lecture 16
11/04/19

laminar Flow through pipe-

→ Head loss
→ Power consumption

(a) Head loss

$$h_f = \frac{f.L.V^2}{2gD}$$

$$f = \frac{64}{Re}$$

$$h_f = \frac{32.\mu.V.L}{\rho.g\,D^2}$$

$$\boxed{h_f \propto V}$$

My wrong thinking

$$\cancel{h_f = \frac{f.L.V^2}{2gD}}$$

$$\cancel{\boxed{h_f \propto V^2}}$$

$$V = \frac{Q}{\frac{\pi}{4}D^2}$$

$$h_f = \frac{128}{\pi}.\frac{\mu.Q.L}{\rho g\,D^4}$$

$$\boxed{h_f \propto Q}$$

②

My wronge thinking

$$\cancel{h_f = \frac{8Q^2}{\pi^2 g}\cdot\frac{f.L}{D^5}} \qquad \boxed{h_f \propto Q^2}$$

Power consumption: $\dot{m} g h_f$

$$\left\{ \begin{array}{l} h_f \propto Q \\ h_f = \underset{\text{constant}}{c}\,Q \end{array} \right.$$

$= (\rho Q . g)(cQ)$

$= \underbrace{(c\rho g)}_{\text{const.}} Q^2$

Power consumption $\propto Q^2$

Pb (12)

$P_{initial} = C . Q^2$

$P_{initial} = c(1.2Q)^2$

$$\% \text{ change} = \frac{c(1.2Q)^2 - cQ^2}{cQ^2} \times 100$$

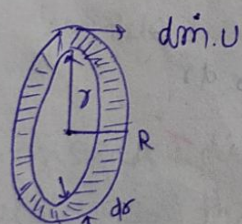
$= 44$

Correction factor:-

→ Momentum correction factor (β)

→ Kinetic energy correction factor (α)

① M.C.F = $\dfrac{\text{momentum per sec across a particular sec}^n \text{ calculated on the basis of (actual velocity)}}{\text{mom. per sec across the same section calculated on the basis of (Mean velocity)}}$



$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\beta = \frac{\int_0^R d\dot{m}.u}{\dot{m}\,\bar{U}} = \int_0^R \frac{\cancel{\rho}(2\cancel{\pi}.r.dr).u.u}{\cancel{\rho}(\cancel{\pi} R^2)\bar{U}.\bar{U}}$$

$$= \frac{2}{R^2.\bar{U}^2}\int_0^R u^2.r.dr \qquad \left(\bar{U} = \frac{U_{max}}{2}\right)$$

$$= \frac{2}{R^2\left(\frac{U_{max}}{2}\right)^2}\int_0^R U_{max}^2\left(1 - \frac{r^2}{R^2}\right).r.dr$$

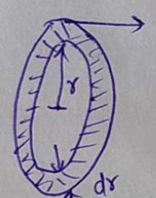
$$\beta = \frac{8}{R^2}\int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 . r\,dr$$

$$= \frac{8}{R^2}\int_0^R \left[r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right] dr$$

$$= \frac{8}{R^2}\left[\frac{R^2}{2} + \frac{R^6}{6R^4} - \frac{2R^4}{4R^2}\right] = \frac{8}{R^2}\times\frac{R^2}{6} = \frac{8}{6} = 1.33$$

$\boxed{\beta = 1.33}$ [For laminar pipe flow]

$$\alpha = \frac{\text{K.E. per sec across a particular sec}^n\text{ calculated on the basis of actual velocity}}{\text{K.E. per sec across a same sec}^n\text{ calculated on the basis of [mean velocity]}}$$



$$U = U_{max}\left[1 - \frac{r^2}{R^2}\right]$$

$$\alpha = \frac{\int_0^R \frac{1}{2}\, d\dot{m} u^2}{\frac{1}{2}\dot{m}\,\bar{U}^2} = \frac{\int_0^R \rho\,(2\pi r\, dr).U.U^2}{\rho(\pi R^2)\bar{U}.\bar{U}^2}$$

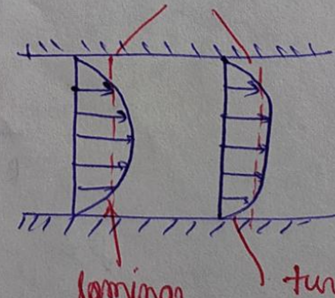
$$= \frac{2}{R^2\bar{U}^3}\int_0^R U^3.r.dr$$

$$= \frac{2}{R^2\left(\frac{U_{max}}{2}\right)^3}\int_0^R U_{max}^3\left(1 - \frac{r^2}{R^2}\right)^3.r.dr$$

$$\alpha = \frac{16}{R^2}\int_0^R \cancel{U_{max}^3}\left(1 - \frac{r^2}{R^2}\right)^3.r.dr$$

$$\alpha = \frac{16}{R^2}\times\frac{R^2}{8} = 2$$

$\boxed{\alpha = 2}$



$\dot{m} = \int d\dot{m}$

&

$\rho.A.\bar{U}$

$\dot{m}\bar{U} \neq \int d\dot{m}\, U$

$\frac{1}{2}\dot{m}\bar{U}^2 \neq \int \frac{1}{2}\, d\dot{m}\, U^2$

(4) • α, β value will be less in turbulant Pipe Flow.

(Pb) (20)

(2) $\frac{P}{\rho_{oil} \cdot g} = -\frac{(13.6)(10^3)\cancel{g}(0.02)}{(750)\cancel{g}} = -0.362\,m$

$V = \frac{Q}{A} = \frac{0.07}{\frac{\pi}{4}(0.15)^2} \times 1.1 = 0.879\,m$

$Z = \frac{\alpha V^2}{2g} = \frac{(3.96)^2}{2\times 9.81} \times 1.1 = 0.879\,m$

$\frac{P}{\rho_{oil} g} + \frac{\alpha V^2}{2g} + Z = -0.362 + 0.879 + 0.12$

$= 0.637\,m$ Ans.

Shear Velocity (V^*)

$\sqrt{\frac{\tau_0}{\rho}} = V^* = \bar{U}\sqrt{\frac{f}{8}}$ m/s

$\bar{U}$ = mean velocity of Flow

(Shear velocity is Fictitious quantity)

New chapter (8)

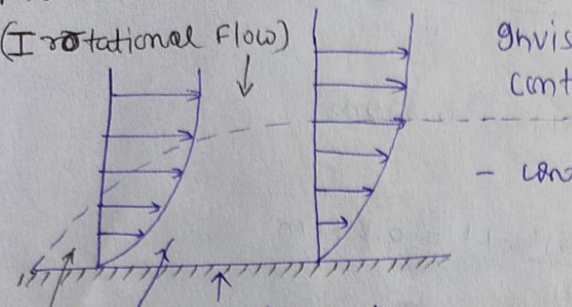
$\boxed{\text{Boundary layer theory}}$: When a real fluid flow over a solid body, the fluid particle at the surface of body flow with the same velocity as with the surface to satisfied no slip condition so relative velocity of the fluid particle at the surface of solid body is '0'. away from the solid body. in the transverse dirn the velocity of fluid particle increases gradually so the velocity gradient exist in this reason closed to boundary due to velocity gradient viscous stesses ($\tau = \mu \frac{du}{dy}$) exist in this reason this narrow reason is known as boundary region.

As we move away from the surface there is a developement of velocity in the transverse dirn of flow therefore, slowly viscous shear stress are decreasing and the velocity will increasing and after travelling the distance in the transverse dirn almost viscous shear stress are closed to '0'

(5) and the velocity is almost constant

Flow over flate Plate :

(Irrotational Flow)



Inviscid region of fluid flow ($F_{vis} \approx 0$)
Continuity eqn • Euler momentum eqn

- continuity eqn • Navier - Stoke momentum eqn

U_∞ = free stream velocity of flow

Rotational [$F_{vis} \neq 0$] Flow

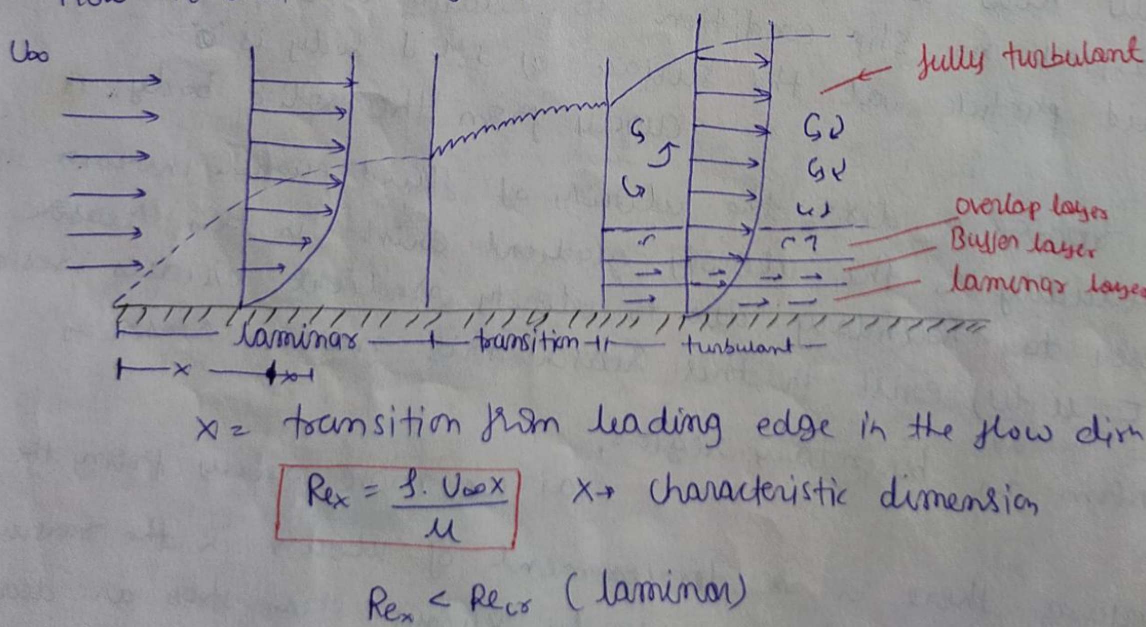
No slip condn

(Discover by Prandtl)

Development of boundary layer over a flat plate :

It was experimently found that upto a certain distance the flow in the boundary layer is laminar and as the boundary layer goes instability occurs and the flow changes from laminar to turbulant through transition

Note Fully developed flow can not be achieved in external Flow it can be only in internal flow (pipe flow)



x = transition from leading edge in the flow dirn

$$\boxed{Re_x = \frac{\rho \cdot U_\infty x}{\mu}}$$ x → characteristic dimension

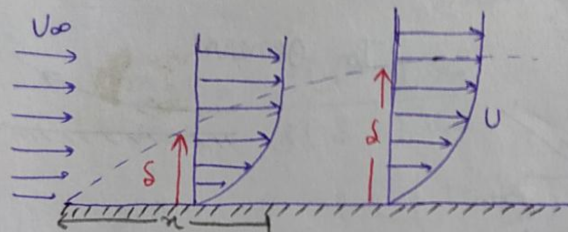
$Re_x < Re_{cr}$ (laminar)

$Re_x > Re_{cr}$ (turbulant)

Re_{cr} = critical reynold No.

For Flate Plate $\boxed{Re_{cr} = 5 \times 10^5}$

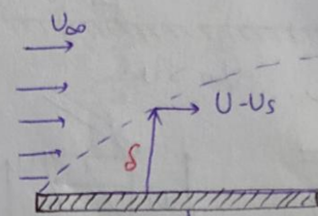
⑥ • **Boundary layer thickness (δ) :-** It is ⊥ distance from the surface of solid body measured at a particular point (x), at which the velocity of fluid particle is 99% of free stream velocity



At $y=\delta$

$U \simeq 0.99\,U_\infty$

Extra



U_s = velocity of surface

At $y=\delta$

$U-U_s \approx 0.99(U_\infty - U_s)$

• **Displacement thickness : (δ^*)**

$$\delta^* = \int_0^\delta \left[1-\frac{U}{U_\infty}\right]dy$$

• **Kinetic energy thickness : (δ^{**})**

$$\delta^{**} = \int_0^\delta \frac{U}{U_\infty}\left[1-\frac{U^2}{U_\infty^2}\right]dy$$

• **Momentum thickness (θ)**

$$\theta = \int_0^\delta \frac{U}{U_\infty}\left[1-\frac{U}{U_\infty}\right]\cdot dy$$

Shape factor = $\dfrac{\text{Displacement thickness}}{\text{Momentum thickness}}$

Pb ① Given,

$$\frac{U}{U_\infty} = \frac{y}{\delta}$$

Determine $\delta^*, \theta, \delta^{**}$

$$\delta^* = \int_0^\delta \left[1-\frac{U}{U_\infty}\right]dy$$

$$= \int_0^\delta \left(1-\frac{y}{\delta}\right)dy$$

$$= \left(y - \frac{y^2}{2\delta}\right)_0^\delta = \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{U}{U_\infty}\left(1-\frac{U}{U_\infty}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy$$

$$= \int_0^\delta \left(\frac{y}{\delta}-\frac{y^2}{\delta^2}\right)dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta}{2}-\frac{\delta}{3} = \frac{\delta}{6}$$

$$\delta^{**} = \int_0^\delta \frac{U}{U_\infty}\left(1-\frac{U^2}{U_\infty^2}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y^2}{\delta^2}\right)dy$$

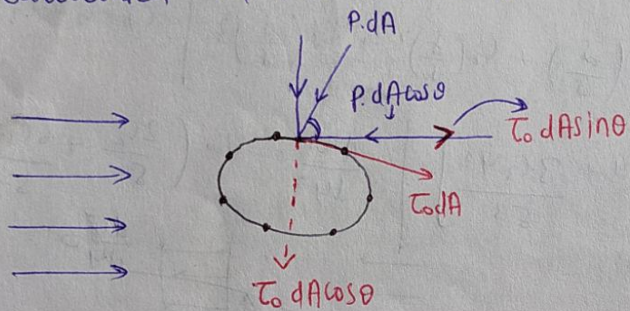
$$= \int_0^\delta \left(\frac{y}{\delta}-\frac{y^3}{\delta^3}\right)dy$$

$$= \left[\frac{y^2}{2\delta}-\frac{y^4}{4\delta^3}\right]_0^\delta$$

$$= \frac{\delta}{2}-\frac{\delta}{4}$$

$$= \boxed{\frac{\delta}{4}}$$

Flow over submerge body: When a fluid is Flowing over a stationary body, a force is exerted by the fluid on the body. similarly when a body is moving in a stationary fluid, force is exerted by the fluid on the body also when the body at fluid both are move at different velocity, a force is exerted by the fluid on the body (calculated on the basis of relative velocity) (Imp for ESE purpose)



Drag force
(Force applied on the fluid over the body in the dirn of flow

$$F_D = \underbrace{\oint P.dA\cos\theta}_{\text{Pressure drag}} + \underbrace{\oint \tau_0.dA\sin\theta}_{\text{skin-friction drag}}$$

Pressure drag
(or)
Form drag

Lift force (F_L) (force applied on the fluid over a body in the transverse dirn)

$$F_L = \oint P.dA.\sin\theta + \oint \tau_0 dA\cos\theta$$

also

Projected Area

$$F_D = C_D \times \frac{1}{2}\rho A \times V_\infty^2$$

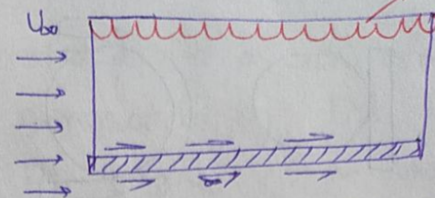
$$F_L = C_L \times \frac{1}{2}\rho \times A \times V_\infty^2$$

lift coefficient

Drag on different type of body:-

streamlined body

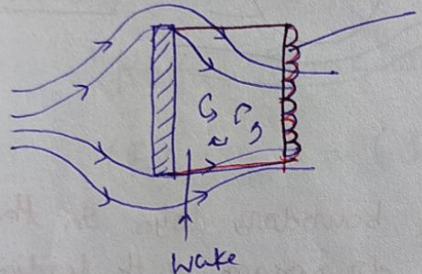
A body whose shape concides with stream line, when placed in a flow, known as stream line body



[Projected Area in the transverse dirn of fluid) Flow

Only Skin friction drag

② Bluff body

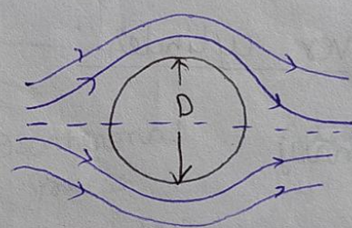


Projected area (in the dirn of flow)

Pressure drag ⇒ max

③ Flow over a sphere →

$Re = \frac{\rho U_\infty D}{\mu}$ $\left[C_D = \frac{24}{Re}\right]$



($Re \leq 0.2$) Stoke law is valid

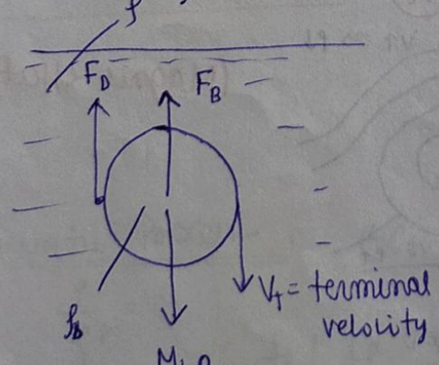
$$F_D = C_D \times \frac{1}{2} \rho \cdot A \cdot U_\infty^2$$

$$= \frac{24}{Re} \times \frac{1}{2} \times \rho \times \frac{\pi D^2}{4} \times U_\infty^2$$

$$= \frac{3\pi \rho . D^2 . U_\infty^2}{\frac{\rho U_\infty . D}{\mu}},$$

$$\boxed{F_D = 3\pi \mu . D . U_\infty}$$

(Pb) -



ρ = density of fluid

ρ_b = density of body

$$F_B + F_D = M_b g$$

$$F_D = M_b g - F_B$$

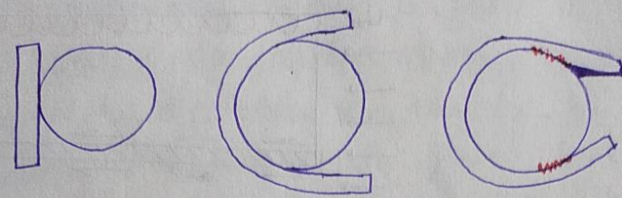
$$3\pi \mu . D V_T = \rho_b \forall_{sphere} . g - \rho \forall_{sphere} . g$$

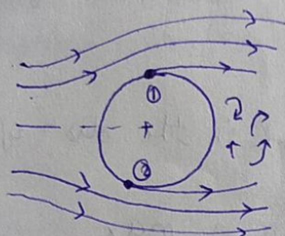
$$3\pi \mu . D V_T = (\rho_b - \rho) \frac{4}{3} \pi \left(\frac{D}{2}\right)^3 . g$$

$$3\mu . D . V_T = (\rho_b - \rho) \frac{D^3}{6} . g$$

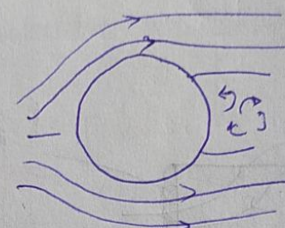
$$\boxed{V_T = \frac{(\rho_b - \rho) D^2 . g}{18\mu}}$$

In general





1,2. Flow separation points



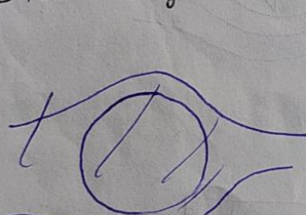
$\boxed{C_D \uparrow\uparrow}$

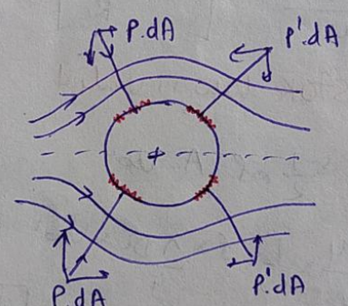
W.B.

(24) total drag is reduced if the boundary layer on the surface of a cylinder separates further downstream of the leading posit.

Flow over cylinder:

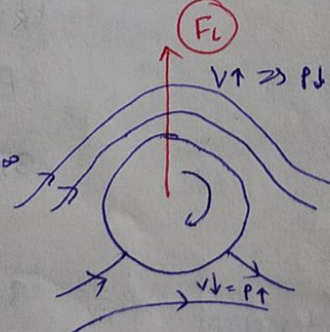
Stationary





$\boxed{F_L = 0}$

Rotating



(Magnus effect)

$F_L \neq 0$

Magnus effect

If the cylinder is rotated in a uniform stream there will be automatic generation of lift force this effect is was firstly seen by magnus known as magnus effect

Lift theorum :- (direct formula pucha gaya hai)

"Kutta - Jonkowski"

$$(F_L)_{cylinder} = (\rho V_\infty \Gamma . L)$$

$L \to$ length of cylinder

$\Gamma =$ circulation

Imp.

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + g.Z = \text{const}$$

Incompressible Flow

$\rho = \text{const}$

$$\frac{P}{\rho} + \frac{V^2}{2} + gZ = \text{const}$$

Divide by g

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{const}}$$

Isothermal Process

$T = \text{const}$

$$\boxed{\frac{P}{\rho} = C_1}$$

$$\int \frac{dP}{\rho} = C_1 \int \frac{dP}{P} = \frac{P}{\rho} \ln P$$

then,

$$\frac{P}{\rho} \ln P + \frac{V^2}{2} + gZ = \text{const}$$

Divide by g

$$\frac{P}{\rho g} \ln P + \frac{V^2}{2g} + Z = \text{const}$$

Adiabatic Process

Adiabatic Process -

(γ = Gamma)

$$\frac{P}{\rho^\gamma} = C_2$$

$$\int \frac{dP}{\rho} = C_2^{\frac{1}{\gamma}} \int \frac{dP}{P^{\frac{1}{\gamma}}} = \left(\frac{P}{\rho^\gamma}\right)^{\frac{1}{\gamma}} \cdot \frac{P^{-\frac{1}{\gamma}+1}}{-\frac{1}{\gamma}+1} = \frac{P^{\frac{1}{\gamma}} . P^{-\frac{1}{\gamma}+1}}{\rho . \frac{\gamma-1}{\gamma}} = \frac{\gamma}{\gamma-1} . \frac{P}{\rho}$$

① theory

① Boussinesq's theory :

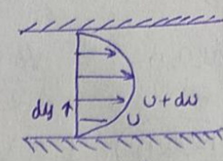
$$\tau_t = \eta \left(\frac{d\bar{u}}{dy}\right)$$

η = Eddy-viscosity

(Imp hai exam me puch sakta hai) (Flow property)

$$\tau_v = \mu \frac{du}{dy}$$

In general



μ = dynamic viscosity

(Fluid property)

Ex: for H_2O

P = 1 atm, $T = 20°C$

μ_w = 0.01 Pascal

② Reynold's theory

A/c to this

$\tau_t = \rho \overline{u'v'}$ $\{ \overline{u'v'} \neq 0 \}$

$\rho \overline{u'v'}$ are known as Reynold's stresses

③ Prandtl's mixing length theory : $L \Rightarrow$ [mixing length]

A/c to this the mixing length is defined as the avg. distance travelled by a particle in jumping from one layer to another, till it attains the velocity of other layer.

$L \rightarrow$ mixing ~~theory~~ length

$$u' \approx v' \approx L\left(\frac{d\bar{u}}{dy}\right)$$

A/c to Reynold's

$$\tau_t = \rho \cdot \overline{u'} \cdot \overline{v'}$$

$$\boxed{\tau_t = \rho L^2 \left(\frac{d\bar{u}}{dy}\right)^2}$$

④ universal velocity distribution eqn :- (2 mark question) bhi ban sakta question

$\tau = \tau_t$

A/c to Reynold theory

$$\tau = \rho \cdot L^2 \left(\frac{du}{dy}\right)^2$$

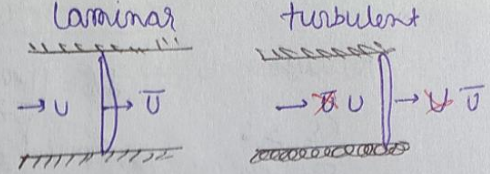
[For convenience, the bar denoting time avg of velocity is ornated (हटा दिया है)

$\left(\frac{d\bar{u}}{dy}\right) = \left(\frac{du}{dy}\right)$ because due to turbulence notation it is temporal mean)]

Near to wall

$\tau \approx \tau_0$

then, $\tau_0 = \rho L^2 \left(\frac{du}{dy}\right)^2$



$\Rightarrow \sqrt{\frac{\tau_0}{\rho}} = l \cdot \frac{du}{dy}$ [Prandt assume $l = Ky$ Karmann constant]

$\boxed{K = 0.4}$ *** imp note

V^*

$V^* = 0.4y \frac{du}{dy} = \frac{4}{10} \cdot y \cdot \frac{du}{dy} \Rightarrow 2.5 V^* \frac{dy}{y} = du$

grit it

$U = 2.5 V^* \log_e(y) + c$

$U = 5.75 V^* \log_{10} y + c$ --- (1)

Apply boundary cond

At $y = y'$, $U = 0$ (ये इस लिए ही है क्योंकि यही value assume ki hai) ($y=0$ nahi) hoga kuki $\log(0)$ undefined.

$0 = 5.75 V^* \log_{10} y' + c$

$c = 5.75 V^* \log_{10} y'$

By eq (1)

$U = 5.75 V^* \log_{10} \frac{y}{y'}$

$$\boxed{\frac{U}{V^*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)}$$

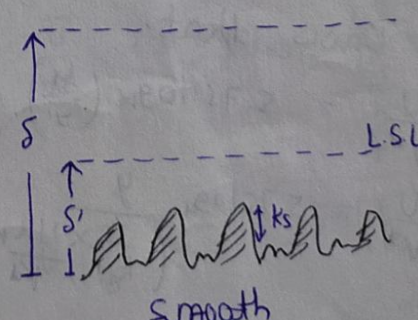
Hydrodynamics boundries:

smooth

$\frac{K_s}{\delta'} < 0.25$

transition

$0.25 < \frac{K_s}{\delta'} < 6$

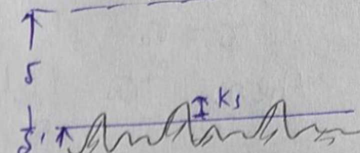


(By Nikuradse experiment)

Rough

$\frac{K_s}{\delta'} > 6$

$\delta' = \frac{11.6\nu}{V^*} \rightarrow \frac{m^2/s}{m/s}$



K_s = Surface Roughness

LSL → Laminar sub layer

$\sqrt{\frac{\tau_0}{\rho}} = V^* = \bar{U}\sqrt{\frac{f}{8}}$

$\bar{U}$ → mean velocity of pipe

PS-(32)

$K_s = 0.15mm$

$\tau_0 = 4.9\ N/m^2$

$\nu = 0.07$ stokes

Sol = $\frac{ks}{\delta'} = \frac{ks}{\frac{11.6\nu}{V^*}} = \frac{Ks \cdot V^*}{11.6\nu} \rightarrow \frac{Ks}{11.6}\sqrt{\frac{\tau_0}{\rho}}$

$= \frac{0.5 \times 10^{-3}}{11.6 \times 0.07 \times 10^{-4}} \sqrt{\frac{4.9}{10^3}}$

$= 0.905$ (transit)

Velocity distribution

Smooth

By nikuradse Exp:

$\boxed{y' = \frac{\delta'}{107}}$ $\boxed{\delta' = \frac{11.6\nu}{V^*}}$

we know that,

$\frac{U}{V^*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$

$\frac{U}{V^*} = 5.75\log_{10} \frac{y}{\left(\frac{1}{107} \times \frac{11.6\nu}{V^*}\right)}$

$= 5.75 \log_{10} \frac{yV^*}{\nu} + 5.75 \log_{10} \frac{(107)}{(11.6)}$

$\frac{U}{V^*} = 5.75\log_{10} \frac{yV^*}{\nu} + 5.5$

$\nu \rightarrow m^2/s$

Rough

we know that,

$\frac{U}{V^*} = 5.75\log_{10}\left(\frac{y}{y'}\right)$

$\frac{U}{V^*} = 5.75\log_{10}\left(\frac{y}{\frac{ks}{30}}\right)$

$= 5.75 \log_{10}\frac{y}{ks} + 5.75\log_{10}30$

$= 5.75 \log_{10}\left(\frac{y}{ks}\right) + 8.5$

common mean velocity distribution eqn:

(Valid for smooth and rough pipes)

$$\frac{U-\bar{U}}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$\bar{U}$ = mean velocity of pipe flow

$R \rightarrow$ Radius of pipe

Pb - 33 At $y = ??$, $U = \bar{U}$

$$\frac{\bar{U}-\bar{U}}{V^*} \rightarrow 0 = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$$\boxed{y = 0.2228R}$$

Friction law

Smooth $f = f^n(Re)$

transition $f = f^n\left(Re, \frac{K_s}{D}\right)$

Rough $f = f^n\left(\frac{K_s}{D}\right)$

$\frac{K_s}{D}$ = Relative roughness

Smooth

valid ($4000 < Re < 1$ lakh)

$$\boxed{f = \frac{0.316}{Re^{\frac{1}{4}}}}$$

$$\frac{U}{V^*} = 5.75 \log_{10} \frac{yV^*}{\nu} + 5.5$$

$$\frac{U}{V^*} - \frac{\bar{U}}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$$\frac{\bar{U}}{V^*} = 5.75 \log_{10} \frac{\frac{yV^*}{\nu}}{\frac{y}{R}} + 1.75$$

$$\frac{\bar{U}}{\bar{U}\sqrt{\frac{f}{8}}} = 5.75 \log_{10} \frac{RV^*}{\nu} + 1.75$$

Rough

$$\frac{U}{V^*} = 5.75 \log_{10}\left(\frac{y}{K_s}\right) + 8.5$$

$$\frac{U}{V^*} - \frac{\bar{U}}{V^*} = 5.75 \log_{10} \frac{y}{R} + 3.75$$

$$\frac{\bar{U}}{V^*} = 5.75 \log_{10} \frac{\frac{y}{K_s}}{\frac{y}{R}} + 4.75$$

$$\frac{\bar{U}}{\bar{U}\sqrt{\frac{f}{8}}} = 5.75 \log_{10}\left(\frac{R}{K_s}\right) + 4.75$$

$$\frac{1}{\sqrt{f}} = \frac{5.75}{\sqrt{8}} \log_{10}\left(\frac{D}{2K_s}\right) + 1.68$$

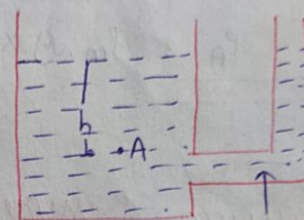
② **Simple Manometer** → It uses to measure the Pressure at a Point

[Free surface → the surface of fluid that is subjected to zero Parallel shear stress]

ⓐ Piezometer
ⓑ U-tube manometer
ⓒ Single column manometer

ⓐ **Piezometer**

$$P_A = \rho . g . h$$



Dia sufficiently large to neglect capillary effect

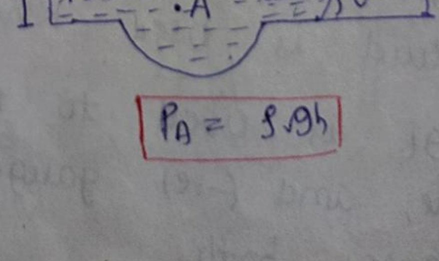
It is a single vertical tube, One end is open in atmosphere and the 2nd end is connected to that placed where the Pressure as to measure

Limitations

① Uses to measure moderate (+ve) gauge pressure of liquid.

Sensitivity of manometer :-

For a small gauge Pressure it is better to use inclined single tube the reading L recorded on a scale of incline tube manometer. is ~~Hise~~ ~~th~~

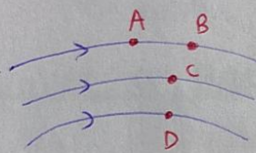
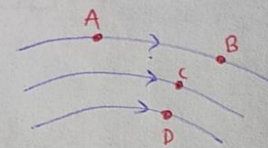


$$L = \frac{h}{\sin\theta}$$

$$\boxed{P_A = \rho g h}$$

hieigher than vertical tube manometer by a factor of

Application -

Irrotational Flow	Rotational flow
	
$E_A = E_B = E_C = E_D$	$E_A = E_B \neq E_C \neq E_D$
- Bernaullie eqn is valid for the Point lies on the same of different stream line	- Bernaullie eqn is only valid when the point around the same stream line

Head Form (energy per unit wt)

It is representation of energy in terms of height of fluid column.

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{const.}$$

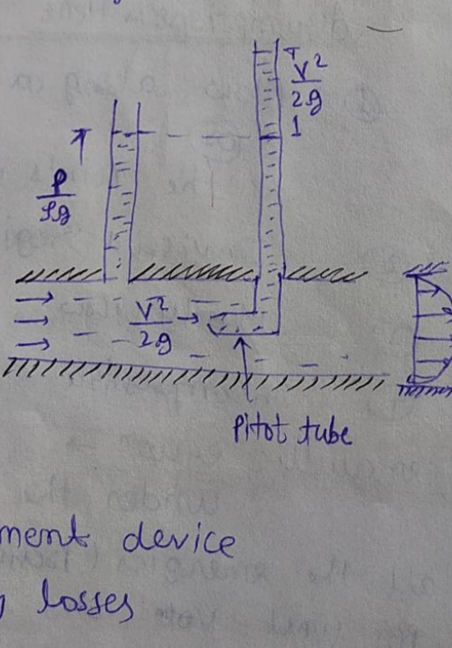
$\frac{P}{\rho g}$ = Pressure head, $\frac{V^2}{2g}$ = velocity head / Kinematic head / dynamic head

Z - Datum head

$$\frac{\cancel{m}\cancel{g} Z}{\cancel{m}\cancel{g}} = Z$$

total head = $\frac{P}{\rho g} + \frac{V^2}{2g} + Z$

Piezometric head = $\frac{P}{\rho g} + Z$



Pitot tube

Flow measurement devices :-

① Venturimeter : ① Discharge measurement device
② minimum energy losses

$\longrightarrow C_d = \frac{Q_{actual}}{Q_{theoritical}}$

Coefficient of discharge

$C_d \approx 0.95 - 0.99$

$\frac{F_D}{D^2 V^2 \rho}$, $\frac{\mu}{\rho . V . D}$

Solution- $m = 4+1$, $n = 3$
$= 5$

π_1 and π_2

① $\pi_1 = [D^a . V^b . \rho^c] . \mu$

Dimension

$M^0 L^0 T^0 = [L]^a [LT^{-1}]^b [ML^{-3}]^c \cancel{[ML^{-3}]} [ML^{-1}T^{-1}] = M^c . L^{(a+b-3c)} T^{-b} [ML^{-1}T^{-1}]$

$= M^{c+1} . L^{a+b-3c-1} . T^{-b-1}$

compare, $c+1=0$, $c=-1$

$+b+1=0$, $b=-1$

$a+b+3(-1)=0$

$a-1+3-1=0$ $\boxed{a=-1}$

② $\pi_1 = D^{-1} . V^{-1} \rho^{-1} . \mu$

$\boxed{\pi_1 = \frac{\mu}{\rho . V . D}}$

② $\pi_2 = [D^a . V^b . \rho^c] . F_D$

Dimension,

$= [L]^a (LT^{-1})^b (ML^{-3})^c [MLT^{-2}]$

$= [M^c L^{a+b-3c} . T^{-b}] [MLT^{-2}]$

$= M^{c+1} . L^{a+b-3c+1} . T^{-b+2}$

compare, $c+1=0$, $\boxed{c=-1}$

$-b-2=0$, $\boxed{b=-2}$

$a+b+1-3c=0$

$\boxed{a=-2}$

$\pi_2 = D^{-2} V^{-2} \rho^{-1} . F_D$

$\pi_2 = \frac{F_D}{D^2 . V^2 . \rho}$

A/c to Buckinghams

$\pi_2 = K fn(\pi_1)$

$\frac{F_D}{D^2 V^2 \rho} = K . fn\left(\frac{\mu}{\rho V D}\right)$

$$\boxed{F_D = K D^2 . V^2 . \rho \, fn\left(\frac{\mu}{\rho V D}\right)}$$

(Pb) $f_D = fn(D, V, \rho, \mu)$ (Not in short note)

$m=5$, $n=3$

π_1 and π_2

$\pi_1 = [D^a . V^b . \mu^c] . \rho$

Dimensions

$M^0 L^0 T^0 = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c \cancel{[ML^{-3}]} [ML^{-3}]$

$= (M^c . L^{a+b-c} . T^{-b-c}) [ML^{-3}]$

$= M^{c+1} . L^{a+b-c-3} . T^{-b-c}$

compare

$C+1=0, \ C=-1$

$-b-C=0$

$-b+1=0, b=1$

$a+b-C-3=0$

$a+1+1-3, a=-1$

$\pi = D^{1} V^{1} \mu^{-1} \rho$

$$\boxed{\pi_1 = \frac{\rho V D}{\mu}}$$

$\pi_2 = [D^a V^b \mu^c] F_D$

Dimension

$M^0 L^0 T^0 = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c [MLT^{-2}]$

$= M^{c+1} . L^{a+b-c} . T^{-b-c} [MLT^{-2}]$

$= M^{c+1} . L^{a+b-c+1} . T^{-b-c-2}$

Compare,

$c+1=0, \quad c=-1$

$a+b-c+1=$

$-b-c-2=0, \quad -b-(-1)-2=0$

$-b+1-2=0$

$-b-1=0$

$\boxed{b=-1}$ $\boxed{a=-1}$

(2) $\pi_2 = D^{-1} V^{-1} \mu^{-1} F_D$

$$\boxed{\pi_2 = \frac{F_D}{DV\mu}}$$

A/c to Buckingham's π method

$\pi_2 = K fn(\pi_1)$

$\frac{F_D}{DV\mu} = K fn\left(\frac{\rho.V.D}{\mu}\right)$

$F_D = K.D.V.\mu \, fn\left(\frac{\rho.V.D}{\mu}\right)$

Math-Extra, $F_D = K.D.V.\mu \, fn\left(\frac{\rho.V.D}{\mu}\right)$

$F_D = K D^2 V^2 \rho \left(\frac{\mu}{\rho.V.D}\right)$ fn(Re)

$F_D = K \rho^2$

Model and Simplitude study

generally to see the performance of any prototype, to perform the experiment on its model rather than to make prototype. generally (Not always) the size of model is much smaller than the Prototype a model should be similar to the Prototype. a model can be considered similar to the Prototype when the model has following 3 type of similarities

1. Geometric similarity
2. Kinematic similarity
3. dynamic similarity

(b) elastic pipe:-

K.E. of water (Before closure of valve) $= \frac{1}{2} mV^2 = \frac{1}{2} \rho (A.L)^2 V^2$ --- (1)

(2) Strain energy stored in water (After closure of valve) $= \left[\frac{1}{2} \times \text{stress} \times \text{strain}\right] \times \text{volm}$

$$= \left[\frac{1}{2} \times P \times \frac{P}{K}\right] A.L$$

$$= \frac{1}{2} \times \frac{P^2 \times (A \times L)}{K} \text{ --- (2)}$$

Strain energy stored in the pipe material (After closure of valve)

$$\underbrace{\frac{1}{2E}\left[\sigma_l^2 + \sigma_c^2 - 2\sigma_l \cdot \sigma_c \cdot \gamma\right]}_{\text{strain energy per unit volm}} \cdot \pi . D . L . t$$

$\sigma_l = \frac{P.D}{4t}$, $\sigma_c = \frac{P.D}{2t}$, $\gamma = \frac{1}{4}$

$$= \frac{1}{2E}\left[\frac{P^2 . D^2}{16t^2} + \frac{P^2 D^2}{4t^2} - 2\left(\frac{PD}{4t}\right)\left(\frac{PD}{2t}\right) \cdot \frac{1}{4}\right] \pi . D . L . t$$

$$= \frac{1}{2E}\left[\frac{P^2 D^2}{16t^2} + \frac{P^2 D^2}{4t^2} - \frac{P^2 D^2}{16t^2}\right] \pi . D . L . t$$

$$= \frac{1}{2E} \frac{P^2 D^2 . (\pi) D . L . t)}{4 t^2} = \frac{1}{2} \times \frac{P^2 D (A L)}{\cancel{AE} At} \text{ --- (3)}$$

By eq (1) (2) and (3)

$$\cancel{\frac{1}{2}} \frac{P^2}{K} (\cancel{A}.L) + \cancel{\frac{1}{2}} \frac{P^2 D (A.\cancel{L})}{E.L} = \cancel{\frac{1}{2}} \rho (\cancel{A}L) V^2$$

$$\frac{P^2}{K}\left(1 + \frac{DK}{Et}\right) = \rho V^2 \quad \text{Divide by } \rho g$$

$$P^2 = \frac{\rho . K . V^2}{1 + \frac{DK}{Et}}$$

$$\boxed{\frac{P}{\rho g} = \frac{V}{g}\sqrt{\frac{K}{\rho\left(1 + \frac{DK}{Et}\right)}}}$$

** Gradual closure = $\frac{P}{\rho g} = \frac{V}{g} \cdot \frac{L}{T}$

Sudden closure = Rigid pipe $\frac{P}{\rho g} = \frac{V}{g}\sqrt{\frac{K}{\rho}}$

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elastic pipe $\frac{P}{} = \frac{V}{} \sqrt{\frac{K}{}}$