CBSE Sample Paper -02
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours | Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.
2. Express $0.\overline{6}$ as rational number in simplest form.
3. If $\sec^2\theta(1+\sin\theta)=k,$ then find they value of k.
4. Evaluate: $\dfrac{\tan 26^o}{\cot 64^o}$

SECTION – B

5. Find maximum value of $\dfrac{1}{\sec\theta}, 0^o \le \theta \le 90^o.$
6. Find the HCF of 96 and 404 by prime factorisation method. Hence, find their LCM.
7. In a $\Delta ABC, if\ \angle C = 90^o$, prove that $\sin^2 A + \sin^2 B = 1.$
8. Sum of two numbers if 35 and their difference is 13. Find the numbers.
9. The number of students absent in a school was recorded every day for 147 days and the raw data was presented in the form of the following frequency table.

No.of students absent	5	6	7	8	9	10	11	12	13	15	18	20
No. of days	1	5	11	14	16	13	10	70	4	1	1	1

Obtain the median and describe what information it conveys.

10. In an isosceles $\Delta ABC,$ if AC=BC and $AB^2 = 2AC^2$ then find $\angle C.$

SECTION – C

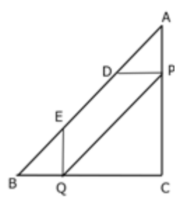
11. Show that there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

12. E and F are points on the sides PQ and PR respectively of a ΔPQR. Show that EF||QR. If PQ=1.28 cm, PR=2.56 cm, PE=0.18 cm and PE=0.36 cm.

13. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km, the charge paid is Rs 75 and for a journey of 15 km, the charge paid is Rs 110. What will a person have to pay for travelling a distance of 25 km?

14. If $\sin A=\frac{3}{4}$, Calculate cos A and tan A.

15. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.



16. Find the zeros of the polynomial $f(u) = 4u^2 + 8u$ and verify the relationship between the zeros and its coefficients.

17. Evaluate the following: $\frac{\sin^2 20^o+\sin^2 70^o}{\cos^2 20^o+\cos^2 70^o}+\left[\frac{\sin(90^o-\theta.\sin\theta}{\tan\theta}+\frac{\cos(90^o-\theta.\cos\theta}{\cot\theta}\right]$

18. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

19. Solve: $ax + by = c$

    $bx + ay = 1 + c$

20. If $\sec\theta=x+\frac{1}{4x}$, prove that $\sec\theta+\tan\theta=2x\ or\ \frac{1}{2x}$.

SECTION – D

21. Let a, b, c and $p$be rational numbers such that p is not a perfect cube. If $a+bp^{\frac{1}{3}}+cp^{\frac{2}{3}}=0$, then prove that $a = b = c = 0$.

22. Prove that: $\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$.

23. The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3CD (see Fig. 4.40). Prove that $2AB^2 = 2AC^2 + 2BC^2$.

24. A frequency distribution of the life times of 400 T.V. picture tubes tested in a company is given below. Find the average life of a tube.

Life time (in hours)	Frequency	Life time (in hours)	Frequency
300-399	14	800-899	62
400-499	46	900-999	48
500-599	58	1000-1099	22
600-699	76	1100-1199	6
700-799	68		

25. Prove that: $\frac{1}{(\cos ecx + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\cos ecx - \cot x)}$.

26. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

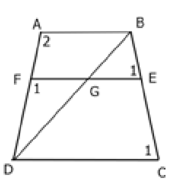
27. Solve the following system of linear equations graphically.

$x - y = 1$

$2x + y = 8$

Shade the area bounded by these two lines and y-axis. Also, determine this area.

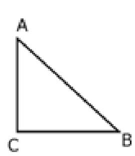
28. In trapezium ABCD, AB || DC and DC = 2AB. A line EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{\text{BE}}{\text{EC}} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE = 10AB.



29. In a ΔABC, right angled at C and ∠A = ∠B,

(i) Is cosA = cosB? (ii) Is tanA = tanB?

What about other trigonometric ratios for ∠A and ∠B. Will they be equal?



30. A sweet seller has 420 kajuburfis and 130 badamburfis. She wants to stack them in such a way that each stack has the same number and they take up the least area of the tray. What is the number of burfis that can be placed in each stack for this purpose?

31. In a housing society, people decided to do rainwater harvesting. Rainwater is collected in the underground tank at the rate of 30 cm^3/sec. Taking volume of water collected in x seconds as y cm^3.

a. Form a linear equation.

b. Write it in standard form as $ax + by + c = 0$.

c. Which values are promoted by the members of this society?

CBSE Sample Paper -02
SUMMATIVE ASSESSMENT –I
Class – X
Mathematics

Time allowed: 3 hours **Answers** Maximum Marks: 90

SECTION – A

1. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 18^2$

$\therefore$ The give triangle is not a right triangle.

2. Let $x = 0.\overline{6}$.

Then, $x = 0.666$ ... (i)

$\therefore$ $10x = 6.666$ ... (ii)

On subtracting (i) from (ii), we get

$9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

Thus, $0.\overline{6} = \frac{2}{3}$

3. $\sec^2\theta(1+\sin\theta)(1-\sin\theta)$

$= \sec^2\theta(1-\sin^2\theta)$ $[(a+b)(a-b) = a^2 - b^2]$

$= \sec^2\theta.\cos^2\theta = 1$ $[\because \cos^2\theta + \sin^2\theta = 1]$

Therefore, k = 1.

4. $\frac{\tan 18^o}{\cot 64^o}$

$= \frac{\sin(90^o - 64^o)}{\cot 64^o} = \frac{\cot 64^o}{\cot 64^o} = 1$

SECTION – B

5. $\frac{1}{\sec\theta}, 0^o \leq \theta \leq 90^o$. (Given)

$\because$ $\sec\theta$ in the denominator.

$\therefore$ The min. value of $\sec\theta$ will return max. value for $\frac{1}{\sec\theta}$.

But the min. value of $\sec\theta$ is sec 0 = 1.

Hence, the max. value of $\frac{1}{\sec^o} = \frac{1}{1} = 1$.

6. We have,

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$404 = 2 \times 2 \times 101 = 2^2 \times 101$

$\therefore$ HCF $= 2^2 = 2 \times 2 = 4$

Now, HCF × LCM = Product of the numbers

$\Rightarrow$ $4 \times \text{LCM} = 96 \times 404$

$\Rightarrow$ LCM $= \frac{96 \times 404}{4} = 96 \times 101 = 9696$

7. Since $\angle C = 90^o$

$\therefore$ $\angle A + \angle B = 180^o - \angle C = 90^o$

Now, $\sin^2 A + B + \sin^2 B = \sin^2 A + \sin^2 (90^o - A)$

$= \sin^2 A + \cos^2 A = 1.$

8. Let the two numbers be x and y. Then,

$x + y = 35$

$x - y = 13$

Adding equations (i) and (ii), we get

$2x = 48 \quad \Rightarrow \quad x = 24$

Subtracting equation (ii) from equation (i), we get

$2y = 22 \quad \Rightarrow \quad y = 11$

Hence, the two numbers are 24 and 11.

9. Calculation of median

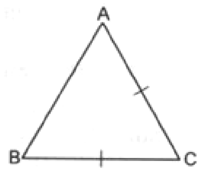
x_i	5	6	7	8	9	10	11	12	13	15	18	20
f_i	1	5	11	14	16	13	10	70	4	1	1	1
cf	1	6	17	31	47	60	70	140	144	145	146	147

We have,

$N = 147 \quad \Rightarrow \quad \frac{N}{2} = \frac{147}{2} = 73.5$

The cumulative frequency just greater than $\frac{N}{2}$ is 140 and the corresponding value of variable x is 12. Thus, the median = 12. This means that for about half the number of days, more than 12 students were absent.

10.



$AB^2 = 2AC^2 \quad (Given)$

$AB^2 = AC^2 + AC^2$

$AB^2 + AC^2 BC^2 \quad (\because AC = BC)$

Hence AB is the hypotenuse and ΔABC is a right angle Δ.

So, $\angle C = 90^o$

SECTION – C

11. If possible, let there be a positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational equal to $\frac{a}{b}$ (say), where a, b are positive integers. Then,

$$\frac{a}{b} = \sqrt{n-1}+\sqrt{n+1} \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{\left\{\sqrt{n+1}+\sqrt{n-1}\right\}\left\{\sqrt{n+1}-\sqrt{n-1}\right\}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{(n+1)-(n-1)} = \frac{\sqrt{n+1}-\sqrt{n-1}}{2}$$

$$\Rightarrow \quad \frac{2b}{a} = \sqrt{n+1}-\sqrt{n-1} \quad \text{...(ii)}$$

Adding (i) and (ii) and subtracting (ii) from (i), we get

$$2\sqrt{n+1} = \frac{a}{b}+\frac{2b}{a} \text{ and } 2\sqrt{n-1} = \frac{a}{b}-\frac{2b}{a}$$

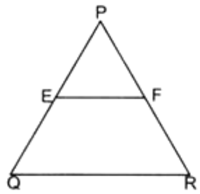
$$\Rightarrow \quad \sqrt{n+1} = \frac{a^2+2b^2}{2ab} \text{ and } \sqrt{n-1} = \frac{a^2-2b^2}{2ab}$$

$$\Rightarrow \quad \sqrt{n+1} \text{ and } \sqrt{n-1} \text{ arerationals} \qquad \left[\begin{array}{l}\because a,b \text{ are integers} \\ \therefore \frac{a^2+2b^2}{2ab} \text{ and } \frac{a^2-2b^2}{2ab} \text{ are rationals.}\end{array}\right]$$

$\Rightarrow$ (n + 1) and (n – 1) are perfect squares of positive integers.

This is not possible as any two perfect squares differ at least by 3. Thus, there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

12. We have,



$PQ = 1.28\, cm.\ PR = 2.56\ cm$
$PE = 0.18 cm,\ PF = 0.36 cm$

$Now,\ EQ = PQ - PF = 1.28 - 0.18 = 1.10\, cm$
$and \quad FR = PR - PF = 2.56 - 0.36 = 2.20 cm$

$$Now, \frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$$

$$and, \frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55} \quad \therefore \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF||QR [By the converse of basic proportionality Theorem]

13. Let the fixed charges of taxi be Rs*x* per km and the running charges be Rs*y* km/hr.

According to the given condition, we have

$x + 10y = 75$ ...(i)

$x + 15y = 110$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$-5y = -35 \quad \Rightarrow \quad y = 7$

Putting $y = 7$ in equation (i), we get $x = 5$.

∴ Total charges from travelling a distance of 25 km

$= x + 25y$

$= 5 + 25 \times 7 =$ Rs 180

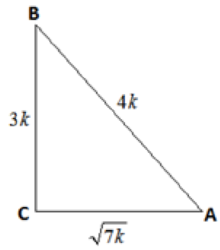
14. Let us first draw a right ΔABC in which $\angle C = 90^o$.

Now, we know that

$$\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AB} = \frac{3}{4}$$

Let BC = 3k and AB = 4k, where k is a positive number.

Then, by Pythagoras Theorem, we have



$AB^2 = BC^2 + AC^2$

$\Rightarrow (4k)^2 = (3k)^2 + AC^2$

$\Rightarrow 16k^2 - 9k^2 = AC^2 \quad \Rightarrow 7k^2 = AC^2$

$\therefore \quad AC = \sqrt{7}k$

$\therefore \quad \cos A = \dfrac{AC}{AB} = \dfrac{\sqrt{7}k}{4k} = \dfrac{\sqrt{7}}{4}$

and $\quad \tan A = \dfrac{BC}{AC} = \dfrac{3k}{\sqrt{7}k} = \dfrac{3}{\sqrt{7}}.$

15. In ΔABC, we have

DP || BC and EQ || AC

$\therefore \quad \dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AP}}{\text{PC}}$ and $\dfrac{\text{BE}}{\text{EA}} = \dfrac{\text{BQ}}{\text{QC}}$

$\Rightarrow \quad \dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AP}}{\text{PC}}$ and $\dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{BQ}}{\text{QC}}$ [∵ EA = ED + DA = ED + BE = BD, ∴ AD = BE]

$\Rightarrow \quad \dfrac{\text{AP}}{\text{PC}} = \dfrac{\text{BQ}}{\text{QC}}$

⇒ In aΔABC, P and Q divide sides CA and CB respectively in the same ratio.

⇒ PQ || AB.

16. We have,

$f(u) = 4u^2 + 8u$

$= 4u(u + 2)$

The zeros of $f(u)$ are given by

$f(u) = 0$

$\Rightarrow \quad 4u(u + 2) = 0$

$\Rightarrow \quad u = 0$ or $u + 2 = 0$

$\Rightarrow \quad u = 0$ or $u = -2$

Hence, the zeros of f(u) are:

$\alpha = 0$ and $\beta = -2$

Now, $\alpha + \beta = 0 + (-2) = -2$ and $\alpha\beta = 0 \times -2 = 0$

Also, $-\dfrac{\text{Coefficient of } u}{\text{Coefficient of } u^2} = -\dfrac{8}{4} = -2$

And, $\dfrac{\text{Constant term}}{\text{Coefficient of } u^2} = \dfrac{0}{2} = 0$

$\therefore$ Sum of the zeros = $-\dfrac{\text{Coefficient of } u}{\text{Coefficient of } u^2}$

And, Product of the zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } u^2}$

17. we have $\dfrac{\sin^2 20^o + \sin^2 70^o}{\cos^2 20^o + \cos^2 70^o} + \left[\dfrac{\sin(90^o - \theta.\sin\theta}{\tan\theta} + \dfrac{\cos(90^o - \theta.\cos\theta}{\cot\theta}\right]$

$$= \frac{\sin^2 20^o + \sin^2(90^o - 20^o)}{\cos^2 20^o + \sin^2 20^o} + \left[\frac{\cos\theta.\sin\theta}{\tan\theta} + \frac{\cos\theta.\sin\theta}{\cot\theta}\right]$$

$$= \frac{\sin^2 20^o + \cos^2 20^o}{\cos^2 20^o + \sin^2 20^o} + \left[\frac{\cos\theta.\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta.\sin\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$= \frac{1}{1} + [\cos^2\theta + \sin^2\theta] = 1 + 1 = 2.$$

18. Here, median = 28.5 and n = 60

Now, we have

Class interval	Frequency (f_i)	Cumulative frequency (Cf)
0-10	5	5
10-20	X	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	Y	40 + x + y
50-60	5	45 + x4 - y
Total	$\sum f_i = 60$	

Since the median is given to be 28.5, thus the median class is 20-30.

$\dfrac{n}{2} = 30$, l=20, cf = 5+x and f =20

Median = $l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$

$28.5 = 20 + \left[\frac{30-(5+x)}{20}\right] \times 10$

$28.5 = 20 + \frac{25-x}{20} \times 10$

$28.5 = 20 + \frac{25-x}{2}$

57 = 40 + 25 - x

57 = 65 – x

x= 65-57 = 8

Also, n = $\sum fi = 60$

45 + x + y = 60

45 + 8 + y = 60

Therefore, y = 60 – 53

y = 7

Hence, x = 8 and y = 7

19. The given system of equations may be written as

$ax + by - c = 0$

$bx + ay - (1 + c) = 0$

By cross multiplication, we have

$$\frac{x}{b\times -(1+c) - a\times -c} = \frac{-y}{a\times -(1+c) - b\times -c} = \frac{1}{a\times a - b\times b}$$

$$\Rightarrow \quad \frac{x}{-b(1+c)+ac} = \frac{-y}{-a(1+c)+bc} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{ac-bc-b} = \frac{y}{ac-bc+a} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{c(a-b)-b} = \frac{y}{c(a-b)+a} = \frac{1}{(a-b)(a+b)}$$

$$\Rightarrow \quad x = \frac{c(a-b)-b}{(a-b)(a+b)} \text{ and } y = \frac{c(a-b)+a}{(a-b)(a+b)}$$

$$\Rightarrow \quad x = \frac{c}{a+b} - \frac{b}{(a-b)(a+b)} \text{ and } y = \frac{c}{a+b} + \frac{a}{(a-b)(a+b)}$$

Hence, the solution of the given system of equation is

$$x = \frac{c}{a+b} - \frac{b}{a^2-b^2} \text{ and } y = \frac{c}{a+b} + \frac{a}{a^2-b^2}$$

20. Let $\sec\theta + \tan\theta = \lambda$ (i)

We know that, $\sec\theta + \tan\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\lambda(\sec\theta - \tan\theta) = 1$

$(\sec\theta - \tan\theta) = \dfrac{1}{\lambda}$ (ii)

Adding equations (i) and (ii), we get

$2\sec\theta = \lambda + \dfrac{1}{\lambda} \quad \Rightarrow \quad 2\left(x + \dfrac{1}{4x}\right) = \lambda + \dfrac{1}{\lambda}$

$\Rightarrow \quad 2x\dfrac{1}{2x} = \lambda + \dfrac{1}{\lambda}$

On comparing, we get $\lambda = 2x \text{ } or \text{ } \lambda + \dfrac{1}{2x}$

$\Rightarrow \sec\theta + \tan\theta = 2x \text{ } or \text{ } \dfrac{1}{2x}$.

SECTION – D

21. We have,

$a + bp^{\frac{1}{3}} + cp^{\frac{2}{3}} = 0$ ...(i)

Multiplying both sides by $p^{\frac{1}{3}}$, we get

$ap^{\frac{1}{3}} + bp^{\frac{2}{3}} + cp = 0$ ...(ii)

Multiplying (i) by b and (ii) by c and subtracting, we get

$\left(ab + b^2p^{\frac{1}{3}} + bcp^{\frac{2}{3}}\right) - \left(acp^{\frac{1}{3}} + bcp^{\frac{2}{3}} + c^2p\right) = 0$

$\Rightarrow \quad \left(b^2 - ac\right)p^{\frac{1}{3}} + ab - c^2p = 0$ [$\because p^{\frac{1}{3}}$ is irrational]

$\Rightarrow \quad b^2 - ac = 0$ and $ab - c^2p = 0$

$\Rightarrow \quad b^2 = ac$ and $ab = c^2p$

$\Rightarrow \quad b^2 = ac$ and $a^2b^2 = c^4p^2$

$\Rightarrow \quad a^2(ac) = c^4p^2$ [Putting $b^2 = ac$ in $a^2b^2 = c^4p^2$]

$\Rightarrow \quad a^3c - c^4p^2 = 0$

$\Rightarrow \quad \left(a^3 - c^3p^2\right)c = 0$

$\Rightarrow \quad a^3 - c^3p^2 = 0$ or $c = 0$

Now, $a^3 - c^3p^2 = 0$

$\Rightarrow \quad p^2 = \frac{a^3}{c^3}$

$\Rightarrow \quad (p^2)^{\frac{1}{3}} = \left(\frac{a^3}{c^3}\right)^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2 = \left\{\left(\frac{a}{c}\right)^3\right\}^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2 = \left\{\left(\frac{a}{c}\right)^3\right\}^{\frac{1}{3}}$

$\Rightarrow \quad \left(p^{\frac{1}{3}}\right)^2 = \frac{a}{c}$

This is not possible as $p^{\frac{1}{3}}$ is irrational and $\frac{a}{c}$ is rational.

$\therefore \quad a^3 - c^3p^2 \neq 0$ and hence $c = 0$

Putting $c = 0$ in $b^2 - ac = 0$, we get $b = 0$.

Putting $b = 0$ and $c = 0$ in $a + bp^{\frac{1}{3}} + cp^{\frac{2}{3}} = 0$, we get $a = 0$.

Hence, $a = b = c = 0$.

22. L.H.S = $\tan^2 A - \tan^2 B$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\cos^2 B}$

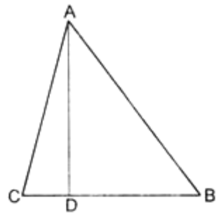
$= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A\cos^2 B}$

$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B}$

Also, $\frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B} = \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A\cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$ =R.H.S

23. We have, DB = 3CD



Now, BC = BD + CD

$\Rightarrow BC = 3CD + CD = 4CD$ (Given DB = 3CD)

$\therefore CD = \frac{1}{4}BC$

Now, in right-angled triangle ABD, we have

$AB^2 = AD^2 + DB^2$ ...(i)

Again, in right-angled triangle ΔADC, we have

$AC^2 = AD^2 = CD^2$ ...(ii)

Subtracting (ii) from (i), we have

$AB^2 - AC^2 = DB^2 - CD^2$

$\Rightarrow AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2$

$\Rightarrow AB^2 - AC^2 = \frac{1}{2}BC^2$

$\therefore 2AB^2 - 2AC^2 = BC^2 \quad \Rightarrow 2AB^2 = 2AC^2 + BC^2$

24. Here, the class intervals are formed by exclusive method. If we make the series an inclusive, one of the mid-values remain same. So, there is no need to convert the series into an inclusive form.

Let the assumed mean be A = 749.5 and h = 100.

Calculation of mean

Life time (in hours)	Frequency f_i	Mid-values x_i	$d_i = x_i - A$ $= x_i - 749.5$	$u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 749.5}{100}$	$f_i u_i$
300-399	14	349.5	–400	–4	–56
400-499	46	449.5	–300	–3	–138
500-599	58	549.5	–200	–2	–116

600-699	76	649.5	−100	−1	−76
700-799	68	749.5	0	0	0
800-899	62	849.5	100	1	62
900-999	48	949.5	200	2	96
1000-1099	22	1049.5	300	3	66
1100-1199	6	1149.5	400	4	24
$N=\sum f_i = 400$ $\sum f_i u_i = -138$					

We have N = 400, A = 749.5, h = 100 and $\sum f_i u_i = -138$

$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

$= 749.5 + 100 \times \left(\frac{-138}{400}\right)$

$= 749.5 - \frac{138}{4}$

= 749.5 – 34.5 = 715

Thus, the average life time of a tube is 715 hours.

25. In order to show that,

$$\frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}$$

It is sufficient to show,

$$\frac{1}{\operatorname{cosec} x + \cot x} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{2}{\sin x} \quad \text{.......................... (i)}$$

Now, LHS of above is

$$\frac{1}{\operatorname{cosec} x + \cot x} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{(\operatorname{cosec} x - \cot x) + (\operatorname{cosec} x + \cot x)}{(\operatorname{cosec} x - \cot x)(\operatorname{cosec} x + \cot x)}$$

$$\frac{2\operatorname{cosec} x}{\operatorname{cosec}^2 x - \cot^2 x}\left[\because (a+b)(a-b) = a^2 - b^2\right]$$

$$= \frac{2\operatorname{cosec} x}{1} = \frac{2}{\sin x}$$

RHS of (i)

Hence, $\dfrac{1}{(\operatorname{cosec}x+\cot x)}+\dfrac{1}{(\operatorname{cosec}x-\cot x)}=\dfrac{1}{\sin x}+\dfrac{1}{\sin x}$

Or, $\dfrac{1}{(\operatorname{cosec}x+\cot x)}-\dfrac{1}{\sin x}=\dfrac{1}{\sin x}-\dfrac{1}{(\operatorname{cosec}x-\cot x)}$

26. If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then the remainder should be zero.

On dividing, we get

$$
\begin{array}{l}
x^2+1 \overline{\big) \; x^4+x^3+8x^2+ax+b \;} \; (x^2+x+7 \\
\qquad\; x^4 \qquad +x^2 \\
\qquad\; - \qquad\;\; - \\
\hline
\qquad\qquad x^3+7x^2+ax+b \\
\qquad\qquad x^3 \qquad\quad +x \\
\qquad\qquad + \qquad\quad\; - \\
\hline
\qquad\qquad\qquad 7x^2+x(a-1)+b \\
\qquad\qquad\qquad 7x^2 \qquad\qquad +7 \\
\qquad\qquad\qquad - \qquad\qquad\;\; - \\
\hline
\qquad\qquad\qquad\qquad x(a-1)+b-7
\end{array}
$$

$\therefore$ Quotient = $x^2 + x + 7$ and Remainder = $x(a - 1) + (b - 7)$

Now, remainder = 0

$\Rightarrow$ $x(a - 1) + (b - 7) = 0$

$\Rightarrow$ $x(a - 1) + (b - 7) = 0x + 0$

$\Rightarrow$ $a - 1 = 0$ and $b - 7 = 0$

$\Rightarrow$ $a = 1$ and $b = 7$

27. We have,

$x - y = 1$

$2x + y = 8$

Graph of the equation $x - y = 1$:

We have,

$x - y = 1 \quad \Rightarrow \quad y = x - 1$ and $x = y + 1$

Putting $x = 0$, we get $y = -1$

Putting $y = 0$, we get $x = 1$

Thus, we have the following table for the points on the line $x - y = 1$:

x	0	1
y	-1	0

Graph of the equation $2x + y = 8$:

We have,

$2x + y = 8 \quad \Rightarrow \quad y = 8 - 2x$ and $x = \dfrac{8-y}{2}$

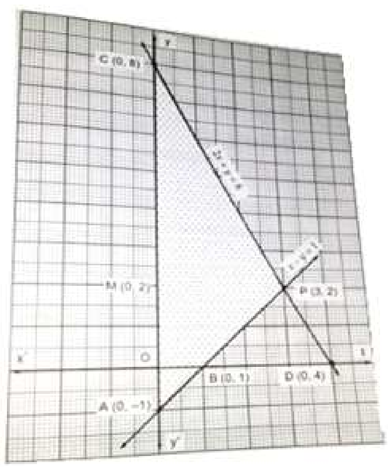
Putting $x = 0$, we get $y = 8$

Putting $y = 0$, we get $x = 4$

Thus, we have the following table for the points on the line $2x + y = 8$:

x	0	8
y	8	0

Plotting points A(0, –1), B(1, 0) on the graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $x - y = 1$.



Plotting points C(0, 8), D(4, 0) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 8$.

Clearly, the two lines intersect at P(3, 2). The area enclosed by the lines represented by the given equations and the y-axis is shaded.

Now, required area = Area of the shaded region

= Area of ΔPAC

$= \dfrac{1}{2}(\text{Base} \times \text{Height})$

$= \dfrac{1}{2}(\text{AC} \times \text{PM})$ [$\because$ PM = x-coordinate of P = 3]

$= \dfrac{1}{2}(9 \times 3)$ = 13.5 sq. units

28. In ΔDFG and ΔDAB, we have

$\angle 1 = \angle 2$ [$\because$ AB || DC || EF, $\therefore \angle 1$ and $\angle 2$ are corresponding angles]

$\angle FDG = \angle ADB$ [Common]

So, by AA-criterion of similarity, we have

$\Delta DFG \sim \Delta DAB \Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$ ...(i)

In trapezium ABCD, we have

EF || AB || DC

$\therefore \frac{AF}{DF} = \frac{BE}{EC}$

$\Rightarrow \frac{AF}{DF} = \frac{3}{4}$ $\left[\because \frac{BE}{EC} = \frac{3}{4}(\text{Given})\right]$

$\Rightarrow \frac{AF}{DF} + 1 = \frac{3}{4} + 1$ [Adding 1 on both sides]

$\Rightarrow \frac{AF + DF}{DF} = \frac{7}{4}$

$\Rightarrow \frac{AD}{DF} = \frac{7}{4} \Rightarrow \frac{DF}{AD} = \frac{4}{7}$ ...(ii)

From (i) and (ii), we get

$\frac{FG}{AB} = \frac{4}{7} \Rightarrow FG = \frac{4}{7}AB$ ...(iii)

In ΔBEG and ΔBCD, we have

$\angle BEG = \angle BCD$ [Corresponding angles]

$\angle B = \angle B$ [Common]

$\therefore \Delta BEG \sim \Delta BCD$ [By AA-criterion of similarity]

$\Rightarrow \frac{BE}{EC} = \frac{EG}{CD}$

$\Rightarrow \frac{3}{7} = \frac{EG}{CD}$ $\left[\because \frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC}{BE} + 1 = \frac{4}{3} + 1 \Rightarrow \frac{BC}{BE} = \frac{7}{3}\right]$

$\Rightarrow EG = \frac{3}{7}CD = \frac{3}{7} \times 2AB = \frac{6}{7}AB$ ...(iv)

Adding (iii) and (iv), we get

$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB$

$\Rightarrow \quad EF = \frac{10}{7}AB$

$\Rightarrow \quad 7EF = 10AB$

29. We have,

$\angle A = \angle B$

$\Rightarrow \quad BC = AC$ [$\because$ Sides opposite to equal angles are equal]

Let BC = AC = x (say)

Using Pythagoras theorem in ΔACB, we have

$AB^2 = AC^2 + BC^2$

$= x^2 + x^2$

$\Rightarrow \quad AB = \sqrt{2}x$

(i) We have,

$$\cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$$\cos B = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$\therefore \quad \cos A = \cos B$

(ii) We have,

$$\tan A = \frac{BC}{AC} = \frac{x}{x} = 1$$

$$\tan B = \frac{AC}{BC} = \frac{x}{x} = 1$$

$\therefore \quad \tan A = \tan B$

Now, $\sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\sin B = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\therefore \quad \sin A = \sin B$

$$\cot A = \frac{AC}{BC} = \frac{x}{x} = 1 \text{ and } \cot B = \frac{BC}{AC} = \frac{x}{x} = 1$$

$\therefore \quad \cot A = \cot B$

$$\sec A = \frac{AB}{AC} = \frac{\sqrt{2}x}{x} = \sqrt{2} \text{ and } \sec B = \frac{AB}{BC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$$

$\therefore \quad \sec A = \sec B$

$$\text{cosecA} = \frac{AB}{BC} = \frac{\sqrt{2}x}{x} = \sqrt{2} \text{ and cosecB} = \frac{AB}{AC} = \frac{\sqrt{2}x}{x} = \sqrt{2}$$

$\therefore$ cosecA = cosecB

30. The area of the tray that is used up in stacking the burfis will be least if the seet seller stacks maximum number of burfis in each stack. Since each stack must have the same number of burfis, therefore, the number of stacks will be least if the number of burfis in each stack is equal to the HCF of 420 and 130.

In order to find the HCF of 420 and 130, let us apply Euclid's division lemma to 420 and 130 to get

$420 = 130 \times 3 + 130$ ...(i)

$$\left[\begin{array}{r} 3 \\ 130\overline{)\ 420} \\ \underline{-390} \\ 30 \end{array}\right]$$

Let us now consider the divisor 130 and the remainder 30 and apply division lemma to get

$130 = 30 \times 4 + 10$ ...(ii)

$$\left[\begin{array}{r} 4 \\ 30\overline{)\ 130} \\ \underline{-120} \\ 10 \end{array}\right]$$

Considering now divisor 30 and the remainder 10 and apply division lemma, we get

$30 = 3 \times 10 + 0$ ...(iii)

$$\left[\begin{array}{r} 3 \\ 10\overline{)\ 30} \\ \underline{-30} \\ 0 \end{array}\right]$$

Since, the remainder at this stage is zero. Therefore, last divisor 10 is the HCF of 420 and 130. Hence, the sweet seller can make stacks of 10 burfis of each kind to cover the least area of the tray.

31. Rate at which rainwater is collected in the tank = 30 cm^3/sec

Time for which water is collected = x seconds

Total amount of water collected = $y$cm3

a. According to the given condition, linear equation formed is $y = 30x$

b. The equation in standard form is $30x - y + 0 = 0$

c. Values promoted by the members of the society are environmental protection and co-operation.