ALTERNATING CURRENT

CHAPTER – 7

# ALTERNATING CURRENT

As we have seen earlier a rotating coil in a magnetic field, induces an alternating emf and hence an alternating current. Since the emf induced in the coil varies in magnitude and direction periodically, it is called an alternating emf. The significance of an alternating emf is that it can be changed to lower or higher voltages conveniently and efficiently using a transformer. Also the frequency of the induced emf can be altered by changing the speed of the coil. This enables us to utilize the whole range of electromagnetic spectrum for one purpose or the other. For example domestic power in India is supplied at a frequency of 50 Hz. For transmission of audio and video signals, the required frequency range of radio waves is between 100 KHz and 100 MHz. Thus owing to its wide applicability most of the countries in the world use alternating current.

**AC circuit containing pure resistance**

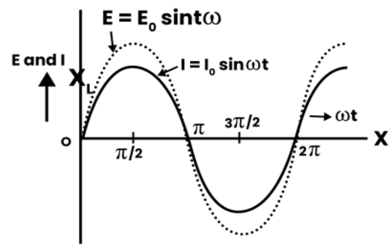

Let at any instant t, the current in the circuit = I.

Potential difference across the resistance = IR

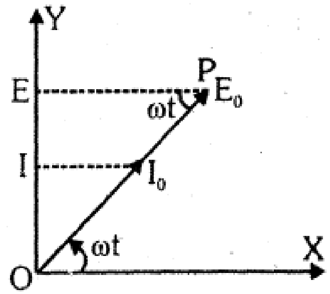
with the help of Kirchhoff's circuital law E – I R = 0 ⇒ $E_0 \sin \omega t = IR$

$\Rightarrow I = \frac{E_0}{R} \sin \omega t = I_0 \sin\omega t$ ($I_0$ = peak or maximum value of current)

Alternating current developed in a pure resistance is also of the sinusoidal nature. In a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. are as shown in figure.




a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors $E_0$ and $I_0$ are in the same direction, making an angle ωt with OX. Their projection on Y–axis represent
the instantaneous values of alternating current and voltage.




i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$

Since $I_0 = \frac{E_0}{R}$, hence $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \Rightarrow I_{rms} = \frac{E_{rms}}{R}$

## AC circuit containing pure inductance


A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin \omega t$

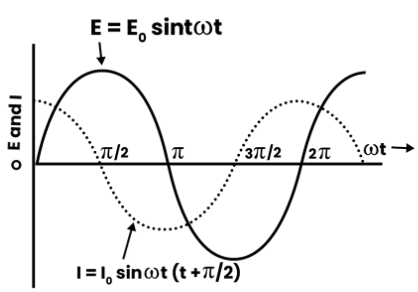
When a.c. flows through the circuit, emf induced across inductance $= -L\frac{dI}{dt}$

**Note**

Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other than inductance so with the help of Kirchhoff's circuital law $E + \left(-L\frac{dI}{dt}\right) = 0 \Rightarrow E = L\frac{dI}{dt}$ so we get $I = \frac{E_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$

Maximum current $I_0 = \frac{E_0}{\omega L} \times 1 = \frac{E_0}{\omega L}$



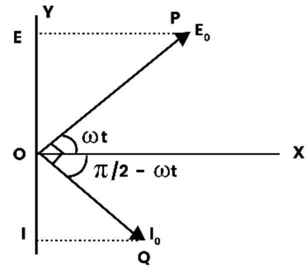

Hence, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$ or alternating emf leads the a.c. by a phase angle of $\frac{\pi}{2}$.

Expression $I_0 = \frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I} = R$.

This non–resistive opposition to the flow of A.C. in a circuit is called the inductive reactance ($X_L$) of the circuit.

$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.

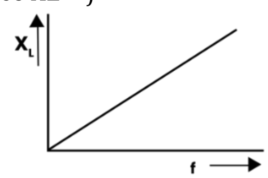



**Unit of $X_L$ :** ohm

($\omega L$) = Unit of L × Unit of ($\omega = 2\pi f$) = henry × sec−1

$= \frac{volt}{\text{ampere/sec}} \times sec^{-1} = \frac{volt}{ampere} = ohm$

Inductive Reactance $X_L \propto f$




Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

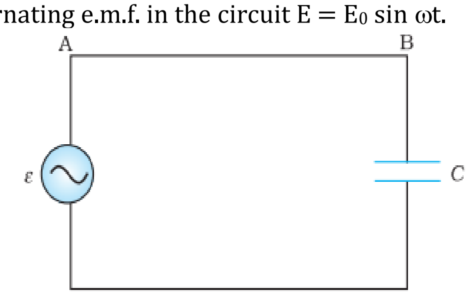
**For d.c. circuit, f = 0**

$\therefore X_L = \omega L = 2\pi f L = 0$

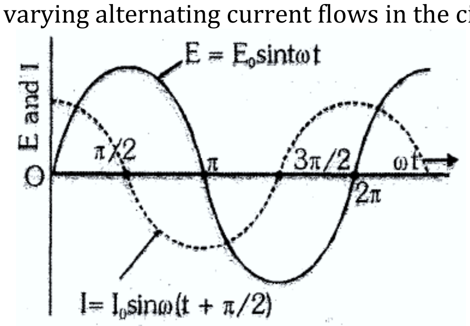
Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.

## AC circuit containing pure Capacitance

A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$.



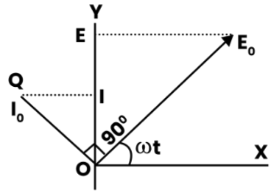

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.




The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time. Let at any instant t charge on the capacitor = q

Instantaneous potential difference across the capacitor E = q/C

$\Rightarrow$ q = CE $\Rightarrow$ q = $CE_0$ sin ωτ



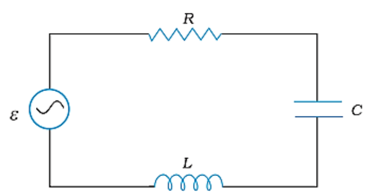

The instantaneous value of current $I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega\, t)$

= $CE_0$ω cosω

$\Rightarrow I = \frac{E_0}{(1/\omega C}\sin\left(\omega t\frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ where I0 = ω$CE_0$

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of π/2. The alternative emf lags behinds the alternating current by a phase angle.

## Ac Voltage Applied to A Series LCR Circuit




A circuit containing a series combination of an resistance R, a toil of inductance Land a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of $E_0$, as shown in figure.

wt. As L, C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

**However, voltage across each element bears a different phase relationship with the current.**

Let at any instant of time t the current in the circuit is I.

Let at this time t the potential difference across L, C, and R

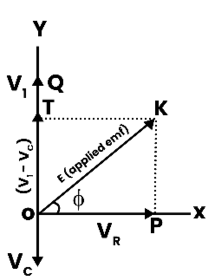
$V_L = I X_L$, $V_C = I X_C$ and $V_R = IR$

Now, $V_R$ is in phase with current I but $V_L$ leads I by 90° While $V_C$ lags behind I by 90°.

The vector OP: represents $V_R$ (which is in phase with I) the vector OQ represent $V_L$ (which leads I by 90°) and the vector OS represents $V_C$ (which legs behind I by 90°)

$V_L$ and $V_C$ are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be ($V_L - V_C$) which is represented by OT. Finally, the vector OK represents the resultant of $V_R$ and ($V_L - V_C$), that is, the resultant of all the three ≃ applied e.m.f.




Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R_2 + (X_L - X_C)^2}$

$\Rightarrow I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

Impedance $Z = \sqrt{R2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ $\tan\phi = \frac{X_L - X_C}{R}$

## Resonance

A circuit is said to be resonant when the natural frequency of circuit Is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

**Series Resonance**

**(a) At Resonance**

(i) $X_L = X_C$

(ii) $V_L = V_C$

(iii) ϕ = 0 (V and I in same phase)

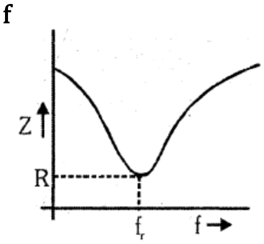
(iv) $Z_{min}$ = R (impedance minimum)

(v) $I_{max} = \frac{V}{R}$ (current maximum)

**(b) Resonance frequency**

$X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 \frac{1}{LC} \Rightarrow \omega_\rho = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$

**(c) Variation with f**

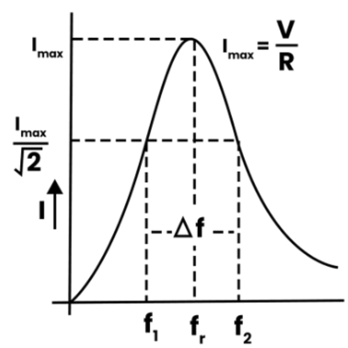



(i) If f < $f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At f = $f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If f > $f_r$, then $X_L > X_C$ circuit nature is inductive, ϕ (positive)

**(d) Variation of I with f**




as f increase, Z first decrease then increase as f increase, I first increase then decreases

At resonance, impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

## Half power frequencies

The frequencies at which, power become half of its maximum value is called half power frequencies

**Band width** $= \Delta f = f_2 - f_1$

**Quality factor Q :** Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$

(i) It represents the sharpness of resonance.

(ii) It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$

**Magnification**

At resonance VL or $V_C$ = QE (where E = supplied voltage)

So at resonance  Magnification factor = Q–factor

**Sharpness of resonance**

Sharpness $\propto$ Quality factor $\propto$ Magnification factor

R decrease $\Rightarrow$ Q increase $\Rightarrow$ Sharpness increases

## Power In an AC Circuit

In case of steady current, the rate of doing work is given by, P = VI

In an alternating circuit, current and voltage both vary with time, so the work done by the source in time interval dt is given by

dw = Vidt

Suppose in an ac, the current is leading the voltage by an angle $\varphi$. Then we can write $V = V_m \sin \omega t$ and

$I = i_m \sin(\omega t + \varphi)$

$dw = V_m I_m \sin \omega t \sin(\omega t + \varphi) dt$

$dw = V_m i_m (\sin^2 \omega t \cos \varphi + \sin \omega t \cos \omega t \sin \varphi) dt$

The total work done in a complete cycle is

$W = V_m i_m \cos \varphi \int_0^T \sin^2 \omega t dt + V_m i_m \sin \varphi \int_0^T \sin \omega t \cos \omega t dt$

$W = \frac{1}{2} V_m i_m \cos \varphi \int_0^T (1 - \cos 2\omega t) dt + \frac{1}{2} V_m i_m \sin \varphi \int_0^T \sin 2\omega t dt$

$W = \frac{1}{2} V_m i_m T \cos \varphi$

The average power delivered by the source is, therefore

P = W/T

$P = \frac{1}{2} V_m i_m \cos \varphi$

$P = \frac{V_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos \varphi$

$P = V_{rms} i_{rms} \cos \varphi$

This can also be written as,

$P = I^2 Z \cos \phi$

Here, Z is impedance, the term $\cos \varphi$ is known as power factor

It is said to be leading if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags voltage by 60° (as $^{-1}\cos^{-1} 0.5 = 60°$). the product of $V_{rms}$ and $i_{rms}$ gives the apparent power. While the true power is obtained by multiplying the apparent power by the power factor $\cos \varphi$.

(i) Resistive circuit: For $\varphi = 0°$, the current and voltage are in phase. The power is thus, maximum.

(ii) purely inductive or capacitive circuit: For $\varphi = 90°$, the power is zero. The current is then stated as wattless. Such a case will arise when resistance in the circuit is zero. The circuit is purely inductive or capacitive

(iii) LCR series circuit: In an LCR series circuit, power dissipated is given by $P = I^2 Z \cos \phi$ were

$\varphi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

So, $\phi$ may be non-zero in a $RL$ or $RC$ or $RCL$ circuit. Even in such cases, power is dissipated only in the resistor.

(iv) Power dissipated at resonance in LCR circuit: At resonance $XC - XL = 0$, and $\phi = 0$. Therefore, $\cos \phi = 1$ and $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit (through $R$ ) at resonance

**Q.** In an L-C-R A.C. series circuit $L = 5H, \omega = 100$ rad s$^{-1}$, $R = 100\Omega$ and power factor is 0.5. Calculate the value of capacitance of the capacitor

**Sol.** Power factor

$\cos \delta = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2}}$

$\text{Cos } \delta = 0.5$

Squaring on both side

$\cos^2 \delta = \frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2}$

$\frac{1}{4} = \frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2}$

$R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2 = 4R^2$

$\left(\omega L - \frac{1}{\omega L}\right)^2 = 3R^2$

$\omega L - \frac{1}{\omega C} = \sqrt{3} R$

$\omega L - \sqrt{3} R = \frac{1}{\omega C}$

$C = \frac{1}{\omega}\left(\frac{1}{\omega L - \sqrt{3} R}\right)$

$C = \frac{1}{100}\left(\frac{1}{100 \times 5 - \sqrt{3} \times 100}\right)$

$C = \frac{10^{-2}}{500 - 173.2} = \frac{10^{-2}}{326.8} = 30.6 \times 10^{-6}$ F

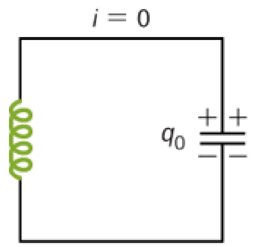
$C = 30.6 \mu F$

## LC Oscillations

We know that a capacitor and an inductor can store electrical and magnetic energy, respectively.

When a capacitor (initially charged) is connected to an inductor, the charge on the capacitor and the current in the circuit exhibit the phenomenon of electrical oscillations similar to oscillations in mechanical systems.

Let a capacitor be charged $q_m (at\ t = 0)$ and connected to an inductor as shown in Fig.




$t = 0, T$

The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit. Let $q$ and $i$ be the charge and current in the circuit at time $t$.
Since di/dt is positive, the induced emf in $L$ will have polarity as shown, i.e.,

$$v_b < v_a.$$

According to Kirchhoff's loop rule,

$$\frac{q}{C} - L\frac{di}{dt} = 0$$
$$i = -(\mathrm{d}q/\mathrm{d}t)$$

in the present case (as $q$ decreases, $i$ increases).
Therefore, above equation becomes:

$$\frac{d^2q}{dt^2} - \frac{1}{LC}q = 0$$

Comparing above equation with standard equation for oscillation

$$\frac{d^2x}{dt^2} - \omega_0^2 x = 0$$

The charge, therefore, oscillates with a natural frequency
$\omega_0 = \frac{1}{\sqrt{LC}}$
and varies sinusoidally with time as

$$q = q_m \cos(\omega_0 t + \varphi)$$

where,
$qm$ is the maximum value of $q$ and $\phi$ is a phase constant.
Since $q = qm$ at $t = 0$,

we have $\cos\phi = 1$ or $\phi = 0$.
Therefore, in the present case,

$$q = q_m \cos(\omega_0 t)$$
$$\text{current } I = i_m \sin(\omega_0 t)$$
$$\text{here } i_m = q_m \omega_0$$

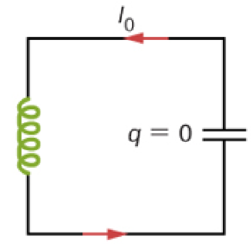
Initially capacitor is fully charged, it stores energy in the form of electric field

$$U_E = \frac{1}{2}CV^2$$

At $t = 0$, the switch is closed and the capacitor starts to discharge As the current increases, it sets up a magnetic field in the inductor and thereby, some energy gets stored in the inductor in the form of magnetic energy:

$$U_B = \frac{1}{2}Li^2.$$

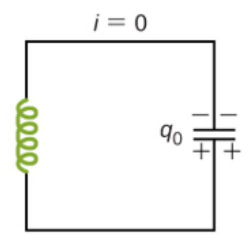
As the current reaches its maximum value $i_m$, (at $t = T/4$ ) all the energy is stored in the magnetic field:




$t = \frac{T}{4}$

$$U_B = \frac{1}{2}Li^2$$

The capacitor now has no charge and hence no energy.
The current now starts charging the capacitor, this process continues till the capacitor is fully charged (at $t = T/2$ B ut it is charged with a polarity opposite to its initial state The whole process just described will now repeat itself till the system reverts to its original state.




$t = \frac{T}{2}$

Thus, the energy in the system oscillates between the capacitor and the inductor.

Note that the above discussion of LC oscillations is not realistic for two reasons:

(i) Every inductor has some resistance. The effect of this resistance is to introduce a damping effect on the charge and current in the circuit and the oscillations finally die away.

(ii) Even if the resistance were zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves (discussed in the next chapter). In fact, radio and TV transmitters depend on this radiation.

**Q.** A capacitor of capacitance $25\mu F$ is charged to $300$ V. It is then connected across a $10$mH inductor. The resistance of circuit is negligible

(a) Fins the frequency of oscillation of the circuit

(b) Find the potential difference across capacitor and magnitude of circuit current 1.2 ms after the inductor a capacitor are connected

(c) Find the magnetic energy and electric energy at $t = 0$ and $t = 1.2$ ms.

**Sol.** (a) The frequency of oscillation of the circuit is $f = \frac{1}{2\pi\sqrt{LC}}$

Substituting the given values, we have $f = \frac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}} = \frac{10^3}{\pi} Hz$

(b) Charge across the capacitor at time $t$ will be $q = qo\cos\omega_0 t$ and $I = -q\omega_0 \sin\omega_0 t$

Here $q_0 = CV_0 = (25\times10^{-6})(300) = 7.5\times10^{-3}$C

Now, charge in the capacitor after $t = 1.25\times10^{-3}$ s is

$q = (7.5\times10^{-3})\cos(2\pi\times318.3)(1.2\times10^{-3})C = 5.53\times10^{-3}C$

∴ P.D across capacitor,

$V = \frac{|q|}{C} = \frac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2$ volt

The magnitude of current in the circuit at $t = 1.2\times10^{-3}$ s is

$|i| = q\omega_0 \sin\omega_0 t$

$|i| = (7.5\times10^{-3})(2\pi)(318.3)\sin(2\pi\times318.3)(1.2\times10^{-3})A = 10.13$ A

(c) At $t = 0$, Current in the circuit is zero. Hence $U_L = 0$

Charge on the capacitor is maximum

Hence

$U_C = \frac{1}{2}\frac{q_0^2}{C} \Rightarrow U_C = \frac{1}{2}\frac{(7.5\times10^{-3})^2}{25\times10^{-6}} = 1.125$ J

At $t = 1.25$ ms, $q = 5.53\times10^{-3}$C

$U_C = \frac{1}{2}\frac{q_0^2}{C} \quad U_C = \frac{1}{2}\frac{(5.53\times10^{-3})^2}{25\times10^{-6}} = 0.612J$

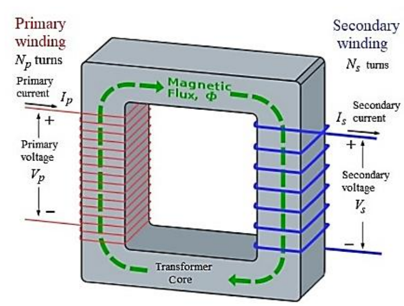
## Transformer

An electrical device that can change the A.C. current is known as a transformer.

Principle – A transformer works on the principle of mutual induction. Mutual induction is the phenomenon by which when the amount of magnetic flux linked with a coil changes, an E.M.F. is induced in the neighboring coil.

**Construction** –

A transformer is made up of a rectangular iron core. Two coils, a primary (P) coil with two sides P1 and P2, and a secondary (S) coil with two sides S1 and S2. Both these coils are insulated from the Ferro-magnetic iron core. The source of the alternate current is connected to the primary winding and the output is obtained through the secondary winding which is connected in parallel to a resistance R.




**Working –**

For an ideal transformer, we consider that resistances of the primary and secondary coils are negligible.

Let the E.M.F. of the alternate current supplied by the A.C source be

$E_P = E_0 \sin\omega t$

Let's assume that the primary winding to be a pure inductance, so here $I_p$ will lag behind the voltage $E_P$ by 90°. Thus, the power factor for primary coil becomes,

$\cos\phi = \cos 90° = 0$

Let that the number of turns in primary wire be $N_P$ and secondary wire be $N_S$

According to faraday law, the induced E.M.F. through one turn of both the coils will be the same.

Let the flux through one turn be $\phi$, the flux through the primary coil be $\phi_P$ and the flux through the secondary coil be $\phi_S$.

So $\phi_p = N_P\phi$

$\phi_S = N_S\phi$

We also know by Faraday's law

$E = \frac{d\phi}{dt}$

So for the primary coil this equation becomes

$E_S = \frac{d\phi_S}{dt}$ ( Equation 1)

And for the secondary coil this equation becomes

$E_P = \frac{d\phi_P}{dt}$ ( Equation 2)

Dividing equation 1 by equation 2

$$\frac{E_S}{E_P} = \frac{d\phi_S}{d\phi_P} = \frac{N_S\phi}{N_P\phi}$$

$$E_S = E_p\frac{N_S}{N_p} \text{( Equation 3)}$$

We know that

$P = VI$

Here $P =$ Power, $V =$ Voltage, I= current

For primary coil this equation becomes

$P_P = E_P I_P$ ( Equation 4)

For secondary coil this equation becomes

$P_S = E_S I_S$ ( Equation 5)

For an ideal transformer no energy is lost, so

$P_p = P_S$

$$E_P I_P = E_S I_S$$

$$I_S = I_p\frac{E_p}{E_s}\left(\because \frac{E_p}{E_S} = \frac{N_p}{N_S}\right)$$

$$I_S = I_p\frac{N_p}{N_S}$$

**Note**

A transformer that increases the A.C. voltage is known as a step-up transformer ( $N_S > N_p$) and the transformer that decreases the A.C. voltage is known as a step down transformer $(N_S < N_p)$. Additionally, an iron core is used because it is a ferromagnetic material which helps in increasing the strength of the magnetic field

**Q.** In an ideal step-up transformer input voltage is 110 V and current flowing in the secondary is 10 A. If transformation ratio is 10 , calculate output voltage, current in primary, input and out put power

**Sol.** Transformer ratio

(i) $r = \frac{N_S}{N_P} = 10$ $\qquad \frac{E_S}{E_P} = \frac{N_S}{N_P}$

$E_S = E_P\frac{N_S}{N_P} = 110 \times 10 = 1100 \text{ V}$

(ii) Output voltage $E_S = 1100 \text{ V}$

(iii) $E_P I_P = E_S I_S$ $\qquad I_P = \frac{E_S}{E_P} I_S = 10 \times 10 = 100 \text{ A}$

Input power = Output power for ideal transformer $\qquad E_S I_S = E_p I_p = (1100)(10) = 11000 \text{ W}$

# SUMMARY

- **Alternating Current:**
  The current whose magnitude changes with time and direction reverses periodically is called alternating current. a) Alternating emf E and current I at any time am given by:
  $E = E_0 \sin \omega t$
  $E_0 = NBA\omega$
  Where
  $I = I_0 \sin(\omega t - \Phi)$
  Where $I_0 = \frac{NBA\omega}{R}$
  $\omega = 2\pi n = \frac{2\pi}{T}$
  Where T is the time period.
- **Values of Alternating Current and Voltage**
  (a) Instantaneous value:
  It is the value of alternating current and voltage at an instant t.
  (b) Peak value:
  Maximum values of voltage $E_0$ and current $I_0$ in a cycle are called peak values.
  (c) Mean value:
  For complete cycle,
  $< E >= \frac{I}{T}\int_0^T Edt = 0$
  $< I >= \frac{I}{T}\int_0^T Edt = 0$
  Mean value for half cycle: $E_{mean} = \frac{2E_0}{\pi}$
  (d) Root – mean- square (rms) value:
  $E_{rms} = (< E^2 >)^{1/2} = \frac{E_0}{\sqrt{2}} = 0.707E_0 = 70.7\%E_0$
  $I_{rms} = (< I^2 >)^{1/2} = \frac{I_0}{\sqrt{2}} = 0.707I_0 = 70.7\%I_0$
  RMS values are also called apparent or effective values.
- **Phase difference Between the EMF (Voltage) and the Current in an AC Circuit**
  (a) For pure resistance:
  The voltage and the current are in same phase i.e. phase difference $\Phi = 0$
  (b) For pure inductance:
  The voltage is ahead of current by $\frac{\pi}{2}$ i .e. phase difference $\Phi = +\frac{\pi}{2}$.
  (c) For pure capacitance:
  The voltage lags behind the current by $\frac{\pi}{2}$ i.e. phase difference $\Phi = -\frac{\pi}{2}$
- **Reactance:**
  Reactance
  (a) $X = \frac{E}{I} = \frac{E_0}{I_0}\frac{E_{rms}}{I_{rms}} \pm \pi/2$
  Inductive reactance
  (b) $X_L = \omega L = 2\pi nL$
  Capacitive reactance
  (c) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi nC}$
- **Impedance:**
  Impedance is defined as,
  $Z = \frac{E}{I} = \frac{E_0}{I_0}\frac{E_{rms}}{I_{rms}}\phi$
  Where $\Phi$ is the phase difference of the voltage E relative to the current I.
  (a) For L – R series circuit:
  $Z_{RL} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega L^2}$
  $\tan\phi = \left(\frac{\omega L}{R}\right) or \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$
  (b) For R – C series circuit:
  $Z_{RC} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$
  $\tan\phi = \frac{1}{\omega CR}$ Or $\phi = tan^{-1}\left(\frac{1}{\omega CR}\right)$
  (c) For L – C series circuit:
  $Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$
  $= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
  $\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$ Or $\phi = tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$
- **Conductance:**
  Reciprocal of resistance is called conductance.
  $G = \frac{1}{R} mho$
- **Power in and AC Circuit:**
  (a) Electric power = (current in circuit) x (voltage in circuit)
  P = IE
  (b) Instantaneous power:
  $P_{inst} = E_{inst} X I_{inst}$
  (c) Average power:
  $P_{av} = \frac{1}{2}E_0 I_0 \cos\phi = E_{rms}I_{rms} \cos\phi$
  (d) Virtual power (apparent power):
  $= \frac{1}{2}E_0 I_0 = E_{rms}I_{rms}$
- **Power Factor:**
  (a) Power factor
  $\cos\phi = \frac{P_{av}}{P_v} = \frac{R}{Z}$
  (b) For pure inductance
  Power factor, cos $\phi$ =1
  (c) For pure capacitance
  Power factor, cos $\phi$ =0
  (d) For LCR circuit
  Power factor, $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$
  $X = \left(\omega L - \frac{1}{\omega C}\right)$
- **Wattless Current:**
  The component of current differing in phase by $\frac{\pi}{2}$ relative to the voltage, is called wattles current.
- **The rms value of wattless current:**

$= \frac{I_0}{\sqrt{2}} sin\, \phi$

$= I_{rms}\, sin\, \phi = \frac{I_0}{\sqrt{2}}\left(\frac{X}{Z}\right)$

- **Choke Coil:**
  - **(**a) An inductive coil used for controlling alternating current whose self- inductance is high and resistance in negligible, is called choke coil.
  - (b) The power factor of this coil is approximately zero.
- **Series Resonant Circuit**
  - **(**a) When the inductive reactance (XL) becomes equal to the capacitive reactance (XC) in the circuit, the total impedance becomes purely resistive (Z=R).
  - (b) In this state, the voltage and current are in same phase ( $\Phi$ = 0), the current and power are maximum and impedance is minimum. This state is called resonance.
  - (c) At resonance,

    $\omega_r L = \frac{1}{\omega_r C}$

    Hence, resonance frequency is,

    $f_r = \frac{1}{2\pi\sqrt{LC}}$
  - (d) In resonance, the power factor of the circuit is one.
- **Half – Power Frequencies:**

  Those frequencies $f_1$ and $f_2$ at which the power is half of the maximum power (power at resonance), i.e., $f_1$ and $f_2$ are called half – power frequencies.

  $P = \frac{1}{2} P_{max}$

  $I = \frac{I_{max}}{\sqrt{2}}$

  $\therefore P = \frac{P_{max}}{2}$
- **Band – Width:**
  - **(**a) The frequency interval between half – power frequencies is called band – width.

    Bandwidth $\Delta f = f_2 - f_1$
  - (b) For a series LCR resonant circuit,

    $$\Delta f = \frac{1}{2\pi}\frac{R}{L}$$
- **Quality Factor (Q):**

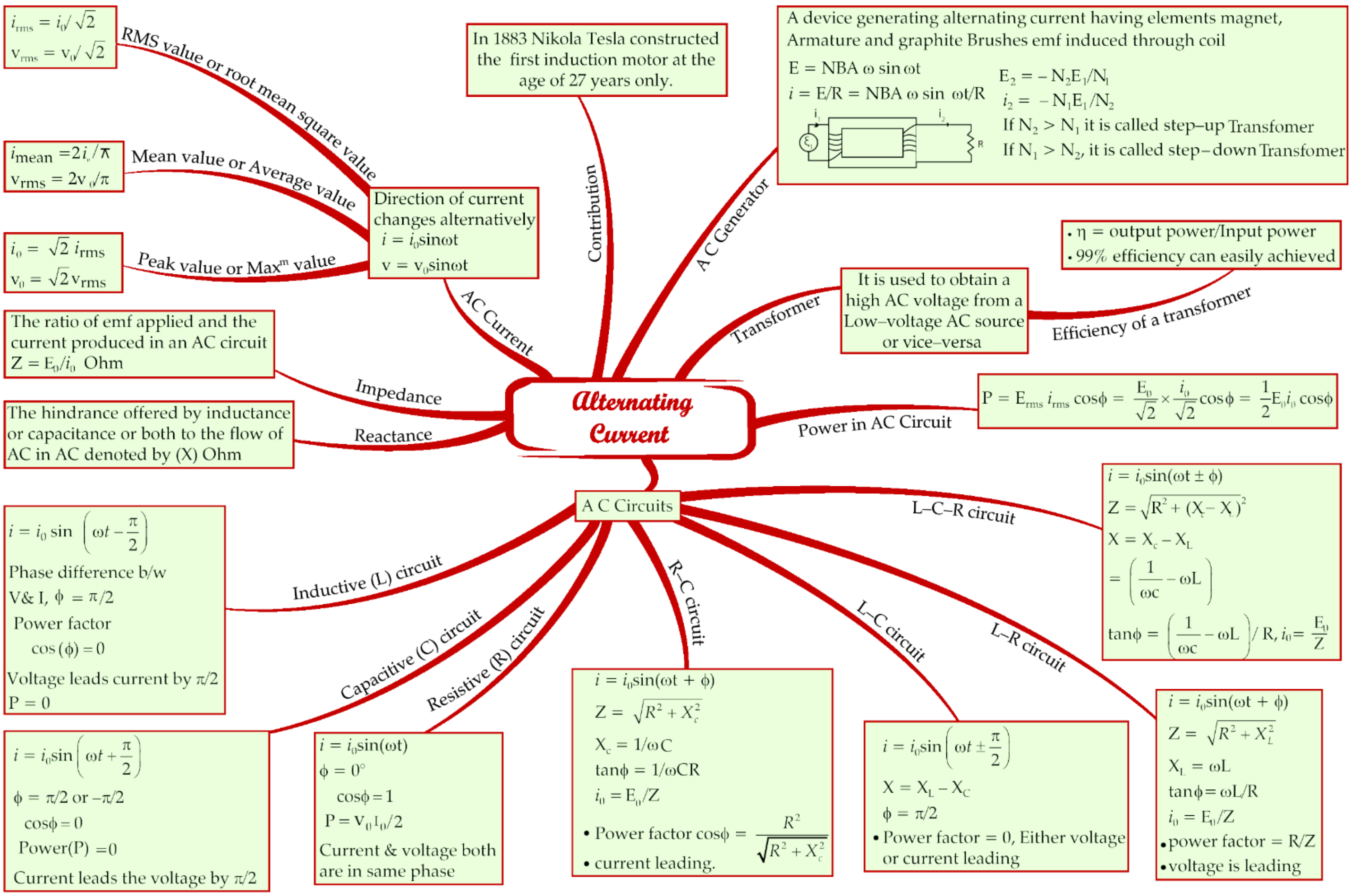
  $Q = 2\pi \times \frac{\text{Maximumenergystored}}{\text{Energydissipatedpercycle}}$

  $= \frac{2\pi}{T} \times \frac{\text{Maximumenergystored}}{\text{Mean powerdissipated}}$

  $Or$

  $Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$

# MIND MAP

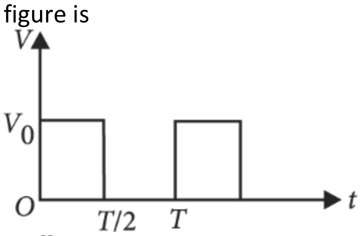

Alternating Current
$i_{rms} = i_0/\sqrt{2}$
$v_{rms} = v_0/\sqrt{2}$
RMS value or root mean square value
$i_{mean} = 2i_0/\pi$
$v_{rms} = 2v_0/\pi$
Mean value or Average value
$i_0 = \sqrt{2}\, i_{rms}$
$v_0 = \sqrt{2}\, v_{rms}$
Peak value or Max$^m$ value
Direction of current changes alternatively
$i = i_0 \sin\omega t$
$v = v_0 \sin\omega t$
AC Current
In 1883 Nikola Tesla constructed the first induction motor at the age of 27 years only.
Contribution
A C Generator
A device generating alternating current having elements magnet, Armature and graphite Brushes emf induced through coil
$E = NBA\,\omega \sin\omega t$
$i = E/R = NBA\,\omega \sin\omega t/R$
$E_2 = -N_2E_1/N_1$
$i_2 = -N_1E_1/N_2$
If $N_2 > N_1$ it is called step–up Transfomer
If $N_1 > N_2$, it is called step–down Transfomer
Transformer
It is used to obtain a high AC voltage from a Low–voltage AC source or vice–versa
Efficiency of a transformer
• η = output power/Input power
• 99% efficiency can easily achieved
The ratio of emf applied and the current produced in an AC circuit $Z = E_0/i_0$ Ohm
Impedance
The hindrance offered by inductance or capacitance or both to the flow of AC in AC denoted by (X) Ohm
Reactance
Power in AC Circuit
$P = E_{rms}\, i_{rms} \cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} E_0 i_0 \cos\phi$
A C Circuits
Inductive (L) circuit
$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$
Phase difference b/w V& I, $\phi = \pi/2$
Power factor
$\cos(\phi) = 0$
Voltage leads current by $\pi/2$
$P = 0$
Capacitive (C) circuit
$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
$\phi = \pi/2$ or $-\pi/2$
$\cos\phi = 0$
Power(P) = 0
Current leads the voltage by $\pi/2$
Resistive (R) circuit
$i = i_0 \sin(\omega t)$
$\phi = 0°$
$\cos\phi = 1$
$P = V_0 I_0/2$
Current & voltage both are in same phase
R–C circuit
$i = i_0 \sin(\omega t + \phi)$
$Z = \sqrt{R^2 + X_c^2}$
$X_c = 1/\omega C$
$\tan\phi = 1/\omega CR$
$i_0 = E_0/Z$
• Power factor $\cos\phi = \frac{R^2}{\sqrt{R^2 + X_c^2}}$
• current leading.
L–C circuit
$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$
$X = X_L - X_C$
$\phi = \pi/2$
• Power factor = 0, Either voltage or current leading
L–R circuit
$i = i_0 \sin(\omega t + \phi)$
$Z = \sqrt{R^2 + X_L^2}$
$X_L = \omega L$
$\tan\phi = \omega L/R$
$i_0 = E_0/Z$
• power factor = R/Z
• voltage is leading
L–C–R circuit
$i = i_0 \sin(\omega t \pm \phi)$
$Z = \sqrt{R^2 + (X_c - X_L)^2}$
$X = X_c - X_L$
$= \left(\frac{1}{\omega c} - \omega L\right)$
$\tan\phi = \left(\frac{1}{\omega c} - \omega L\right) / R,\ i_0 = \frac{E_0}{Z}$

# PRACTICE EXERCISE

## MCQ

**Q1.** In a series resonant LCR circuit, the voltage across R is 100 volts and R = 1 $k\Omega$ with C = $2\mu F$. The resonant frequency $\omega$ is 200 rad/s. At resonance, the voltage across L is
(a) $2.5 \times 10^{-2}$ V
(b) 40 $V$
(c) 250 $V$
(d) $4 \times 10^{-3}$ $V$

**Q2.** The r.m.s. value of potential difference $V$ shown in the figure is




(a) $\frac{V_0}{\sqrt{3}}$
(b) $V_0$
(c) $\frac{V_0}{\sqrt{2}}$
(d) $\frac{V_0}{2}$

**Q3.** An alternating voltage V = $V_0$ sin $\omega t$ is applied across a circuit. As a result, a current $I = I_0$ sin $(\omega t - \frac{\pi}{2})$ flows in it. The power consumed per cycle is
(a) Zero
(b) $0.5 V_0 I_0$
(c) 0.707 $V_0$ $I_0$
(d) 1.414 $V_0 I_0$

**Q4.** A coil of inductance 300 mH and resistance 2$\Omega$ is connected to a source of voltage 2V. The current half of its steady state value in
(a) 0.1 s
(b) 0.05 s
(c) 0.3 s
(d) 0.15 s

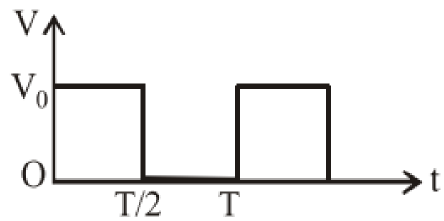
**Q5.** A bulb is rated at 100 V, 100 W, it can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.
(a) $\frac{\pi}{\sqrt{3}} H$
(b) 100 H
(c) $\frac{\sqrt{2}}{\pi} H$
(d) $\frac{\sqrt{3}}{\pi} H$

**Q6.** Two cables of copper are of equal lengths. One of them has a single wire of area of cross-section $A$, while other has 10 wires of cross-sectional area $A/10$ each. Give their suitability for transporting A.C. and D.C.
(a) only multiple strands for A.C., either for D.C.
(b) only multiple strands for A.C., only single strand for D.C.
(c) only single strand for D.C., either for A.C.
(d) only single strand for A.C., either for D.C.

**Q7.** Large transformers, when used for some time, become hot and are cooled by circulating oil. The heating of transformer is due to
(a) heating effect of current alone
(b) hysteresis loss alone
(c) both the hysteresis loss and heating effect of current
(d) None of the above

**Q8.** An ideal coil of 10H is connected in Series with a resistance of 5$\Omega$ and a battery of 5V. 2second after the connection is made, the current flowing in ampere in the circuit is
(a) $(1 - e^{-1})$(b) $(1 - e)$
(c) $e$(d) $e^{-1}$

**Q9.** For an LCR series circuit with an A.C. source of angular frequency $\omega$
(a) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
(b) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
(c) Power factor of circuit will be unity if capacitive reactance equals inductive reactance
(d) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$




**Q10.** The r.m.s. value of potential V difference V shown in the figure is
(a) $V_0$
(b) $V_0/\sqrt{2}$
(c) $V_0/2$
(d) $V_0$ $/\sqrt{3}$

**Q11.** Which of the following statements is/are incorrect?
(a) If the resonance is less sharp, not only is the maximum current less, the circuit is close to resonance for a larger range $\Delta\omega$ of frequencies and the tuning of the circuit will not be good.
(b) Less sharp the resonance less is the selectivity of the circuit or vice-versa.
(c) If quality factor is large, i.e., R is low or L is large, the circuit is more selective.
(d) Below resonance, voltage leads the current while above it, current leads the voltage.

**Q12.** A lamp consumes only 50% of peak power in an a.c. circuit. What is the phase difference between the applied voltage and the circuit current?
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$

**Q11.** A step-down transformer reduces 220 V to 110 V. the primary draws 5 ampere of current and secondary supplies 9 ampere. The efficiency of transformer is
(a) 20%
(b) 44%
(c) 90%
(d) 100%

**Q12.** A coil has resistance 30 ohm and inductive reactance 20 ohm at 50 Hz frequency. If an ac source, of 200-volt, 100 Hz, is connected across the coil, the current in the coil will be
(a) $4.0A$
(b) 8.0A

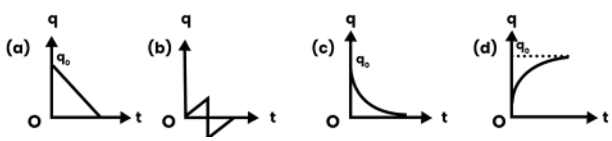
(c) $\frac{20}{\sqrt{13}}A$ (d) $2.0A$

**Q13.** A.C. power is transmitted from a power house at a high voltage as
(a) the rate of transmission is faster at high voltages
(b) it is more economical due to less power loss
(c) power cannot be transmitted at low voltages
(d) a precaution against theft of transmission lines

**Q14.** A transformer has an efficiency of 80%. It works at 4 kW and 100 V. If secondary voltage is 240 V, the current in primary coil is
(a) 0.4 A (b) 4 A
(c) 10 A (d) 40 A

**Q15.** A 12 Ω resistor and a 0.21 Henry inductor are connected in series to an a.c. source operating at 20 volts, 50 cycle. The phase angle between the current and source voltage is
(a) $30^0$ (b) $40^0$
(c) $80^0$(d) $90^0$

**Q16.** In LCR series circuit fed by a DC source, how does the amplitude of charge oscillations vary with time during discharge?




**Q17.** What is the value of inductance L for which the current is maximum in a series LCR circuit with
C = 10 $\mu F$ and $\omega = 1000s^{-1}$?
(a) 1 mH
(b) cannot be calculated unless R is known
(c) 10 mH
(d) 100 mH

**Q18.** The voltage of an ac source varies with time according to the equation V = 100 sin 100 $\pi t$ cos 100 $\pi t$ cos 100 $\pi t$ where t is in seconds and V is in volt. Then
(a) the peak voltage of the source is 100 volt
(b) the peak voltage of the source is 50 volt
(c) the peak voltage of the source is 100/ $\sqrt{2}$ volt
(d) the frequency of the source is 50 Hz

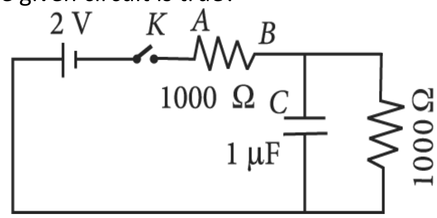
**Q19.** A 100 $\mu F$ capacitor in series with a $40\Omega$ resistance is connected to a 110 V, 60 Hz supply.
What is the maximum current in the circuit?
(a) 3.24 A (b) 4.25 A
(c) 2.25 A (d) 5.20 A

**Q20.** The core of any transformer is laminated so as to
(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

**Q23.** In an A.C. circuit, $I_{\text{rms}}$ and $I_0$ are related as
(a) $I_{\text{rms}} = \pi I_0$ (b) $I_{\text{rms}} = \sqrt{2}\, I_0$
(c) $I_{\text{rms}} = I_0/\pi$ (d) $I_{\text{rms}} = I_0/\sqrt{2}$

**Q24.** The time constant of $C$-$R$ circuit is
(a) $1/CR$ (b) $C/R$
(c) $CR$ (d) $R/C$

**Q25.** When the key $K$ is pressed at time $t$ = 0, then which of the following statement about the current $I$ in the resistor $AB$ of the given circuit is true?




(a) $I$ oscillates between 1 mA and 2 mA
(b) At $t$ = 0, $I$ = 2 mA and with time it goes to 1 mA
(c) $I$ = 1 mA at all $t$
(d) $I$ = 2 mA at all $t$.

## ASSERTION AND REASONING

**Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

**Q1.** Assertion: Average value of ac over a complete cycle is always zero.
Reason: Average value of ac is always defined over half cycle

**Q2.** Assertion: The voltage and current in a series AC circuit are given by $V = V_0 \sin \omega t$ and $i = i_0 \cos \omega t$. The power dissipated in the circuit is zero.
Reason: Power in AC circuit is given by $P = \frac{V_0 i_0}{2} \cos \phi$.

**Q3.** Assertion: The alternating current lags behind the emf by a phase angle of $\frac{\pi}{2}$, when AC flows through an inductor.
Reason: The inductive reactance increases as the frequency of AC source increases.

**Q4.** Assertion: The inductive reactance limits amplitude of the current in a purely inductive circuit. Reason: The inductive reactance is independent of the frequency of the current

**Q5.** Assertion: A laminated core is used in transformers to increase eddy currents. Reason: The efficiency of a transformer increases with increase in eddy currents.

## VERY SHORT ANSWER QUESTIONS

**Q1.** Define capacitor reactance. Write its SI units?

**Q2.** Why is the use of ac voltage preferred over dc voltage? Give two reasons.

**Q3.** What is wattless current?

**Q4.** Mention the two characteristic properties of the material suitable for making core of a transformer.

## SHORT ANSWER QUESTIONS

**Q1.** Explain why the reactance provided by a capacitor to an alternating current decrease with increasing frequency.

**Q2.** Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

## NUMERICAL TYPE QUESTIONS

**Q1.** A charged 40 $\mu$F capacitor is connected to a 16 mH inductor. What is the angular frequency of free oscillations of the circuit?

**Q2.** In an AC circuit, the instantaneous values of emf and current are $\varepsilon$ = 200 sin 314$t$ V and $I$ = sin (314$t$ + $\pi$/3) A. Then find the average power consumed.

**Q3.** In series *R-L-C* circuit, $L$ = 1.00 mH, $C$ = 1.00 nF and $R$= 200 Ω. For, the source applied with $V_m$ = 100 V, then determine the resonant frequency.

**Q4.** If an AC main supply is given to be 220 V. What would be the average emf during a positive half-cycle?

**Q5.** A resistor and a capacitor are connected in series with an a.c. source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, find the applied voltage.

**Q6.** The instantaneous value of current in an A.C. circuit is $\mathrm{I} = 2\sin(100\,\pi t + \pi/3)\mathrm{A}$. Then find the initial time at which current will be maximum.

**Q7.** In an L – R circuit, the value of $L$ is $\left(\frac{0.4}{\pi}\right)$ henry and the value of $R$ is 30 ohms. If in the circuit, an alternating *emf* of 200 volts at 50 cycles per sec is connected, then determine the impedance of the circuit and current.

**Q8.** One 10 V, 60 W bulb is to be connected to 100 V line. Then find the value of self-inductance that required for induction coil ($f$ = 50 Hz)

**Q9.** If the voltage in an ac circuit is represented by the equation, $V = 220\sqrt{2}\sin(314\mathrm{t} - \phi)$, then determine the Average voltage.

**Q10.** A series $LCR$ circuit is connected to an ac voltage source. When $L$ is removed from the circuit, the phase difference between current and voltage is $\pi/3$. If instead $C$ is removed from the circuit, the phase difference is again $\pi/3$ between current and voltage. Then determine the value of power factor of the circuit.

# HOMEWORK EXERCISE

## MCQ

**Q1.** A coil of self-inductance $L$ is connected in series with a bulb $B$ and an $AC$ source. Brightness of the bulb decreases when
(a) a capacitance of reactance $X_C = X_L$ is included in the same circuit.
(b) an iron rod is inserted in the coil.
(c) frequency of the $AC$ source is decreased.
(d) number of turns in the coil is reduced.

**Q2.** A circuit when connected to an AC source of 12 V gives a current of 0.2 A. The same circuit when connected to a DC source of 12 V, gives a current of 0.4 A. The circuit is
(a) series $LR$ (b) series $RC$
(c) series $LC$ (d) series $LCR$

**Q3.** An ac voltage is applied to a resistance $R$ and an inductor $L$ in series. If $R$ and the inductive reactance are both equal to 3 Ω, the phase difference between the applied voltage and the current in the circuit is
(a) $\pi/6$ (b) $\pi/4$
(c) $\pi/2$ (d) zero

**Q4.** What is the value of inductance $L$ for which the current is maximum in a series $LCR$ circuit with $C$ = 10 μF and ω = 1000 $s^{-1}$?
(a) 1 mH
(b) cannot be calculated unless $R$ is known
(c) 10 mH
(d) 100 mH

**Q5.** The instantaneous values of alternating current and voltages in a circuit are given as
$i = \frac{1}{\sqrt{2}}\sin(100\,\pi t)$ ampere
$e = \frac{1}{\sqrt{2}}\sin\left(100\,\pi t + \frac{\pi}{3}\right)$ volt
The average power in watts consumed in the circuit is
(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

**Q6.** In an a.c. circuit the e.m.f. $(\varepsilon)$ and the current $(i)$ at any instant are given respectively by
$e = E_0 \sin \omega t, i = I_0 \sin(\omega t - \phi)$
The average power in the circuit over one cycle of a.c. is
(a) $\frac{E_0 I_0}{2}\cos\phi$ (b) $E_0 I_0$
(c) $\frac{E_0 I_0}{2}$ (d) $\frac{E_0 I_0}{2}\sin\phi$

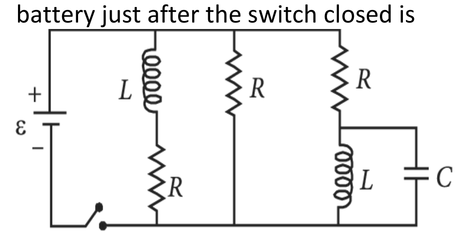
**Q7.** The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an $L$-$C$-$R$ circuit. The power factor of this circuit is
(a) 0.4 (b) 0.5
(c) 0.8 (d) 1.0

**Q8.** A transistor-oscillator using a resonant circuit with an inductor $L$ (of negligible resistance) and a capacitor $C$ in series produces oscillations of frequency $f$. If $L$ is doubled and $C$ is changed to $4C$, the frequency will be
(a) $f/2$(b) $f/4$
(c) $8f$(d) $f/2\sqrt{2}$

**Q9.** The primary of a transformer when connected to a dc battery of 10 volt draws a current of 1 mA. The number of turns of the primary and secondary windings are 50 and 100 respectively. The voltage in the secondary and the current drawn by the circuit in the secondary are respectively
(a) 20 V and 2.0 mA (b) 10 V and 0.5 mA
(c) Zero volt and therefore no current(d) 20 V and 0.5 mA

**Q10.** The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux ϕ linked with the primary coil is given by $\phi = \phi_0 + 4t$, where ϕ is in webers, $t$ is time in seconds and $\phi_0$ is a constant, the output voltage across the secondary coil is
(a) 120 volts (b) 220 volts
(c) 30 volts (d) 90 volts

**Q11.** Figure shows a circuit that contains three identical resistors with resistance $R$ = 9.0 Ω each, two identical inductors with inductance $L$ = 2.0 mH each, and an ideal battery with emf $\varepsilon$ = 18 V. The current $i$ through the battery just after the switch closed is




(a) 0.2 A (b) 4 A
(c) 0 ampere (d) 2 mA

**Q12.** A 40 μF capacitor is connected to a 200 V, 50 Hz ac supply. The r.m.s value of the current in the circuit is, nearly
(a) 1.7 A (b) 2.05 A
(c) 2.5 A (d) 25.1 A

**Q13.** In an ac circuit an alternating voltage $200\sqrt{2}\ \sin 100t$ volts is connected to a capacitor of capacity 1 μF. The r.m.s. value of the current in the circuit is
(a) 10 mA (b) 100 mA
(c) 200 mA (d) 20 mA

**Q14.** A series $R$-$C$ circuit is connected to an alternating voltage source. Consider two situations
(i) When capacitor is air filled.
(ii) When capacitor is mica filled.
Current through resistor is $i$ and voltage across capacitor is $V$ then
(a) $i_a > i_b$ (b) $V_a = V_b$
(c) $V_a < V_b$ (d) $V_a > V_b$

**Q15.** The value of quality factor is
(a) $\frac{\omega L}{R}$ (b) $\frac{1}{\omega RC}$
(c) $\sqrt{LC}$ (d) $L/R$

## ASSERTION AND REASONING

**Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

Q1. Assertion: A capacitor is connected to a direct current source. Its reactance is infinite.
Reason: Reactance of a capacitor is given by $X_C = \frac{1}{\omega C}$

**Q2.** Assertion: In a purely inductive or capacitive circuit, the current is referred to as wattless current.
Reason: No power is dissipated in a purely inductive or capacitive circuit even though a current is flowing in the circuit.

**Q3.** Assertion: The power in an ac circuit is minimum if the circuit has only a resistor.
Reason: Power of a circuit is independent of the phase

**Q4.** Assertion: When the frequency of the AC source in an LCR circuit equals the resonant frequency, the reactance of the circuit is zero, and so there is no current through the inductor or the capacitor.
Reason: The net current in the inductor and capacitor is zero.

**Q5.** Assertion: In series LCR resonance circuit, the impedance is equal to the ohmic resistance.
Reason: At resonance, the inductive reactance exceeds the capacitive reactance.

## VERY SHORT ANSWER QUESTIONS

**Q1.** Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

**Q2.** Define power factor. State the conditions under which it is (i) maximum and (ii) minimum.

**Q3.** When an ac source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.

**Q4.** When an $ac$ source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero.

**Q5.** The current flowing through a pure inductor of inductance 2mH is $i = 15\cos 300t$ ampere. What is the (i) rms and (ii) average value of current for a complete cycle?

## SHORT ANSWER QUESTIONS

**Q1.** State the underlying principle of a transformer. How is the large-scale transmission of electric energy over long distances done with the use of transformers?

**Q2.** State the principle of working of a transformer. Can a transformer be used to step up or step down a dc voltage? Justify your answer.

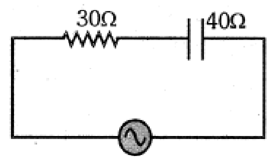
## NUMERICAL TYPE QUESTIONS

**Q1.** If E = 20 sin (100π t) volt then calculate value of E at t = $\frac{1}{600} s$

**Q2.** If Phase Difference between E and I is $\frac{\pi}{4}$ and f = 50 Hz then calculate time difference.

**Q3.** A 50 W, 100 V lamp is to be connected to an AC main of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp?

**Q4.** Calculate the impedance of the circuit shown in the figure.




**Q5.** When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduce to 2 A. Calculate reactance of the coil

**Q6.** In LCR circuit with an AC source R = 300 Ω, C = 20 μF, L = 1.0 H, $E_{rms}$ = 50V and f = 50/π Hz. Find RMS current in the circuit.

**Q7.** A coil, a capacitor and an A C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a battery of emf 12 V, and internal resistance 40, then calculate the current through the coil.

**Q8.** An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0
(a) What is the total energy stored initially.
(b) What is the natural frequency of the circuit.
(c) At what minimum time is the energy stored is completely magnetic.
(d) At what minimum time is the total energy shared equally between inductor and th capacitor.

**Q9.** A circuit is set up by connecting inductance $L = 100\text{mH}$, resistor $R = 100\Omega$ and a capacitor of reactance $200\Omega$ in series. An alternating emf of $150\sqrt{2}\,\text{V}, 500/\pi\text{Hz}$ is applied across this series combination. Calculate the power dissipated in the resistor.

**Q10.** A capacitor of unknown capacitance, a resistor of 100Ω and an inductor of self inductance $L = \left(\frac{4}{\pi^2}\right)$ henry is connected in series to an ac source of 200 V and 50 Hz. Calculate the value of the capacitance and impedance of the circuit when the current is in phase with the voltage. Calculate the power dissipated in the circuit.

# PRACTICE EXERCISE SOLUTIONS

## MCQ

**S1.** (c) Across resistor, I = $\frac{V}{R} = \frac{100}{1000} = 0.1\ A$

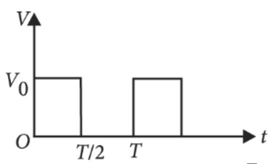
At resonance,

$X_L = X_C = \frac{1}{\omega C} = \frac{1}{200 \times 2 \times 10^{-6}} = 2500$

Voltage across L is

$I\ X_L = 0.1 \times 2500 = 250\ V$

**S2.** (c)




$V = V_0$ for $0 \le t \le \frac{T}{2}$

$T = 0$ for $\frac{T}{2} \le t \le T$

$$V_{\text{rms}} = \left[\frac{\int_0^T V^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{\int_0^{\frac{T}{2}} V_0^2 dt + \int_{\frac{T}{2}}^T (0) dt}{2}\right]^{1/2}$$

$$= \left[\frac{V_0^2}{T}[t]_0^{T/2}\right]^{1/2} = \left[\frac{V_0^2}{T}\left(\frac{T}{2}\right)\right]^{1/2} = \left[\frac{V_0^2}{2}\right]^{1/2}$$

$\therefore V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

**S3.** (a) The phase angle between voltage V and current I is $\frac{\pi}{2}$. Therefore, powe factor $\cos\phi = \cos\left(\frac{\pi}{2}\right) = 0$. Hence the power consumed is zero.

**S4.** (a) The charging of inductance given by,

$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

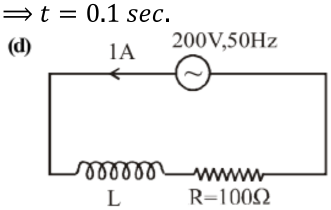
$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{Rt}{L}}\right) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$

Taking log on both the sides,

$-\frac{Rt}{L} = log\ 1 - log 2$

$\Rightarrow$ t = $\frac{L}{R}\ log 2 = \frac{300 \times 10^{-3}}{2} \times 0.69$

$\Rightarrow t = 0.1\ sec.$




**S5.** From the rating of the bulb, the resistance of the bulb can be calculated.

$R = \frac{V_{rms}^2}{P} = 100\Omega$

For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $I_{rms} = \frac{V_{rms}}{Z}$

$\therefore 1 = \frac{200}{\sqrt{100^2 + (2\pi 50.L)^2}}$

$L = \frac{\sqrt{3}}{\pi} H$

**S6.** (a) The major portion of the A.C. flows on the surface of the wire. So, where a thick wire is required, a number of thin wires are joined together to give an equivalent effect of a thick wire. Therefore, multiple strands are suitable for transporting A.C. Similarly multiple strands can also be used for D.C.

**S7.** (c) Heat loss in a magnetic circuit is due to two reasons one is hysteresis and other is due to ohmic loss due to eddy current. Laminations and circulating oil are some of methods to avoid heating. Hysteresis loss is entirely material property while eddy current loss depends on geometry of core and amount of current.

**S8.** (a) $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

(When current is in growth in LR circuit)

$= \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times 2}\right)$

= $(1 - e^{-1})$

**S9.** (c) The circuit will have inductive nature if

$\omega > \frac{1}{\sqrt{LC}}\left(\omega L > \frac{1}{\sqrt{LC}}\right)$

Hence (a) is false. Also, if circuit has inductive nature the current will lag behind voltage.

Hence (d) is also false.

If $\omega = \frac{1}{\sqrt{LC}}\left(\omega L = \frac{1}{\omega C}\right)$ the circuit resistance nature.

Hence (b) is false.

Power factor

$\cos\phi = \frac{2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1\ if\ \omega L = \frac{1}{\omega C}$

**S10.** (b) $V_{rms} = \sqrt{\frac{\left(\frac{T}{2}\right)V_0{}^2 + 0}{T}} = \frac{V_0}{\sqrt{2}}$.

**S11.** (d) Option (d) is false because the reason why the voltage leads the current is because $\frac{1}{C\omega} > L\omega$ and if the voltage lags, the inductive reactance is greater than the capacitive reactance.

**S12.** (b) $P = \frac{1}{2}V_0 i_0\ cos\ \phi \Rightarrow P = P_{peak}.cos\ \phi$

$\Rightarrow \frac{1}{2}\left(P_{peak}\right) = \left(P_{peak}\right)cos\phi \Rightarrow cos\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

**S13.** (c) $\eta = \frac{E_S I_S}{E_P I_P} \therefore \eta = \frac{110 \times 9}{220 \times 5} = 0.9 \times 100\% = 90\%$

**S14.** (a) If $\omega = 50 \times 2\pi$ then $\omega L = 20\Omega$

If $\omega'$ = 100 × $2\pi$ then $\omega' L$ = 40Ω

Current flowing in the coil is

$I = \frac{200}{z} = \frac{200}{\sqrt{R^2 + (\omega' L)^2}} = \frac{200}{\sqrt{(30)^2 + (40)^2}}$

I = 4A.

**S15.** (b) AC power is transmitted from a power house at a high voltage as it is more economical due to less power loss.

**S16.** (d) As $E_P\, I_P = P_i$ $\therefore I_P = \frac{P_i}{E_P} = \frac{4000}{100} = 40A$. m

**S17.** (c) The phase angle is given by
$\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.21}{12} = 5.5$

S18. (c) since the change around capacitor or an inductor always varies in exponential terms the graph has a curve and also due to resistor there is conversion of electrical energy to heat energy which give that graph has to be strictly decreasing and will eventually stop at zero. option (C) satisfies exponential curve, strictly decreasing and stops at zero.

**S19.** (d) Condition for which the current is maximum in a series LCR circuit is,
$\omega = \frac{1}{\sqrt{LC}}$
$1000 = \frac{1}{\sqrt{L(10 \times 10^{-6})}}$
$\Longrightarrow = L = 100\ mH$

**S20.** (b) V = 50 × 2 sin $100\pi$ cos $100\pi t = 50$ sin $200\ \pi t$
$\Rightarrow V_0 = 50 Volts$ and v = 100 Hz

**S21.** (a) Here, C = $100\ \mu F = 100 \times 10^{-6}\ F, R = 40\Omega, V_{rms} = 110\ V$, f = 60 Hz
Peak voltage,
$V_0 = \sqrt{2}\,.V_{rms} = 100\sqrt{2} = 155.54\ V$
Circuit impedance,
$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$
$= \sqrt{40^2 + \frac{1}{(2 \times \pi \times 60 \times 100 \times 10^{-6})^2}}$
$= \sqrt{40^2 + \frac{1}{(2\times \pi \times 60 \times 100 \times 10^{-6})^2}}$
$= \sqrt{1600 + 703.60} = \sqrt{2303.60} = 48\Omega$
Hence, maximum current in coil,
$I_0 = \frac{V_0}{Z} \frac{155.54}{48} = 3.24\ A$

**S22.** (a) Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decrease thereby decreasing the eddy current losses.

**S23.** (d) The relation between $I_{\text{rms}}$ and $I_0$is
$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

**S24.** (c) The time constant for $R$–$C$. circuit, $\tau = CR$
Growth of charge in a circuit containing capacitance and resistance is given by the formula,
$q = q_0(1 - e^{-t/CR})$
$CR$ is known as time constant in this formula.

**S25.** (b) Initially, the current will pass through the capacitor (and not through the resistance which is parallel to the capacitor). So effective resistance in the circuit is $R_{AB}$. Therefore the current in the resistor is 2 mA. After some time, the capacitor will become fully charged and will be in its steady state. Now no current will pass through the capacitor and the effective resistance of the circuit that is
$(1000 + 1000) = 2000\ \Omega$.
Therefore, final current in the resistor
$= \frac{V}{R} = \frac{2}{2000} = 1 \times 10^{-3}\text{A} = 1$ mA

## ASSERTION AND REASONING

**S1.** (b) The means or average value of alternating current or e.m.f during a half cycle is given by $I_m = 0.636 I_0$ or $E_m = 0.636 E_0$ During the next half cycle, the mean value of ac will be equal in magnitude but opposite in direction. For this reason, the average value of ac over a complete cycle is always zero. So, the average value is always defined over a half cycle of ac.

**S2.** (a) $V = V_0 \sin\omega t i = i_0 \cos\omega t = i_0 \sin(\omega t + \pi/2)$ $\therefore$ $\phi = \frac{\pi}{2}$, and $\cos\phi = 0$.

**S3.** (b) In case of inductive circuit emf leads current by $\pi/2$ rad

**S4.** (c) The inductive reactance limits the amplitude of current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.
i.e. $I_0 = \frac{\varepsilon_0}{X_L}$

**S5.** (d) Large eddy currents are produced in non-laminated iron core of the transformer by the induced emf, as the resistance of bulk iron core is very small. By using thin iron sheets as core, the resistance is increased. Laminating the core substantially reduces the eddy currents. Eddy current heats up the core of the transformer. More the eddy currents greater is the loss of energy and the efficiency goes down.

## VERY SHORT ANSWER QUESTIONS

**S1.** The imaginary/virtual resistance offered by a capacitor to the flow of an alternating current is called capacitor reactance, $X_C = \frac{1}{\omega C}$. Its SI unit is ohm.

**S2.** (i) The generation of ac is more economical than dc.
(ii) Alternating voltage can be stepped up or stepped down as per requirement during transmission from power generating station to the consumer.
(iii) Alternating current in a circuit can be controlled by using wattless devices like the choke coil.
(iv) Alternating voltages can be transmitted from one place to another, with much lower energy loss in the transmission line.

**S3.** When pure inductor and/or pure capacitor is connected to ac source, the current flows in the circuit, but with no power loss; the phase difference between voltage and current is . 2 r Such a current is called the wattless current.

**S4.** Two characteristic properties: (i) Low hysteresis loss (ii) Low coercivity

## SHORT ANSWER QUESTIONS

**S1.** A capacitor does not allow flow of direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this changing voltage (or charge). Thus, a capacitor will pass more current through it if the voltage is changing at a faster rate, i.e., if the frequency of supply is higher. This implies that the reactance offered by a capacitor is less with increasing frequency; it is given by $1/\omega C$

**S2.** An inductor opposes flow of current through it by developing an induced emf according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and vice versa. Since the induced emf is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change is faster, i.e., if the frequency is higher. The reactance of an inductor, therefore, is proportional to the frequency, being given by L.

## NUMERICAL TYPE QUESTIONS

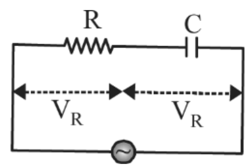
**S1.** Given, $C$ = 40 $\mu$F = 40 × $10^{-6}$ F,
and $L$ = 16 mH = 16 × $10^{-3}$ H
Angular frequency of oscillating circuit,
$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16\times10^{-3})(40\times10^{-16})}}$
$= \frac{10^4}{8} = 1.25 \times 10^3\ s^{-1}$

**S2.** Given, $\varepsilon = 200$ sin 314$t$ and $I$ = sin (314$t$ + $\pi$ / 3)
$\therefore \quad \phi = \frac{\pi}{3}, V_m = 200\ V, I_m = 1A$
$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}}$
$i_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1}{\sqrt{2}}$
$\therefore$ Average power consumed is,
$P = V_{rms}\, i_{rms} \cos\phi = \frac{200}{\sqrt{2}} \frac{1}{\sqrt{2}} \cos \pi/3$
$= \frac{200}{2} \times \frac{1}{2} = 50\ W$

**S3.** Given, $L$ = 1 mH = 1 × $10^{-3}$ H and $C$ = 1 nF = 1 × $10^{-9}$ F
Resonant frequency of $R$-$L$-$C$ series circuit,
$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3}\times1\times10^{-9}}}$
$= 1 \times 10^6$ rad /s

**S4.** Given, $V_{rms} = 220\ V$
$V_{av} = \frac{2}{\pi} V_m = \frac{2}{\pi} \times (V_{rms} \times \sqrt{2}) = \frac{2\sqrt{2}}{\pi} V_{rms}$
$= \frac{2\sqrt{2}}{\pi} \times 220 = 198\ V$

**S5.**

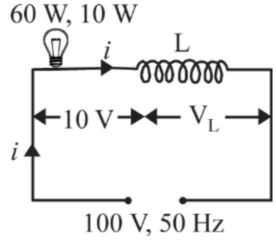



Let the applied voltage be V, volt.
Here, $V_R = 12$ V, $V_C = 5$V
$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(12)^2 + (5)^2} = 13$ V

**S6.** Current will be max at first time when
$100\,\pi t + \pi/3 = \pi/2 \Rightarrow 100\,\pi t = \pi/6 \quad \Rightarrow t = 1/600\ s.$

**S7.** $Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (2\pi f L)^2}$
$= \sqrt{(30)^2 + \left(2\pi \times 50 \times \frac{0.4}{\pi}\right)^2} = \sqrt{900 + 1600} = 50\Omega$
$i = \frac{V}{Z} = \frac{200}{50} = 4$ ampere

**S8.** Current through the bulb $i = \frac{P}{V} = \frac{60}{10} = 6$A



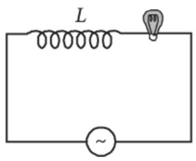

$V = \sqrt{V_R^2 + V_L^2}$
$(100)^2 = (10)^2 + V_L^2 \Rightarrow V_L = 99.5$ Volt
Also $V_L = iX_L = i \times (2\pi v L)$
$\Rightarrow \quad 99.5 = 6 \times 2 \times 3.14 \times 50 \times L \Longrightarrow L = 0.052$ H.

**S9.** As in case of ac,
$V = V_0 \sin(\omega t - \phi)$
The peak value $V_0 = 220\sqrt{2} = 311$ V
and as in case of ac,
$V_{rms} = \frac{V_0}{\sqrt{2}}; V_{rms} = 220$ V

**S10.** When $L$ is removed
$\tan\phi = \frac{|X_C|}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_C}{R}$ ...(i)
When $C$ is removed,
$\tan\phi = \frac{|X_L|}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_L}{R}$ ...(ii)
From (i) and (ii), $X_C = X_L$.
Since, $X_L = X_C$, the circuit is in resonance.
$Z = R$
Power factor, $\cos\phi = \frac{Z}{R} = \frac{R}{R} = 1$

## HOMEWORK EXERCISE SOLUTIONS

**S1.** **(b)**




The situation is as shown in the figure.
As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron, increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied AC voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the brightness of the light bulb decreases.

**S2.** (a) When circuit is connected to an AC source of 12 V, gives a current of 0.2 A.

∴ Impedance, $Z = \frac{12}{0.2} = 60\ \Omega$

When the same circuit is connected to a DC source of 12 V, gives a current of 0.4 A.

∴ Resistance, $R = \frac{12}{0.4} = 30\ \Omega$

As, power factor, $\cos\phi = \frac{R}{Z} = \frac{30}{60} = \frac{1}{2} = \cos 60°$

⇒ $\phi = 60°$, *i.e.*, current lags behind the emf.

So, we can conclude that the circuit is a series $LR$.

**S3.** (b) Here, $R = 3\ \Omega, X_L = 3\ \Omega$

The phase difference between the applied voltage and the current in the circuit is

$\tan\phi = \frac{X_L}{R} = \frac{3\Omega}{3\Omega} = 1$ or $\phi = \tan^{-1}(1) = \frac{\pi}{4}$

**S4.** (d) In series $LCR$, current is maximum at resonance.

∴ Resonant frequency, $\omega = \frac{1}{\sqrt{LC}}$

∴ $\omega^2 = \frac{1}{LC}$ or, $L = \frac{1}{\omega^2 C}$

Given $\omega = 1000\ \text{s}^{-1}$ and $C = 10\ \mu\text{F}$

∴ $L = \frac{1}{1000 \times 1000 \times 10 \times 10^{-6}} = 0.1\ \text{H} = 100\ \text{mH}$

**S5.** (d) Given : $i = \frac{1}{\sqrt{2}}\sin(100\pi t)$ ampere

Compare it with $i = i_0 \sin(\omega t)$, we get

$i_0 = \frac{1}{\sqrt{2}}\text{A}$

Given : $e = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right)$ volt

Compare it with $e = e_0 \sin(\omega t + \phi)$, we get

$e_0 = \frac{1}{\sqrt{2}}\text{V}, \phi = \frac{\pi}{3}$

∴ $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{1}{2}\text{A}$ and $e_{\text{rms}} = \frac{e_0}{\sqrt{2}} = \frac{1}{2}\text{V}$

Average power consumed in the circuit,

$P = i_{\text{rms}} e_{\text{rms}} \cos\phi$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\cos\frac{\pi}{3} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}\text{W}$

**S6.** (a) Average power $= \frac{E_0 I_0}{2}\cos\phi$

**S7.** (d) $X_L = 31\ \Omega, X_C = 25\ \Omega, R = 8\ \Omega$

Impedance of series $LCR$ is

$Z = \sqrt{(R^2) + (X_L - X_C)^2}$

$= \sqrt{(8)^2 + (31 - 25)^2} = \sqrt{64 + 36} = 10\ \Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8$

**S8.** (d) Frequency of $LC$ oscillation $= \frac{1}{2\pi\sqrt{LC}}$

or, $\frac{f_1}{f_2} = \left(\frac{L_2C_2}{L_1C_1}\right)^{1/2} = \left(\frac{2L \times 4C}{L \times C}\right)^{1/2} = (8)^{1/2}$

∴ $\frac{f_1}{f_2} = 2\sqrt{2} \Rightarrow f_2 = \frac{f_1}{2\sqrt{2}}$ or, $f_2 = \frac{f}{2\sqrt{2}}$ $(\because f_1 = f)$

**S9.** (c) Transformer cannot work on dc.

∴ $V_s = 0$ and $I_s = 0$

**S10.** (a) No. of turns across primary $N_p$ = 50

Number of turns across secondary $N_s$ = 1500

Magnetic flux linked with primary, $\phi = \phi_0 + 4t$

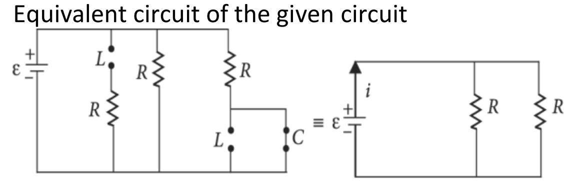
∴ Voltage across the primary,

$V_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t) = 4$ volt

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$ or $V_s = \left(\frac{1500}{50}\right) \times 4 = 120\ \text{V}$

**S11.** (b) At time, $t$ = 0 *i.e.*, when switch is closed, inductor in the circuit provides very high resistance (open circuit) while capacitor starts charging with maximum current (low resistance).

Equivalent circuit of the given circuit




Current drawn from battery,

$i = \frac{\varepsilon}{(R/2)} = \frac{2\varepsilon}{R} = \frac{2 \times 18}{9} = 4\ \text{A}$

**S12.** (c) Here, $C = 40\mu\text{F} = 40 \times 10^{-6}$ F

$\text{V}_{\text{rms}} = 200\ \text{V}; \upsilon = 50\ \text{Hz}$

The value of the current, $I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{\frac{1}{\omega C}} = \varepsilon_{\text{rms}}\omega C$

or $I_{\text{rms}} = 200\ (2\pi \times 50) \times (40 \times 10^{-6}) = 2.51$ A $(\because \omega = 2\pi\upsilon)$

**S13.** (d) The given equation of alternating voltage is

$e = 200\sqrt{2}\sin 100t$ ...(i)

The standard equation of alternating voltage is

$e = e_0 \sin\omega t$ ...(ii)

Comparing (i) and (ii), we get

$e_0 = 200\sqrt{2}\ \text{V}, \omega = 100\ \text{rad s}^{-1}$

The capacitive reactance is

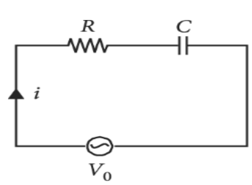
$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 1 \times 10^{-6}}\Omega$

The r.m.s. value of the current in the circuit is

$i_{\text{r.m.s.}} = \frac{v_{\text{r.m.s.}}}{X_C} = \frac{e_0/\sqrt{2}}{1/\omega C} = \frac{(200\sqrt{2}/\sqrt{2})}{(1/100 \times 10^{-6})}$

$= 200 \times 100 \times 10^{-6}\text{A} = 2 \times 10^{-2}\text{A} = 20\ \text{mA}$

**S14.** (d)



Current through resistor,
$i$ = Current in the circuit
$= \frac{V_0}{\sqrt{R^2+X_C^2}} = \frac{V_0}{\sqrt{R^2+(1/\omega C)^2}}$
Voltage across capacitor, $V = iX_C$
$= \frac{V_0}{\sqrt{R^2+(1/\omega C)^2}} \times \frac{1}{\omega C} = \frac{V_0}{\sqrt{R^2\omega^2C^2+1}}$
As $C_a < C_b$
$\therefore$ $i_a < i_b$ and $V_a > V_b$

**S15.** (a b) Quality factor, $Q = \frac{\omega L}{R}$
Since $\omega^2 = \frac{1}{LC}$
$\therefore$ Quality factor, $Q = \frac{1}{\omega RC}$

## ASSERTION AND REASONING

**S1.** (a) As $X_C = \frac{1}{\omega C}$, so for $\omega = 0, X_C \rightarrow \infty$.

**S2.** (a) In a purely inductive or capacitive circuit, power factor, $\cos\phi = 0$ and no power is dissipated even though a current is flowing in the circuit. In such cases, current is referred to as wattless current.

**S3.** (d) Power in a series ac circuit consisting of $L, C$ and $R$ is given by
$P = I_{\text{rms}}V_{\text{mss}}\cos\phi$ where $\phi = \tan^{-1}\left(\frac{|X_L - X_C|}{R}\right)$
For a purely resistive circuit $X_L = 0$ and $X_C = 0$
Therefore, $\tan\phi = 0$ or $\phi = 0$ and thereby $\cos\phi = 1$ and $P = IV$.
The power is maximum as $\cos\phi$ is maximum. Power depends on the phase angle through the power factor $\cos\phi$

**S4.** (d) The currents in capacitor and in inductor are opposite and so net current is zero.

**S5.** (c) In series resonance circuit, inductive reactance is equal to capacitive reactance. i.e. $\omega L = \frac{1}{\omega C}$
$\therefore Z = \sqrt{R^2 + \left(\omega L = \frac{1}{\omega C}\right)^2} = R$

## VERY SHORT ANSWER QUESTIONS

**S1.** An ac current changes direction with the source frequency and the attractive force would average to zero. Thus, the ac ampere must be defined in terms of some property that is independent of the direction of current. Joule's heating effect is such property and hence it is used to define rms value of ac

**S2.** The power factor $(\cos\phi)$ is the ratio of resistance and impedance of an $ac$ circuit $i.e.$,
Power factor, $\cos\phi = \frac{R}{Z}$.
Maximum power factor is 1 when $Z = R$ i.e., when circuit is purely resistive. Minimum power factor is 0 when $R = 0$ i.e., when circuit is purely inductive or capacitive.

**S3.** For an ideal inductor phase difference between current and applied voltage $= \pi/2$
$\therefore$ Power, $P = V_{rms}I_{rms}\cos\phi = V_{rms}I_{rms}\cos\frac{\pi}{2} = 0.$
Thus, the power consumed in a pure inductor is zero.

**S4.** Power dissipated in $ac$ circuit, $P = V_{rms}I_{rms}\cos\phi$
where $\cos\phi = \frac{R}{Z}$
For an ideal capacitor $R = 0$ $\therefore \cos\phi = \frac{R}{Z} = 0$
$\therefore$ $P = V_{rms}I_{rms}\cos\phi = V_{rms}I_{rms} \times 0 = 0$ (zero).
i.e., power dissipated in an ideal capacitor is zero.

**S5.** Peak value of current $(i_0) = 15$ A
(i) $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} = \frac{15}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 7.5\sqrt{2}\mathbf{A}$
(ii) $i_{gv} = \mathbf{0}$

## SHORT ANSWER QUESTIONS

**S1.** The principle of transformer is based upon the principle of mutual induction which states that due to continuous change in the current in the primary coil an emf gets induced across the secondary coil. At the power generating station, the step-up transformers step up the output voltage which reduces the current through the cables and hence reduce resistive power loss. Then, at the consumer end, a step-down transformer steps down the voltage. Hence, the large-scale transmission of electric energy over long distances is done by stepping up the voltage at the generating station to minimize the power loss in the transmission cables.

**S2.** Working of a transformer is based on the principle of mutual induction. Transformer cannot step up or step down a dc voltage. Reason: No change in magnetic flux. Explanation: When dc voltage source is applied across a primary coil of a transformer, the current in primary coil remains same, so there is no change in magnetic flux associated with it and hence no voltage is induced across the secondary coil.

## NUMERICAL TYPE QUESTIONS

**S1.** At t = $\frac{1}{600}s$ $E = 20\,sin\left[100\pi \times \frac{1}{600}\right] = 20\,sin\left[\frac{\pi}{6}\right]$ = $20 \times \frac{1}{2}$ = 10V

**S2.** Q 2p ° T
$\frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T}$
$\Rightarrow$ Time difference = $\frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50\times 8}$ = 2.5 ms

**S3.** resistance of the lamp R = $\frac{V_S^2}{W} = \frac{(100)^2}{50}$ = 200 W and the maximum current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}A$
when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400\,\Omega$ Now as in case of C–R circuit

Z = $\sqrt{R^2 + \frac{1}{(\omega C)^2}}$,

$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \Rightarrow \frac{1}{(\omega\, C)^2}$ = 16 × $10^4$ – $(200)^2$ = 12 × $10^4$ ⇒ $\frac{1}{\omega C} = \sqrt{12} \times 10^2$

⇒ C = $\frac{1}{100\pi \times \sqrt{12} \times 10^2} F$

$= \frac{100}{\pi\sqrt{12}} \mu F$ = 9.2mF

**S4.** Z = $\sqrt{R^2 + (X_C)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500}$ = 50 W

**S5.** For 10 V D.C. Q V = IR

Resistance of coil R = $\frac{10}{2.5}$ = 4W

For 10 V A.C. V = IZ ⇒ $Z = \frac{V}{I} = \frac{20}{10}$ = 5W

Q $Z = \sqrt{R^2 + X_L^2}$ = 5 ⇒ $R^2 + X_L^2 = 25$

⇒ $X_L^2 = 5^2 - 4^2$ ⇒ $X_L$ = 3 W

**S6.** $I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}}$

$= \frac{50}{\sqrt{300^2 + \left[2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}}\right]^2}}$

ss $I_{rms} = \frac{50}{\sqrt{(300)^2 + \left[100 - \frac{10^3}{2}\right]^2}} = \frac{50}{100\sqrt{9+16}} = \frac{1}{10}$

= 0.1A

**S7.** At resonance current is maximum, I = $\frac{V}{R}$

⇒Resistance of coil R = $\frac{V}{I} = \frac{24}{6}$ = 4 W

When coil is connected to battery,

suppose I current flow through it then I = $\frac{E}{R+r} = \frac{12}{4+4}$ = 1.5 A

**S8.** (a) $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0 J$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 10^3$ rad/sec

⇒ f = 159 Hz

(c) Qq = $q_0$ cos wt

Energy stored is completely magnetic (i.e. electrical energy is zero, q = 0)

at $t = \frac{T}{4}$, where $T = \frac{1}{f}$ = 6.3 ms

(d) Energy is shared equally between L and C when charge on capacitor become $\frac{q_0}{\sqrt{2}}$

so, at $t = \frac{T}{8}$, energy is shared equally between L and C

**S9.** Here,L=100×$10^{-3}$H, $R = 100\Omega, X_C = 200\Omega, V_{rms} = 150\sqrt{2}$ V

$v = \frac{500}{\pi}$ Hz.

Inductive reactance $X_L = \omega L = 2\pi v L$

$= 2\pi \frac{500}{\pi} \times 100 \times 10^{-3} = 100\Omega$

Impedance of circuit

$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(100)^2 + (200 - 100)^2} = \sqrt{20000} = 100\sqrt{2}\Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{150\sqrt{2}}{100\sqrt{2}} = \frac{3}{2}$

Power dissipated $(I_{rms})^2 R = \frac{9}{4} \times 100 = 225\mathbf{W}$

**S10.** Capacitance, $C = \frac{1}{L\omega^2}$

$= \frac{1}{\frac{4}{\pi^2}(2\pi \times 50)^2}\ \text{F} = \frac{1}{40000}\ \text{F} = 2.5 \times 10^{-5}\ \text{F}$

Since $V$ and $I$ are in same phase

Impedance = Resistance = 100Ω

Power dissipated = $\frac{E_{rms}^2}{2} = \frac{(200)^2}{100}$ W = **400**W