Short Answer Type Questions – II

[3 MARKS]

Que 1. Determine, if the points (1,5), (2,3) and (-2, - 11) are collinear.

Sol. Let A (1,5), B (2,3) and C (-2, - 11) be the given points. Then we have

AB = $\sqrt{(2-1)^2+(3-5)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{(2-2)^2+(-11-3)^2} = \sqrt{16+196} = \sqrt{4 \times 53}$ = 2 $\sqrt{53}$

AC = $\sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{9+256} = \sqrt{265}$

Clearly, AB + BC ≠ AC

∴ A, B, C are not collinear.

Que 2. Find the distance between the following pairs of points:
(i) (-5, 7), (-1, 3) (ii) (a, b), (-a, - b)

Sol. (i) let two given points be A (-5, 7) and B (-1, 3).

Thus, we have x_1 = -5 and x_2 = - 1

Y_1 = 7 and y_2 = 3

∴ AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = $\sqrt{(-1+5)^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32}$ = 4 $\sqrt{2}$ units.

(ii) Let two given points be A (a, b) and B (-a,-b)

Here, x_1 = a and x_2 = - a; y_1 = b and y_2 = - b

∴ AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2}$

= $\sqrt{4a^2+4b^2}$ = 2 $\sqrt{a^2+b^2}$ units.

Que 3. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:
(i) (-1, - 2), (1, 0), (-1, 2), (-3, 0)
(ii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (i) Let A (-1, - 2), B (1,0), C (-1, 2) and D (-3, 0) be the four given points.

Then, using distance formula, we have,

AB = $\sqrt{(1+1)^2+(0+2)^2}\ \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{(-1-1)^2+(2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $2\sqrt{(-3+1)^2+(0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{(-1+3)^2+(-2+0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

AC = $\sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+16} = 4$

And BD = $\sqrt{(-3-1)^2+(0-0)^2} = \sqrt{16} = 4$

Hence, four sides of quadrilateral are equal and diagonal AC and BD are also equal.

∴ Quadrilateral ABCD is a square.

(ii) Let A (4, 5), B (7, 6), C (4, 3) and D (1, 2) be the given points. Then,

AB = $\sqrt{(7-4)^2+(6-5)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(4-7)^2+(3-6)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{(4-1)^2+(5-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

AC = $\sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+4} = 2$

And BD = $\sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Cleary, AB = CD, BC = DA and AC $\neq$ BD

∴ ABCD is a parallelogram.

Que 4. Find the value of y for which the distance between the points p (2, 3) and Q (10, y) is 10 units.

Sol. We have, PQ = 10

⇒ $\sqrt{(10-2)^2+(y+3)^2} = 10$

Squaring both sides, we have

⇒ $(8)^2 + (y + 3)^2 = 100$ ⇒ $(y + 3)^2 = 100 – 64$

⇒ $(y + 3)^2 = 36$ or $y + 3 = \pm 6$

⇒ y + 3 = 6, y + 3 = - 6 or y = 3, y = - 9

Hence, values of y are – 9 and 3.

Que 5. If Q (0, 1) is equidistant from P (5, - 3) and R (x, 6), find the value of x. Also, find the distances QR and PR.

Sol. Since, point Q (0, 1) is equidistant from P (5, - 3) and R (x, 6).

Therefore, QP = QR

Squaring both sides, we have, $QP^2 = QR^2$

⇒ $(5 – 0)^2 + (-3 – 1)^2 = (x – 0)^2 + (6 – 1)^2$

⇒ $25 + 16 = x^2 + 25$

⇒ $x^2 = 16$ ∴ $x = \pm 4$

Thus, R is (4, 6) or (-4, 6).

Now, QR = $\sqrt{(4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Or, QR = $\sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

And PR = $\sqrt{(4-5)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$

Or, PR = $\sqrt{(-4-5)^2+(6+3)^2} = \sqrt{81+81} = 9\sqrt{2}$

Que 6. Find the point on the x – axis which is equidistant from (2, - 5) and (-2, 9).

Sol. Let P (x, 0) be any point on x-axis.

Now, P (x, 0) is equidistant from point A (2, -5) and B (-2, 9)

∴ AP = BP

⇒ $\sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$

Squaring both sides, we have

$(x – 2)^2 + 25 = (x + 2)^2 + 81$

⇒ $x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$ ⇒ $- 8x = 56$

∴ $x = \frac{56}{-8} = -7$

∴ The point on the x-axis equidistant from given points is (-7, 0).

Que 7. Find the relation between x and y, if the points (x, y), (1, 2) and (7, 0) are collinear.

Sol. Given points are A (x, y), B (1, 2) and C (7, 0)

These points will be collinear if the area of the triangle formed by them is zero.

Now, ar (ΔABC) = $\frac{1}{2}[x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)]$

⇒ $0 = \frac{1}{2}[x(2 – 0) + 1(0 – y) + 7(y – 2)]$

$\Rightarrow 0 = \frac{1}{2}(2x - y + 7y - 14)$ $\Rightarrow$ $2x + 6y - 14 = 0$

$\Rightarrow x + 3y = 7$, which is the required relation between x and y.

Que 8. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Sol. Let P (x, y) be equidistant from the point A (3, 6) and B (-3, 4)

i.e., PA = PB

Squaring both sides, we get

$AP^2 = BP^2$

$\Rightarrow (x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow -12x - 4y + 20 = 0$ $\Rightarrow 3x + y - 5 = 0$, which is the required relation.

Que 9. Find the coordinates of the point which divides the line joining of (-1, 7) and (4, -3) in the ratio 2:3.

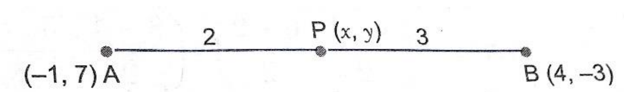

Fig. 6.14

Sol. Let P (x, y) be the required point. Thus, we have

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

Therefore,

$$x = \frac{2\times4+3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1 \quad \text{and,} \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$Y = \frac{2\times(-3)+3\times7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

So, the coordinates of P are (1, 3).

Que 10. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

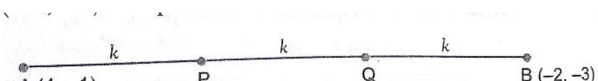

Fig. 6.15

Sol. Let the given points be A (4, - 1) and B (-2, - 3) and points of trisection be P and Q.

Let AP = PQ = QB = k

∴ PB = PQ + QB = k + k = 2k

AP: PB = k: 2k = 1: 2.

Therefore, coordinates of P are

$$\left(\frac{1\times-2+2\times4}{3}, \frac{1\times-3+2\times-1}{3}\right) = \left(2, -\frac{5}{3}\right)$$

Now, AQ = AP + PQ = k + k = 2k

∴ AQ: QB = 2k: k = 2: 1

And, coordinates of Q are

$$\left(\frac{2\times-2+1\times4}{3}, \frac{2\times-3+1\times-1}{3}\right) = \left(0, -\frac{7}{3}\right)$$

Hence, points of trisection are $\left(2, -\frac{5}{3}\right)$ and $\left(0, -\frac{7}{3}\right)$

Que 11. Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Sol. Let the required ratio be k: 1. Then, the coordinates of the point of division is

$$P \quad \left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$$

Since, this point lies on x-axis. Therefore its y-coordinates is zero.

i.e., $\frac{5k-5}{k+1} = 0 \Rightarrow \quad 5k - 5 = 0$

$\Rightarrow \quad 5k = 5 \quad$ or $\quad k = \frac{5}{5} = 1$

Thus, the required ratio is 1: 1 and the point of division is P $\left(\frac{-4\times1+1}{1+1}, \frac{5\times1-5}{1+1}\right)$

i.e., P $\left(-\frac{3}{2}, 0\right)$

Que 12. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

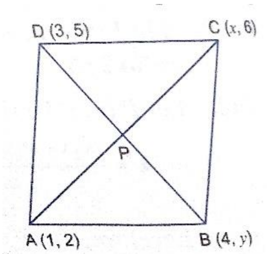

Fig. 6.16

Sol. Let A (1, 2), B (4, y), C (x, 6) and D (3, 5) be the vertices of a parallelogram ABCD.

Since, the diagonals of a parallelogram bisect each other.

∴ $\left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

⇒ $\frac{x+1}{2}, \frac{7}{2}$ ⇒ x + 1 = 7 or x = 6

And 4 = $\frac{5+y}{2}$

⇒ 5 + y = 8 or y = 8 – 5 = 3

Hence, x = 6 and y = 3.

Que 13. Find the coordinates of a points A, where AB is the diameter of a circle whose center is (2,-3) and B is (1, 4).

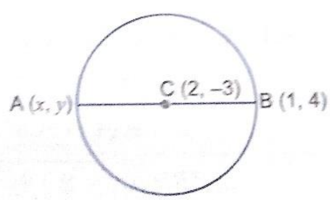

Fig. 6.17

Sol. Let the coordinates of A be (x,y).

Now, C is the center of circle therefore, the coordinates of

C = $\left(\frac{x+1}{2}, \frac{y+4}{2}\right)$ but coordinates of C are given as (2,-3)

∴ $\frac{x+1}{2}$ = 2 ⇒ x + 1 = 4 ∴ x = 3

And $\frac{y+4}{2}$ = - 3 ⇒ y + 4 = - 6 ∴ y = - 10

Hence, coordinates of A are (3,-10).

Que 14. If A and B are (-2,-2) and (2,-4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

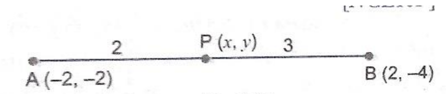

Fig. 6.18

Sol. In Fig. 8.10, we have, AP = $\frac{3}{7}$ AB

$\Rightarrow \frac{AB}{AB} = \frac{3}{7}$ $\quad\Rightarrow\quad$ $\frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \frac{AP+PB}{AP} = \frac{7}{3}$ $\quad\Rightarrow\quad$ $\frac{AP}{AP} + \frac{PB}{AP}\ \frac{7}{3}$

$\Rightarrow 1+\frac{PB}{AP} = \frac{7}{3}$ $\quad\Rightarrow\quad$ $\frac{PB}{AP} + \frac{7}{3} - 1 = \frac{4}{3}$

$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$ $\quad\Rightarrow\quad$ AP: PB = 3: 4

Let P (x, y) be the point which divides the join of A (-2, -2) and B (2, - 4) in the ratio 3: 4.

$\therefore x = \frac{3\times 2+4\times -2}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$ and y = $\frac{3\times -4+4\times -2}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$

Hence, the coordinates of the point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

Que 15. Find the coordinates of the points which divide the line segment joining A (-2,2) and B (2, 8) into four equal parts (Fig. 6.19).

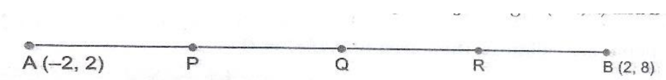

Fig. 6.19

Sol. Let P, Q, R be the points that divide the line segment joining A (-2,2) and B (2,8) into four

equal parts.

Since, Q divides the line segment AB into two equal parts, i.e., Q is the mid-point of AB.

$\therefore$ Coordinates of Q are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ i.e., (0,5)

Now, P divides AQ into two equal parts i.e., P is the mid-point of AQ.

$\therefore$ Coordinates of P are $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$ i.e., $\left(-1, \frac{7}{2}\right)$

Again, R is the mid-point of QB.

$\therefore$ Coordinates of R are $\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$ i.e., $\left(1, \frac{13}{2}\right)$.

Que 16. Find the area of a rhombus if its vertices (3, 0), (4, 5), (-1, 4) and (-2,-1) are taken in order.

Sol. Let A (3, 0), B (4, 5), C (-1, 4) and D (-2,-1) be the vertices of a rhombus.

Therefore, its diagonals

AC = $\sqrt{(-1-3)^2+(4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

And BD = $\sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

∴ Area of rhombus ABCD = $\frac{1}{2}$ x (Product of length of diagonals)

= $\frac{1}{2}$ x AC x BD = $\frac{1}{2}$ x 4 $\sqrt{2}$ x 6 $\sqrt{2}$ = 24 sq units.

Que 17. Find the area of the triangle whose vertices are: (-5,-1), (3,-5), (5, 2)

Sol. Let A (x_1, y_1) = (-5,-1), B (x_2, y_2) = (3, - 5), C (x_3, y_3) = (5, 2)

∴ Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [- 5 (- 5 – 2) + 3 (2 + 1) + 5 (-1 + 5)]

= $\frac{1}{2}$ (35 + 9 + 20) = $\frac{1}{2}$ x 64 = 32 sq units.

Que 18. If the points A (0, 2) is equidistant from the points B (3, p) and C (p, 5), find p. Also find the length of AB.

Sol. Given that A (0, 2) is equidistant from B (3, p) and C (p, 5)

∴ AB = AC

Or $AB^2 = AC^2$

⇒ $(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

⇒ $3^2 + p^2 + 4 - 4p = p^2 + 9$ ⇒ 4 – 4p = 0

⇒ 4p = 4 ⇒ p = 1

Length of AB = $\sqrt{(3-0)^2+(1-2)^2}$

= $\sqrt{3^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

Que 19. If the points A (-2, 1), B (a, b) and C (4, - 1) are collinear and a – b = 1, find the value of a and b.

Sol. Since the given points are collinear, then area of ΔABC = 0

$\Rightarrow \quad \frac{1}{2}[x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)] = 0$

Given, $x_1 = -2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$, $x_3 = 4$, $y_3 = -1$

Putting the value,

$$\frac{1}{2}[-2(b + 1) + a (-1 –1) + 4(1 – b)] = 0$$

$\Rightarrow \quad -2b – 2 – 2a + 4 – 4ab = 0 \quad \Rightarrow \; 2a + 6b = 2$

$\Rightarrow \quad a + 3b = 1 \quad$ …(i)

Given, $a – b = 1$ …(ii)

Subtracting (i) from (ii), we have

$$- 4b = 0 \Rightarrow b = 0$$

Subtracting the value of b in (ii), we have a = 1

Que 20. If the point P (k – 1, 2) is equidistant from the points A (3, k) and B (k, 5), find the value of k.

Sol. Since P is equidistant from A and B.

$$AP = BP \text{ or } AP^2 = BP^2$$

$$[3 – (k – 1)]^2 + (k – 2)^2 = [k – (k – 1)]^2 + (5 -)^2$$

$$(3 – k + 1)^2 + (k – 2)^2 = [k – k + 1)^2 + (3)^2$$

$$(4 – k)^2 + (k – 2)^2 = (1)^2 + (3)^2 \quad \Rightarrow \quad 16 + k^2 - 8k + k^2 + 4 – 4k = 1 + 9$$

$$2k^2 – 12k + 20 = 10 \quad \Rightarrow \quad k^2 – 6k + 10 = 5$$

$$K^2 – 6k + 5 = 0 \quad \Rightarrow \quad k^2 – 5k – k + 5 = 0$$

$$K (k – 5) – 1(k – 5) = 0 \quad \Rightarrow \quad k = 1 \text{ or } k = 5$$

Que 21. Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x-axis. Also find the coordinates of the points of division.

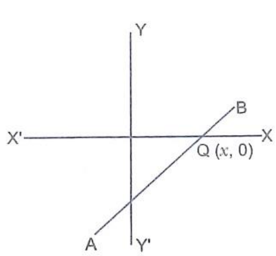

Fig. 6.20

Sol. Here, points Q is on x axis so its ordinate is *O.*

Let ratio be k: 1 and coordinates of points Q be (x, 0)

So, $Q_y = \frac{(my_2+my_1)}{m+n}$

We are given that A (3, -3) and B (-2,7)

$\therefore \quad 0 = \frac{k\times 7+1\times(-3)}{k+1}$

$0 = \frac{7k-3}{k+1} \quad \Rightarrow \; 0\,(K + 1) = 7K - 3$

$\Rightarrow \quad 7K = 3 \quad \Rightarrow \quad K = \frac{3}{7}$

K: 1 = 3: 7

Now, $Q_x = \frac{(mx_2+nx_1)}{m+n}$

$\Rightarrow \quad Q_x = \frac{\left(\frac{3}{7}\times -2\right)+1\times 3}{\frac{3}{7}+1} = 1.5$

Que 22. Find the value of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.

Sol. Points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear

$\therefore$ Area of ΔABC = 0

$\Rightarrow \quad \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

$\Rightarrow \quad \frac{1}{2}\left[(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)\{2k - (2k + 3)\}\right] = 0$

$\Rightarrow \quad \frac{1}{2}\left[(k + 1)(-3k + 3) + 3k(3k) + (5k - 1)(2k - 2k - 3)\right] = 0$

$\Rightarrow \quad \frac{1}{2}\left[-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3\right] = 0$

$\Rightarrow \quad \frac{1}{2}\left[6k^2 - 15k + 6\right] = 0 \quad \Rightarrow \quad 6k^2 - 15k + 6 = 0$

$\Rightarrow \quad 2k^2 - 5k + 2 = 0 \quad \Rightarrow \quad 2k^2 - 4k - k + 2 = 0$

$\Rightarrow \quad (k - 2)(2k - 1) = 0$

If k – 2 = 0. Then k = 2

If 2k – 1 = 0. Then k = $\frac{1}{2}$

$\therefore$ k = 2, $\frac{1}{2}$

Que 23. If the points P (x, y) is equidistant from the points A (a+b, b-a) and B (a-b, a+b). Prove that bx = ay.

Sol. Given. PA = PB or $(PA)^2 = (PB)^2$

$(a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow (a + b)^2 + x^2 – 2ax – 2bx + (b – a)^2 + y^2 – 2by + 2ay$

$= (a – b)^2 + x^2 – 2ax + 2bx + (a + b)^2 + y^2 – 2ay – 2by$

$\Rightarrow$ $4ay = 4bx$ or $bx = ay$

Hence proved.

Que 24. If the point C (-1, 2) divides internally the line segment joining the points A (2, 5) and B (x, y) in the ratio of 3: 4, find the value of $x^2 + y^2$.

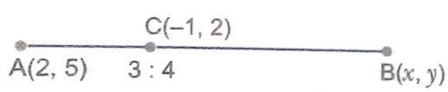

Fig. 6.21

Sol. $\frac{3x+4(2)}{7}$ = -1 $\Rightarrow x = -5$

$\frac{3y+4(5)}{7} = 2 \Rightarrow y = -2$

$\therefore x^2 + y^2 = (-5)^2 + (-2)^2 = 29$