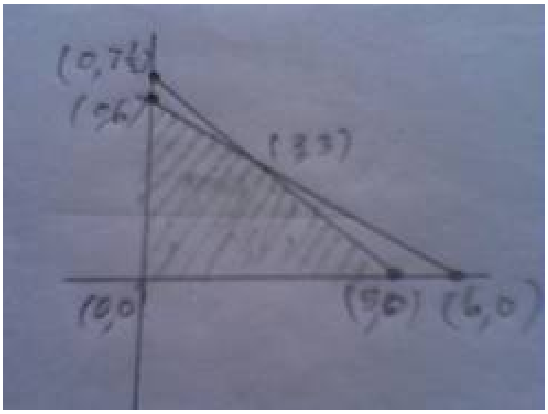
Linear Programming

Q.1. A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and two hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs.30 per trunk on the first type of trunk and Rs.25 per trunk on the second type. Formulate a Linear Programming Problem to find out how many trunks of each type he must make each day to maximize his profit.

Solution : 1



Fig

Let x unit of type A and y unit of type B be produced each day. \

Therefore , $z = 30x + 25y$

Under the restrictions :

$3x + 3y \le 18$

$3x + 2y \le 15$

$x \ge 0$, $y \ge 0$.

Corner points of shaded portion are : (0,0),(0,6),(5,0),(3,3).

Maximum z is at (3,3) .

Therefore , x = 3 and y = 3 .

Q.2. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs50 per kg to purchase Food I and Rs70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

Solution : 2

Let the mixture contain x kg of Food I and y kg of Food II. Then $x \geq 0, y \geq 0$.

Resources	Food I (x)	Food II (y)	Requirement
Vitamin A (unit/kg)	2	1	8
Vitamin C (unit/kg)	1	2	10
Cost (Rs/kg)	50	70	

Since the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C, then we have the constraints as :

$2x + y \geq 8$ and $x + 2y \geq 10$.

Total cost Z of purchasing x kg of food I and y kg of food II is given by

$Z = 50x + 70y$

Hence , the mathematical formulation of the problem is :

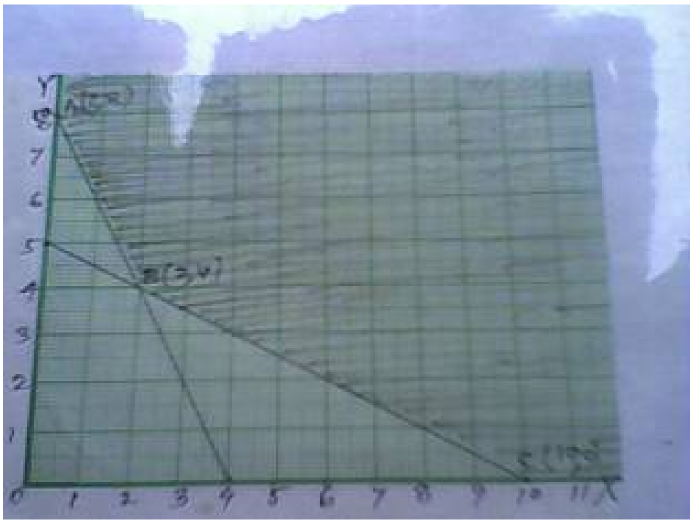
Minimize $Z = 50x + 70y$ ---------------------------- (1)

Subject to the constraints :

$2x + y \geq 8$ ------------------------------- (2)

$x + 2y \geq 0$ ------------------------------- (3)

$x, y \geq 0$ ------------------------------- (4)



from the graph , the feasible region is unbounded. Let us evaluate Z at the corner points A(0,8), B(2,4) and C(10,0).

Corner Point	Z = 50 x + 70 y	
(0,8)	560	
(2,4)	380	Minimum
(10,0)	500 <-	

Hence, the minimum value of Z is Rs380.

Q.3. A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs10,500 and Rs9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, not more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ?

Solution : 3

Let x hectare of land be allocatedto crop X and y hectare to crop Y.

Clearly $x \geq 0$ and $y \geq 0$.

Profit per hectare on crop X = Rs10,500,

Profit per hectare on crop Y = Rs9,000,

Therefore total profit = Rs(10,500 x + 9,000 y).

The mathematical formulation of the problem is as follows :

Maximize Z = 10,500 x + 9,000 y

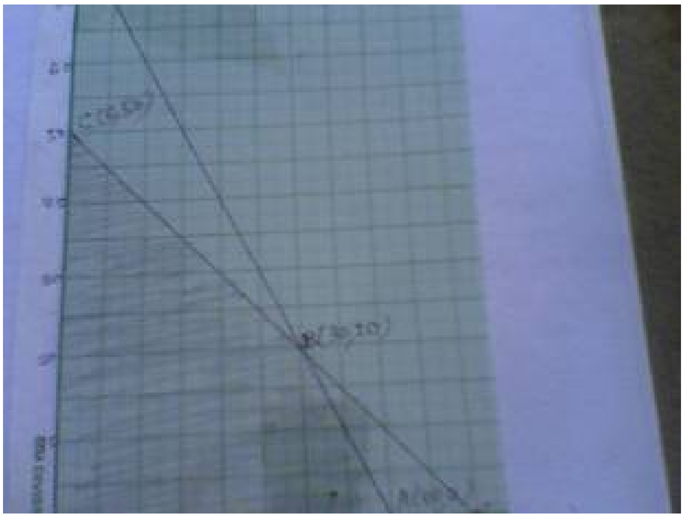
subject to the constraints :

$x + y \le 50$ (constraint related to land) ----------------- (1)

$20x + 10y \le 800$ (constraint related to use of herbicide)

Or, $2x + y \le 80$ --- (2)

And $x \ge 0$, $y \ge 0$ (non negative constraint) -------------------- (3)



In the graph we see that the feasible region is bounded. The corner points are O (0, 0), A (40, 0), B (30, 20) and C (0, 50). Let us evaluate the objective function Z = 10,500 x + 9,000 y at these vertices to find which one gives the maximum profit.

Corner Point	Z = 10,500 x + 9,000 y
O (0, 0)	0
A (40, 0)	4,20,000
B (30, 20)	4,95,000 ← Maximum
C (0, 50)	4,50,000

Hence , society will get the maximum profit of Rs4,95,000 by allocating 30 hectare for crop X and 20 hectare for crop Y.

Q.4. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs5 per kilogram and rice costs Rs4 per kilogram.

Solution : 4

Let the cereal contains x kg of bran and y kg of rice. Clearly $x \geq 0$ and $y \geq 0$.

According to the hypothesis the linear programming problem is :

Minimize $Z = 5x + 4y$

Subject to the constraints :

$(80/1000)\ x + (100/1000)\ y \geq (88/1000)$

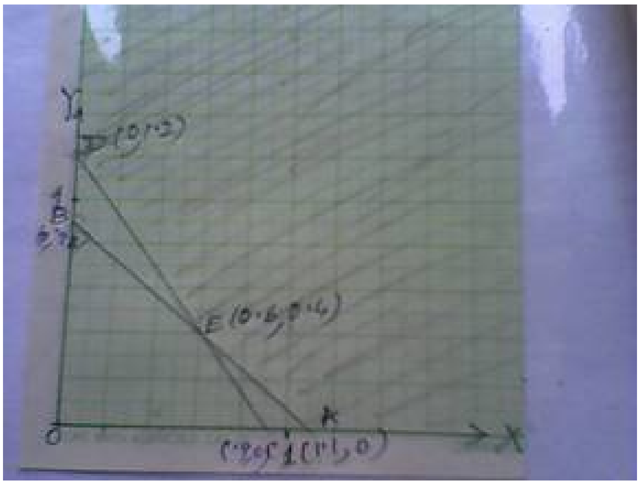
Or, $20\ x + 25\ y \geq 22$ ----------------- (1)

And $(40/1000)\ x + (30/1000)\ y \geq (36/1000)$

Or, $20\ x + 15\ y \geq 18$ ------------------ (2)

And also $x \geq 0$, $y \geq 0$ ------------------- (3)

Now the lines $20\ x + 25\ y = 22$ and $20x + 15\ y = 18$ are drawn.



These lines meet at E (0.6, 0.4). The feasible region is shaded and it is an unbounded region with vertices A (1.1, 0), E (0.6, 0.4) and D (0, 1.2).

Corner Point	$Z = 5\ x + 4\ y$	
A (1.1, 0)	$5 \times 1.1 + 4 \times 0 = 5.5$	

E (0.6, 0.4)	$5 \times 0.6 + 4 \times 0.4 = 4.6 \leftarrow$	Minimum
D (0, 1.2)	$5 \times 0 + 4 \times 1.2 = 4.8$	

Therefore, minimum cost of producing this cereal is Rs4.60 per kg.

Q.5. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs5,760 to invest and has space for at most 20 items. A fan cost him Rs360 and a sewing machine Rs240. He expects to sell a fan at a profit of Rs22 and a sewing machine at a profit of Rs18. Assuming that he can sell all the atoms that he buys, how should he invest his money to maximize the profit ? Solve graphically and find the maximum profit.

Solution : 5

Let x be the number of fans and y the number of sewing machines (bought and sold). According to the hypothesis , the linear programming problem is :

Maximize $Z = 22x + 18y$

Subject to the constraints as :

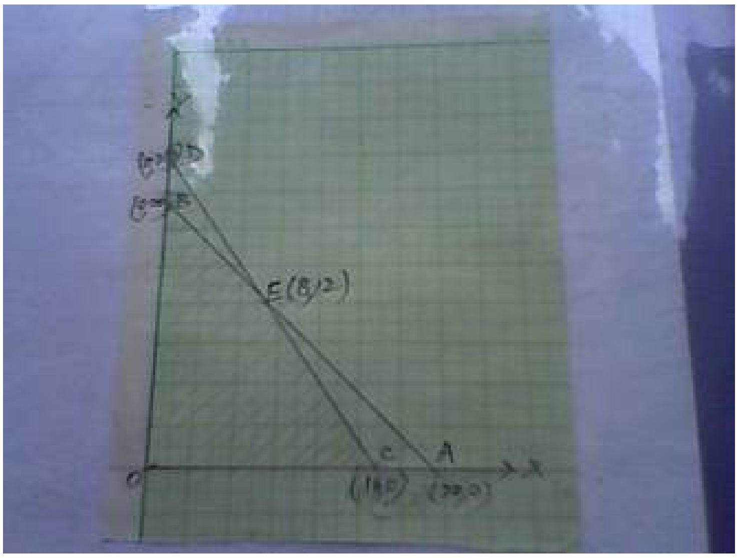
$x + y \le 20$ --------------------------- (1)

And $360x + 240y \le 5760$

Or, $3x + 2y \le 48$ --------------------------- (2)

And also $x \ge 0$, $y \ge 0$ --------------------------- (3)

Now the lines $x + y = 20$ and $3x + 2y = 48$ is drawn.



The lines meet at E (8, 12). The feasible region is shaded. We see that the feasible region is bounded. The corner points are C (16, 0), E (8, 12) and B (0, 20).

Corner Point	$Z = 22x + 18y$	
C (16, 0)	$22 \times 16 + 18 \times 0 = 352$	
E (8, 12)	$22 \times 8 + 18 \times 12 =$ **392** ←	Maximum
D (0, 24)	$22 \times 0 + 18 \times 20 = 360$	

Therefore, for maximum profit he should buy and sell 8 fans and 12 sewing machines.

His maximum profit is Rs392.