Vibration of Strings and Air Columns

Superposition of waves and superposition of waves and superposition principle

When two or more waves arrive simultaneously in certain region of a medium, the particles of the medium are subjected to two or more simultaneous displacements, one due to each wave. The resultant displacement is the vector sum of full the displacements (because (displacement is a vector) and a new wave motion is produced.
(a) Superposition of waves. The phenomena of intermixing of two or more waves to produce a new wave, is called superposition of waves.
(b) Superposition principle. The superposition principle states that the resultant displacement of a particle is equal to the vector sum of the individual displacements given to it by the superposing waves.

$$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} + ... + \vec{y_n}$$

Stationary waves

Definition. Superposition of two travelling waves of same frequency and same amplitude and travelling with same velocity in opposite directions, produces stationary waves.

Nodes and Antinodes. In some regions, particles of medium have zero displacement and maximum strain. These regions are called nodes. They are denoted by N in Fig.
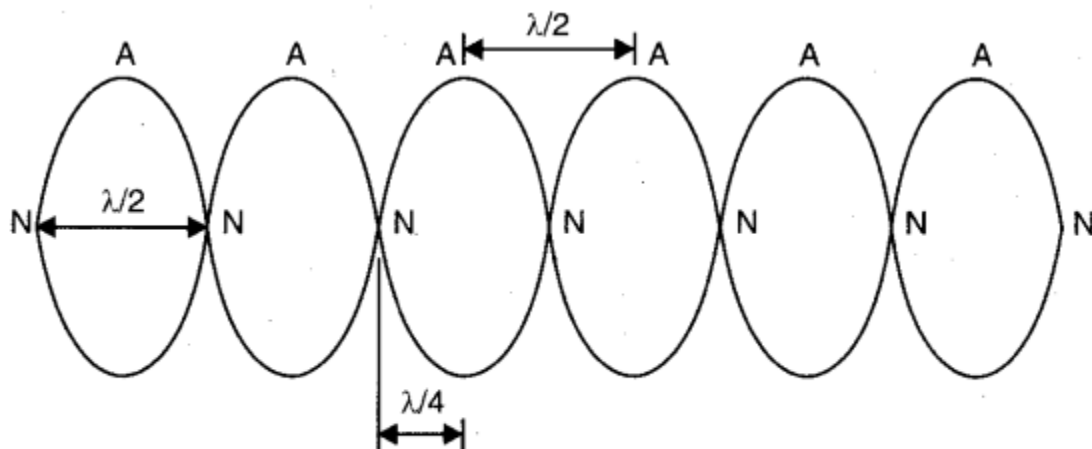

Fig. Stationary wave—nodes, antinodes.

In some other regions, particles of medium have maximum displacement and zero strain. These regions are called antinodes. They are denoted by A in Fig. They are situated between nodes.

The distance between two consecutive nodes or two consecutive antinodes is equal to half the wavelength of the stationary wave. The distance between a node and the nearest antinode, is equal to one-quarter of the wavelength of the stationary wave. (i.e., NN = AA = $\lambda/2$, NA = $\lambda/4$).

Production of transverse stationary waves in stretched string

Let a string be fixed at its ends A and B [Fig.]. Let it be plucked at C in the middle [Fig]. A disturbance is produced.

When the string is left free, it moves downward and disturbances move towards ends A and B. These disturbances are reflected from the fixed ends A and B and travel back in the string. Along the string, incident and reflected disturbances (waves) get superposed. Their superposition produces stationary waves in the string. The stationary waves are transverse because the string moves from C to D and back from D to C, whereas waves travel along the string from A to B and B to A.
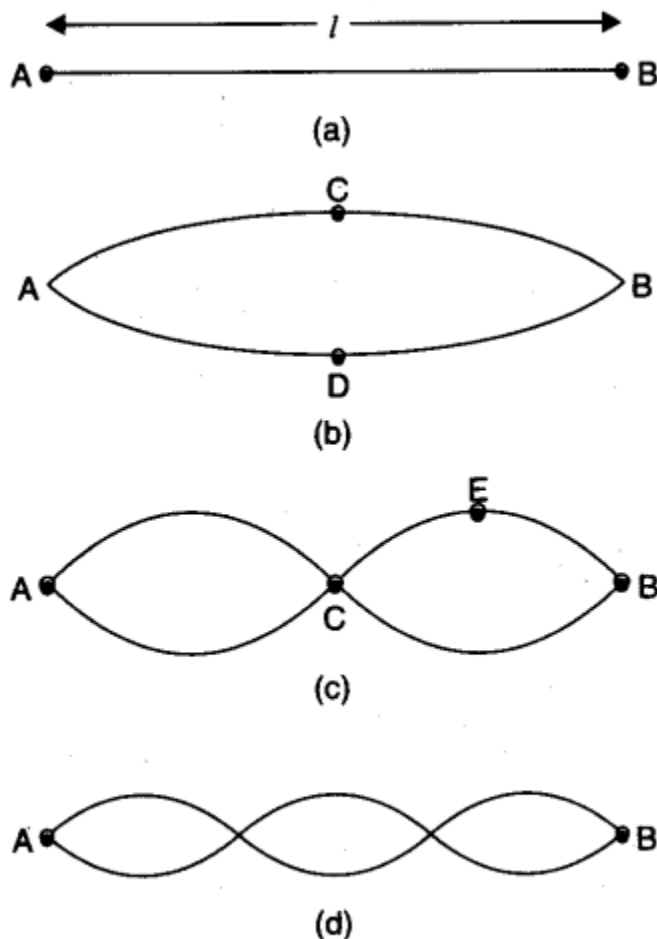

Fig. Different modes of vibration of a string.

Nodes are produced at the fixed ends A and B, where displacement is zero and strain is maximum. Antinode is produced in the middle (where the string was plucked) because there the displacement is maximum and strain is zero.

If the string be held at C and plucked at E—middle point of C and B [Fig. (c)] and then left free, it vibrates in two segments. Similarly, it can be made to vibrate in three segments [Fig. (d)]. Figs. (b), (c) and (d) represent different modes of vibration of a string.

Fundamental, harmonics and overtones

When a body is vibrated, it vibrates with more than one frequencies. All the frequencies are an integral multiple of some least frequency.

The least frequency is called first harmonic or fundamental. Frequency double than the fundamental is called second harmonic, triple the fundamental is called third harmonic and so on. In general, a frequency n times the fundamental is called nth harmonic.

All frequencies, other than the fundamental are called overtones. They are called first, second, third overtones in order of their increasing frequency after the fundamental. Thus second harmonic becomes first overtone and third harmonic becomes second overtone. In absence of second harmonic, third harmonic becomes first overtone.

Time period and frequency of transverse vibrations of a stretched string

Let a string of length l and mass per unit length m be fixed at its two ends and stretched by a tension T [Fig.(a)]. Let it be plucked in the middle at C and left free [Fig.(b)]. Transverse stationary waves are produced in it and they travel along the length of the string. The velocity of these transverse waves is given by the formula,

$$v = \sqrt{\frac{T}{m}}$$

$$\nu = \frac{v}{2l}$$

or

$$\boxed{\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

This frequency is called **first harmonic** or **fundamental** for the string vibrating in **one** segment [Fig. 18.02 (b)].

When string vibrates in **two** segment [Fig. 18.02 (c)], then length of each segment becomes $l/2$

and

$$\nu_1 = \frac{1}{\frac{2l}{2}}\sqrt{\frac{T}{m}} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2\nu$$

This frequency is called **second harmonic**. It becomes **first overtone.**

When string vibrates in **three segments** [Fig. 18.02 (d)], then length of each segment becomes $l/3$

and

$$\nu_2 = \frac{1}{\frac{2l}{3}}\sqrt{\frac{T}{m}} = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3\nu$$

This frequency is called **third harmonic**. It becomes **second overtone.**

In general, when the string vibrates in p segments, then

$$\nu = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

which becomes pth harmonic.

Therefore, in a stretched string, even and odd both types harmonics produced.

Laws of transverse vibrations of a stretched string

We know that a string of length l having mass per unit length m and stretched with a

tension T, has fundamental frequency of vibration v given by

$$v = \frac{l}{2l} \sqrt{\frac{T}{m}}$$

From above, we derive **three** laws of vibrations of a stretched string. The laws are :

1. **Law of Length.** The frequency of vibration of a string varies inversely as its length (provided its mass per unit length and tension remain constant).

i.e., $$v \propto \frac{1}{l} \qquad (m \text{ and } T \text{ constant}).$$

2. **Law of Tension.** The frequency of vibration of a string varies directly as the square root of its tension, (provided its length and mass per unit length remain constant),

i.e., $$v \propto \sqrt{T} \qquad (l \text{ and } m \text{ constant})$$

3. **Law of Mass.** The frequency of vibration of a string varies inversely as the square root of its mass per unit length (provided its length and tension remain constant).

i.e., $$v \propto \frac{1}{\sqrt{m}} \qquad (l \text{ and } T \text{ constant}).$$

We have also, $$v = \frac{1}{D} \qquad \text{(Law of diameter)}$$

and $$v = \frac{1}{\sqrt{\rho}} \qquad \text{(Law of density)}.$$

Production of longitudinal stationary waves in clamped rods

Introduction. When a clamped rod is rubbed along its length by a resined cloth, longitudinal waves are produced in it. These waves reflect from clamped r region and get superposed with the incident waves. As a result, longitudinal stationary waves are produced in the clamped rod.

Different Cases
1. Rod clamped at one end. The clamped end becomes node (N) and free end becomes antinode (A) as shown in Fig.
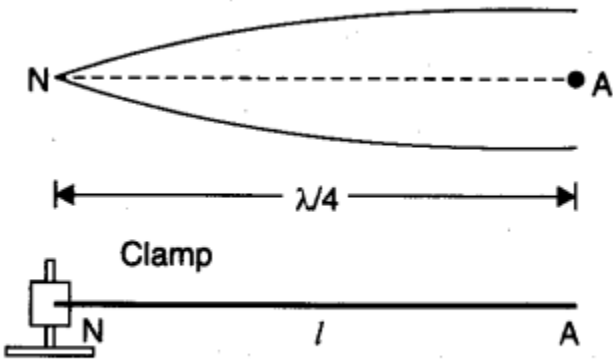
Fig.   Rod clamped at one end.

The length (l) of the rod is equal to quarter wavelength of the longitudinal waves

i.e., $\qquad l = \dfrac{\lambda}{4}$ or $\lambda = 4l.$

2. Rod clamped in the middle. The clamped part becomes node (N) and both the free ends antinode (A, A), as shown in Fig.

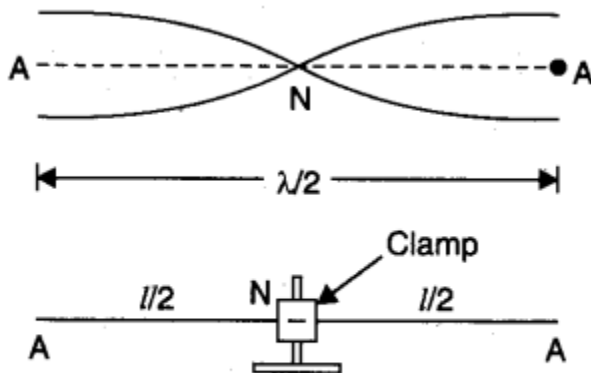**Fig.**   Rod clamped in the middle.

The length (l) of the rod is equal to half wavelength of the longitudinal waves

i.e., $\qquad l = \dfrac{\lambda}{2}$ or $\lambda = 2l.$

3. Rod clamped at both the ends. The clamped ends become node (N, N) and the middle point becomes antinode (A), as shown in Fig.
The length (l) of the rod is equal to half wavelength of the longitudinal waves.

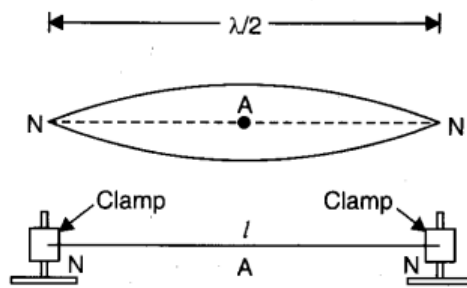*i.e.,* $$l = \lambda/2 \quad \text{or} \quad \lambda = 2l.$$

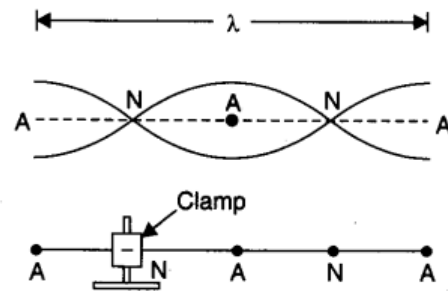**Fig.**   Rod clamped at both the ends.   **Fig.**   Rod clamped at one quater
length from one end.

4. Rod clamped at a point at quarter length from one end. The clamped point becomes node (N), and the two free ends become antinodes (A, A). Besides one more antinode is formed in the middle and one more node is formed at a point one quarter length away from the other end. It is shown in Fig.
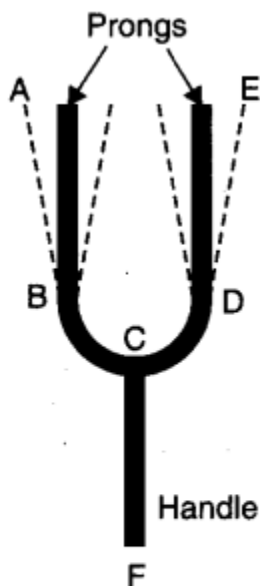
The length (l) of the rod is equal to the wavelength of the longitudinal wave i.e., l=λ .
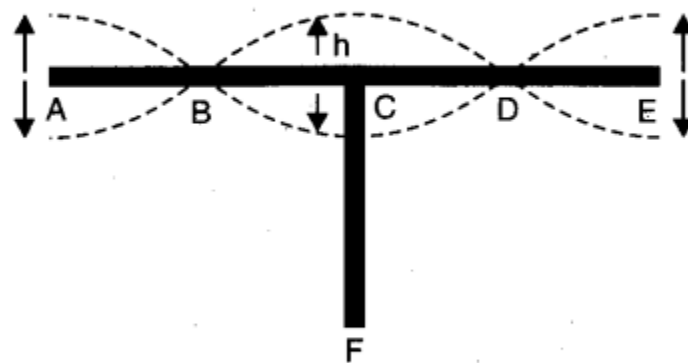
Tuning fork

Introduction. It is a source of a standard frequency very useful in sound experiment where a frequency standard is needed.

Principle. It works on the principle of vibration of a clamped (or supported) rod.

Diagram.

(a) Tuning fork.   (b) Mode of vibration of a tuning fork.

Fig.   Tuning fork and its mode of vibration.

Construction. It is a U-shaped bar ABODE made of steel with a handle CF attached at

the bend (C). Thus, it has a shape of letter Y. Its free ends A and E are called prongs. Its frequency is written near the bend.

Working. When one of the prong is struck with a soft rubber pad, the prongs begin to vibrate. This sets the whole tuning fork into vibrations as shown in Fig.

From Fig., it is quite clear that vibrations of prongs A and E are transverse (perpendicular to AC to EC), whereas vibrations of handle CF are longitudinal (along CF). It is for this reason that vibrations of prongs stop just by touching them by a finger, whereas vibrations of handle continue even when it is held tightly in hand.

Application. When vibrated, tuning fork vibrates with its frequency (written near the bend). It then produces in air sound waves of same frequency as its own frequency. These vibrations of known frequency can be used either with a sonometer for verifying laws of transverse vibrations of a stretched string or with a resonance tube for determination of velocity of sound in air.

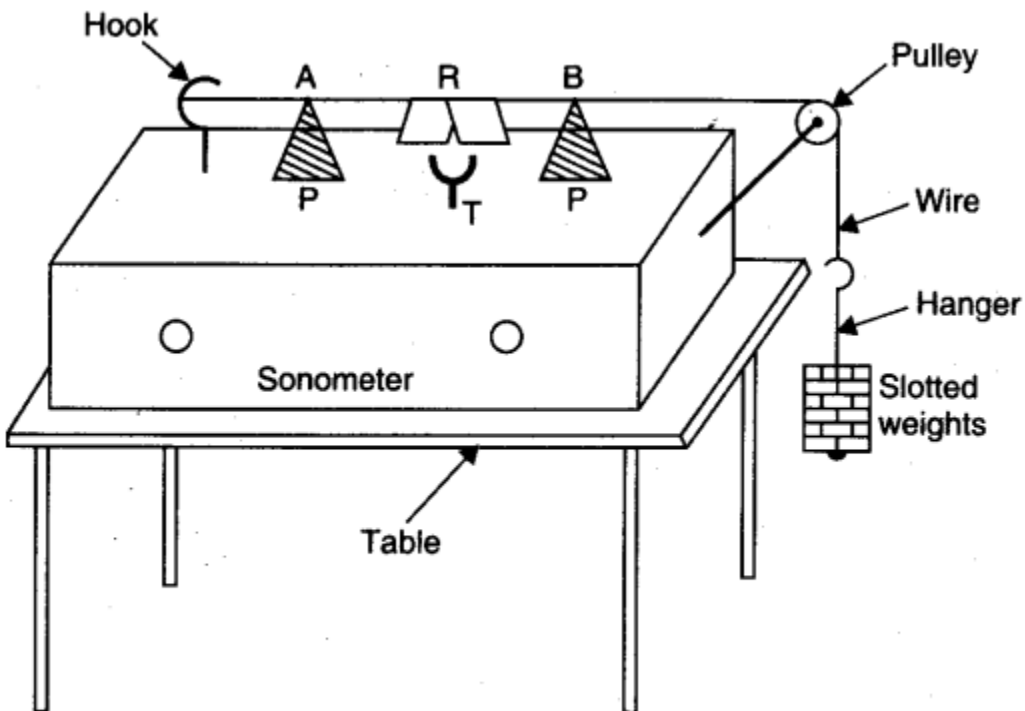## Sonometer
Diagram.

Fig.   Sonometer in experimental set up.

Construction. It is a hollow and rectangular wooden box of length more than a metre. It has a hook fixed over its board at one end and a friction less pulley at the other end. A string has one end tied to the hook and passes over the pulley. From other end of the string is suspended a hanger in which slotted weights can be added. The weights give tension to the string. Two wooden prisms (bridges) P, P are put over the board. The length of the string is adjusted between edges of these two prisms. Two holes are provided in one of the long vertical wall. These holes allow the vibrations in the air

inside the sonometer to outside air to be received by the experiementer.

Working. A known weight is suspended from the hanger to give a known tension T to the string. The tuning fork (T) of unknown frequency v is vibrated and put over sonometer board. The length of string between edges is so adjusted that it vibrates in resonance with tuning fork. (When this happens, a paper rider R kept over the string in the middle of the edges falls down). The length l of the string between the edges is noted.

Calculation. If m be the mass per unit length of the string, then velocity of transverse waves through string is given by

$$v = \sqrt{\frac{T}{m}}.$$

The length l of the string is equal to half the wavelength of stationary waves having frequency equal to frequency (v) of the timing fork.

Wavelength of stationary waves, $\qquad\qquad \lambda = 2l$

Frequency of stationary waves, $\qquad\qquad v = \dfrac{v}{\lambda} = \dfrac{\sqrt{T/m}}{2l}$

or $$\boxed{v = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

The value of v gives unknown frequency of the tuning fork.

Free, forced and resonant vibrations

(a) Free vibrations. Vibrations of a vibrating body, continued by itself, are called free vibrations. In this case, the body vibrates with its own frequency, called natural frequency.

Example. In sonometer experiment, vibrations of tuning fork are free vibrations.

(b) Forced vibrations. Vibrations of a vibrating body under a periodic force, are called forced vibrations. In this case the body vibrates with the frequency of the applied periodic force and only so long as the force acts. Once the periodic force is withdrawn, the body either stops vibrating or vibrates with its own frequency.

Example. In sonometer experiment, vibrations of sonometer board are forced vibrations.

(c) Resonant vibrations. Vibrations of a vibrating body, under the influence of a second vibrating body of same frequency kept near it, are called resonant vibrations. The phenomena is called resonance. In this case, the first vibrating body will continue vibrating even when the second body is removed.

Example. In sonometer experiment, vibrations of sonometer wire, when paper rider falls, are resonant vibrations. (The wire is in resonance with the tuning fork. Both have same frequency).