CBSE Test Paper 05
Chapter 7 Integrals

1. $\int \sqrt{x^2+a^2}\,dx$ is equal to

   a. $\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log(x+\sqrt{(x^2+a^2)}$
   b. $\tan^{-1}(x^2+x+2)+C$
   c. $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log(x+\sqrt{(x^2+a^2)}$
   d. $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

2. If f (x) be a function such that $\frac{d}{dx}(f(x))=\log x$, then f (x) is equal to

   a. x log (x e) + C
   b. $x\log\left(\frac{x}{e}\right)$ + C
   c. $x\log\left(\frac{e}{x}\right)$ + C
   d. $\frac{\log x}{x}$ + C

3. $\int e^x(f(x)+f'(x))$ is equal to

   a. $e^x+f(x)+C$
   b. $e^x(f(x)-f'(x))+C$
   c. $e^{f(x)}+f(e^x)+C$
   d. $e^x f(x)$ + C

4. $\int \sqrt{a^2-x^2}$ dx is equal to

   a. $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2-x^2}\right|+C$
   b. $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$
   c. $\frac{x}{2}\sqrt{a^2-x^2}-\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$
   d. $\frac{a}{2}\sqrt{a^2-x^2}+\frac{x^2}{2}\log\left|x+\sqrt{a^2-x^2}\right|+C$

5. $\int$log x dx is equal to

   a. $\frac{1}{2}(\log x)^2$ + C
   b. x log x – x + C
   c. $\frac{1}{x}$ + C

d. $x + log\, x + C$

6. Evaluate, $\int \sec^2(7-4x)dx$.

7. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$.

8. Evaluate $\int_0^{\pi/2} e^x(\sin x - \cos x)dx$.

9. Integrate the following function $\frac{1}{\sqrt{x^2+2x+2}}$

10. Show that $\int_0^a f(x) \,.\, g(x)\, dx = 2\int_0^a f(x)\, dx$

11. Evaluate $\int\limits_{-1}^{2} f(x)\, dx$, where f(x) = |x + 1| + |x| + |x + 1|.

12. Evaluate $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$.

13. Find $\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$.

14. Find $\int \frac{5}{(x+1)(x^2+9)} dx$

15. Find $\int \frac{dx}{3x^2+13x-10}$

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Solution

1. c. $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log(x+\sqrt{(x^2+a^2)}$

   Explanation: = $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log(x+\sqrt{(x^2+a^2)}$

   Standard Formulae

   Can be done by By-Part taking '1' as the second function and $\sqrt{a^2+x^2}$as first function.

2. b. $x\log\left(\frac{x}{e}\right)$ + C

   Explanation: Integrating both sides, we get

   $\int\frac{d}{dx}(f(x))dx=\int\log x\ dx\ we\ get\ f(x)=\int\log xdx$

   $\int\log xdx=x\log x-x=x(\log x-1)=x(\log x-\log e)=x\{\log\left(\frac{x}{e}\right)\}$

3. d. $e^x f(x)$ + C

   Explanation: $\int e^x(f(x)+f'(x))dx=\int e^x\ f(x)dx+\int e^x\ f'(x)dx$

   $\Rightarrow f(x)e^x-\int f'(x)e^xdx+\int f'(x)e^xdx+C$

   (Using By Part, taking f(x) as I function)

   $\Rightarrow f(x)e^x+C$

   It is a standard formula.

4. b. $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$

   Explanation: Put x = asin t then, dx = a cost dt

   $\Rightarrow\int a^2.cos^2t\ dt=a^2\int\frac{1+cos\ 2t}{2}\ dt$

   $=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$

5. b. x log x – x + C

   Explanation: $\int 1.(logx)dx=logx.x-\int\frac{1}{x}.xdx$

   (Using By Part, Taking log x as I and 1 as II function)

   $\Rightarrow log\ x.x-x+C$

6. Let $I=\int\sec^2(7-4x)dx$

   put 7 - 4x = t

   $\Rightarrow\quad -4dx=dt\Rightarrow dx=\frac{-1}{4}dt$

   $\therefore\quad I=\frac{-1}{4}\int\sec^2 tdt=\frac{-1}{4}\tan t+C$

$= -\frac{\tan(7-4x)}{4} + C$

7. Let I = $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$
   $= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx - \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$
   $= \int \sec^2 x dx - \int \cos ec^2 x dx$
   = tan x + cot x + C
8. Let $I = \int_0^{\pi/2} e^x$ (sin x - cos x) dx
   $\Rightarrow \quad I = -\int_0^{\pi/2}$ e^x (cos x - sin x) dx
   Now, consider, f(x) = cos x
   then f'(x) = -sin x
   Now, by using $\int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + C$,
   we get, $I =$ -$[e^x \cos x]_0^{\pi/2}$
   $= -e^{\pi/2} \cos \frac{\pi}{2} + e^0 \cos(0)$
   = 0 + 1(1) = 1
9. $\int \frac{1}{\sqrt{x^2+2x+2}} dx$
   $= \int \frac{1}{\sqrt{x^2+2x+1+1}} dx$
   $= \int \frac{1}{\sqrt{(x+1)^2+(1)^2}} dx$
   $= \log\left|(x+1) + \sqrt{(x+1)^2 + (1)^2}\right| + c$
   $= \log\left|(x+1) + \sqrt{x^2+2x+2}\right| + c$
10. $I = \int_0^a f(x) \,.\, g(x)\, dx$
    $= \int_0^a f(a-x) \,.\, g(a-x)\, dx \;\; [by\, P_4]$
    $= \int_0^a f(x) \,.\, [4 - g(x)] dx$[From given]
    $= 4\int_0^a f(x) \,.\, dx - \int_0^a f(x) \,.\, g(x)\, dx$
    $I = 4\int_0^a f(x) \,.\, dx - I$
    $2I = 4\int_0^a f(x) \,.\, dx$
    $I = 2\int_0^a f(x)\, dx$
11. We can redefine f as $f(x) = \begin{cases} 2-x, \; if \; -1 < x \leqslant 0 \\ x+2, \; if \; 0 < x \leqslant 1 \\ 3x, \; if \; 1 < x \leqslant 2 \end{cases}$
    Therefore, $\int\limits_{-1}^{2} f(x)\, dx = \int\limits_{-1}^{0} (2-x)\, dx + \int\limits_{0}^{1} (x+2)\, dx + \int\limits_{1}^{2} 3x dx \, (by P_2)$
    $= \left(2x - \frac{x^2}{2}\right)^0 + \left(\frac{x^2}{2} + 2x\right)_0^1 + \left(\frac{3x^2}{2}\right)_1^2$

$= 0 - \left(-2 - \frac{1}{2}\right) + \left(\frac{1}{2} + 2\right) + 3\left(\frac{4}{3} - \frac{1}{2}\right) = \frac{5}{2} + \frac{5}{2} + \frac{9}{2} = \frac{19}{2}$

12. Given I = $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$

$\left[\begin{array}{l} \because \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \\ \text{and } 1+\cos x = 2\cos^2\frac{x}{2} \end{array}\right]$

$\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} \underset{I}{x} \underset{II}{\sec^2} \frac{x}{2} dx + \int_0^{\pi/2} \tan\frac{x}{2} dx$

$\Rightarrow I = \frac{1}{2}\Big\{\left[x \int \sec^2\frac{x}{2} dx\right]_0^{\pi/2}$

$- \int_0^{\pi/2}\left[\frac{d}{dx}(x)\int\left(\sec^2\frac{x}{2}dx\right)\right]dx\Big\}$

$+ \int_0^{\pi/2} \tan\frac{x}{2} dx$

$\Rightarrow I = \frac{1}{2}\left\{\left[x \cdot \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2} - \int_0^{\pi/2}\frac{\tan\frac{x}{2}}{\frac{1}{2}}dx\right\}$

$+ \int_0^{\pi/2} \tan\frac{x}{2} dx$

[Integration by parts]

$= \left[x \cdot \tan\frac{x}{2}\right]_0^{\pi/2} - \int_0^{\pi/2}\tan\frac{x}{2}dx + \int_0^{\pi/2}\tan\frac{x}{2}dx$

$= \frac{\pi}{2} \cdot \tan\frac{\pi}{4} - 0$

$\therefore \quad I = \frac{\pi}{2} \quad \left[\because \tan\frac{\pi}{4} = 1\right]$

13. According to the question, $I = \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$= \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2.2\sin x\cos x}} [\because sin2x = 2sin\ x\ cos\ x]$

$= \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}}$

$= \int_0^{\pi/4} \frac{dx}{(\cos^3 x)(2)\sqrt{\sin x\cos x}}$

$= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^3 x\cos^{1/2} x\sin^{1/2} x}$

$= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^{7/2} x\sin^{1/2} x}$

Dividing numerator and denominator by $cos^4 x$

$= \frac{1}{2}\int_0^{\pi/4} \frac{\frac{1}{cos^4 x}dx}{\frac{\cos^{7/2} x\sin^{1/2} x}{cos^4 x}} [\because \frac{1}{cosx} = secx]$

$= \frac{1}{2}\int_0^{\pi/4} \frac{\sec^4 x}{\cos^{\frac{7}{2}-4} x\sin^{1/2} x}dx$

$= \frac{1}{2}\int_0^{\pi/4} \frac{\sec^4 x}{\cos^{-1/2} x\sin^{1/2} x}dx$

$= \frac{1}{2}\int_0^{\pi/4} \frac{\sec^2 x.sec^2 x}{\frac{\sin^{1/2}x}{\cos^{1/2}x}} dx$

$= \frac{1}{2}\int_0^{\pi/4} \frac{\sec^2 x\left(1+\tan^2 x\right)}{\tan^{1/2} x} dx$[

$sec^2 x - tan^2 x = 1 \implies sec^2 = 1 + tan^2 x \; and \; \frac{sinx}{cosx} = tanx$]

Put $tanx = t \Rightarrow sec^2 xdx = dt$

Lower Limit, when x = 0, then t = tan 0 = 0

Upper Limit, when x = $\frac{\pi}{4}$, then t = tan$\frac{\pi}{4}$ = 1

$\therefore \quad I = \frac{1}{2}\int_0^1 \left(\frac{1+t^2}{t^{1/2}}\right) dt$

$\therefore \quad I = \frac{1}{2}\int_0^1 \frac{1}{t^{1/2}} dt + \frac{1}{2}\int_0^1 \frac{t^2}{t^{1/2}} dt$

$= \frac{1}{2}\int_0^1 \left(t^{-1/2} + t^{3/2}\right) dt$

$= \frac{1}{2}\left[2t^{1/2} + \frac{2}{5}t^{5/2}\right]_0^1$

$= \frac{2}{2}\left[t^{1/2} + \frac{1}{5}t^{5/2}\right]_0^1$

$= \left[t^{1/2} + \frac{1}{5}t^{5/2}\right]_0^1$

$= (1)^{1/2} + \frac{1}{5}(1)^{5/2} - 0 = 1 + \frac{1}{5} = \frac{6}{5}$

14. Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9}$

$\Rightarrow 5x = A(x^2+9) + (Bx+c)(x+1)$

On comparing coefficients of x^2 and x, we get,

0 = A + B ....(i)

5 = C+B ....(ii)

Eqn. (i) - Eqn (ii), we get

A - C = -5 ....(iii)

Compairing the constant terms

9A + C = 0 ....(iv)

Eqn (iii) + Eqn(iv)

$\Rightarrow (A - C + 5) - (9A + C) \Rightarrow 10A = -5 \Rightarrow A = -\frac{1}{2}$

B = 1/2 [by (i)]

$C = \frac{9}{2}$ [by (ii)]

$I = \int \frac{5x}{(x+1)(x^2+9)} = \int \frac{-1/2}{(x+1)} + \int \frac{\frac{1}{2}x + \frac{9}{2}}{x^2+9} dx$

$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x}{x^2+9} dx + \frac{9}{2}\int \frac{dx}{x^2+9}$

$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{2x}{x^2+9} dx + \frac{9}{2}\int \frac{dx}{x^2+3^2}$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{9}{2}.\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$
$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

15. $I = \int \frac{dx}{3\left[x^2+\frac{13}{3}x-\frac{10}{3}\right]}$

$$= \int \left(\frac{dx}{3\left[x^2+\frac{13}{3}x+\left(\frac{13}{6}\right)^2-\frac{10}{3}-\left(\frac{13}{6}\right)^2\right]}\right)$$
$$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{13}{6}\right)^2-\left(\frac{17}{6}\right)^2}$$

Put $x + \frac{13}{6} = t$

dx = dt

$$\therefore I = \frac{1}{3}\int \frac{dt}{t^2-\left(\frac{17}{6}\right)^2}$$
$$= \frac{1}{3\times2\times\frac{17}{6}}\log\left|\frac{t-\frac{17}{6}}{t+\frac{17}{6}}\right| + c$$

Putting the value of t, we get

$$I = \frac{1}{17}\log\left|\frac{x+\frac{13}{6}-\frac{17}{6}}{x+\frac{13}{6}+\frac{17}{6}}\right| + c$$
$$= \frac{1}{17}\log\left|\frac{6x-4}{6x+30}\right| + c$$
$$= \frac{1}{17}\log\left|\frac{3x-2}{x+5}\right| + c[\therefore log\left(\frac{2}{6}\right) \text{ is absorbed by constant c}]$$