# Rational Numbers

- **Rational numbers on number line**

  Rational numbers can be represented on number line in the similar manner like fractions and integers.

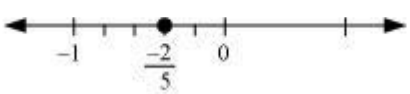
  Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

  **Example:** Represent $-\frac{2}{5}$ on number line.

  **Solution:** The given rational number is negative. Therefore, it will lie to the left of 0.

  The space between –1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$ .

  Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.

- To find rational numbers between any two given rational numbers, firstly we have to make their denominators same and then find the respective rational numbers.

  **Example:** Find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$ .

  **Solution:** The L.C.M. of 6 and 8 is 24.

  Now, we can write

  $$\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$$
  $$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

  Therefore, some of the rational numbers between $\frac{4}{24}\left(\frac{1}{6}\right)$ and $\frac{21}{24}\left(\frac{7}{8}\right)$ are $\frac{5}{24}, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}, \frac{9}{24}, \frac{10}{24}, \frac{11}{24}, \frac{12}{24}, \frac{13}{24}, \frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{17}{24}, \frac{18}{24}, \frac{19}{24}, \frac{20}{24}$

- Natural numbers are a collection of all positive numbers starting from 1.
- Whole numbers are a collection of all natural numbers including 0.
- Integers are the set of numbers comprising of all the natural numbers 1, 2, 3 ... and their negatives −1, −2, −3 ..., and the number 0.
- Rational numbers are the numbers that can be written in $\frac{p}{q}$ form, where *p* and *q* are integers and q ≠ 0
- **Closure property**

  - Whole numbers are closed under addition and multiplication. However, they are **not** closed under subtraction and division.
  - Integers are also closed under addition, subtraction and multiplication. However, they are **not** closed under division.
  - Rational numbers:

1. Rational numbers are closed under addition.

   Example: $\frac{2}{5}+\frac{3}{2}=\frac{19}{10}$ is a rational number.

2. Rational numbers are closed under subtraction.

   Example: $\frac{1}{5}-\frac{3}{4}=\frac{-11}{20}$ is rational number.

3. Rational numbers are closed under multiplication.

   Example: $\frac{2}{3}\times\left(\frac{-3}{5}\right)=\frac{-2}{5}$ is a rational number.

4. Rational numbers are **not** closed under division.

   Example: 2 ÷ 0 is not defined.

- **Commutativity**

  - Whole numbers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
  - Integers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
  - Rational numbers:

1. Rational numbers are commutative under addition.

Example:
$\frac{2}{3}+\left(\frac{-3}{2}\right)=\left(\frac{-3}{2}\right)+\left(\frac{2}{3}\right)=\frac{-5}{6}$

2. Rational numbers are not commutative under subtraction.

Example :
$\left(\frac{3}{4}\right)-\left(\frac{5}{2}\right)=\left(\frac{-7}{4}\right)$ and $\frac{5}{2}-\frac{3}{4}=\frac{7}{4}$
$\therefore\left(\frac{3}{4}\right)-\left(\frac{5}{2}\right)\neq\left(\frac{5}{2}\right)-\left(\frac{3}{4}\right)$

3. Rational numbers are commutative under multiplication.

Example:
$\left(\frac{3}{4}\right)\times\left(\frac{-2}{6}\right)=\left(\frac{-2}{6}\right)\times\left(\frac{3}{4}\right)=\frac{-1}{4}$

4. Rational numbers are not commutative under division.

2 ÷ 5 ≠ 5 ÷ 2

- **Associativity**

  - Whole numbers are associative under addition and multiplication. However, they are **not** associative under subtraction and division.

  - Integers are associative under addition and multiplication. However, they are **not** associative under subtraction and division.

  - Rational numbers:

1. Rational numbers are associative under addition.

Example:
$\left(\frac{2}{3}+\frac{1}{3}\right)+1=\frac{2}{3}+\left(\frac{1}{3}+1\right)=2$

2. Rational numbers are **not** associative under subtraction.

Example:
$\left(\frac{2}{3}-\frac{1}{3}\right)-1=\frac{-2}{3}$
$\frac{2}{3}-\left(\frac{1}{3}-1\right)=\frac{4}{3}$
$\therefore\left(\frac{2}{3}-\frac{1}{3}\right)-1\neq\frac{2}{3}-\left(\frac{1}{3}-1\right)$

3. Rational numbers are associative under multiplication.

Example:
$\left(\frac{2}{3} \times \frac{1}{3}\right) \times 1 = \frac{2}{3} \times \left(\frac{1}{3} \times 1\right) = \frac{2}{9}$

4. Rational numbers are **not** associative under division.

   Example:
   $\left\{\frac{2}{7} \div \left(\frac{-1}{14}\right)\right\} \div \frac{3}{7} = \frac{-28}{3}$
   $\frac{2}{7} \div \left\{\left(\frac{-1}{14}\right) \div \frac{3}{7}\right\} = \frac{-12}{7}$
   $\therefore \frac{2}{7} \div \left\{\left(\frac{-1}{14}\right)\right\} \div \frac{3}{7} \neq \frac{2}{7} \div \left\{\left(\frac{-1}{14}\right) \div \frac{3}{7}\right\}$

- 0 is the additive identity of whole numbers, integers, and rational numbers.

  ∴ 0 + $a$ = $a$ + 0 = $a$, where $a$ is a rational number

- 1 is the multiplicative identity of whole numbers, integers, and rational numbers.

  $a \times 1 = 1 \times a = a$

- Rational numbers are distributive over addition and subtraction.

  i.e., for any rational numbers $a$, $b$, and $c$, $a\,(b + c) = ab + ac$, $a(\,b - c) = ab - ac$

- Additive inverse of a number is the number, which when added to a number, gives 0. It is also called the negative of a number.

  a + (– a) = (– a) + = 0
  ∴Additive inverse of $\frac{2}{5}$ is $\left(\frac{-2}{5}\right)$

- Reciprocal or multiplicative inverse of a number is the number, which when multiplied by the number, gives 1. Therefore, the reciprocal of $a$ is $\frac{1}{a}$. $\left[a \times \frac{1}{a} = 1\right]$

  ∴ Reciprocal of $\frac{-2}{3}$ is $\frac{-3}{2}$