7} \times (3.5)^2\right\} \; cm^2 \; \Rightarrow \; A_2 = \left\{\frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\} \; cm^2 \; = \; \frac{77}{24} cm^2$$

∴ Area of the shaded region = $A_1 - A_2 = \left(\frac{77}{6} - \frac{77}{24}\right) \; cm^2$

$$= \frac{77}{24} \times (4 - 1) \; cm^2 \; = \frac{77}{8} cm^2 \; = 9.625 \; cm^2$$

Que 10. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 AM and 9.35 AM.

Sol. We have,

Angle described by the minute hand in one minute = 6 °

∴ Angle described by the minute hand in 35 minutes = (6 × 35)° = 210°

∴ Area swept by the minute hand in 35 minutes = Area of a sector of a circle of radius 10 cm

$$= \left\{\frac{210°}{360°} \times \frac{22}{7} \times (10)^2\right\} \; cm^2 \; = \; 183.3 \; cm^2$$

Que 11. Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use$\boldsymbol{\pi = 3.14}$)

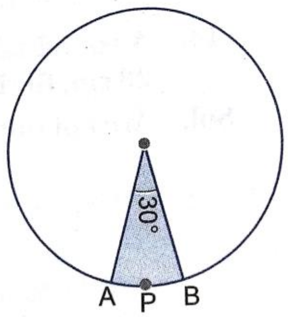

Fig. 12.16

Sol. Area of the sector OAPB = $\frac{\theta}{360°} \times \pi r^2$

$$= \frac{30°}{360°} \times 3.14 \times 4 \times 4 \; cm^2$$

$$= \frac{12.56}{3} cm^2 \; = \; 4.19 \; cm^2 \; (approx.)$$

Area of the corresponding major sector = πr^2 – Area of sector OAPB

= (3.14 × 4 × 4 – 4.19) cm^2 = (50.24 – 4.19) cm^2

= 46.05 cm^2 = 46.4 cm^2 (approx.)

Que 12. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use$\pi = 3.14$ and$\sqrt{3} = 1.73$)

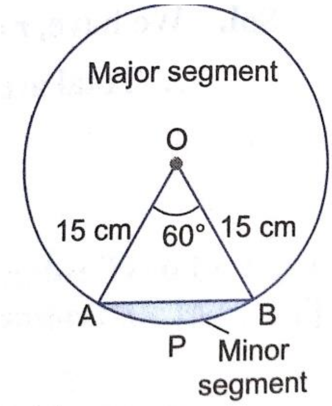

Fig. 12.17

Sol. We have, r = 15 cm and θ = 60°

Given segment is APB

$\therefore$ Area of minor segment $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

$$= \frac{60}{360} \times 3.14 \times 15 \times 15 - \frac{1}{2} \times 15 \times 15 \times \sin 60°$$

$$= \frac{1}{6} \times 3.14 \times 15 \times 15 - \frac{1}{2} \times 15 \times 15 \times \frac{\sqrt{3}}{2}$$

$$= 225 \left(\frac{3.14}{6} - \frac{1.73}{4}\right) = 225 \left(\frac{6.28 - 5.19}{12}\right)$$

$$= \frac{225 \times 1.09}{12} = \frac{245.25}{12} = 20.44\ cm^2$$

Now, area of major segment = Area of circle – Area of minor segment

$= \pi(15)^2 - 20.44 = 3.14 \times 225 - 20.44$

$= 706.5 - 20.44 = 686.06\ cm^2$

Que 13. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use$\pi = 3.14$ and $\sqrt{3} = 1.73$)

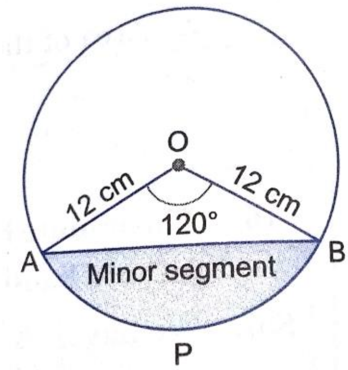

Fig. 12.18

Sol. We have, r = 12 cm and $\theta = 120°$

Given segment is APB

Now, area of the corresponding segment of circle

= Area of the minor segment

$= \frac{\theta}{360} \times \pi r^2 \ - \ \frac{1}{2} r^2 \sin\theta$

$= \frac{120}{360} \times 3.14 \times (12)^2 \ - \frac{1}{2} \times (12)^2 \sin 120°$

$\left[\because \ \sin 120° = \sin (180° - 60°) = \ \sin 60° = \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{3} \times 3.14 \times 144 - \frac{1}{2} \times 144 \times \frac{\sqrt{3}}{2} = 144 \left[\frac{3.14}{3} - \frac{\sqrt{3}}{4}\right] \ = 144 \left[\frac{12.56 - 3\sqrt{3}}{12}\right]$

$= 12\ (12.56 - 3 \times 1.73) \ = 12\ (12.56 - 5.19) \ = 12 \times 7.37 \ = 88.44\ cm^2$

Que 14. A round table cover has six equal designs as shown in Fig. 12.19. If the radius of the cover is 28 cm, find the cost of making the design at the rate of ₹ 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)

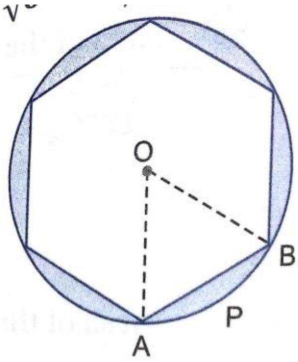

Fig. 12.19

Sol. Area of one design = Area of the sector OAPB – Area of ΔAOB

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \sin 60°$$

$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \frac{\sqrt{3}}{2}$$

$$= 28 \times 28 \left(\frac{11}{21} - \frac{1.7}{4}\right) = 28 \times 28 \times \frac{8.3}{84} = 77.47\ cm^2$$

∴ Area of 6 such designs = 77.47 × 6 = 464.8 cm^2

Hence, cost of making such designs = 464.8 × 0.35 = ₹ 162.69

Que 15. Find the area of the shaded region in Fig. 12.20, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

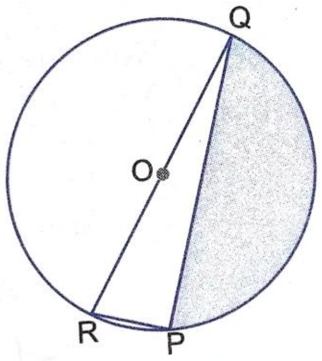

Fig. 12.20

Sol. Here, ROQ is the diameter if given circle, therefore $\angle RPQ = 90°$

Now, in right angled triangle $\Delta PRQ, we\ have$

$RQ^2 = RP^2 + PQ^2$ (by Pythagoras Theorem)

⇒ $RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$

∴ $RQ = \sqrt{625} = 25\ cm$

Therefore, radius r = $\frac{25}{2}\ cm$

Now, area of shaded region

= Area of the semi-circle – Area of ΔRPQ

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} PQ \times RP = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - \frac{1}{2} \times 24 \times 7$$

$$= \left(\frac{6875}{28} - 84\right) cm^2 = \left(\frac{6875 - 2352}{28}\right) cm^2 = \frac{4523}{28}\ cm^2 = 161.54\ cm^2$$