Electronics Engineering
(Afternoon Session)
Exam Date- 15-02-2025

SECTION - A GENERAL APTITUDE

Q.1 Here are two analogous groups, Group-I and Group-II, that list words in their decreasing order of intensity. Identify the missing word in Group-II.

Group-I: Abuse → Insult → Ridicule

Group-II: ________ → Praise → Appreciate

(a) Extol
(b) Prize
(c) Appropriate
(d) Espouse

Ans. (a)

End of Solution

Q.2 Had I learnt acting as a child, I ________ a famous film star.

Select the most appropriate option to complete the above sentence.

(a) will be
(b) can be
(c) am going to be
(d) could have been

Ans. (d)

End of Solution

Q.3 The 12 musical notes are given as C, $C^{\#}$, D, $D^{\#}$, E, F, $F^{\#}$, G, $G^{\#}$, A, $A^{\#}$. Frequency of each note is $\sqrt[12]{2}$ times the frequency of the previous note. If the frequency of the note C is 130.8 Hz, then the ratio of frequencies of notes $F^{\#}$ and C is:

(a) $\sqrt[6]{2}$
(b) $\sqrt{2}$
(c) $\sqrt[4]{2}$
(d) 2

Ans. (b)

Given, Common ratio $= \sqrt[12]{2} = (2)^{1/12}$

$F^{\#} \rightarrow 7^{th}$ term

$C \rightarrow 1^{st}$ term

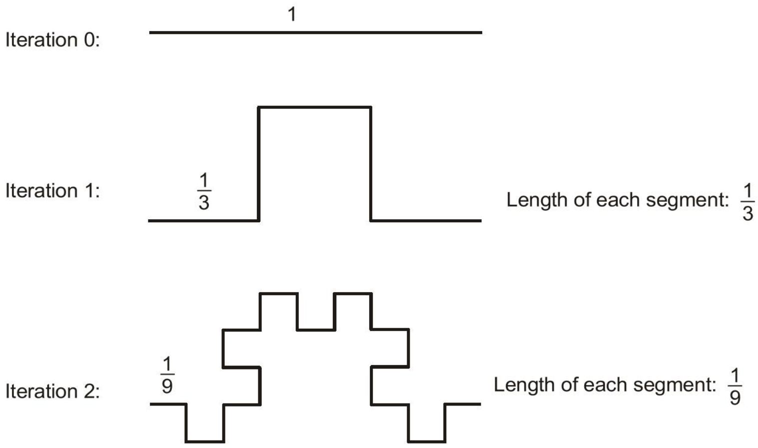
∴ The ratio of frequencies of notes $F^{\#}$ and C is,

$$= \frac{7^{th} \text{ term}}{1^{st} \text{ term}} = \frac{130.8 \times [2^{1/12}]^6}{130.8} = 2^{1/2} = \sqrt{2}$$

End of Solution

Q.4 The following figures show three curves generated using an iterative algorithm. The total length of the curve generated after 'Iteration n' is:

Note: The figures shown are representative.



(a) $\left(\frac{5}{3}\right)^{\frac{n}{2}}$ (b) $\left(\frac{5}{3}\right)^{n}$

(c) $\left(\frac{5}{3}\right)^{2n}$ (d) $\left(\frac{5}{3}\right)^{n(2n-1)}$

Ans. **(b)**

Iteration 0 : [1 segment × 1] = 1

Iteration 1 : $\left[5 \text{ segments} \times \frac{1}{3}\right] = \frac{5}{3}$

Iteration 2 : $\left[25 \text{ segments} \times \frac{1}{9}\right] = \frac{25}{9} = \left(\frac{5}{3}\right)^2$

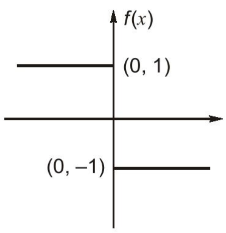
⋮

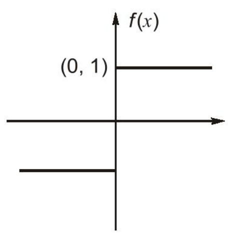
Iteration n : $\text{Length} = \left(\frac{5}{3}\right)^n$

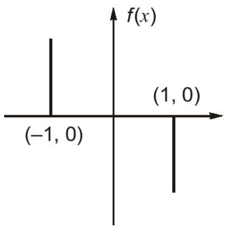
End of Solution

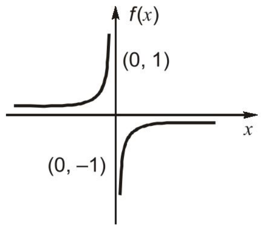
Q.5 Which one of the following plots represents $f(x) = -\frac{|x|}{x}$, where x is a non-zero real number?

Note: The figures shown are representative.

(a)


(b)


(c) 

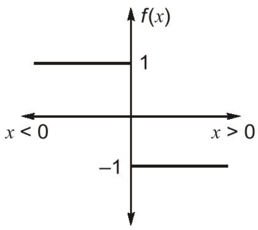
(d) 

Ans. **(a)**

$$f(x) = \frac{-|x|}{x}, \ \forall \ x \in R$$

$$f(x) = \begin{cases} \dfrac{-(-x)}{x} = 1; & x < 0 \\ \dfrac{-x}{x} = -1; & x \geq 0 \end{cases}$$

Graph:



End of Solution

Q.6 Identify the option that has the most appropriate sequence such that a coherent paragraph is formed:

P. Over time, such adaptations lead to significant evolutionary changes with the potential to shape the development of new species.

Q. In natural world, organisms constantly adapt to their environments in response to challenges and opportunities.

R. This process of adaptation is driven by the principle of natural selection, where favorable traits increase an organism's chances of survival and reproduction.

S. As environments change, organisms that can adapt their behavior, structure and physiology to such changes are more likely to survive.

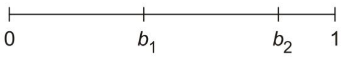
(a) P → Q → R → S
(b) Q → S → R → P
(c) R → S → Q → P
(d) S → P → R → Q

Ans. **(b)**

End of Solution

Q.7 A stick of length one meter is broken at two locations at distances of b_1 and b_2 from the origin (0), as shown in the figure. Note that $0 < b_1 < b_2 < 1$. Which one of the following is NOT a necessary condition for forming a triangle using the three pieces?

Note: All lengths are in meter. The figure shown is representative.



(a) $b_1 < 0.5$
(b) $b_2 > 0.5$
(c) $b_2 < b_1 + 0.5$
(d) $b_1 + b_2 < 1$

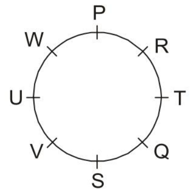
Ans. (d)

End of Solution

Q.8 Eight students (P, Q, R, S, T, U, V, and W) are playing musical chairs. The figure indicates their order of position at the start of the game. They play the game by moving forward in a circle in the clockwise direction.

After the 1st round, 4th student behind P leaves the game. After 2nd round, 5th student behind Q leaves the game. After 3rd round, 3rd student behind V leaves the game. After 4th round, 4th student behind U leaves the game. Who all are left in the game after the 4th round?

Note: The figure shown is representative.



(a) P; T; Q; S
(b) V; P; T; Q
(c) W; R; Q; V
(d) Q; T; V; W

Ans. (d)

According to GATE key.

End of Solution

Q.9 The table lists the top 5 nations according to the number of gold medals won in a tournament; also included are the number of silver and the bronze medals won by them. Based only on the data provided in the table, which one of the following statements is INCORRECT?

Nation	Gold	Silver	Bronze
USA	40	44	41
Canada	39	27	24
Japan	20	12	13
Australia	17	19	16
France	16	26	22

(a) France will occupy the third place if the list were made on the basis of the total number of medals won.

(b) The order of the top two nations will not change even if the list is made on the basis of the total number of medals won.

(c) USA and Canada together have less than 50% of the medals awarded to the nations in the above table.

(d) Canada has won twice as many total medals as Japan.

Ans. **(c)**

Nation	Gold	Silver	Bronze
USA	40	44	41
Canada	39	27	24
Japan	20	12	13
Australia	17	19	16
France	16	26	22

Total medals awarded to the nations = 376

Medals awarded to USA = 125

Medals awarded to Canada = 90

Medals awarded to USA and Canada = 90 + 125 = 215

$$\therefore \qquad \frac{215}{376} \times 100 = 57.18\%$$

$\Rightarrow$ USA and Canada together have 57.18% of the total medals awarded to the nations.

End of Solution

Q.10 An organization allows its employees to work independently on consultancy projects but charges an overhead on the consulting fee. The overhead is 20% of the consulting fee, if the fee is up to . 5,00,000. For higher fees, the overhead is . 1,00,000 plus 10% of the amount by which the fee exceeds . 5,00,000. The government charges a Goods and Services Tax of 18% on the total amount (the consulting fee plus the overhead). An employee of the organization charges this entire amount, i.e., the consulting fee, overhead, and tax, to the client. If the client cannot pay more than . 10,00,000, what is the maximum consulting fee that the employee can charge?

(a) 7,01,438
(b) 7,24,961
(c) 7,51,232
(d) 7,75,784

Ans. **(b)**

Let, consultation fee = X

Case (i):

If $X \leq 5{,}00{,}000$

$$\text{Overhead} = 20\% \text{ of } X = 0.2X$$

$$\text{Total cost} = X + 0.2X = 1.2X$$

$$\text{Tax} = 18\% \text{ of } 1.2X = \left(\frac{18}{100}\right)1.2X = 0.216X$$

Total amount paid by client = $1.2X + 0.216X = 1.416X$

Given that, the client can only pay 10,00,000.

$$\Rightarrow \qquad 1.416X = 10{,}00{,}000$$

$$\Rightarrow \qquad X = 7{,}06{,}215$$

Case (ii):

If $X > 500000$

$$\text{Overhead} = 1,00,000 + 10\%[X - 500000]$$
$$= 1,00,000 + 0.1[X - 500000]$$
$$\text{Total cost} = X + 100000 + 0.1[X - 500000]$$
$$= 1.1X + 50000$$

$$\text{Tax} = \frac{18}{100}(1.1X + 50000) = 0.198X + 9000$$

Total amount paid by client = $1.1X + 50000 + 0.198X + 9000 = 1.298X + 59000$

Given that, the client can only pay 1000000

$\Rightarrow \quad 1.298X + 59000 = 1000000$

$X = 724,961$

$\therefore$ Maximum consultation fee that the client can afford = 724961.

End of Solution

■■■■

Q.11 Consider the matrix A below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of α, β and γ, is the rank of A at least three?

(i) $\alpha = 0$ and $\beta = \gamma \neq 0$

(ii) $\alpha = \beta = \gamma = 0$

(iii) $\beta = \gamma = 0$ and $\alpha \neq 0$

(iv) $\alpha = \beta = \gamma \neq 0$

(a) Only (i), (iii), and (iv)
(b) Only (iv)
(c) Only (ii)
(d) Only (i) and (iii)

Ans. (a)

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \alpha \end{bmatrix}$$

Required, $\rho(A) \geq 3$

$\Rightarrow$ $\rho(A) = 3$ or 4

(i) For $\alpha = 0$ and $\beta = \gamma \neq 0$

$$A \cong \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & \beta \neq 0 \end{bmatrix}$$

$\because$ $R_3 = R_4$

$\Rightarrow$ $\rho(A) = 3 \geq 3$

$\therefore$ (i) is true.

(ii) For $\alpha = \beta = \gamma = 0$

$$A \cong \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2 \not\geq 3$$

$\therefore$ (ii) is false.

(iii) For $\beta = \gamma = 0$ and $\alpha \neq 0$

Then, $$A \cong \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 3 \geq 3 \qquad (\because \alpha \neq 0)$$

$\therefore$ (iii) is true.

(iv) For $\alpha = \beta = \gamma \neq 0$

Then, $$A \cong \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \alpha \\ 0 & 0 & 0 & \alpha \end{bmatrix}$$

$$A \cong \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \rho(A) = 4 \geq 3$$

$\therefore$ (iv) is true.

End of Solution

Q.12 Consider the following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(a) Only (ii) converges
(b) Only (ii) and (iii) converge
(c) Only (iii) converges
(d) All three converge

Ans. **(b)**

(i) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

By P-test: $P = \frac{1}{2} < 1$

$\therefore$ Series is divergent.

(ii) $$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$$

As $n \to \infty$

$$= 1$$

$\therefore$ It is convergent.

(iii) $\sum_{n=1}^{\infty} \frac{1}{n!}$

By ratio test:

$$\lim_{n\to\infty} \frac{u_n}{u_n+1} = \lim_{n\to\infty} \frac{\frac{1}{n!}}{\frac{1}{(n+1)!}} = \infty > 1$$

∴ It is convergent.

End of Solution

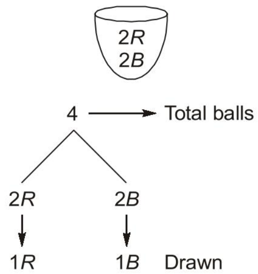
Q.13 A pot contains two red balls and two blue balls. Two balls are drawn from this pot randomly without replacement.

What is the probability that the two balls drawn have different colours?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

Ans. **(a)**



∴ $$P(E) = \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} = \frac{4}{6} = \frac{2}{3}$$

End of Solution

Q.14 Consider a frequency-modulated (FM) signal

$$f(t) = A_c \cos(2\pi f_c t + 3\sin(2\pi f_1 t) + 4\sin(6\pi f_1 t)),$$

where A_c and f_c are, respectively, the amplitude and frequency (in Hz) of the carrier waveform. The frequency f_1 is in Hz, and assume that $f_c > 100 f_1$.

The peak frequency deviation of the FM signal in Hz is _______.

(a) $15 f_1$ (b) $12 f_1$

(c) $4 f_1$ (d) $2 f_1$

Ans. **(a)**

$$f(t) = A_c \cos[2\pi f_c t + 3\sin(2\pi f_1 t) + 4\sin(6\pi f_1 t)]$$

General expression of angle modulated signal,

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

From comparison: $\phi(t) = 3\sin 2\pi f_1 t + 4\sin 6\pi f_1 t$

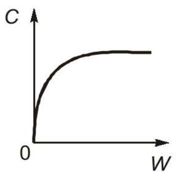
Peak frequency deviation:

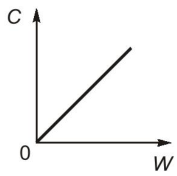
$$\Delta f = \left|\frac{1}{2\pi}\frac{d}{dt}\phi(t)\right|_{max} = \left|\frac{1}{2\pi}[3 \times 2\pi f_1 \cos 2\pi f_1 t + 4 \times 6\pi f_1 \cos 6\pi f_1 t]\right|_{max}$$

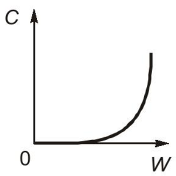
$$= \left|3f_1 \cos 2\pi f_1 t + 12 f_1 \cos 6\pi f_1 t\right|_{max} = 3f_1 + 12f_1 = 15f_1$$

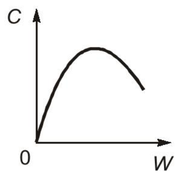
End of Solution

Q.15 Consider an additive white Gaussian noise (AWGN) channel with bandwidth W and noise power spectral density $\frac{N_o}{2}$. Let P_{av} denote the average transmit power constraint. Which one of the following plots illustrates the dependence of the channel capacity C on the bandwidth W (keeping P_{av} and N_0 fixed)?

(a)


(b)


(c)


(d)


Ans. **(a)**

We know that, $C = B\log_2\left(1 + \frac{S}{N}\right)$

Given average transmitted power = P_{av} and channel bandwidth = W.

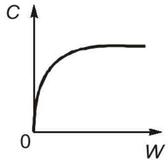
$$C = W\log_2\left(1 + \frac{P_{av}}{N_o W}\right)$$

Given P_{av} and N_o are fixed.

Let, $\frac{P_{av}}{N_o} = K$ (fixed)

$$C = W\log_2\left(1 + \frac{K}{W}\right)$$

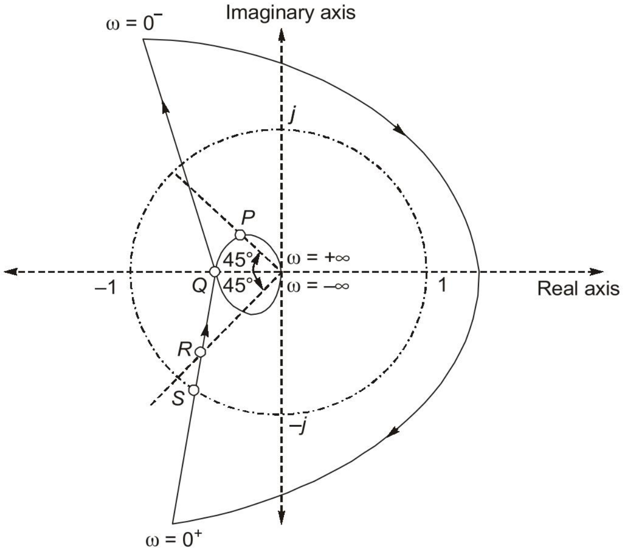
Evaluating C for various W values with fixed K value, plot between C and W will be as below:



End of Solution

Q.16 The Nyquist plot of a system is given in the figure below. Let ω_P, ω_Q, ω_R, and ω_S be the positive frequencies at the points *P*, *Q*, *R*, and *S*, respectively.

Which one of the following statements is TRUE?



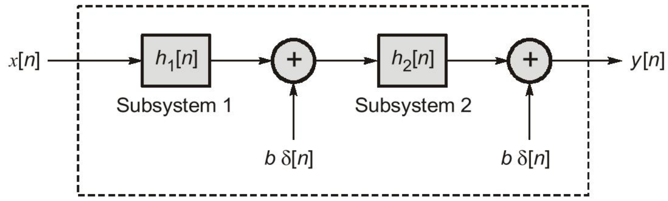
(a) ω_S is the gain crossover frequency and ω_P is the phase crossover frequency
(b) ω_Q is the gain crossover frequency and ω_R is the phase crossover frequency
(c) ω_Q is the gain crossover frequency and ω_S is the phase crossover frequency
(d) ω_S is the gain crossover frequency and ω_Q is the phase crossover frequency

Ans. (d)

End of Solution

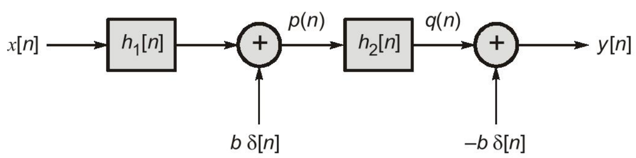
Q.17 Consider the discrete-time system below with input $x[n]$ and output $y[n]$. In the figure, $h_1[n]$ and $h_2[n]$ denote the impulse responses of LTI Subsystems 1 and 2, respectively. Also, $\delta[n]$ is the unit impulse, and $b > 0$.

Assuming $h_2[n] \neq \delta[n]$, the overall system (denoted by the dashed box) is _______.



(a) linear and time invariant
(b) linear and time variant
(c) nonlinear and time invariant
(d) nonlinear and time variant

Ans. (d)



$\therefore \quad p(n) = [x(n) * h_1(n)] + b\delta(n)$

$\Rightarrow \quad q(n) = h_2(n) * p(n) = h_2(n) * [x(n) * h_1(n) + b\delta(n)]$

$= x(n) * h_1(n) * h_2(n) + bh_2(n) \qquad [\because h_2(n) * \delta(n) = h_2(n)]$

$\Rightarrow \quad y(n) = q(n) - b\delta(n)$

$= [x(n) * h_1(n) * h_2(n) + b \cdot h_2(n)] - b\delta(n)$

$\therefore \quad y(n) = x(n) * h_1(n) * h_2(n) + \underbrace{bh_2(n) - b\delta(n)}_{f(n)}$

where, $f(n) = bh_2(n) - b\delta(n)$

Because of $f(n)$ in the output $y(n)$, the system is non-linear and time-variant.

End of Solution

Q.18 Consider a continuous-time, real-valued signal $f(t)$ whose Fourier transform

$F(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-j\omega t)dt$ exists.

Which one of the following statements is always TRUE?

(a) $|F(\omega)| \leq \int_{-\infty}^{\infty} |f(t)|dt$

(b) $|F(\omega)| > \int_{-\infty}^{\infty} |f(t)|dt$

(c) $|F(\omega)| \leq \int_{-\infty}^{\infty} f(t)dt$

(d) $|F(\omega)| \geq \int_{-\infty}^{\infty} f(t)dt$

Ans. **(a)**

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

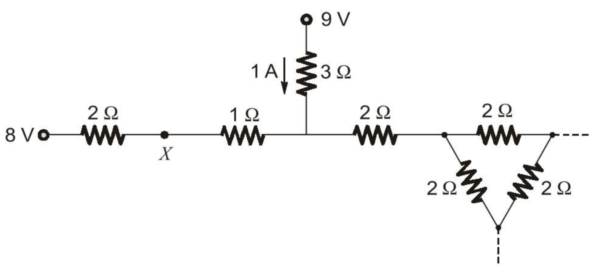
$$\Rightarrow \quad |F(\omega)| = \left|\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\right| \leq \int_{-\infty}^{\infty} \left|f(t)e^{-j\omega t}\right|dt$$

$$\Rightarrow \quad |F(\omega)| \leq \int_{-\infty}^{\infty} |f(t)|dt$$

End of Solution

Q.19 Consider a part of an electrical network as shown below. Some node voltages, and the current flowing through the 3 Ω resistor are as indicated.

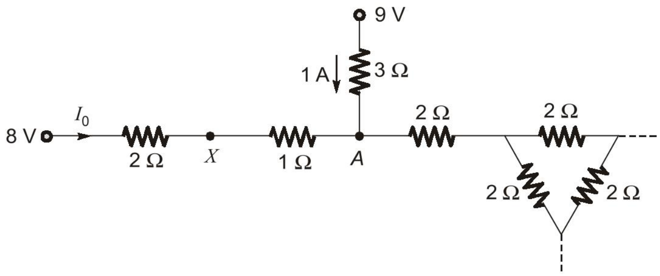
The voltage (in Volts) at node X is ______.



(a) $\frac{20}{3}$ (b) $\frac{32}{3}$

(c) $\frac{22}{3}$ (d) $\frac{2}{3}$

Ans. (a)

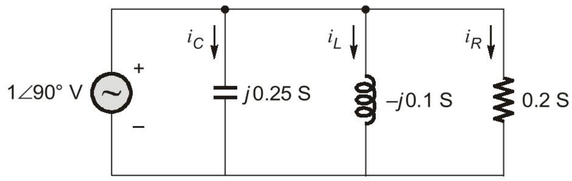


Voltage at node A is = 9 – (3 × 1) = 6 V

$$I_D = \frac{8-6}{2+1} = \frac{2}{3}\text{ A} \quad ; \quad V_x = 8-\left(2\times\frac{2}{3}\right) = \frac{20}{3}\text{ V}$$

End of Solution

Q.20 Let i_C, i_L, and i_R be the currents flowing through the capacitor, inductor, and resistor, respectively, in the circuit given below. The AC admittances are given in Siemens(S). Which one of the following is true?



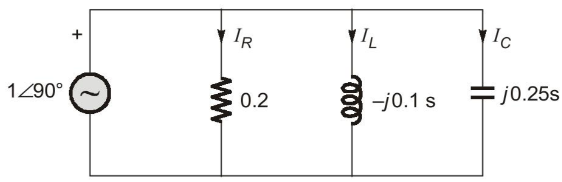
(a) $i_C = 0.25\angle 180°$ A, $i_L = 0.1\angle 0°$ A, $i_R = 0.2\angle 90°$ A

(b) $i_C = 4\angle 180°$ A, $i_L = 10\angle 0°$ A, $i_R = 5\angle 90°$ A

(c) $i_C = 0.25\angle 270°$ A, $i_L = 0.1\angle 90°$ A, $i_R = 0.2\angle 90°$ A

(d) $i_C = 4\angle 90°$ A, $i_L = 10\angle 270°$ A, $i_R = 5\angle 0°$ A

Ans. (a)



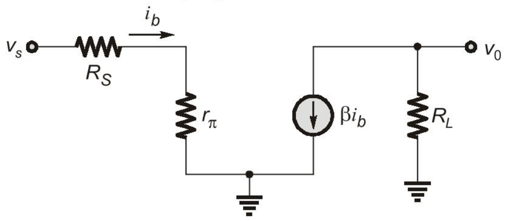
$I_R = 1\angle 90\ (0.2) = 0.2\angle 90°$

$I_L = 1\angle 90\ (0.1\angle -90) = 0.1\angle 0°$

$I_C = 1\angle 90\ (0.25\angle 90) = 0.25\angle 180°$

End of Solution

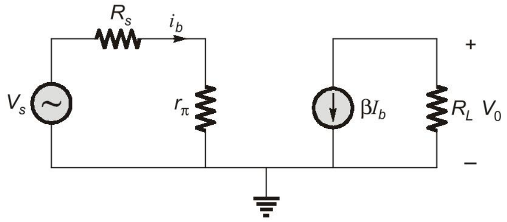
Q.21 A simplified small-signal equivalent circuit of a BJT-based amplifier is given below. The small-signal voltage gain V_o/V_s (in V/V) is ___________.



(a) $\dfrac{-\beta R_L}{R_s + r_\pi}$ (b) $\dfrac{+\beta R_L}{R_s}$

(c) $\dfrac{-\beta R_L}{R_s}$ (d) $\dfrac{+\beta R_L}{R_s + r_\pi}$

Ans. **(a)**

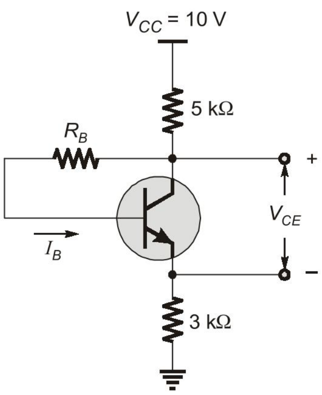


$$\frac{V_0}{V_s} = \frac{-\beta I_b \times R_L}{I_b(R_s + r_\pi)}$$

$$\frac{V_0}{V_s} = \frac{-\beta R_L}{R_s + r_\pi}$$

End of Solution

Q.22 The ideal BJT in the circuit given below is biased in the active region with a β of 100. If I_B is 10 μA, then V_{CE} (in Volts, rounded off to two decimal places) is _______.

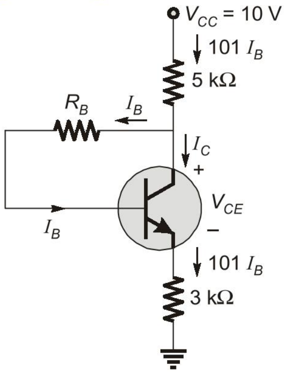


(a) 4.95 (b) 3.03

(c) 1.92 (d) 3.73

Ans. **(c)**

Since, $I_C = \beta I_B = 100I_B$; $I_E = 101I_B$,



$$I_B = 0.01 \text{ mA}$$

By KVL:

$$10 = 5 \times 101I_B + V_{CE} + 3 \times 101I_B$$

$$V_{CE} = 10 - 808 \times 0.01$$

$$V_{CE} = 1.92 \text{ V}$$

End of Solution

Q.23 A 3-input majority logic gate has inputs *X*, *Y* and *Z*. The output *F* of the gate is logic '1' if two or more of the inputs are logic '1'. The output *F* is logic '0' if two or more of the inputs are logic '0'.

Which one of the following options is a Boolean expression of the output *F*?

(a) $XY + YZ + ZX$ (b) $X \oplus Y \oplus Z$

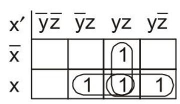
(c) $X + Y + Z$ (d) XYZ

Ans. **(a)**

Given inputs are,

X	*Y*	*Z*	*F*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

From 3-variable k-map:

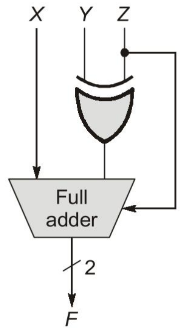


$\therefore$ $F = yz + xy + xz$

End of Solution

Q.24 A full adder and an XOR gate are used to design a digital circuit with inputs *X*, *Y*, and *Z*, and output *F*, as shown below. The input *Z* is connected to the carry-in input of the full adder.

If the input *Z* is set to logic '1', then the circuit functions as ________ with *X* and *Y* as inputs.



(a) an adder
(b) a subtractor
(c) a multiplier
(d) a binary to Gray code converter

Ans. (b)

End of Solution

Q.25 Consider the function $f: \mathbb{R} \to \mathbb{R}$ defined as

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

Which of the following statements is/are correct?

(Here, $\mathbb{R}$ is the set of real numbers.)

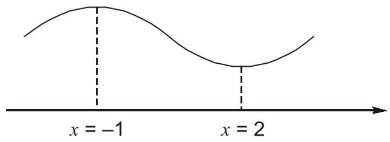
(a) f has no global maximizer
(b) f has no global minimizer
(c) $x = -1$ is a local minimizer of f
(d) $x = 2$ is a local maximizer of f

Ans. (a, b)

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$
$$f'(x) = 6x^2 - 6x - 12$$
$$f'(x) = 0$$
$$6x^2 - 6x - 12 = 0$$
$$6(x^2 - x - 2) = 0$$
$$6(x - 2)(x + 1) = 0$$
$$x = 2$$
$$x = -1$$
$$f''(x) = 12x - 6$$
$$f''(x)\big|_{x=2} = 12 \times 2 - 6$$
$$= 18 > 0$$
$$x = 2 \Rightarrow \text{local minima}$$
$$f''(x)\big|_{x=-1} = 12 \times -1 - 6$$
$$= -12 - 6$$

$= -18 < 0$

$x = -1 \Rightarrow$ local maxima



So function has neither global maxima nor global minima.

End of Solution

Q.26 Consider the unity-negative-feedback system shown in Figure (i) below, where gain $K \geq 0$. The root locus of this system is shown in Figure (ii) below.

For what value(s) of K will the system in Figure (i) have a pole at $-1 + j1$?

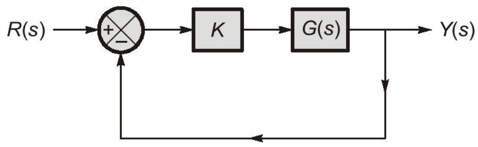


Figure (i)

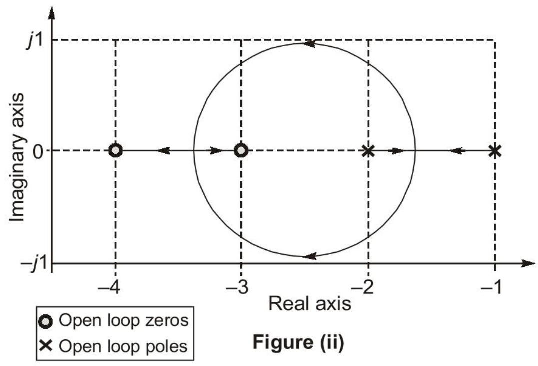


Figure (ii)

(a) $K = 5$ (b) $K = \frac{1}{5}$

(c) For no positive value of K (d) For all positive values of K

Ans. (c)

Since, the given pole $-1 + j1$ is not on the Root locus, hence the system has no positive value of K at $-1 + j1$.

End of Solution

Q.27 Let $x[n]$ be a discrete-time signal whose z-transform is $X(z)$.

Which of the following statements is/are TRUE?

(a) The discrete-time Fourier transform (DTFT) of $x[n]$ always exists

(b) The region of convergence (RoC) of $X(z)$ contains neither poles nor zeros

(c) The discrete-time Fourier transform (DTFT) exists if the region of convergence (RoC) contains the unit circle

(d) If $x[n] = \alpha\delta[n]$, where $\delta[n]$ is the unit impulse and α is a scalar, then the region of convergence (RoC) is the entire z-plane

Ans. **(c, d)**

End of Solution

Q.28 Consider a message signal $m(t)$ which is bandlimited to $[-W, W]$, where W is in Hz. Consider the following two modulation schemes for the message signal:

- Double sideband-suppressed carrier (DSB-SC):

$$f_{DSB}(t) = A_c m(t) \cos(2\pi f_c t)$$

- Amplitude modulation (AM):

$$f_{AM}(t) = A_c(1 + \mu\, m(t)) \cos(2\pi f_c t)$$

Here, A_c and f_c are the amplitude and frequency (in Hz) of the carrier, respectively. In the case of AM, μ denotes the modulation index.

Consider the following statements:

(i) An envelope detector can be used for demodulation in the DSB-SC scheme if $m(t) > 0$ for all t.

(ii) An envelope detector can be used for demodulation in the AM scheme only if $m(t) > 0$ for all t.

Which of the following options is/are correct?

(a) (i) is TRUE
(b) (i) is FALSE
(c) (ii) is TRUE
(d) (ii) is FALSE

Ans. **(a, d)**

If $m(t) > 0$ for all t, envelope detector can be used for demodulation of DSB-SC signal. Even for $m(t) < 0$, envelope detector can be used for demodulation of AM signal.

End of Solution

Q.29 Which of the following statements is/are TRUE with respect to an ideal opamp?

(a) It has an infinite input resistance
(b) It has an infinite output resistance
(c) It has an infinite open-loop differential gain
(d) It has an infinite open-loop common-mode gain

Ans. **(a, c)**

x

End of Solution

Q.30 Which of the following statements is/are TRUE with respect to ideal MOSFET-based DC-coupled single-stage amplifiers having finite load resistors?

(a) The common-gate amplifier has an infinite input resistance
(b) The common-source amplifier has an infinite input resistance
(c) The input and output voltages of the common-source amplifier are in phase
(d) The input and output voltages of the common-drain amplifier are in phase

Ans. **(b, d)**

End of Solution

Q.31 Which of the following can be used as an n-type dopant for silicon?

Select the correct option(s).

(a) Arsenic
(b) Boron
(c) Gallium
(d) Phosphorous

Ans. **(a, d)**

x

End of Solution

Q.32 The function $y(t)$ satisfies

$$t^2 y''(t) - 2ty'(t) + 2y(t) = 0,$$

where $y'(t)$ and $y''(t)$ denote the first and second derivatives of $y(t)$, respectively. Given $y'(0) = 1$ and $y'(1) = -1$, the maximum value of $y(t)$ over $[0, 1]$ is ________. (rounded off to two decimal places).

Ans. **(0.25)**

Cauchy's Euler differential equation,

$$x = e^t$$

$$t = e^u$$

$$\ln t = u$$

$$t^2 y''(t) - 2ty'(t) + 2y(t) = 0$$

$$D(D-1)y - 2Dy + 2y = 0$$

$$(D^2 - D - 2D + 2)y = 0$$

$$(D^2 - 3D + 2)y = 0$$

A.E.

$$m^2 - 3m + 2 = 0$$

$$m = 1, 2$$

$$y = C_1 e^u + C_2 e^{2u}$$

$$y = C_1 t + C_2 t^2$$

$$\frac{dy}{dt} = C_1 + C_2 \times 2t$$

at $t = 0$, $\frac{dy}{dt} = 1$

$\Rightarrow$ $C_1 = 1$

at $t = 1$, $\frac{dy}{dt} = -1$

$$-1 = 1 + C_2 \times 2 \times 1$$

$$-1 = 1 + 2C_2$$

$$-2 = 2C_2$$

$$C_2 = -1$$

$$y = C_1 t + C_2 t^2$$

$$y = t - t^2$$

$$\frac{dy}{dt} = 1 - 2t = 0$$

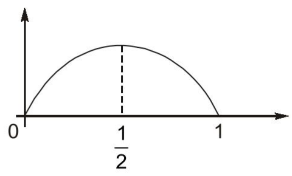
$$t = \frac{1}{2}$$

$$\frac{d^2y}{dt^2} = -2 < 0$$

$$t = \frac{1}{2} \Rightarrow \text{maxima point}$$

$$y\left(\frac{1}{2}\right) = \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$



End of Solution

Q.33 The generator matrix of a (6,3) binary linear block code is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The minimum Hamming distance d_{min} between codewords equals _____ (answer in integer).

Ans. (3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}_{\substack{3 \times 6 \\ K \times n}}$$

Method 1:

Since, $K = 3$, $2^K = 8$ distinct message blocks possible i.e. 000 to 111.

Find corresponding codewords $[c] = [d] \cdot [G]$.

Find Hamming weight of each of the codewords.

Smallest possible Hamming weight of non zero codeword equals to d_{min}.

By evaluating above procedure $d_{min} = 3$.

Method 2:

Minimum number of columns of G that sum to zero equals to d_{min}.

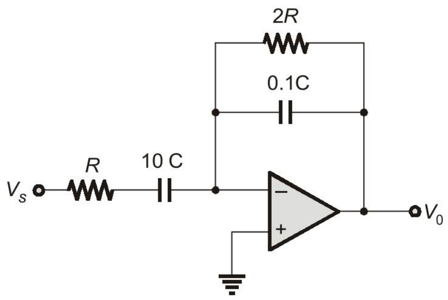
Sum of 1^{st}, 2^{nd} and 6^{th} columns (or) 4^{th}, 5^{th} and 6^{th} columns (or) 2^{nd}, 3^{rd} and 5^{th} columns equals to zero.

For above G, minimum number of columns of G matrix that sum to zero equals to 3 so that $d_{min} = 3$.

End of Solution

Q.34 All the components in the bandpass filter given below are ideal. The lower –3 dB frequency of the filter is 1 MHz.

The upper –3 dB frequency (in MHz, rounded off to the nearest integer) is ________.



Ans. **(50)**

We know that,

$$f_L = \frac{1}{2\pi R_1 C_1}$$

$$10^6 = \frac{1}{2\pi \times R \times 10C}$$

$$\frac{1}{RC} = 2\pi \times 10^7$$

$$f_H = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi \times 2R \times 0.1C}$$

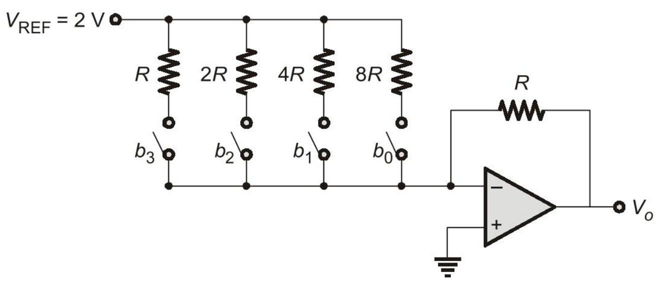
$$= \frac{1}{0.4\pi} \times \frac{1}{RC} = \frac{2\pi \times 10^7}{0.4\pi}$$

$$f_H = 5 \times 10^7 \text{ Hz} = 50 \text{ MHz}$$

End of Solution

Q.35 A 4-bit weighted-resistor DAC with inputs b_3, b_2, b_1, and b_0 (MSB to LSB) is designed using an ideal opamp, as shown below. The switches are closed when the corresponding input bits are logic '1' and open otherwise.

When the input $b_3\, b_2\, b_1\, b_0$ changes from 1110 to 1101, the magnitude of the change in the output voltage V_O (in mV, rounded off to the nearest integer) is ________.



Ans. **(250)**

For the given 4-bit weighted-resistor DAC with inputs b_3, b_2, b_1 and b_0 (MSB to LSB), the output voltage is,

$$V_o = -R\left[\frac{b_0}{8R}+\frac{b_1}{4R}+\frac{b_2}{2R}+\frac{b_3}{R}\right]V_{REF}$$

$$\Rightarrow \quad V_o = -\left[\frac{b_0}{8}+\frac{b_1}{4}+\frac{b_2}{2}+\frac{b_3}{1}\right]\times 2$$

For input 1110:

$$V_{o_1} = -\left[0+\frac{1}{4}+\frac{1}{2}+\frac{1}{1}\right]\times 2 = -3.5$$

For input 1101:

$$V_{o_2} = -\left[\frac{1}{8}+0+\frac{1}{2}+\frac{1}{1}\right]\times 2 = -3.25$$

The magnitude of the change in the output voltage is,

$$V_o = |-3.5-(-3.25)| = 0.25 = 250\text{ mV}$$

End of Solution

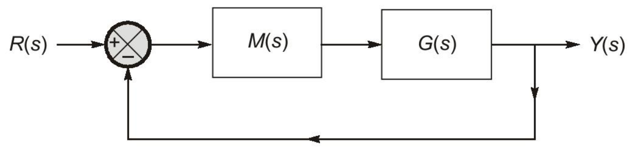
Q.36 Let $G(s) = \frac{1}{10s^2}$ be the transfer function of a second-order system. A controller $M(s)$ is connected to the system $G(s)$ in the configuration shown below.

Consider the following statements.

(i) There exists no controller of the form $M(s) = \frac{K_I}{s}$, where K_I is a positive real number, such that the closed loop system is stable.

(ii) There exists at least one controller of the form $M(s) = K_P + sK_D$, where K_P and K_D are positive real numbers, such that the closed loop system is stable.

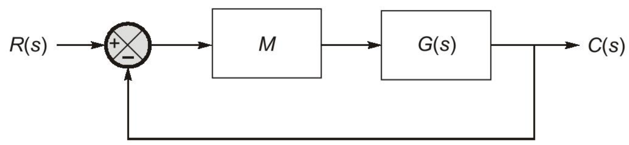
Which one of the following options is correct?



(a) (i) is TRUE and (ii) is FALSE
(b) (i) is FALSE and (ii) is TRUE
(c) Both (i) and (ii) are FALSE
(d) Both (i) and (ii) are TRUE

Ans. **(d)**

Given,



and $G(s) = \frac{1}{s^2}$

(i) Given, $M = \frac{K}{s}$, then closed loop system,

$$\frac{C(s)}{R(s)} = \frac{M\,G(s)}{1+M\,G(s)} = \frac{\frac{K}{s}\times\frac{1}{s^2}}{1+\frac{K}{s}+\frac{1}{s^2}} = \frac{K}{s^3+K}$$

∴ For all values of 'K', the closed loop system is unstable.

(ii) For controller, $M = K_P + K_D s$

Let, $M = 1 + s$

∴ $$\frac{C(s)}{R(s)} = \frac{(1+s)\times\frac{1}{s^2}}{1+\frac{s+1}{s^2}} = \frac{(s+1)}{s^2+s+1}$$

∴ Closed loop system is stable.

∴ Both statements are correct.

End of Solution

Q.37 Consider the polynomial $p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$. Let ($L$, I, R) be defined as follows.

L is the number of roots of $p(s)$ with negative real parts.

I is the number of roots of $p(s)$ that are purely imaginary.

R is the number of roots of $p(s)$ with positive real parts.

Which one of the following options is correct?

(a) $L = 2$, $I = 2$ and $R = 1$
(b) $L = 3$, $I = 2$ and $R = 0$
(c) $L = 1$, $I = 2$ and $R = 2$
(d) $L = 0$, $I = 4$ and $R = 1$

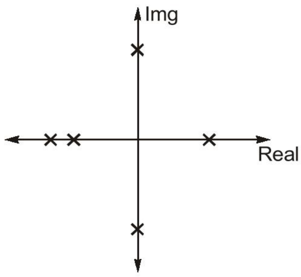
Ans. (a)

Given

$$p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$$

By Routh's Hurwitz criteria

	s^5	1	3	2
	s^4	7	–33	–40
	s^3	$\frac{54}{7}$	$\frac{54}{7}$	
→	s^2	–40	–40	
	s^1	0(–80)	0	
	s^0	–40		



Auxiliary equation is

$$A(s) = -40s^2 - 40$$

$$\frac{dA(s)}{ds} = -80s$$

From the above table we get

$$L = 2,\ I = 2 \text{ and } R = 1$$

End of Solution

Q.38 Consider a continuous-time finite-energy signal $f(t)$ whose Fourier transform vanishes outside the frequency interval $[-\omega_c, \omega_c]$, where ω_c is in rad/sec.

The signal $f(t)$ is uniformly sampled to obtain $y(t) = f(t)\ p(t)$. Here

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \tau - nT_s),$$

with $\delta(t)$ being the Dirac impulse, $T_s > 0$, and $\tau > 0$. The sampled signal $y(t)$ is passed through an ideal lowpass filter $h(t) = \omega_c T_s \dfrac{\sin(\omega_c t)}{\pi \omega_c t}$ with cutoff frequency ω_c and passband gain T_s.

The output of the filter is given by ________________.

(a) $f(t)$ if $T_s < \pi/\omega_c$

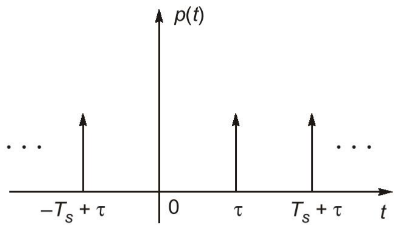
(b) $f(t - \tau)$ if $T_s < \pi/\omega_c$

(c) $f(t - \tau)$ if $T_s < 2\pi/\omega_c$

(d) $T_s f(t)$ if $T_s < 2\pi/\omega_c$

Ans. **(a)**

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \tau - nT_s)$$



For $p(t)$,

$$\text{FS-coefficient: } C_n = \frac{1}{T_s}\int_{T_s} p(t) e^{-jn\omega_s t} dt$$

$$= \frac{1}{T_s}\int_{T_s} \delta(t-\tau) e^{-jn\omega_s t} dt$$

$$= \frac{e^{-jn\omega_s \tau}}{T_s}$$

$$\Rightarrow \quad P(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_s) = 2\pi \sum \frac{e^{-jn\omega_s \tau}}{T_s} \delta(\omega - n\omega_s)$$

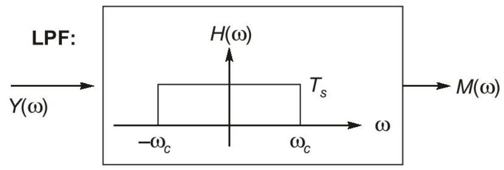
$$= \omega_s \sum e^{-jn\omega_s \tau} \delta(\omega - n\omega_s)$$

Now, $\quad y(t) = f(t) \cdot p(t)$

$$\Rightarrow \quad Y(\omega) = \frac{1}{2\pi}[F(\omega) * P(\omega)] = \frac{1}{2\pi}\left[F(\omega) * \omega_s \sum_{n=-\infty}^{\infty} e^{-jn\omega_s \tau} \delta(\omega - n\omega_s)\right]$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{-jn\omega_s \tau} F(\omega - n\omega_s)$$

$$= \left[\ldots + \frac{e^{-j\omega_s \tau}}{T_s} F(\omega + \omega_s) + \frac{F(\omega)}{T_s} + \frac{e^{-j\omega_s \tau}}{T_s} F(\omega - \omega_s) + \ldots\right]$$



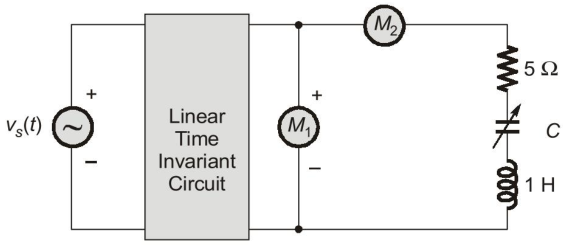
$\therefore \qquad M(\omega) = Y(\omega) \cdot H(\omega) = T_s \cdot \dfrac{F(\omega)}{T_s}$ if $T_s < \dfrac{\pi}{\omega_c}$

$M(\omega) = F(\omega)$ if $T_s < \dfrac{\pi}{\omega_c}$;

$\Rightarrow \qquad m(t)$ = LPF output = f(t) if $T_s < \dfrac{\pi}{\omega_c}$

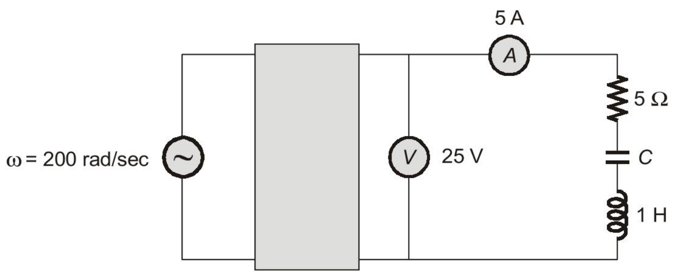
End of Solution

Q.39 In the circuit below, M_1 is an ideal AC voltmeter and M_2 is an ideal AC ammeter. The source voltage (in Volts) is $v_s(t) = 100\cos(200t)$.
What should be the value of the variable capacitor C such that the RMS readings on M_1 and M_2 are 25 V and 5 A, respectively?



(a) 25 μF
(b) 4 μF
(c) 0.25 μF
(d) Insufficient information to find C

Ans. **(a)**



$X_L = \omega L = 200 \times 1 = 200\ \Omega$

$Z = \dfrac{V}{I} = \dfrac{25}{5} = 5\Omega$

$Z = R + j(X_L - X_C)$

$X_L = X_C$

$X_L = X_C = 200\ \Omega$

$$\frac{1}{\omega C} = 200$$

$$\frac{1}{200 \times C} = 200$$

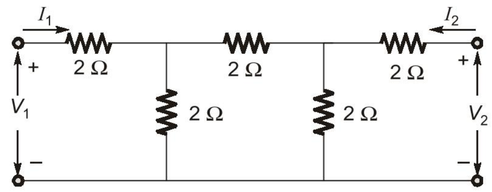
$$C = \frac{1}{200 \times 200} = 25\mu F$$

End of Solution

Q.40 The Z-parameter matrix of a two port network relates the port voltages and port currents as follows:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The Z-parameter matrix (with each entry in Ohms) of the network shown below is __________.



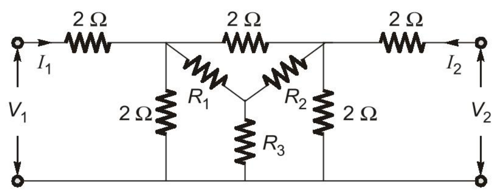
(a) $\begin{bmatrix} \frac{10}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{10}{3} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{2}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{2}{3} \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 2 \\ 2 & 10 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{10}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{10}{3} \end{bmatrix}$

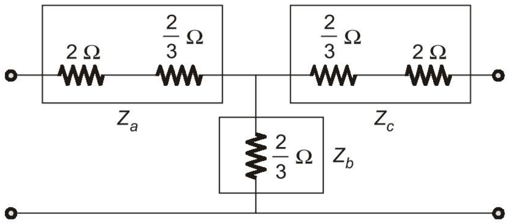
Ans. **(a)**



$$R_1 = \frac{2 \times 2}{2+2+2} = \frac{2}{3}\Omega$$

$$R_1 = \frac{2}{3}\Omega$$

$$R_3 = \frac{2}{3}\Omega$$



$$Z_{11} = Z_a + Z_b = \frac{10}{3}\Omega$$

$$Z_{22} = Z_b + Z_c = \frac{10}{3}\Omega$$

$$Z_{12} = Z_{21} + Z_b = \frac{2}{3}\Omega$$

End of Solution

Q.41 A source transmits symbol S that takes values uniformly at random from the set {–2, 0, 2}. The receiver obtains $Y = S + N$, where N is a zero-mean Gaussian random variable independent of S. The receiver uses the maximum likelihood decoder to estimate the transmitted symbol S.

Suppose the probability of symbol estimation error P_e is expressed as follows:

$$P_e = \alpha P(N > 1),$$

where $P(N > 1)$ denotes the probability that N exceeds1.

What is the value of α?

(a) $\frac{1}{3}$ (b) 1

(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

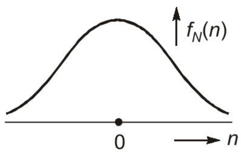
Ans. (d)

S takes the values uniformly at random from the set {–2, 0, 2}

$$P(-2) = P(0) = P(2) = \frac{1}{3}$$

$$Y = S + N$$

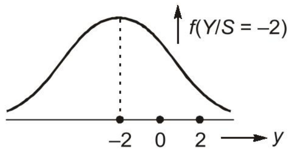
Given N is zero mean Gaussian R.V



$$P_e = P(-2)P_{e-2} + P(0) \cdot P_{e0} + P(2)\, P_{e2}$$

$$= \frac{1}{3}\left[P_{e-2} + P_{e0} + P_{e2}\right]$$

For $S = -2 \rightarrow Y = -2 + N$

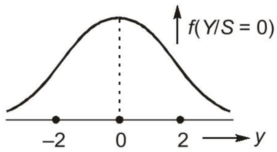


Since ML decoder is used → $P_{e-2} = P(Y > -1)$

$$\because \quad V_{th} = \frac{a_1 + a_2}{2} = \frac{-2+0}{2} = -1$$

$$P_{e-2} = P(-2 + N > -1) = P(N > 1)$$

For $S = 0 \rightarrow Y = N$

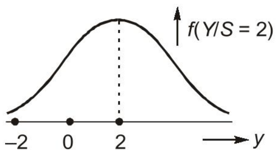


$$P_{e0} = P(Y < -1) \text{ or } P(Y > 1)$$
$$= P(Y < -1) + P(Y > 1)$$
$$= P(N < -1) + P(N > 1)$$

Since N is zero mean GRV, $P(N < -1) = P(N > 1)$

$$P_{e0} = 2P(N > 1)$$

Let $S = 2 \rightarrow Y = 2 + N$



$$P_{e2} = P(Y < 1)$$
$$= P(2 + N < 1)$$
$$= P(N < -1) = P(N > 1)$$

$$P_e = \frac{1}{3}[P(N>1) + 2P(N>1) + P(N>1)]$$

$$= \frac{4}{3}P(N>1)$$

Given, $\quad P_e = \alpha P(N > 1)$

$$\alpha = \frac{4}{3}$$

End of Solution

Q.42 Consider a real-valued random process

$$f(t) = \sum_{n=1}^{N} a_n\, p(t - nT),$$

where $T > 0$ and N is a positive integer. Here, $p(t) = 1$ for $t \in [0, 0.5T]$ and 0 otherwise. The coefficients a_n are pairwise independent, zero-mean unit-variance random variables. Read the following statements about the random process and choose the correct option.

(i) The mean of the process $f(t)$ is independent of time t.

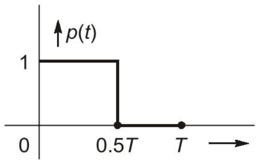
(ii) The autocorrelation function $E[f(t)\ f(t+\tau)]$ is independent of time t for all τ.

(Here, $E[\cdot]$ is the expectation operation.)

(a) (i) is TRUE and (ii) is FALSE
(b) Both (i) and (ii) are TRUE
(c) Both (i) and (ii) are FALSE
(d) (i) is FALSE and (ii) is TRUE

Ans. (a)

Given Random process $f(t) = \sum_{n=1}^{N} a_n\, p(t - nT)$



$\{a_n\}$ specifies set of independent random variables each having zero mean and unit variance.

Mean:

$$E[f(t)] = \sum_{n=1}^{N} E[a_n]\, p(t - nT)$$

Given that $E[a_n] = 0$

$$E[f(t)] = 0$$

$E[f(t)]$ is independent of t.

ACF:

$$\text{ACF}[f(t)] = \sum_{n=1}^{n} \text{ACF}\left[a_n\right] \cdot \text{ACF}\left[p(t - nT)\right]$$

Let $p(t - nT) = g(t)$ where $g(t)$ is a deterministic signal.

$$\text{ACF}[p(t - nT)] = \text{ACF}[g(t)] = \int_{-\infty}^{\infty} g(t) \cdot g(t + \tau)\, dt$$

From observation ACF$[g(t)]$ is function of t.

i.e., ACF$[f(t)]$ is function of t.

End of Solution

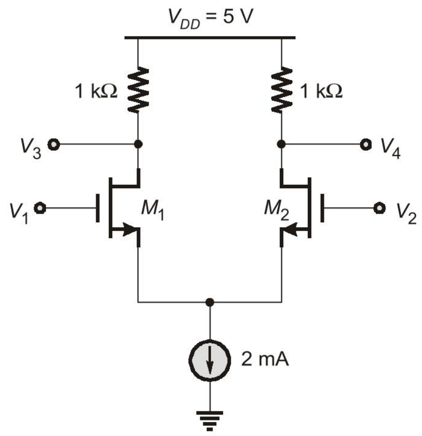
Q.43 The identical MOSFETs M_1 and M_2 in the circuit given below are ideal and biased in the saturation region. M_1 and M_2 have a transconductance g_m of 5 mS.

The input signals (in Volts) are:

$V_1 = 2.5 + 0.01 \sin \omega t$

$V_2 = 2.5 - 0.01 \sin \omega t$

The output signal V_3 (in Volts) is ________.



(a) 3 + 0.05 sin ω*t*
(b) 3 – 0.1 sin ω*t*
(c) 4 + 0.1 sin ω*t*
(d) 4 – 0.05 sin ω*t*

Ans. (d)

$$I_{DS} = \frac{2}{2} = 1\,\text{mA}$$

$$\frac{V_d}{2} = 0.01 \sin \omega t$$

$$V_d = 0.02 \sin \omega t$$

$$V_{CM} = 2.5 \text{ V}$$

Given, $g_m = 5$ ms

$$A_{DM} = \frac{V_0}{V_d} = -\frac{g_m \times R_D}{2} = -\frac{5 \times 1}{2} = -2.5$$

$$V_0 = -2.5 \times V_d = -2.5 \times 0.02 \sin \omega t$$

AC voltage, $V_0 = -0.05 \sin \omega t$

DC voltage, $V_0 = 5 - 1 \times I_{DS} = 4$ V

In given circuit, $A_{CM} = 0$

Hence, common mode input effect is cancelled.

$$V_0 = -0.05 \sin \omega t + 4$$

End of Solution

Q.44 A 10-bit analog-to-digital converter (ADC) has a sampling frequency of 1 MHz and a full scale voltage of 3.3 V.

For an input sinusoidal signal with frequency 500 kHz, the maximum SNR (in dB, rounded off to two decimal places) and the data rate (in Mbps) at the output of the ADC are ________, respectively.

(a) 61.96 and 10
(b) 61.96 and 5
(c) 33.36 and 10
(d) 33.36 and 5

Ans. **(a)**

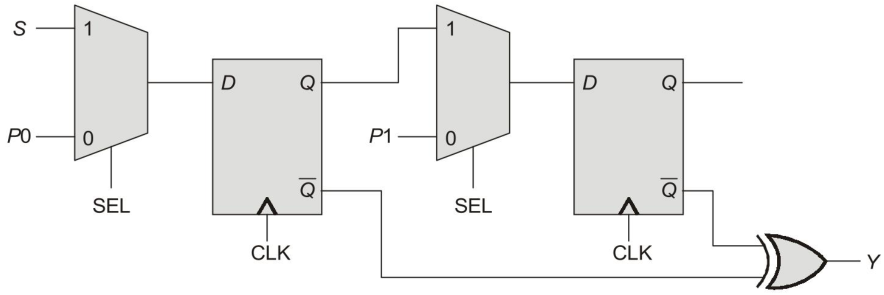
Given

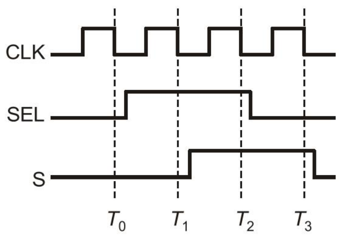
$$n = 10$$
$$f_s = 1 \text{ MHz}$$
$$\text{Full scale voltage} = 3.3 \text{ V}$$
$$f_m = 500 \text{ kHz}$$
$$\text{SNR} = 1.76 + 6.02n$$
$$= 1.76 + 6.02 \times 10 = 61.96 \text{ dB}$$
$$R_b = nf_s = 10 \text{ Mbps}$$

End of Solution

Q.45 A positive-edge-triggered sequential circuit is shown below. There are no timing violations in the circuit. Input $P0$ is set to logic '0' and $P1$ is set to logic '1' at all times. The timing diagram of the inputs SEL and S are also shown below.

The sequence of output Y from time T_0 to T_3 is ________.



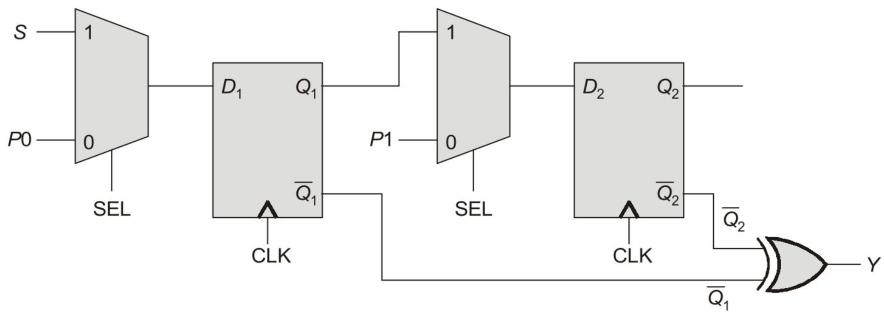


(a) 1011 (b) 0100

(c) 0010 (d) 1101

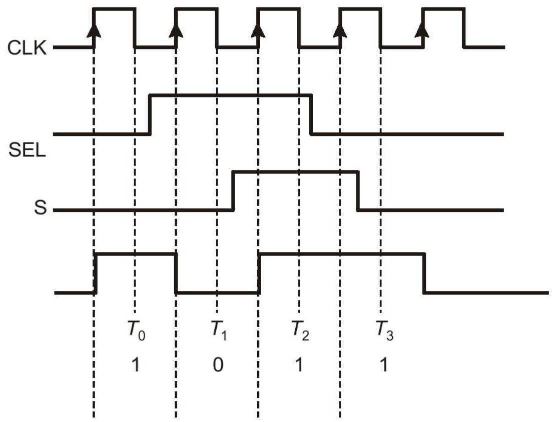
Ans. **(a)**

Given: $P0 = 0$; $P1 = 1$



Output, $Y = \overline{Q_2} \oplus \overline{Q_1}$ ...(i)

Timing Diagram:



End of Solution

Q.46 The intrinsic carrier concentration of a semiconductor is 2.5×10^{16} /m^3 at 300 K. If the electron and hole mobilities are 0.15 m^2/Vs and 0.05 m^2/Vs, respectively, then the intrinsic resistivity of the semiconductor (in kΩ.m) at 300 K is ________.
(Charge of an electron $e = 1.6 \times 10^{-19}$ C.)

(a) 1.65
(b) 1.25
(c) 0.85
(d) 1.95

Ans. **(b)**

Intrinsic resistivity, $\rho_i = \dfrac{1}{n_i q(\mu_p + \mu_n)}$

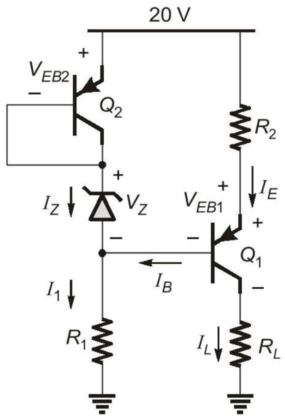
$$= \frac{1}{2.5\times10^{16}\times1.6\times10^{-19}(0.15+0.05)}$$

$$= \frac{1}{2.5\times10^{16}\times1.6\times10^{-19}(0.2)}$$

$\rho_i = 1.25$ kΩ-m

End of Solution

Q.47 In the circuit shown, the identical transistors $Q1$ and $Q2$ are biased in the active region with β =120. The Zener diode is in the breakdown region with $V_Z = 5$ V and $I_Z = 25$ mA. If $I_L = 12$ mA and $V_{EB1} = V_{EB2} = 0.7$ V, then the values of R_1 and R_2 (in kΩ, rounded off to one decimal place) are ______, respectively.



(a) 0.6 and 0.4 (b) 1.4 and 2.5
(c) 14.0 and 25.0 (d) 6.0 and 4.0

Ans. **(a)**

$$I_{C2} = I_L = 12 \text{ mA}$$

$$I_{B2} = \frac{I_{C2}}{\beta} = \frac{12}{120}\text{mA} = 0.1\,\text{mA}$$

KCL: $I_1 = I_Z + I_{B2} = 25.1$ mA

KVL: $20 = 0.7 + 5 + I_1 \times R_1$

$R_1 = 0.5697\ \text{k}\Omega \cong 0.6\ \text{k}\Omega$

$I_2 = I_{E2} = (1 + \beta)\, I_{B2} = 12.1$ mA

KVL: $20 = I_2 R_2 + 0.7 + I_1 \times R_1$

$$R_2 = \frac{20 - 0.7 - 14.3}{12.1} = 0.4\ \text{k}\Omega$$

End of Solution

Q.48 The electron mobility μ_n in a non-degenerate germanium semiconductor at 300 K is 0.38 m^2/Vs.

The electron diffusivity D_n at 300 K (in cm^2/s, rounded off to the nearest integer) is ________.

(Consider the Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K and the charge of an electron $e = 1.6 \times 10^{-19}$ C.)

(a) 26 (b) 98
(c) 38 (d) 10

Ans. **(b)**

Given, $\mu_n = 0.38$ m^2/V-sec

$$D_n = \mu_n V_T$$

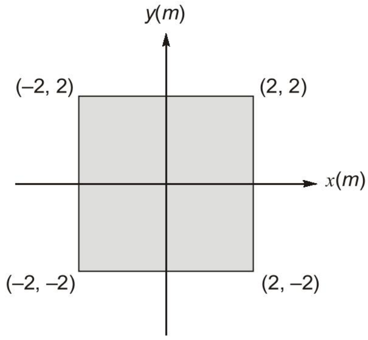
$$= 0.38 \times \frac{T}{11600}$$

$$= 0.38 \times \frac{300}{11600}$$

$$= 0.02586 \times 0.38$$
$$= 0.09826 \text{ m}^2/\text{sec}$$
$$D_n = 98.26 \text{ cm}^2/\text{sec}$$

End of Solution

Q.49 A square metal sheet of 4 m × 4 m is placed on the x-y plane as shown in the figure below.
If the surface charge density (in μC/m²) on the sheet is $\rho_s(x, y) = 4|y|$, then the total charge (in μC, rounded off to the nearest integer) on the sheet is ________.



(a) 16
(b) 85
(c) 64
(d) 256

Ans. (c)

$$Q_{enc} = \int_s \rho_s ds = \int_s 4|y| dx\,dy$$

$$\Rightarrow \quad Q_{enc} = 4 \int_{y=-2}^{2} |y| dy \int_{x=-2}^{2} dx = 4 * 2 * \int_0^2 y dy \cdot x\Big|_{-2}^{2}$$

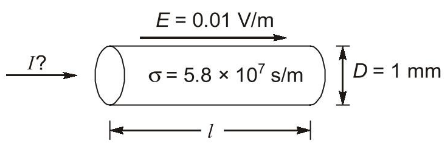
$$\Rightarrow \quad Q_{enc} = 8.\frac{y^2}{2}\Big|_0^2 \cdot 4 = 64\ \mu\text{C}$$

End of Solution

Q.50 An electric field of 0.01 V/m is applied along the length of a copper wire of circular cross-section with diameter 1 mm. Copper has a conductivity of 5.8×10^7 S/m.
The current (in Amperes, rounded off to two decimal places) flowing through the wire is _________.

(a) 0.46
(b) 1.82
(c) 0.58
(d) 1.12

Ans. (a)



As we know that

$$I = \frac{V}{R} = \frac{E.l}{\frac{l}{\sigma A}} = E \cdot \sigma A$$

$$\Rightarrow \qquad I = 0.01 \times 5.8 \times 10^7 \times \frac{\pi}{4}(1 \times 10^{-3})^2 = 0.46 \text{ A}$$

End of Solution

Q.51 Consider a non-negative function $f(x)$ which is continuous and bounded over the interval [2, 8]. Let M and m denote, respectively, the maximum and the minimum values of $f(x)$ over the interval.

Among the combinations of α and β given below, choose the one(s) for which the inequality

$$\beta \le \int_2^8 f(x)dx \le \alpha$$

is guaranteed to hold.

(a) $\beta = 5$ m, $\alpha = 7$ M
(b) $\beta = 6$ m, $\alpha = 5$ M
(c) $\beta = 7$ m, $\alpha = 6$ M
(d) $\beta = 7$ m, $\alpha = 5$ M

Ans. (a)

$f(x) \ge 0$ given in $[2, 8] = [a, b]$

by F.T.I.C

$$m(b - a) \le \int_a^b f(x)dx \le M(b-a)$$

$$m(8 - 2) \le \int_2^8 f(x)dx \le M(8-2)$$

$$6m \le \int_2^8 f(x)dx \le 6M$$

$\because f(x) \ge 0$

$\Rightarrow$ m and $M \ge 0$

$\therefore$ The possible answer is

$$5m \le \int_2^8 f(x)dx \le 7\,M$$

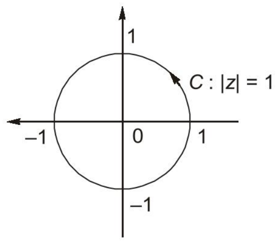
End of Solution

Q.52 Which of the following statements involving contour integrals (evaluated counter-clockwise) on the unit circle C in the complex plane is/are TRUE?

(a) $\oint_C e^z dz = 0$
(b) $\oint_C z^n dz = 0$, where n is an even integer
(c) $\oint_C \cos z\, dz \ne 0$
(d) $\oint_C \sec z\, dz \ne 0$

Ans. **(a, b)**

Given $C : |z| = 1$



(a) $f = e^z$ is always analytic

$\therefore$ by C.I.T

$$\oint_C f(z)dz = 0 \qquad \text{(a) is true}$$

(b) $f(z) = z^n$ is always analytic

$\therefore$ by C.I.T

$$\oint_C z^n dz = 0 \qquad \text{(b) is true}$$

(c) $f(z) = \cos z$ is also always analytic

$\therefore$ by C.I.T

$$\oint_C \cos z\, dz = 0$$

$\therefore$ (c) is false.

(d) $f(z) = \sec z$,

Singularities are given by $\cos z = 0 \Rightarrow z = (2n+1)\frac{\pi}{2}$

i.e., $z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots.$

$\because$ all poles lies out 'C'

$\therefore$ by C.I.T $\oint_C \sec z\, dz = 0$ ($\therefore$ (d) is false)

End of Solution

Q.53 Consider a system where $x_1(t)$, $x_2(t)$, and $x_3(t)$ are three internal state signals and $u(t)$ is the input signal. The differential equations governing the system are given by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t).$$

Which of the following statements is/are TRUE?

(a) The signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ are bounded for all bounded inputs.

(b) There exists a bounded input such that at least one of the signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ is unbounded.

(c) There exists a bounded input such that the signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ are unbounded.

(d) The signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ are unbounded for all bounded inputs.

Ans. **(b)**

From the given matrix, the eigen values are 2, –2, 0

Hence system $x_1(t)$ whose eigen value is positive, it is unbounded for bounded input.

Similarly system $x_3(t)$ is also unbounded for bounded input.

Only system $x_2(t)$ is bounded for bounded input (since its eigen value is negative).

End of Solution

Q.54 The random variable X takes values in {–1, 0, 1} with probabilities

$P(X = -1) = P(X = 1)$ and α and $P(X = 0) = 1 - 2\alpha$, where $0 < \alpha < \frac{1}{2}$.

Let $g(\alpha)$ denote the entropy of X (in bits), parameterized by α.

Which of the following statements is/are TRUE?

(a) $g(0.4) > g(0.3)$
(b) $g(0.3) > g(0.4)$
(c) $g(0.3) > g(0.25)$
(d) $g(0.25) > g(0.3)$

Ans. **(b, c)**

$$X \in \{-1, 0, 1\}$$

$$P(X = -1) = P(X = 1) = \alpha \text{ and } P(X = 0) = 1 - 2\alpha \quad \text{where } 0 < \alpha < \frac{1}{2}$$

$$g(\alpha) = \text{Entropy of } X$$

$$= -\sum_i P(x_i)\log_2 P(x_i)$$

$$= -\{\alpha\log_2\alpha + \alpha\log_2\alpha + (1-2\alpha)\log_2(1-2\alpha)\}$$

$$g(\alpha) = -\{2\alpha \log_2\alpha + (1 - 2\alpha)\log_2(1 - 2\alpha)\}$$

$$g(0.25) = -\{0.5 \log_2 0.25 + 0.5 \log_2 0.5\} = 1.5$$

$$g(0.3) = -\{0.6 \log_2 0.3 + 0.4 \log_2 0.4\} = 1.57$$

$$g(0.4) = -\{0.8 \log_2 0.4 + 0.2 \log_2 0.2\} = 1.52$$

$$g(0.3) > g(0.4) > g(0.25)$$

End of Solution

Q.55 Let $f(t)$ be a periodic signal with fundamental period $T_0 > 0$. Consider the signal $y(t) = f(\alpha t)$, where $\alpha > 1$.

The Fourier series expansions of $f(t)$ and $y(t)$ are given by

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T_0}kT} \quad \text{and} \quad y(t) = \sum_{k=-\infty}^{\infty} d_k e^{j\frac{2\pi}{T_0}\alpha kT}.$$

Which of the following statements is/are TRUE?

(a) $c_k = d_k$ for all k
(b) $y(t)$ is periodic with a fundamental period αT_0
(c) $c_k = d_k/\alpha$ for all k
(d) $y(t)$ is periodic with a fundamental period T_0/α

Ans. **(a, d)**

Given, $f(t) \xleftrightarrow[\text{FTP: } T_0]{} c_k \qquad y(t) \xleftrightarrow[\text{FTP: } \frac{T_0}{\alpha}]{} d_k$

and $y(t) = f(\alpha t)$

By time-scaling property of FS,

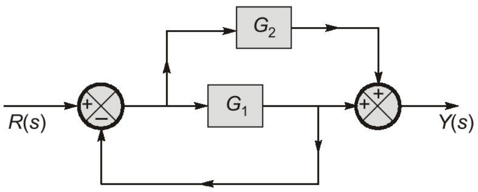
$$d_k = c_k \text{ if } a \text{ is positive}$$

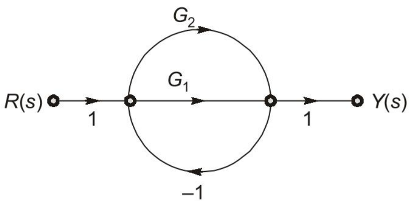
Given that, $\alpha > 1$

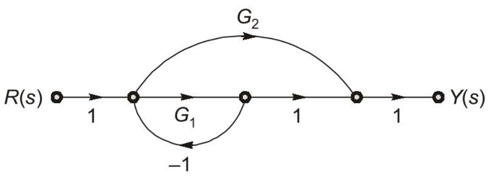
$\therefore$ $d_k = c_k$ for all 'k'

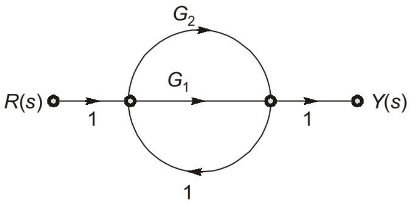
End of Solution

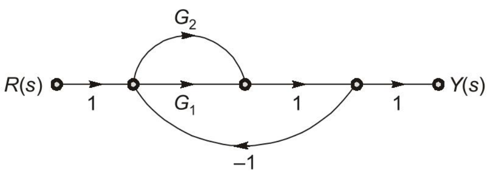
Q.56 Consider a system represented by the block diagram shown below. Which of the following signal flow graphs represent(s) this system? Choose the correct option(s).



(a) 

(b) 

(c) 

(d) 

Ans. **(b)**

From given block diagram,

$$\frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 + G_1}$$

and in options, from option (a),

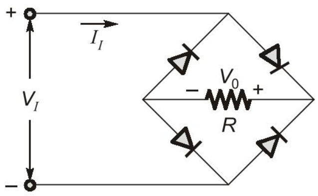
$$\frac{Y(s)}{R(s)} = \frac{G_1 + G_2}{1 + G_1}$$

$\therefore$ Option (b) only satisfies.

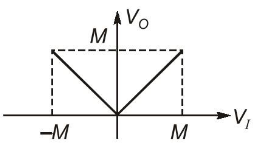
End of Solution

Q.57 All the diodes in the circuit given below are ideal.

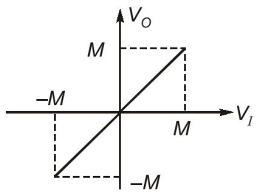
Which of the following plots is/are correct when V_I (in Volts) is swept from $-M$ to M?



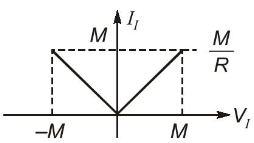
(a)



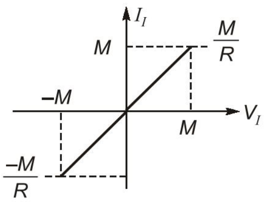
(b)



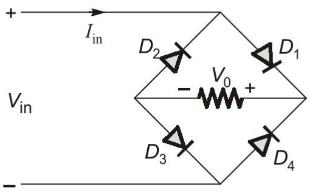
(c)



(d)



Ans. **(a, d)**

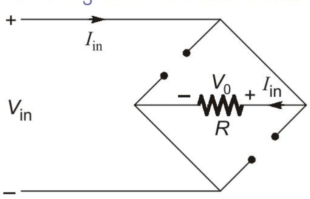


Given, V_{in} is steep of $-M$ to $+M$

For V_0 versus V_{IN}:

Let $V_{IN} = +M$

In the bridge rectifier, D_1 and D_3 are forward bias, and D_2 and D_4 are reverse bias,



$\therefore$ $V_{in} - V_0 = 0 \Rightarrow V_{in} = V_0$

$\therefore$ For $V_{in} = +M \Rightarrow V_0 = M$

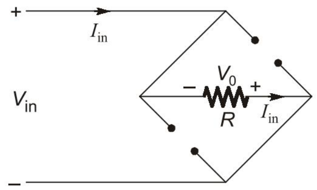
$$-V_{in} + I_{in} R = 0$$

$$I_{in} = \frac{V_{in}}{R} = \frac{M}{R}$$

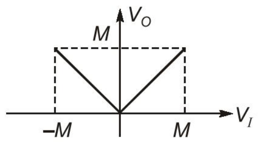
Let $V_{IN} = -M$

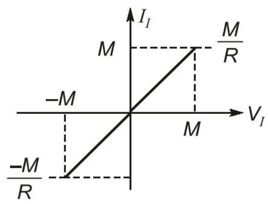
D_1 and D_3 are reverse bias,

D_2 and D_4 are forward bias.



$\therefore$
$$V_{in} + V_0 = 0$$
$$V_o = -V_{in} = -[-M] = M$$
$$-V_{in} + I_{in} \times R = 0$$
$$I_{in} = \frac{V_{in}}{R} = \frac{-M}{R}$$





End of Solution

Q.58 Two fair dice (with faces labeled 1, 2, 3, 4, 5, and 6) are rolled. Let the random variable X denote the sum of the outcomes obtained.

The expectation of X is______ (rounded off to two decimal places).

Ans. (7.0) (7.0 to 7.0)

Sum of two dice = {2, 3, 4, 5, ..., 12}

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(X) = \sum x \cdot P(x)$$

$$= 2\times\frac{1}{36}+3\times\frac{2}{36}+4\times\frac{3}{36}+5\times\frac{4}{36}+6\times\frac{5}{36}+7\times\frac{6}{36}$$

$$+8\times\frac{5}{36}+9\times\frac{4}{36}+10\times\frac{3}{36}+11\times\frac{2}{36}+12\times\frac{1}{36}$$

$$= \frac{252}{36} = 7$$

End of Solution

Q.59 Consider the vectors

$$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 3\sqrt{2} \end{bmatrix}.$$

For real-valued scalar variable x, the value of

$$\min_x \| ax - b \|_2$$

is __________ (rounded off to two decimal places).

$\| \cdot \|_2$ denotes the Euclidean norm, i.e., for $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \| y \|_2 = \sqrt{y_1^2 + y_2^2}$.

Ans. **(3.0) (3.0 to 3.0)**

$$ax - b = \begin{bmatrix} x \\ x - 3\sqrt{2} \end{bmatrix}$$

$$\left\| \begin{matrix} x \\ x - 3\sqrt{2} \end{matrix} \right\| = f(x) = \text{norm}$$

$$f(x) = \sqrt{x^2 + (x - 3\sqrt{2})^2}$$

$$= \sqrt{x^2 + x^2 + 18 - 6\sqrt{2}\,x}$$

$$f(x) = \sqrt{2x^2 - 6\sqrt{2}x + 18}$$

Let, $g(x) = 2x^2 - 6\sqrt{2}x + 18$

$$g'(x) = 4x - 6\sqrt{2} = 0$$

$$x = \frac{6\sqrt{2}}{4} = \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}$$

$$\text{Point of minima} = \frac{3}{\sqrt{2}}$$

$$f(x)\Big|_{x=\frac{3}{\sqrt{2}}} = \sqrt{2\left(\frac{3}{\sqrt{2}}\right)^2 - 6\sqrt{2} \times \frac{3}{\sqrt{2}} + 18}$$

$$= \sqrt{9 - 18 + 18} = 3$$

End of Solution

Q.60 X and Y are Bernoulli random variables taking values in {0, 1}. The joint probability mass function of the random variables is given by:

$P(X = 0, Y = 0) = 0.06$

$P(X = 0, Y = 1) = 0.14$

$P(X = 1, Y = 0) = 0.24$

$P(X = 1, Y = 1) = 0.56$

The mutual information $I(X; Y)$ is ________ (rounded off to two decimal places).

Ans. **(0) (0.0 to 0.0)**

Given, $P(X = 0, Y = 0) = 0.06$

$P(X = 0, Y = 1) = 0.14$

$P(X = 1, Y = 0) = 0.24$

$P(X = 1, Y = 1) = 0.56$

$$[P(X, Y)] = \begin{matrix} & Y=0 & Y=1 & \\ X = 0 & 0.06 & 0.14 & \rightarrow P(X=0) = 0.2 \\ X = 1 & 0.24 & 0.56 & \rightarrow P(X=1) = 0.8 \\ & \downarrow & \downarrow & \\ & P(Y=0)=0.3 & P(Y=1)=0.7 & \end{matrix}$$

From above

$$P(X = 0, Y = 0) = P(X = 0) \cdot P(Y = 0)$$
$$P(X = 0, Y = 1) = P(X = 0) \cdot P(Y = 1)$$
$$P(X = 1, Y = 0) = P(X = 1) \cdot P(Y = 0)$$
$$P(X = 1, Y = 1) = P(X = 1) \cdot P(Y = 1)$$

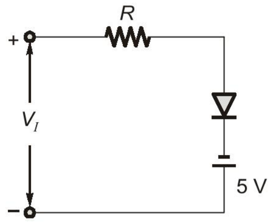
We conclude that X and Y are independent.

So that $I(X; Y) = 0$

End of Solution

Q.61 The diode in the circuit shown below is ideal. The input voltage (in Volts) is given by $V_1 = 10 \sin 100 \pi t$, where time t is in seconds.

The time duration (in ms, rounded off to two decimal places) for which the diode is forward biased during one period of the input is __________.



Ans. **(13.33) (13.32 to 13.34)**

Given, $2\pi ft = 100\pi t$

$$f = 50 \text{ Hz}$$

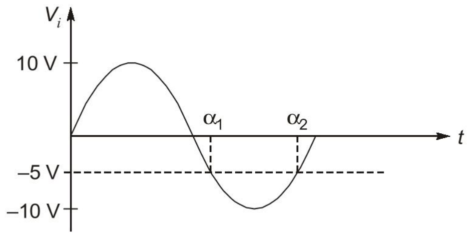
$$T_0 = \frac{1}{f} = 20 \text{ msec}$$

$$V_i = 10 \sin(100\pi t) = 10 \sin\alpha,$$

$$\alpha = 100\pi\, t$$

Diode conducts if $V_i > -5$ V

Diode is OFF if $V_i < -5$ V



$$10 \sin \alpha = -5$$

$$\alpha = \sin^{-1}\left(\frac{-5}{10}\right)$$

$$\alpha = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$100\pi\, t_1 = \frac{7\pi}{6}$$

$$t_1 = 11.66 \text{ msec}$$

$$\alpha_2 = 100\pi t_2 = \frac{11\pi}{6}$$

$$t_2 = 18.33 \text{ msec}$$

Diode is OFF from t_1 to t_2

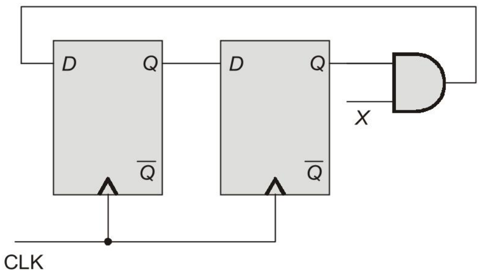
$$t_{OFF} = t_2 - t_1 = 6.66 \text{ msec}$$

$$t_{ON} = T_0 - t_{OFF} = 13.33 \text{ msec}$$

End of Solution

Q.62 In the circuit shown below, the AND gate has a propagation delay of 1 ns. The edge-triggered flip-flops have a set-up time of 2 ns, a hold-time of 0 ns, and a clock-to-Q delay of 2 ns.

The maximum clock frequency (in MHz, rounded off to the nearest integer) such that there are no setup violations is ________.



Ans. **(200) (200 to 200)**

Given,

$$t_{pd} = 2n \text{ sec}$$

$$t_{setup}\ (t_s) = 2n \text{ sec}$$

$$t_{comb} = 1n \text{ sec}$$

$$T_{clkD1} = t_p + t_s + t_{comb} = 2 + 2 + 1 = 5n \text{ sec}$$

$$T_{clkD2} = t_s + t_p = 2 + 2 = 4n \text{ sec}$$

∴ Time required for clock,

$$T_{clk} = \text{Max}\,\{T_{clk_{D1}}, T_{clk_{D2}}\}$$

$$= \text{Max}\ [5\text{ns},\ 4\text{ ns}]$$

$$= 5 \text{ ns}$$

Maximum frequency, $f_{mx} = \dfrac{1}{T_{clk}} = \dfrac{1}{5ns} = 200$ MHz.

End of Solution

Q.63 An ideal p-n junction germanium diode has a reverse saturation current of 10 μA at 300 K. The voltage (in Volts, rounded off to two decimal places) to be applied across the junction to get a forward bias current of 100 mA at 300 K is _____.
(Consider the Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K and the charge of an electron $e = 1.6 \times 10^{-19}$ C.)

Ans. **(0.23) (0.23 to 0.24)**

Given,

Reverse saturation current, $I_0 = 10\ \mu A$

Temperature, $T = 300$ K

Forward bias current, $I_f = 100$ mA

Boltzmann constant, $K_B = 1.38 \times 10^{-23}$ J/K

Charge of electron, $e = 1.6 \times 10^{-19}$ C

We know that,

the forward bias current, $I_f = I_0\left[e^{V/V_T} - 1\right]$

where,
$$V_T = \frac{T}{11600} = \frac{300}{11600} = 25.86 \text{ mV}$$

$$\therefore \quad 100 \times 10^{-3} = 10 \times 10^{-6}\left[e^{\frac{V}{25.86\times 10^{-3}}} - 1\right]$$

$$10^4 = e^{\frac{V}{25.86\times 10^{-3}}}$$

$$9999 = e^{\frac{V}{25.86\times 10^{-3}}}$$

$$\therefore \quad V = 25.86 \times 10^{-3} \ln(9999)$$

$$V = 0.23$$

End of Solution

Q.64 A 50 Ω lossless transmission line is terminated with a load Z_L of $(50 - j75)$ Ω. If the average incident power on the line is 10 mW, then the average power delivered to the load (in mW, rounded off to one decimal place) is __________.

Ans. **(6.4) (6.3 to 6.5)**

$$Z_0 = 50\ \Omega, \quad Z_L = 50 - j75\ \Omega; \quad P_{in} = 10 \text{ mW}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j75 - 50}{50 - j75 + 50} = \frac{-j75}{100 - j75} = \frac{-j3}{4 - j3}$$

$$\Rightarrow \quad |\Gamma| = \frac{3}{5} = 0.6$$

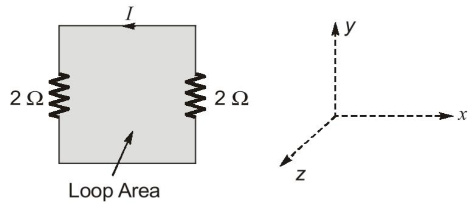
$$\therefore \quad P_{del} = \left\{1 - |\Gamma|^2\right\} P_i$$

$$= [1 - (0.6)^2] * 10 \text{ mW} = 6.4 \text{ mW}$$

End of Solution

Q.65 Two resistors are connected in a circuit loop of area 5 m^2, as shown in the figure below. The circuit loop is placed on the x-y plane.

When a time-varying magnetic flux, with flux-density $B(t) = 0.5t$ (in Tesla), is applied along the positive z-axis, the magnitude of current I (in Amperes, rounded off to two decimal places) in the loop is ________.



Ans. (0.63) (0.62 to 0.63)

Given, $B(t) = 0.5t$

$$\therefore \quad V_{\text{emf}} = -\frac{d\phi}{dt} = \frac{-dB(t)}{dt} \times A = -A\frac{d}{dt}(0.5t) = -A \times 0.5$$

$$\therefore \quad I = \frac{V_{emf}}{R} = \frac{-0.5 \times 5}{4} = -0.625$$

Hence, magnitude of $I = 0.625$ A ≈ 0.63 A

End of Solution

■■■■