CBSE Test Paper 01

CH-13 Limits and Derivatives

1. If G (x) = $\sqrt{25-x^2}$ then $\underset{x \to 1}{Lt} \frac{G(x)-G(1)}{x-1}$ has the value
   a. $\frac{1}{24}$
   b. $-\sqrt{24}$
   c. $\frac{-1}{\sqrt{24}}$
   d. $\frac{1}{5}$
2. $\underset{x \to \frac{\pi}{3}}{Lt} \frac{\sec x-2}{x-\frac{\pi}{3}}$ is equal to
   a. 2
   b. $2+\sqrt{3}$
   c. $\sqrt{3}$
   d. $2\sqrt{3}$
3. The function, $f(x)=(x-a)^2 \cos \frac{1}{x-a} for\ x \neq a$ and f (a) = 0, is
   a. continuous but not derivable at x = 0
   b. derivable at x = a
   c. not continuous at x = a
   d. neither continuous nor derivable at x=a
4. $\underset{x \to 4}{Lt} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} =$
   a. does not exist
   b. 0
   c. $-\frac{1}{3}$
   d. $\frac{1}{3}$
5. $\underset{x \to 0}{Lt} \frac{\sin x^n}{(\sin x)^m}, n>m>0,$ is equal to
   a. $\frac{m}{n}$
   b. 0
   c. 1
   d. $\frac{n}{m}$
6. Fill in the blanks:

   The value of given limit $\lim_{x \to 0} \frac{\cos x}{\pi-x}$ is ________.

7. Fill in the blanks:

   The value of limit $\lim_{r \to 1} \pi r^2$ is ________.

8. font-family: Verdana font-size: 8px Evaluate $\lim_{x \to 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

9. Find the derivative of x at x = 1

10. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$

11. Find the value of $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$.

12. Find the derivative of $2x - \frac{3}{4}$

13. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{px^2+qx+r}$

14. Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$ and if $\lim_{x \to 1}$ f(x) = f(1), then what are the possible values of a and b?

15. The differentiation of sec x with respect to x is sec x tan x.

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Solution

1. (c) $\frac{-1}{\sqrt{24}}$

   Explanation: The equation is in the form of 0/0

   Using L'Hospital rule we have $\frac{\frac{1}{2\sqrt{25-x^2}}\cdot(-2x)}{1}$
   substituting x = 1 we get $\frac{-1}{\sqrt{24}}$
2. (d) $2\sqrt{3}$

   Explanation: Using L'Hospital;

   $\underset{x\to\frac{\pi}{3}}{Lt}\frac{\sec x\tan x}{1}$
   $\Rightarrow 2\sqrt{3}$
3. (b) derivable at x = a

   Explanation: situation x - a = t; then the function will become
   $\Rightarrow \underset{t\to 0}{Lt}\, t^2\cos\frac{1}{t}$
   $\Rightarrow$ 0. Finite number = 0
   f(a) = 0
4. (c) $-\frac{1}{3}$

   Explanation: Equation is in the form of 0/0

   Using L'Hospital rule we get $\frac{-\frac{1}{2\sqrt{5+x}}}{\frac{1}{2\sqrt{5-x}}}$
   Substituting x = 4 we get $\frac{-1}{3}$
5. (b) 0

   Explanation: $\mathrm{lit}_{x\to 0}\frac{\sin x^n}{(\sin x)^m}\cdot\frac{x^{m+n}}{x^{m+n}}$

   $\Rightarrow \underset{x\to 0}{Lt}\frac{\sin x^n}{x^n}\cdot\frac{x^m}{(\sin x)^m}\cdot\frac{x^n}{x^m}$

$\Rightarrow 1.1^m . x^{n-m}$

$\Rightarrow$ 1.0 = 0

6. $\frac{1}{\pi}$

7. π

8. When x = 2, the expression $\frac{x^3-6x^2+11x-6}{x^2-6x+8}$ assume the form $\frac{0}{0}$. Therefore, (x-2) is factor common to numerator and denominator. Factorising the numerator and denominator, we have

   font - family : Verdana font - size : 8px $\therefore \lim_{x\to 2} \frac{x^3-6x^2+11x-6}{x^2-6x+8}$

   $= \lim_{x\to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$

   $= \lim_{x\to 2} \frac{(x-1)(x-3)}{(x-4)} = \frac{(2-1)(2-3)}{(2-4)} = \frac{1}{2}$

9. Here $\frac{d}{dx}$ (x) = 1

   $\therefore$ Derivative of x at x = 1

10. Here $f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x = ax^-4 - bx^{-2} + \cos x$

    $\therefore\ f'(x) = \frac{d}{dx}[ax^{-4} - bx^2 + \cos x] = a\frac{d}{dx}(x^{-4}) - b\frac{d}{dx}(x^{-2}) + \frac{d}{dx}(\cos x)$

    $-ax^{-5} + 2bx^{-3} - \sin x = \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$

    $-4ax^{-5} + 2bx^{-3} - \sin x = \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$

11. We have, $\lim\limits_{x\to 0} \frac{e^{3x}-1}{x} = \lim\limits_{x\to 0} \frac{e^{3x}-1}{x} \times \frac{3}{3}$ [multiplying numerator and denominator by 3]

    $= 3\lim\limits_{x\to 0} \frac{e^{3x}-1}{3x}$ ..... (i)

    Let h = 3x, as x $\to$ 0, then h $\to$ 0

    Then, from Eq. (i), we get

    $\lim\limits_{x\to 0} \frac{e^{3x}-1}{x} = 3\lim\limits_{h\to 0} \frac{e^h-1}{h} = 3(1)\left[\because \lim\limits_{x\to 0} \frac{e^x-1}{x} = 1\right]$

    = 3

12. Here $f(x) = 2x - \frac{3}{4}$

$\therefore\ f'(x) = \frac{d}{dx}\left(2x - \frac{3}{4}\right)$

$= 2\frac{d}{dx}(x) - \frac{d}{dx}\left(\frac{3}{4}\right)$

= 2 × 1 - 0 = 2

13. $f(x) = \frac{ax+b}{px^2+qx+r}$

$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{ax+b}{px^2+qx+r}\right]$

$= \frac{(px^2+qx+r)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(px^2+qx+r)}{(px^2+qx+r)^2}$

$= \frac{(px^2+qx+r)(a)-(ax+b)(2px+q)}{(px^2+qx+r)^2}$

$= \frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{(px^2+qx+r)^2}$

$= \frac{-apx^2-2bpx+ar-bq}{(px^2+qx+r)^2}$

14. We have,

$$f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, x>1 \end{cases}$$

Now, LHL $= \lim\limits_{x\to1^-} f(x)$

$= \lim\limits_{x\to1^-}(a+bx) = \lim\limits_{h\to0}[a+b(1-h)]$ [putting x = 1 - h as x $\to$ 1, then h $\to$ 0]

= a + b

RHL $= \lim\limits_{x\to1^+} f(x)$

$= \lim\limits_{x\to1^+}(b-ax) = \lim\limits_{h\to0}[b-a(1+h)]$ [putting x = 1 + h as x $\to$ 1, then h $\to$ 0]

= b - a

Since, $\lim\limits_{x\to1}$ f(x) = f(1)

$\therefore$ LHL = RHL = f(1)

$\Rightarrow$ a + b = b - a = 4 [$\because$ f(1) = 4, given]

$\Rightarrow$ a + b = 4 ..(i) and b - a = 4 ..(ii)

On solving (i) and (ii), we get

a = 0, b = 4

15. Let f(x) = sec x. Then, f(x + h) = sec (x + h)

$\therefore \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\sec(x+h)-\sec x}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\frac{1}{\cos(x+h)}-\frac{1}{\cos x}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\cos x-\cos(x+h)}{h \cos x \cos(x+h)}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{2 \sin\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x+h-x}{2}\right)}{h \cos x \cos(x+h)}$

$\left[\because \cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)\right]$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{2 \sin\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{h \cos x \cos(x+h)}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos x \cos(x+h)} \times \lim_{h \to 0} \frac{\sin(h/2)}{(h/2)}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\frac{\sin x}{\cos x \cos x} \times 1$ = tan x sec x. $\left[\because \lim_{h \to 0} \frac{\sin(h/2)}{(h/2)} = 1\right]$

Hence, $\frac{d}{dx}$ (sec x) = sec x tan x.