Exercise 6.1

Answer 1E.

(a)

A function *f* is said to be **one-to-one** function if it never takes on the same value twice.

That is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

(b)

To determine the graph of the function whether one-to-one, it must pass the **horizontal line test**.

Horizontal line test: A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Answer 2E.

(A) If f is a one to one function with domain A and range B, then f^{-1} is also a one to one function, defined from B to A as $f^{-1}: B \to A$

$$f^{-1}(y) = x \qquad \text{Where } y = f(x)$$

The domain of f^{-1} = the range of f = B

The domain of f^{-1} = the domain of f = A

(B) If we are given a formula for f and f is one to one then for finding a formula for f^{-1}, we write $y = f(x)$ solve this equation for x (if possible) in terms of y, and at last, interchange x and y. Resulting formula will be $y = f^{-1}(x)$

(C) If we are given the graph of one to one function f, then the graph of f^{-1} is obtained by reflecting the graph of f about the line y = x.

Answer 3E.

A function $f(x)$ is defined by the below table of values:

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

The objective is to determine whether the function $f(x)$ is one – to – one or not.

A function *f* is called a one-to-one function if it never takes on the same value twice; that is,

$f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

From the given table, it can be found that the function $f(x)$ has the same value 2 for two distinct *x* values at $x = 2, \text{ and } 6$.

It means that $2 \neq 6$ but $f(2) = f(6) = 2$.

Therefore, the described function $f(x)$ is not one – to – one.

Answer 4E.

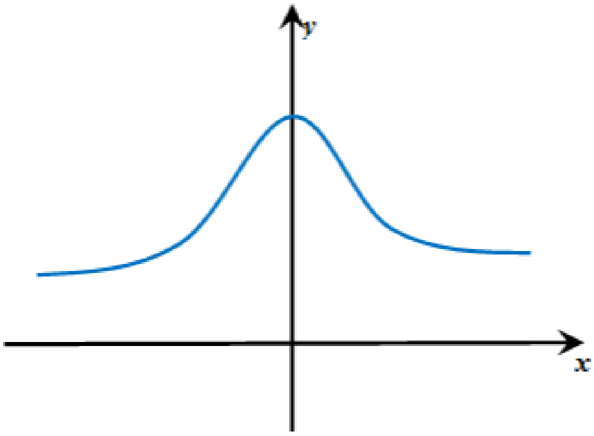
A function f is called a one-to-one if it never takes on the same value twice.

That is, $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Observe the table, for no two values of x, $f(x)$ has the same value.

Therefore the function is not one - to- one.

Consider the following graph:



Determine whether the graph of the function is one-to-one or not.

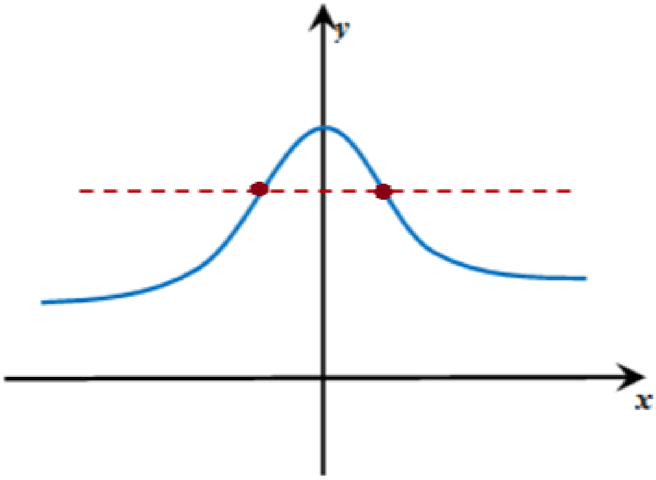
Answer 5E.

To determine whether the graph of the function is one-to-one or not, use horizontal line test.

Recall the horizontal line test.

If a horizontal line intersects a functions graph more than once, the function is not one –to-one.

Draw a horizontal line along with graph as follows:



From the above graph, observe that horizontal line intersect the graph twice.

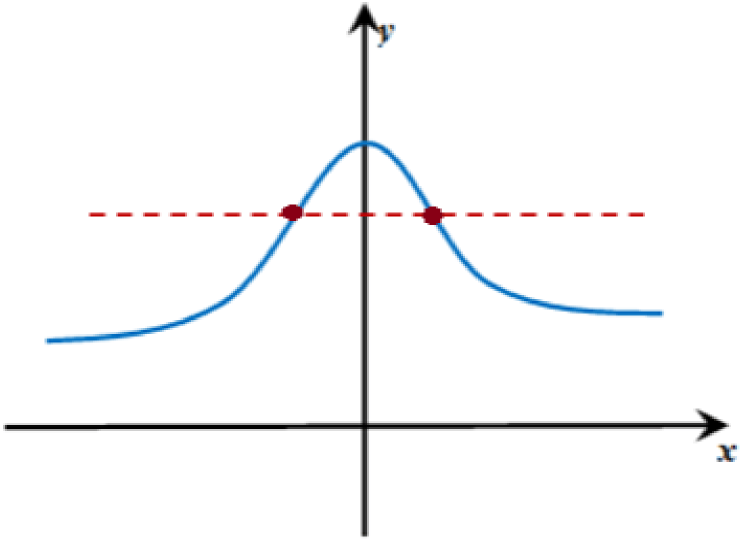
Hence, from the Horizontal Line Test, the graph of the function is **not one-to-one**.

To determine whether the graph of the function is one-to-one or not, use horizontal line test.

Recall the horizontal line test.

If a horizontal line intersects a functions graph more than once, the function is not one –to-one.

Draw a horizontal line along with graph as follows:

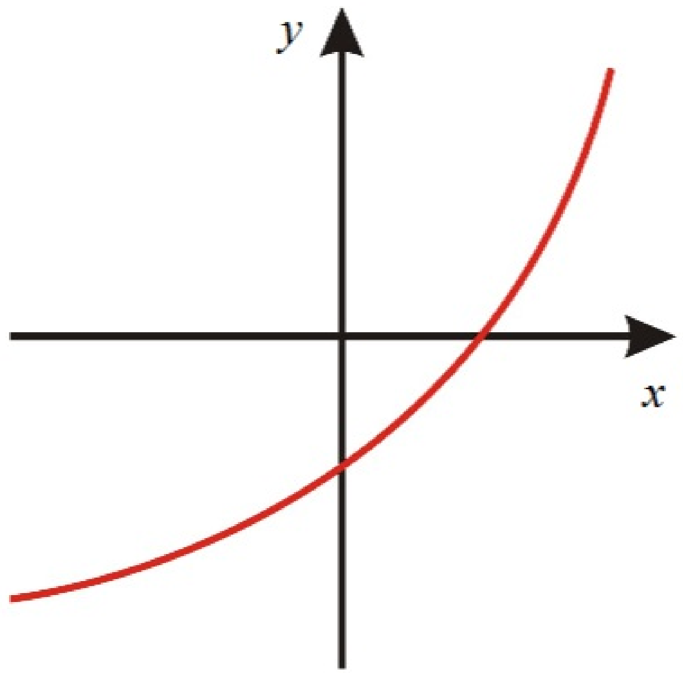


From the above graph, observe that horizontal line intersect the graph twice.

Hence, from the Horizontal Line Test, the graph of the function is not one-to-one.

Answer 6E.

Determine whether the following graph is one-one:



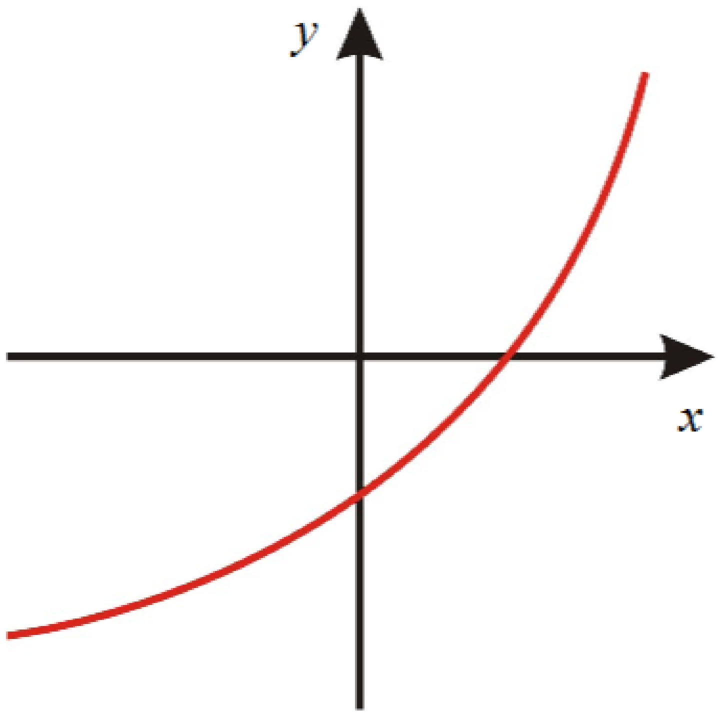
Use the Horizontal line test to find whether the above graph is one-one or not.

According to the horizontal line test if every horizontal line intersects the graph at most one point then the function is one-one.

From the graph it is clear that no horizontal line intersect the graph at more than one point.

I.e. any horizontal line intersects the graph at most one point.

Therefore, the function defined by given graph is one-one.



Answer 7E.

Determine the following function is a one-one function or not.

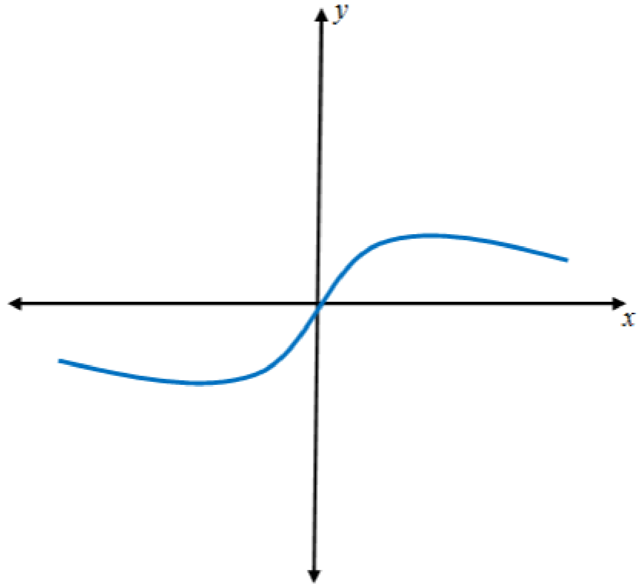
To determine whether the above function is a one-one function or not, use the horizontal line test.

Draw any parallel line on the graph, if it intersects the graph at only one point then the function is one-one, if intersects more than one point on the graph then it is not an one-one function.

From the figure it is obvious that, any parallel line that intersects the curve at only one point so the given function is a one-one function.

Answer 8E.

Consider the following curve:

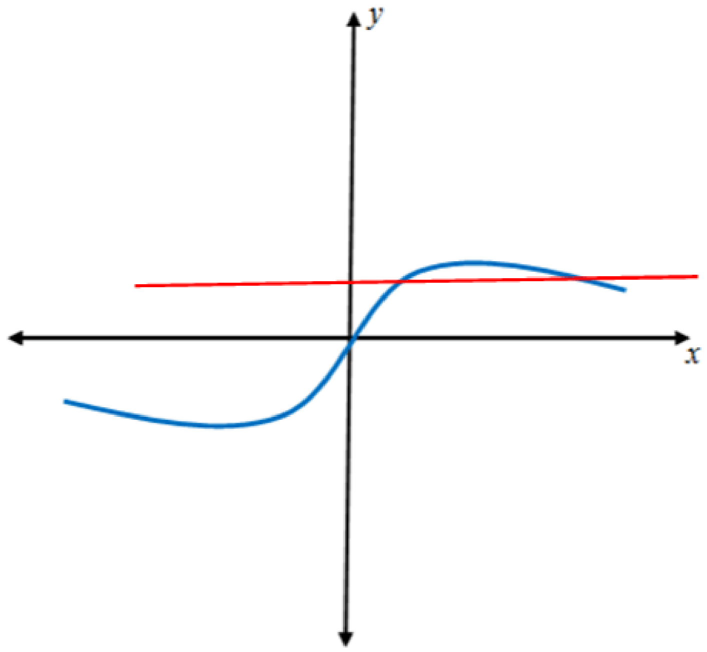


Now to determine whether the given function is one-to-one or not from the given curve.

Horizontal Line Test:

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Horizontal line is shown below:



In the given diagram horizontal line cuts the curve at two points.

Hence the given function is not one-to-one by using horizontal line test.

Answer 9E.

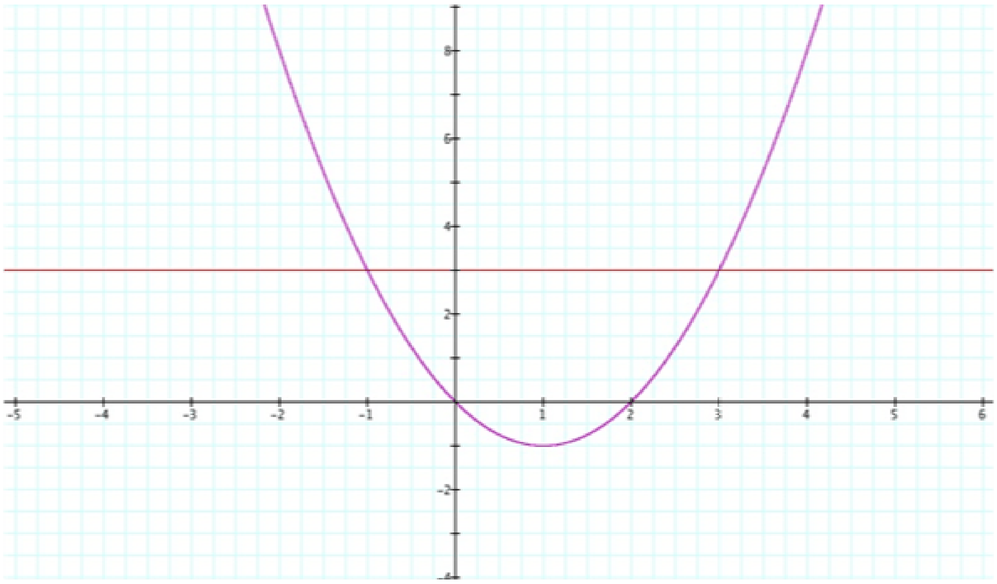
Consider the following function:

$$f(x) = x^2 - 2x$$

To determine whether the function $f(x) = x^2 - 2x$ is one-to-one or not, use Horizontal Line Test.

If a function fails the Horizontal Line Test (a horizontal line goes through more than one point on the curve) then the function is not one-to-one.

The graph of $f(x) = x^2 - 2x$ and a Horizontal line is as shown below:



From the above graph, it is observed that the horizontal line intersects the parabola at two different point.

So, the Horizontal Line Test fails.

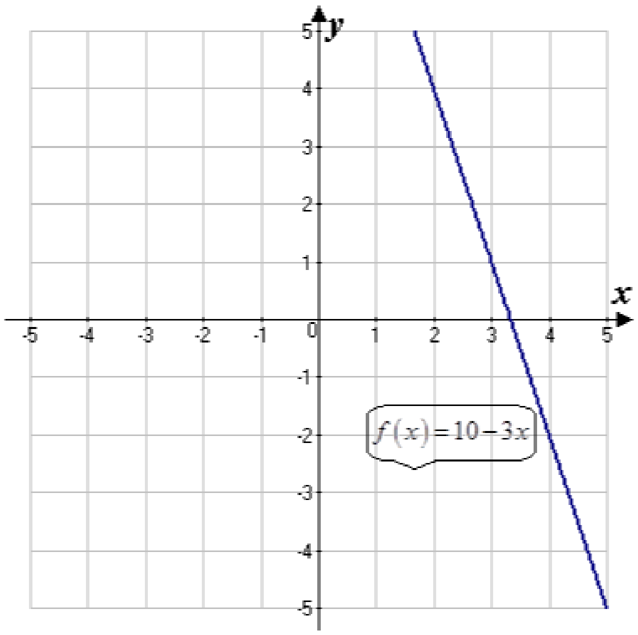
Therefore, the function $f(x) = x^2 - 2x$ is $\boxed{\text{not a one-to-one function}}$.

Answer 10E.

Need to determine whether the function $f(x) = 10 - 3x$ is one-to-one.

If a function fails the Horizontal Line Test (a horizontal line goes through more than one point on the curve) then the function is not one-to-one.

From the graph of $f(x) = 10 - 3x$ below, it is evident that we cannot draw a horizontal line through any two points on our 'curve'.



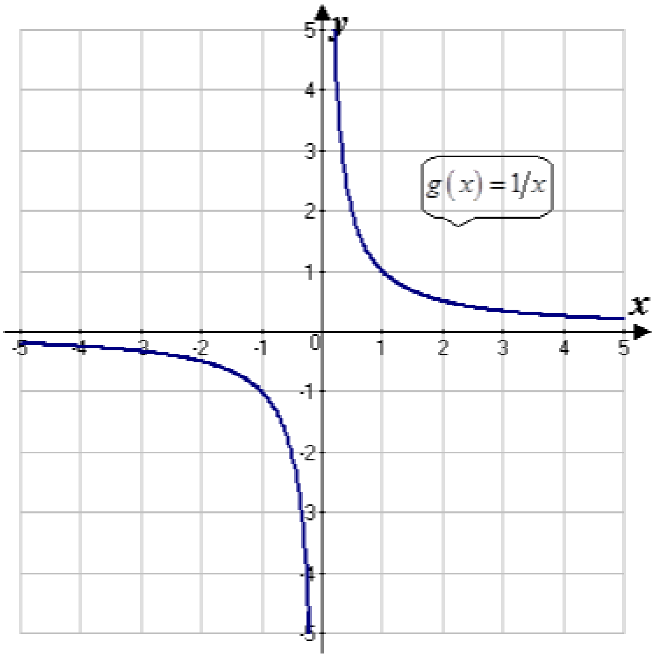
Note that any function that is linear (and not a horizontal line itself) will always be a one-to-one function. So, the function $f(x) = 10 - 3x$ is $\boxed{\text{a one-to-one}}$.

Answer 11E.

Need to determine whether the function $g(x) = 1/x$ is one-to-one.

If a function fails the Horizontal Line Test (a horizontal line goes through more than one point on the curve) then the function is not one-to-one.

From the graph of $g(x)=1/x$ below, it is evident that we cannot draw a horizontal line and hit the curve twice. Therefore, $g(x)$ passes the Horizontal Line Test.



So, the function $g(x)=1/x$ is $\boxed{\text{a one-to-one}}$.

Answer 12E.

The given function is defined as

$$g(x)=|x|$$

Let us take two different values –4 and 4 of x.

Now $g(-4)=|-4|=4$

And $g(4)=|4|=4$

Here, $4 \neq -4$ but $g(4)=g(-4)$

Therefore, the given function g(x) is not one – to – one.

Answer 13E.

Consider the function $h(x)=1+\cos(x)$

This is not one-to-one function.

For $x=\pi$

$$\begin{aligned} h(x) &= 1+\cos(x) \\ h(\pi) &= 1+\cos(\pi) \\ &= 1-1 \\ &= 0 \end{aligned}$$

For $x=-\pi$

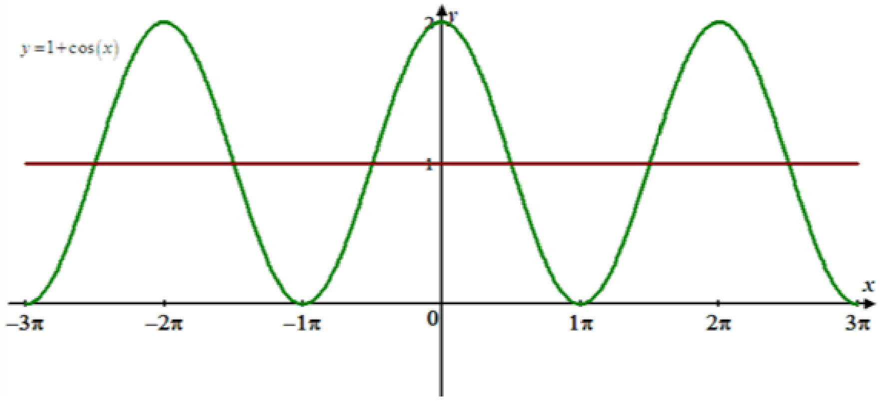
$$\begin{aligned} h(x) &= 1+\cos(x) \\ h(\pi) &= 1+\cos(-\pi) \\ &= 1+\cos(\pi) \qquad (\text{Since } \cos(-\theta)=\cos(\theta)) \\ &= 1-1 \\ &= 0 \end{aligned}$$

For different values of x, get the same function value.

This is contradicting to definition of one-to-one function.

Therefore this is not one-to-one function.

To check Horizontal line test, sketch the graph of the function $h(x)=1+\cos(x)$.



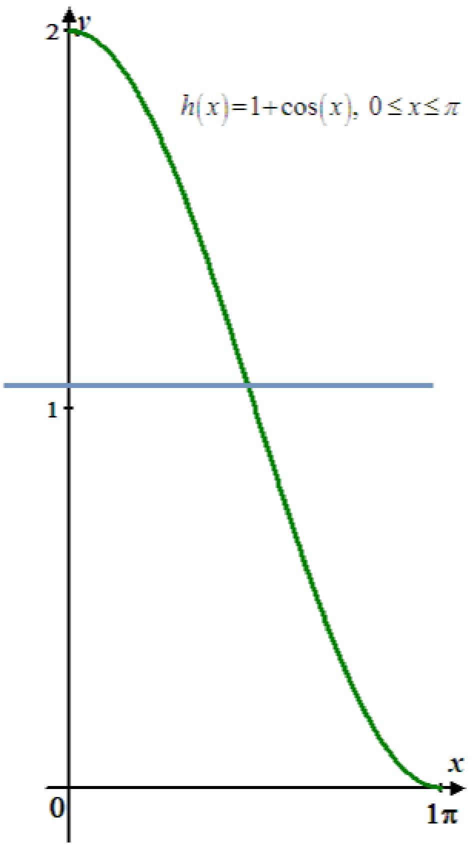
Here horizontal intersects the graph more than one point.

Hence By horizontal line test $h(x)=1+\cos(x)$ is not one-to-one function.

Answer 14E.

Consider the function $h(x)=1+\cos(x)$, $0 \le x \le \pi$.

Horizontal line test: A function is one-to-one function if and only if no horizontal line intersects the graph more than one.

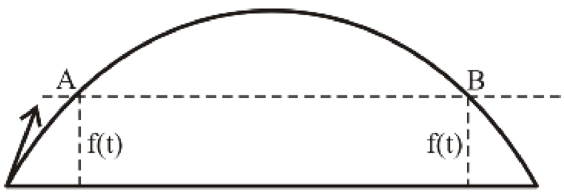


From the above graph Horizontal line intersect the graph more than one point.

So by Horizontal line test the function $h(x)=1+\cos(x)$ is one-to-one function in the interval $0 \le x \le \pi$.

That is for different values of x, function values also different in the interval $0 \le x \le \pi$.

Answer 15E.



When a football is kicked off then the path followed by it is a parabola.
If f(t) is the height of a football at 't' seconds after kickoff then the graph of f(t) will be parabola as shown in figure.
From figure it is clear that, if we draw a horizontal line, it will intersect the graph at two points A and B.
Therefore, by horizontal line test, f(t) is not one – to – one.

Answer 16E.

Given that f is one-to-one

(a) Since $f(6)=17$

Then $\boxed{f^{-1}(17)=6}$

(b) Since $f^{-1}(3)=2$

Then $\boxed{f(2)=3}$

Answer 17E.

Consider the system of equations.

$2x-y=-5$ ......(1)

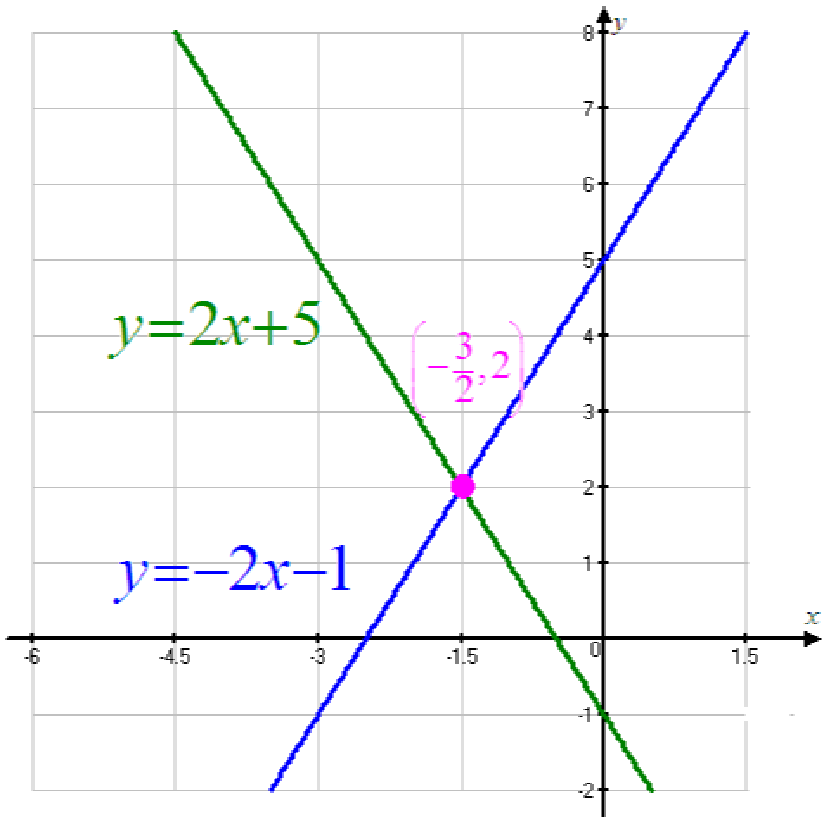
$-2x-y=-1$ ......(2)

Rewrite the equations as follows.

$y=2x+5$

$y=-2x-1$

Sketch the graphs both equations in the same coordinate plane.



From the graph, observe that the intersection point of the lines is $\boxed{\left(\frac{-3}{2},2\right)}$.

Answer 18E.

Consider the function $f(x)=x^5+x^3+x.$

Find the value of $f^{-1}(3).$

Differentiate $f(x)$ with respect to $x.$

$f'(x)=5x^4+3x^2+1.$

It is clear that, $f'(x)=5x^4+3x^2+1>0$ for any value of $x.$

So the graph of the function $y=f(x)$ is increasing and hence f is one-to-one.

Use the definition: If f be a one-to-one function with domain *A* and range *B*.

Then its inverse function f^{-1} had domain *B* and range *A* and is defined by

$f^{-1}(y)=x \Leftrightarrow f(x)=y$ for any y in $B.$

Here, $y=3.$

That is, $f(x)=3.$

Now find the x value corresponding to $f(x)=3.$

$$\begin{aligned} f(x)&=3 \\ x^5+x^3+x&=3 \end{aligned}$$

This equation is satisfied only when $x=1.$

Since f is one-to-once, so it never takes on the same value twice.

Thus, at $x=1,$ the value of $y=f(x)$ is 3.

That means, $f(1)=3.$

Therefore, $f^{-1}(3)=\boxed{1}.$

Find the value of $f\left(f^{-1}(2)\right).$

Use the cancellation equation: $f\left(f^{-1}(x)\right)=x$ for every x in $B.$

Therefore, $f\left(f^{-1}(2)\right)=\boxed{2}.$

Answer 19E.

Given that $h(x)=x+\sqrt{x}.$

Find $h^{-1}(6).$

First write $y=x+\sqrt{x}.$

Solve the equation $y=x+\sqrt{x}$ for $x.$

Subtract x on both sides.

$$y-x=x+\sqrt{x}-x$$

$$y-x=\sqrt{x} \quad \text{Simplify.}$$

$$(y-x)^2=\left(\sqrt{x}\right)^2 \quad \text{Squaring on both sides.}$$

$$y^2+x^2-2xy=x$$

$$y^2+x^2-2xy-x=x-x \quad \text{Subtract } x \text{ on both sides.}$$

$$x^2-x(2y+1)+y^2=0$$

Now compare this equation with the quadratic equation $ax^2+bx+c=0$ and use the formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}.$

Here, $a=1, b=-(2y+1)$ and $c=y^2.$

Substitute these values in the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{(2y+1)\pm\sqrt{(2y+1)^2-4(1)y^2}}{2(1)}$$

$$=\frac{(2y+1)\pm\sqrt{4y^2+1+4y-4y^2}}{2}$$

$$x=\frac{(2y+1)\pm\sqrt{1+4y}}{2}$$

Interchange x and y:

$$y=\frac{(2x+1)\pm\sqrt{1+4x}}{2}$$

Therefore, the inverse function is $h^{-1}(x)=\dfrac{(2x+1)\pm\sqrt{1+4x}}{2}.$

Now substitute $x=6$ in the equation $h^{-1}(x)=\dfrac{(2x+1)\pm\sqrt{1+4x}}{2}.$

$$h^{-1}(6)=\frac{(2(6)+1)\pm\sqrt{1+4(6)}}{2}$$

$$=\frac{(12+1)\pm\sqrt{1+24}}{2}$$

$$=\frac{13\pm\sqrt{25}}{2}$$

$$=\frac{13\pm5}{2}$$

$$=\frac{13+5}{2} \text{ or } \frac{13-5}{2}$$

$$=\frac{18}{2} \text{ or } \frac{8}{2}$$

$$=9 \text{ or } 4$$

Take $h^{-1}(6)=9 \Rightarrow h(9)=6.$

Substitute the point $(9,6)$ in the equation $h(x)=x+\sqrt{x}.$

$$\begin{aligned} h(9) &= 9+\sqrt{9} \\ &= 9+3 \\ &= 12 \end{aligned}$$

The point $(9,6)$ does not satisfies the equation $h(x)=x+\sqrt{x}.$

So, neglect $h^{-1}(6)=9.$

Now take $h^{-1}(6)=4 \Rightarrow h(4)=6.$

Substitute the point $(4,6)$ in the equation $h(x)=x+\sqrt{x}.$

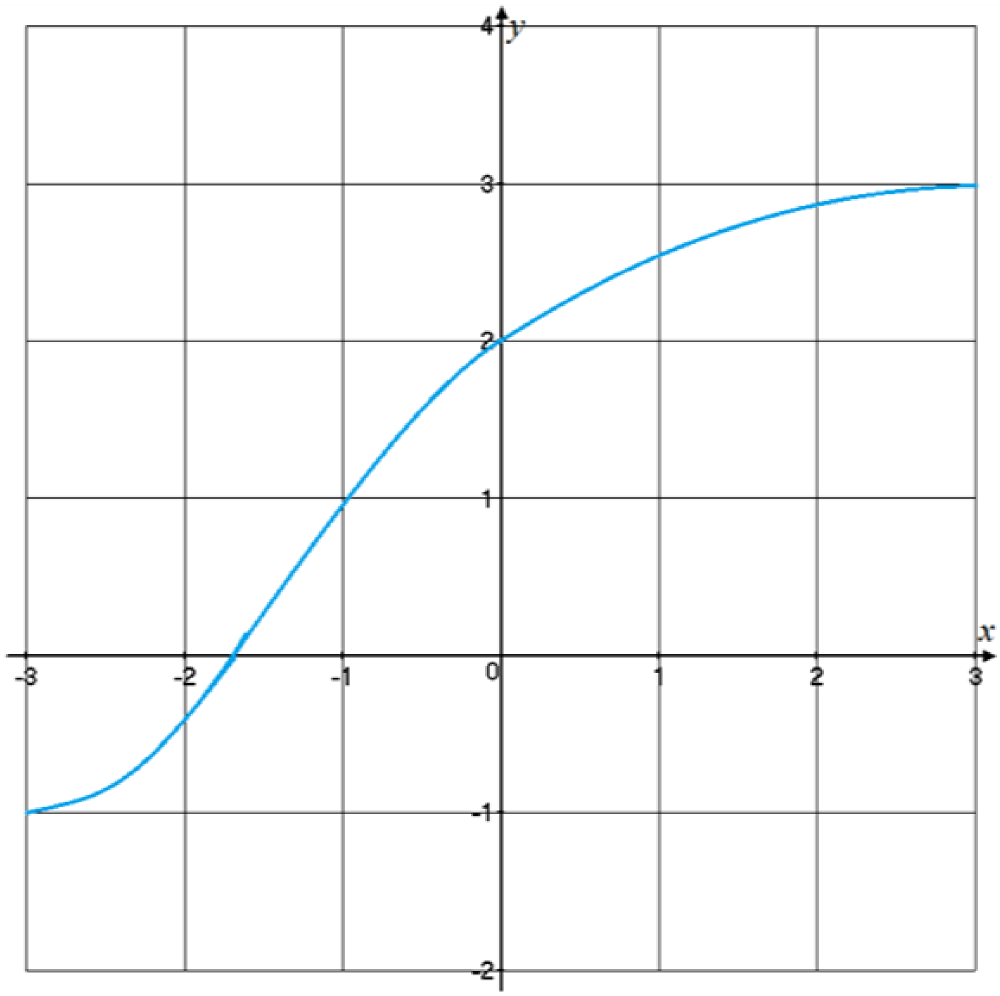
$$\begin{aligned} h(4) &= 4+\sqrt{4} \\ &= 4+2 \\ &= 6 \end{aligned}$$

The point $(4,6)$ satisfies the equation $h(x)=x+\sqrt{x}.$

Therefore, conclude that $h^{-1}(6)=\boxed{4}.$

Answer 20E.

Observe the below graph of the function $y=f(x).$

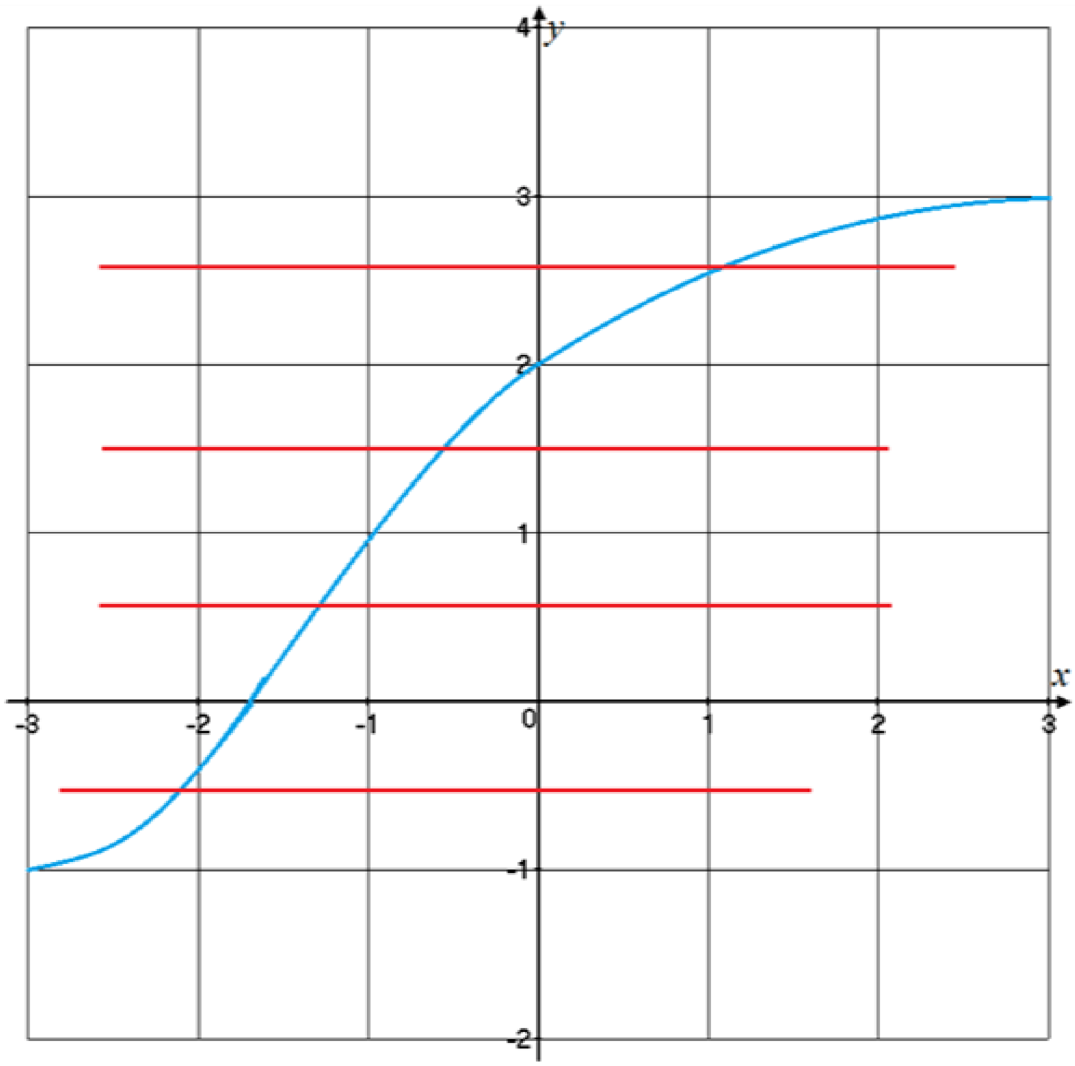


(a)

Determine whether the function $y = f(x)$ is one-to-one.

Use Horizontal Line Test:

Observe the below figure:



From the graph notice that, no horizontal line intersects the graph more than once, so by using Horizontal Line Test, conclude that the function $y = f(x)$ is one-to-one.

(b)

Find the domain of the function f^{-1}.

The domain of the function $y = f(x)$ is $[-3,3]$.

The range of the function $y = f(x)$ is $[-1,3]$.

Since domain of f^{-1} = range of the function f.

Therefore,

The domain of the function $f^{-1}$is $\boxed{[-1,3]}$.

The range of the function $f^{-1}$is $\boxed{[-3,3]}$.

(c)

Find the value of $f^{-1}(2)$.

Use the definition: If f be a one-to-one function with domain *A* and range *B*.

Then its inverse function f^{-1} had domain *B* and range *A* and is defined by
$f^{-1}(y)=x \Leftrightarrow f(x)=y$ for any y in B.

Here, $y=2$.

That is, $f(x)=2$.

Now find the x value corresponding to $f(x)=2$.

From the graph notice that, at $x=0$, the value of $y=f(x)$ is 2.

That implies, $f(0)=2$.

Therefore, $f^{-1}(2)=\boxed{0}$.

(d)

Estimate the value of $f^{-1}(0)$.

Use the definition: If f be a one-to-one function with domain *A* and range *B*.

Then its inverse function f^{-1} had domain *B* and range *A* and is defined by
$f^{-1}(y)=x \Leftrightarrow f(x)=y$ for any y in B.

Here, $y=0$.

That is, $f(x)=0$.

Now find the x value corresponding to $f(x)=0$.

From the graph notice that, the value of $y=f(x)$ is 0 at $x\approx -1.7$.

That implies, $f(-1.7)\approx 0$.

Therefore, $f^{-1}(0)\approx\boxed{-1.7}$.

Answer 21E.

Consider the formula $\boldsymbol{C=\frac{5}{9}(F-32)}$.

Here $\boldsymbol{F\geq -459.67}$

That expresses the Celsius temperature $\boldsymbol{C}$ as a function of the Fahrenheit temperature $\boldsymbol{F}$.

Find the inverse function, interpreting it, and also stating the domain of the inverse function.

Now find the inverse function, you need to solve for $\boldsymbol{F}$.

$C=\frac{5}{9}(F-32)$

$\frac{9}{5}\times C=\frac{5}{9}\times\frac{9}{5}(F-32)$ Multiply each side by $\frac{9}{5}$

$\frac{9}{5}C=(F-32)$

$\frac{9}{5}C+32=F$ Add 32 to each side

The inverse function is $\boxed{\boldsymbol{F=\frac{9}{5}C+32}}$ and this expresses the Fahrenheit temperature $\boldsymbol{F}$ as a function of the Celsius temperature $\boldsymbol{C}$.

The domain of the given function $C=\frac{5}{9}(F-32)$ is $F\geq -459.67$. That means the range of this function can be found by putting this F-value (-459.67) into $C=\frac{5}{9}(F-32)$ to find the lowest point of the range:

$$C=\frac{5}{9}(459.67-32)$$
$$=-273.15$$

So, the range for our function is $C\geq -273.15$.

From the definition of an inverse function, the range of a function is the domain of its inverse function.

The domain of the inverse function is $\boxed{[-273.15,\infty)}$.

Answer 22E.

Squaring on both sides

$$m^2=\frac{m_0{}^2}{1-\frac{v^2}{c^2}}$$

$$m^2\left(1-\frac{v^2}{c^2}\right)=m_0^2$$

$$1-\frac{v^2}{c^2}=\frac{m_0^2}{m^2}$$

$$\frac{v^2}{c^2}=1-\frac{m_0^2}{m^2}$$

$$\frac{v^2}{c^2}=\frac{m^2-m_0^2}{m^2}$$

$$v^2=c^2\left(\frac{m^2-m_0^2}{m^2}\right)$$

$$v=\sqrt{c^2\left(1-\frac{m_0^2}{m^2}\right)}$$

$$v=c\sqrt{1-\left(\frac{m_0}{m}\right)^2}$$

The speed of the particle as a function of mass of the particle and where m_0 is the rest mass of the particle and c is the speed of light in vacuum.

Answer 23E.

We have to find the inverse of the function $f(x)=3-2x$

We write $y=3-2x$

Now we solve this equation for x

$$2x=3-y$$

Or $x=\frac{3-y}{2}$

Interchanging x and y for expressing f^{-1}

$$y=\frac{(3-x)}{2}$$

There fore the inverse function is

$$\boxed{f^{-1}(x)=\frac{(3-x)}{2}}$$

Or $\boxed{f^{-1}(x)=\frac{3}{2}-\frac{x}{2}}$

Answer 24E.

We have to find the inverse of the function $f(x)=\frac{4x-1}{2x+3}$

We write $y=\frac{4x-1}{2x+3}$

We solve this equation for x

$$y(2x+3)=4x-1$$

Or $2yx+3y=4x-1$

Or $4x-2yx=1+3y$

Or $x(4-2y)=1+3y$

Or $x=\frac{1+3y}{(4-2y)}$

Or $x=\frac{1}{2}\frac{(1+3y)}{(2-y)}$

Interchanging x and y

$$y=\frac{1}{2}\cdot\frac{(1+3x)}{(2-x)}$$

Therefore the inverse function $\boxed{f^{-1}(x)=\frac{1}{2}\cdot\frac{(1+3x)}{(2-x)}}$

Answer 25E.

Given $f(x)=1+\sqrt{2+3x}$

i.e.; $y=1+\sqrt{2+3x}$

$\Rightarrow$ $2+3x=(y-1)^2$

$$\Rightarrow \quad x=\frac{(y-1)^2-2}{3}$$

$$=\frac{y^2-2y-1}{3}$$

Therefore $\boxed{f^{-1}(x)=\frac{x^2-2x-1}{3}, x\geq 1}$

Answer 26E.

Consider the function $y=f(x)=x^2-x$

To find the formula for the inverse of the function $y=x^2-x$ when $x\geq\frac{1}{2}$.

Solve the given equation for x:

$x^2-x=y$

$\Rightarrow x^2-x-y=0$, which is a quadratic equation of the form $ax^2+bx+c=0$.

The quadratic formula gives

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-y)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+4y}}{2}$$

$$=\frac{1}{2}\pm\frac{\sqrt{1+4y}}{2}$$

But given that $x\geq\frac{1}{2}$, so we take $x=\frac{1}{2}+\frac{\sqrt{1+4y}}{2}$.

Finally, we interchange x and y:

$$y=\frac{1}{2}+\frac{\sqrt{1+4x}}{2}$$

Or

$$y=f^{-1}(x)=\frac{1}{2}\left(1+\sqrt{1+4x}\right)$$

Hence, the required formula for the inverse of the given function is $\boxed{\frac{1}{2}\left(1+\sqrt{1+4x}\right)}$.

Answer 27E.

We have to find the inverse of the function $f(x)=\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

Here $y=\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

Solving for x

$$y\left(1+\sqrt{x}\right)=\left(1-\sqrt{x}\right)$$

$$y+y\sqrt{x}=1-\sqrt{x}$$

Or $y\sqrt{x}+\sqrt{x}=1-y$

Or $\sqrt{x}(1+y)=1-y$

Or $\sqrt{x}=\left(\dfrac{1-y}{1+y}\right)$ --- (1)

Since $\sqrt{x}$ is a positive square root of x

If $y=-1$ $\sqrt{x}$ is not defined hence $y\neq-1$

If y > 1 and y < -1, then $\left(\dfrac{1-y}{1+y}\right)$ be negative so it must be $=-\sqrt{x}$

This is contradiction with (1)

So $\sqrt{x}=\left(\dfrac{1-y}{1+y}\right)$ when $-1<y\leq1$

Or $x=\left(\dfrac{1-y}{1+y}\right)^2$ when $-1<y\leq1$

Interchanging x and y

The inverse function is

$$\boxed{y=\left(\frac{1-x}{1+x}\right)^2,\quad -1<x\leq1}$$

Answer 28E.

Consider the function $f(x)=2x^2-8x$ where $x\geq2$.

Need to find a formula for the inverse of the function.

First rewrite the function as

$$y=2x^2-8x.$$

$$2x^2-8x-y=0$$

Compare this equation with $ax^2+bx+c=0$, to get

$$a=2,b=-8,c=-y$$

Solve this equation for x:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(-y)}}{2(2)}$$

$$=\frac{8\pm\sqrt{64+8y}}{4}$$

So, $x=\dfrac{8\pm\sqrt{64+8y}}{4}$.

Since $x\geq2$, so take $x=\dfrac{8+\sqrt{64+8y}}{4}$.

Now interchange x and y:

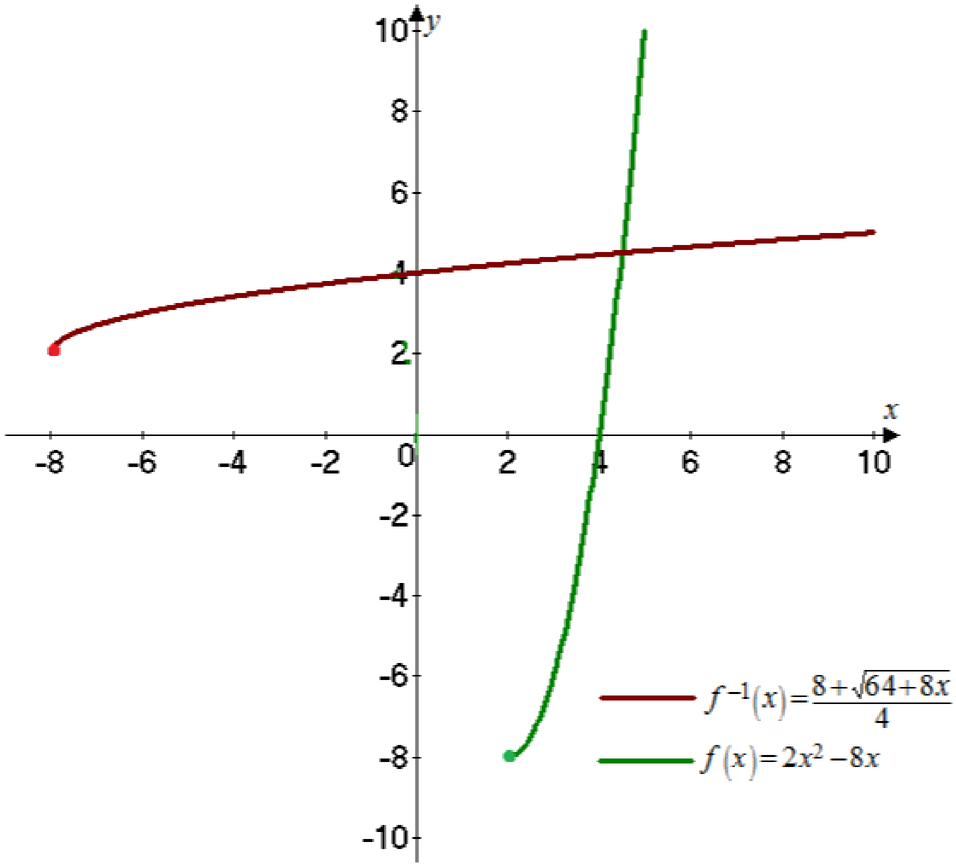
$$y=\frac{8+\sqrt{64+8x}}{4}$$

$$f^{-1}(x)=y=\frac{8+\sqrt{64+8x}}{4}$$

This inverse function is defined only when $64+8x\geq 0$ that is $x\geq -8.$

Hence the inverse of the function $f(x)$ is $\boxed{f^{-1}(x)=\frac{8+\sqrt{64+8x}}{4}, x\geq -8}$

Graphs of the functions f and f^{-1} are given below:



Answer 29E.

Consider the function,

$f(x)=x^4+1, x\geq 0$

To find f^{-1}, take $y=x^4+1$ and solve for x

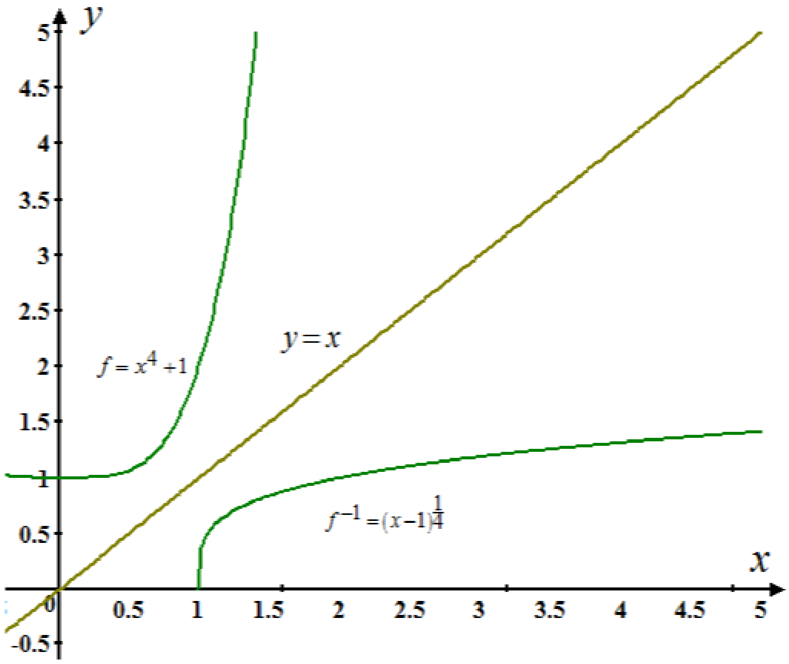
$$y=x^4+1$$
$$x^4=(y-1)$$
$$x=(y-1)^{\frac{1}{4}}$$

Replace x by y and write.

$$y=(x-1)^{\frac{1}{4}}$$

Therefore $f^{-1}=(x-1)^{\frac{1}{4}}$

Figure showing the graphs of f, f^{-1} and $y = x$



Observe that the f and f^{-1} are reflections about the line $y = x$

Answer 30E.

We have to find f^{-1} of $f(x) = \sqrt{x^2 + 2x}$, $x > 0$

We write $y = \sqrt{x^2 + 2x}$ $\quad x > 0$

Solving equation for x

$$y^2 = x^2 + 2x$$

Or $\quad x^2 + 2x - y^2 = 0$

Or $\quad x = \dfrac{-2 \pm \sqrt{4 - 4(-y^2)1}}{2}$

Or $\quad x = \dfrac{-2 \pm \sqrt{4 + 4y^2}}{2}$

Or $\quad x = -1 \pm \sqrt{1 + y^2}$

We cannot take negative value of $\sqrt{1 + y^2}$ because given condition is $x > 0$

So we take $x = -1 + \sqrt{1 + y^2}$

Replacing x and y,

$y = -1 + \sqrt{1 + x^2}$

So $\quad \boxed{f^{-1}(x) = -1 + \sqrt{1 + x^2}}$ $\quad$ for $x > 0$

Now we sketch the curve $f(x)$, $f^{-1}(x)$ and $y = x$ on the same screen for $x > 0$

From figure we see that the graph of $f^{-1}(x)$ is the reflected graph of $f(x)$ about $y = x$

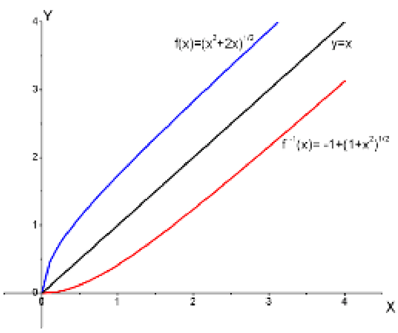
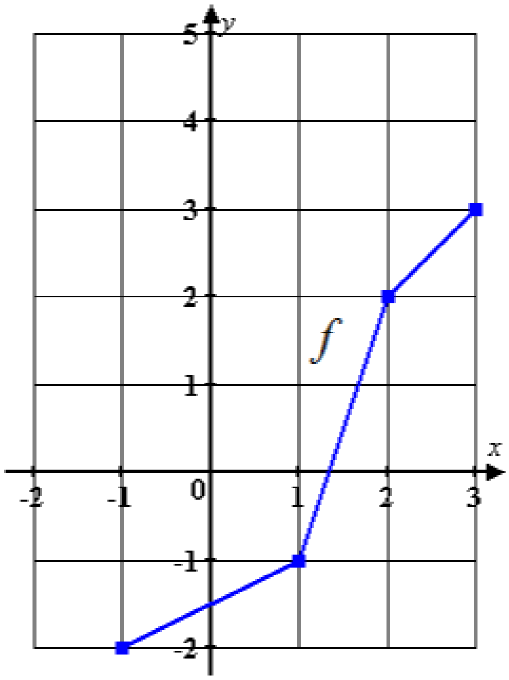


Fig.1

Answer 31E.

Consider the graph,



The object is to graph the inverse of the function f.

Observe that the graph of f passing through the below points.

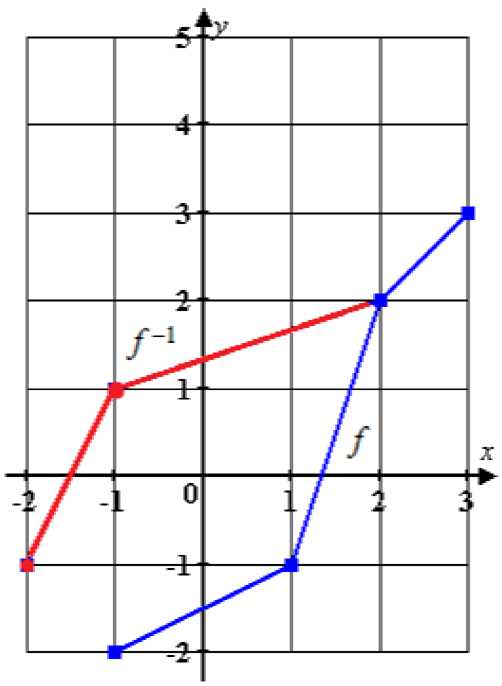
$f = \{(-1,-2),(1,-1),(2,2),(3,3)\}$.

Since the inverse of the function f is obtained by interchanging the x and y coordinates.

So the inverse of f is calculated as,

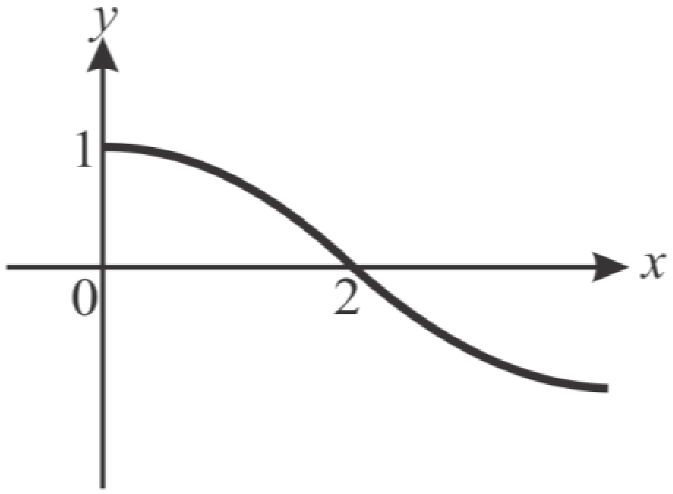
$f^{-1} = \{(-2,-1),(-1,1),(2,2),(3,3)\}$.

The sketch of the graph f^{-1} joining the points $(-2,-1),(-1,1),(2,2),(3,3)$ is shown below:



Answer 32E.

Given the graph of f is:



The objective is to make the graph of f^{-1}.

From the graph, it can be seen that;

$f(0)=1,$

$f(2)=0$

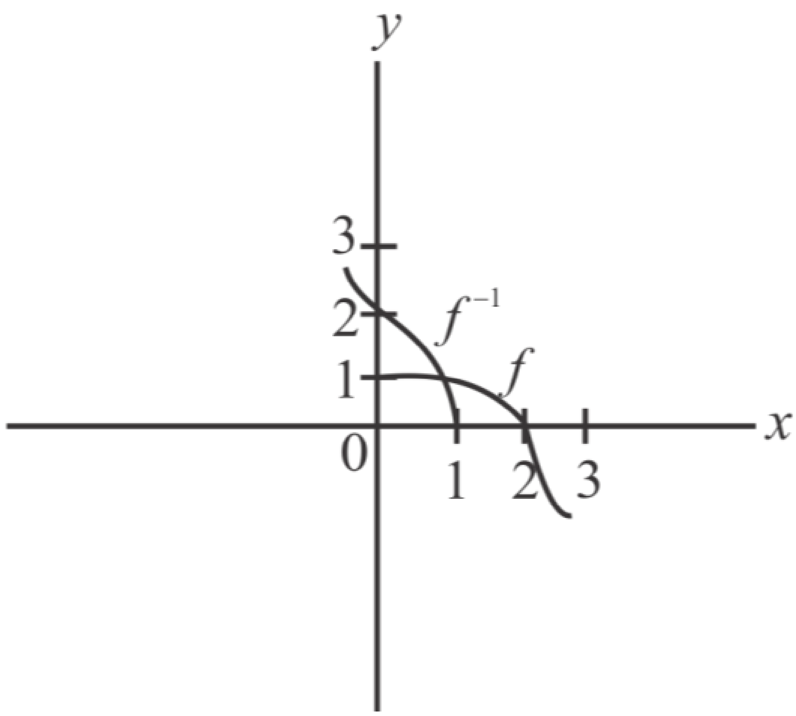
So,

$f^{-1}(1)=0$

$f^{-1}(0)=2$

Hence, now to make a graph that satisfies condition mentioned above;

The graph is as follows;



Answer 33E.

Given $f(x)=\sqrt{1-x^2}, 0 \le x \le 1$

(a) Let $y=\sqrt{1-x^2}$

$\Rightarrow y^2 = 1-x^2$

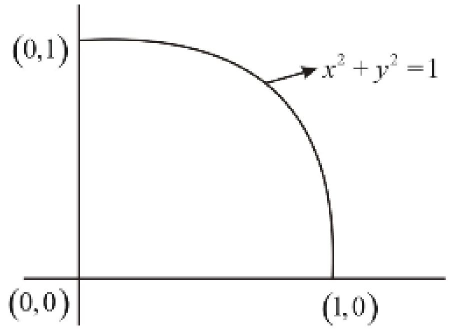
$\Rightarrow x^2+y^2=1$

$\Rightarrow x=\sqrt{1-y^2}, 0 \le y \le 1$

$\Rightarrow \boxed{f^{-1}(x)=\sqrt{1-x^2}, 0 \le x \le 1}$

From $f(x)$ and $f^{-1}(x)$, clearly $f=f^{-1}$

(b) Both the function represents the same graph which is a circle in the positive quadrant.



Answer 34E.

Given $f(x)=\sqrt[3]{1-x^3}$

(a) Let $y=\sqrt[3]{1-x^3}$

$\Rightarrow \quad y^3=1-x^3$

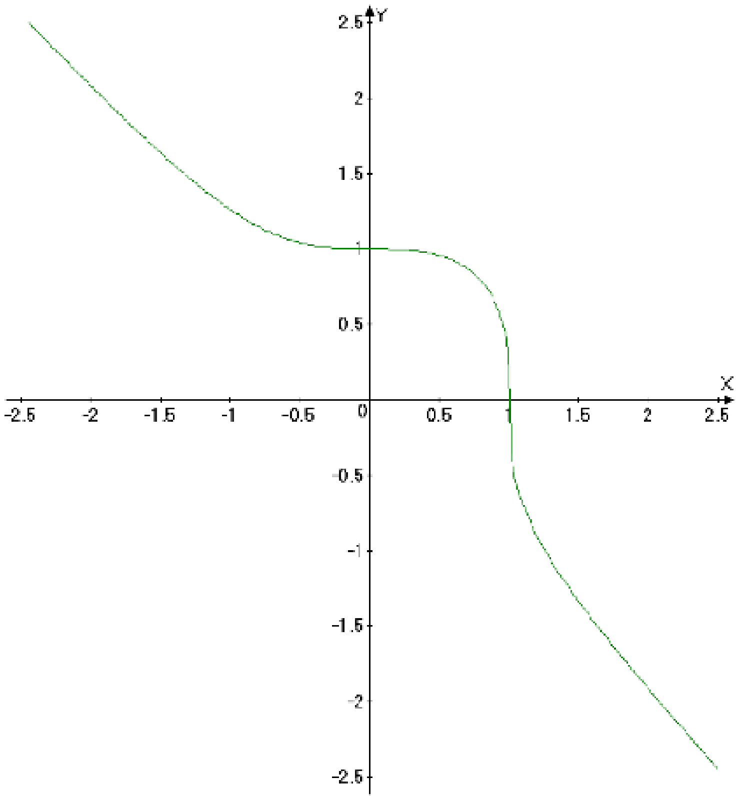
$\Rightarrow \quad x^3+y^3=1$

$\Rightarrow \quad x=\sqrt[3]{1-y^3}$

i.e; $\boxed{f^{-1}(x)=\sqrt[3]{1-y^3}}$

From $f(x), f^{-1}(x)$, clearly $f=f^{-1}$

(b) Both functions represent the same graph which is



Answer 35E.

(A). We have to show $f(x)=x^3$ is one – to – one

Let the two numbers are x_1 *and* x_2 and $x_1 \neq x_2$

So $x_1^3 \neq x_2^3$

Because two different numbers have different cubes

So $\quad f(x_1) \neq f(x_2)$

Then $f(x)=x^3$ is one – to – one

(B). We have $a = 2^3 = 8$

Since $f(2) = 8$

So $f^{-1}(8) = 2$ so $g(8) = f^{-1}(8) = 2$

And $f'(x) = 3x^2$

Then by the given theorem

$$g'(a) = \frac{1}{f'(g(a))}$$

For a = 8

$$g'(8) = \frac{1}{f'(f^{-1}(8))}$$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{3(2^2)}$$

$$\boxed{g'(8) = \frac{1}{12}}$$

(C). $f(x) = x^3$

We write $y = x^3$

Or $x = y^{1/3}$ (solving for x)

We interchange x and y

$y = x^{1/3}$

So $f^{-1}(x) = x^{1/3}$

Or $\boxed{g(x) = x^{1/3}}$ where $g(x) = f^{-1}(x)$

So domain of g(x) = range of $f(x) = R$, the set of real numbers

And Range of g(x) = domain of $f(x) = R$, the set of real numbers

(D). From part (C) we have

$g(x) = x^{1/3}$

Differentiating with respect to x

$$g'(x) = \frac{1}{3}x^{-2/3}$$

For x =a

$$\boxed{g'(a) = \frac{1}{3}a^{-2/3}}$$

Putting a = 8, $g'(8) = \frac{1}{3}(8)^{-2/3}$

$$= \frac{1}{3}\left(\frac{1}{4}\right)$$

$$= \frac{1}{12}$$

So $g'(8) = \frac{1}{12}$,this is same as in part (B).

(E).

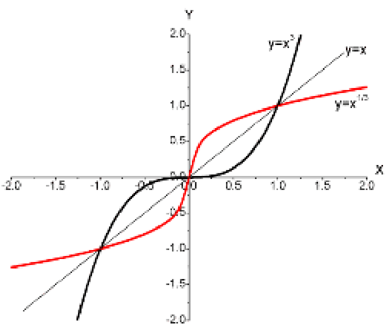


Fig.1

Answer 36E.

(A) We have to show $f(x)=\sqrt{x-2}$ is one to one

Let the two different numbers are x_1, x_2

So $\quad x_1 \neq x_2$

Then $\quad x_1-2 \neq x_2-2$

So $\quad \sqrt{x_1-2} \neq \sqrt{x_2-2}$

And then $\quad f(x_1) \neq f(x_2)$

So $f(x)$ is one to one function

(B) We have $f(x)=\sqrt{x-2}$ and $a=2$

Since $f(6)=\sqrt{6-2}=2$

So $\quad f^{-1}(2)=6 \quad$ so $g(2)=f^{-1}(2)=6$

We have $f'(x)=\dfrac{1}{2\sqrt{x-2}}$

Then by the theorem $g'(a)=\dfrac{1}{f'[g(a)]}$

Putting $a=2$

$$g'(2)=\frac{1}{f'[g(2)]}=\frac{1}{f'(6)}$$

Or $\quad g'(2)=\dfrac{1}{\dfrac{1}{2\sqrt{6-2}}}=\dfrac{1}{1/4}=4$

So $\quad \boxed{g'(2)=4}$

(C) $f(x)=\sqrt{x-2}$

We write $y=\sqrt{x-2}$

Or $\quad y^2=x-2$

Or $\quad x=y^2+2 \qquad$ (solving for x)

We interchange x and y

$\quad y=x^2+2$

So $\quad f^{-1}(x)=(x^2+2)$

Or $\quad \boxed{g(x)=(x^2+2)}$

So domain of $g(x)$ = range of $f(x)=[0,\infty)$

And range of $g(x)$ = domain of $f(x)=[2,\infty)$

(D) From part (C) we have $g(x)=x^2+2$

So $\quad g'(x)=2x$

And then $\boxed{g'(a)=2a}$

Putting $x=2$

So $\quad g'(2)=2\times 2$

Or $\quad \boxed{g'(2)=4}$ which is same as in part (B)

(E)

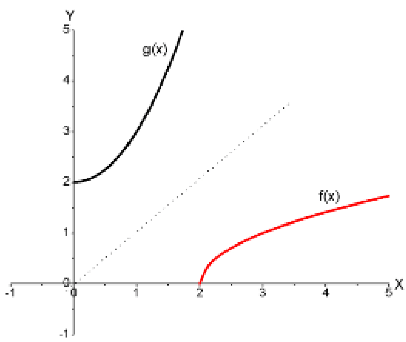


Fig.1

Answer 37E.

(A) We have to show $f(x)=9-x^2$ is one to one in [0, 3]

Let x_1 and x_2 are two difference positive numbers

So $x_1 \neq x_2$

Then $x_1^2 \neq x_2^2$

And $9-x_1^2 \neq 9-x_2^2$

Then $f(x_1) \neq f(x_2)$

So $f(x)$ is one to one function

(B) We have $f(x)=9-x^2$ and $a=8$

Since $f(1)=9-1^2=8$

So $f^{-1}(8)=1$ or $g(8)=f^{-1}(8)=1$

And we have $f'(x)=-2x$

Then by the theorem

$$g'(a)=\frac{1}{f'(g(a))}$$

For a = 8

$$g'(8)=\frac{1}{f'(g(8))}$$

$$=\frac{1}{f'\left[f^{-1}(8)\right]}$$

$$=\frac{1}{f'(1)}$$

$$=\frac{1}{-2(1)}$$

$$=-\frac{1}{2}$$

So $\boxed{g'(8)=-\frac{1}{2}}$

(C) $f(x)=9-x^2$

We write $y=9-x^2$

Solving for x

$$x^2=9-y$$

Or $x=\sqrt{9-y}$

Replacing x and y

$$y=\sqrt{9-x}$$

So $f^{-1}(x)=\sqrt{9-x}$

Or $\boxed{g(x)=\sqrt{9-x}}$

So domain of $g(x)$ = range of $f(x)$ = [0, 9]

Range of $g(x)$ = domain of $f(x)$ = [0, 3]

(D) From part (C) $g(x)=\sqrt{9-x}$

Then $g'(x)=-\frac{1}{2\sqrt{9-x}}$

And so $\boxed{g'(a)=-\frac{1}{2\sqrt{9-a}}}$

For x = 8 $g'(8)=-\frac{1}{2\sqrt{9-8}}=-\frac{1}{2}$

So $\boxed{g'(8)=-\frac{1}{2}}$, this is same as in part (B)

(E)

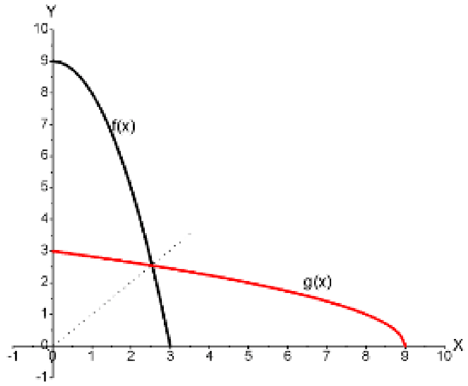


Fig.1

Answer 38E.

(A) We have to show $f(x)=\frac{1}{(x-1)}$, x > 1 is one to one

Let x_1 and x_2 are two different numbers, x_1 *or* $x_2 > 1$

So $x_1 \neq x_2$

Then $x_1 - 1 \neq x_2 - 1$

And $\frac{1}{x_1-1} \neq \frac{1}{x_2-1}$

Then $f(x_1) \neq f(x_2)$

So $f(x)$ is one to one

(B) We have $f(x)=\frac{1}{(x-1)}$ and a = 2

Since $f(1.5)=\frac{1}{(1.5-1)}=\frac{1}{0.5}=2$

So $f^{-1}(2)=1.5$ or $g(2)=f^{-1}(2)=1.5$

And we have $f'(x)=-\frac{1}{(x-1)^2}$

Then by the theorem

$$g'(a)=\frac{1}{f'(g(a))}$$

For a = 2

$$g'(2)=\frac{1}{f'(g(2))}$$

$$=\frac{1}{f'(1.5)}$$

$$=\frac{1}{-1/(1.5-1)^2}$$

Or $\boxed{g'(2)=-0.25}$

(C) $f(x)=\frac{1}{(x-1)}$

We write $y=\frac{1}{(x-1)}$

Solving for x,

$$(x-1)=\frac{1}{y}$$

Or $x=\frac{1}{y}+1$

Replacing x and y

$$y = \frac{1}{x} + 1$$

So $\quad f^{-1}(x) = \frac{1}{x} + 1 \qquad$ or $\qquad \boxed{g(x) = \frac{1}{x} + 1}$

So $\quad$ Domain of $g(x)$ = range of $f(x) = (0, \infty)$

Range of $g(x)$ = domain of $f(x) = (1, \infty)$

(D) We have

$$g(x) = \frac{1}{x} + 1$$

So $\quad g'(x) = -\frac{1}{x^2}$

Then $\quad \boxed{g'(a) = -\frac{1}{a^2}}$

Putting a = 2

$$g'(2) = -\frac{1}{2^2} = -\frac{1}{4}$$

Or $\quad \boxed{g'(2) = -0.25}$ which is same as in part (B)

(E)

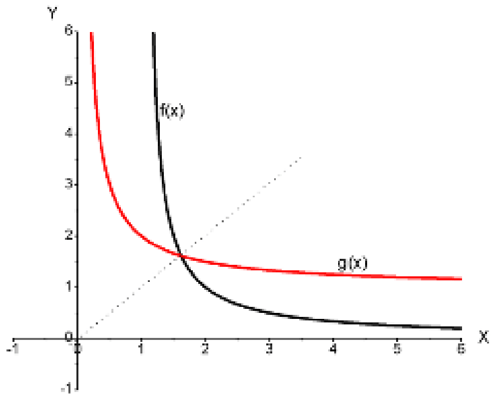


Fig.1

Answer 39E.

Consider the following function:

$$f(x) = 2x^3 + 3x^2 + 7x + 4 \quad a = 4$$

Differentiate the function $f(x) = 2x^3 + 3x^2 + 7x + 4$ with respect to x.

$$f'(x) = 6x^2 + 6x + 7$$

Clearly the function $f(x)$ is one-to-one, because $f'(x) = 6x^2 + 6x + 7 > 0$.

And also $f(x)$ is an increasing function.

By the well-known theorem, $\left(f^{-1}\right)'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$.

Now, find $f^{-1}(4)$.

Using the inspection method, $f(0)=2(0)^3+3(0)^2+7(0)+4=4$

Then $f^{-1}(4)=0$

Therefore, $\left(f^{-1}\right)'(4)=\frac{1}{f'\left(f^{-1}(4)\right)}$

$$\begin{aligned}\left(f^{-1}\right)'(4)&=\frac{1}{f'(0)}\\&=\frac{1}{6(0)^2+6(0)+7}\\&=\frac{1}{7}\end{aligned}$$

Hence, the required value is $\left(f^{-1}\right)'(4)=\frac{1}{7}$.

Answer 40E.

Consider the function $f(x)=x^3+3\sin x+2\cos x$.

Find $\left(f^{-1}\right)'(a)$ at $a=2$

That is finding $\left(f^{-1}\right)'(2)$ of the function $f(x)=x^3+3\sin x+2\cos x$.

Use theorem if f is one-to-one differentiable function with inverse function and $f'\left(f^{-1}(a)\right)\neq 0$, then inverse function is differentiable at a and $\left(f^{-1}\right)'(a)=\frac{1}{f'\left(f^{-1}(a)\right)}$

First check f is a one-to-one function.

It is enough to check $f'(x)>0$.

$f'(x)=3x^2+3\cos x-2\sin x$ ...... (1)

The values of sine and cosine oscillate between -1 and 1.

Here values of $f'(x)$ doesn't going to below zero.

To do that, examine the x-values that give these minimums for sine and cosine.

Examine

We know that $\sin\frac{3\pi}{2}=-1$, and $\cos\pi=-1$.

Now check either of these values make $f'(x)<0$

$$\begin{aligned}f'(x)&=3x^2+3\cos x-2\sin x\\f'\left(\frac{3\pi}{2}\right)&=3\left(\frac{3\pi}{2}\right)^2+3\cos\left(\frac{3\pi}{2}\right)-2\sin\left(\frac{3\pi}{2}\right)\\&=3\frac{9\pi^2}{4}+0-2(-1)\\&=\frac{27\pi^2}{4}+2>0\end{aligned}$$

$$\begin{aligned}f'(x)&=3x^2+3\cos x-2\sin x\\f'(\pi)&=3(\pi)^2+3\cos(\pi)-2\sin(\pi)\\&=3(\pi)^2+3(-1)+0\\&=3(\pi)^2-3>0\end{aligned}$$

Therefore for any value $f'(x)>0$

Therefore f is a one-to-one function.

If f is a one-to-one function then $f(x)=y \Leftrightarrow f^{-1}(y)=x$.

From the above relation, write $f^{-1}(2)=x \Leftrightarrow f(x)=2$.

$$f(x)=x^3+3\sin x+2\cos x$$
$$2=x^3+3\sin x+2\cos x$$

When $x=0$ the value of $f(0)$ is as follows.

$$f(x)=x^3+3\sin x+2\cos x$$
$$f(0)=0^3+3\sin 0+2\cos 0$$
$$=2$$
$$f(0)=2$$

Since $f(0)=2$ that means that $f^{-1}(2)=0$.

Now $\left(f^{-1}\right)'(a)=\dfrac{1}{f'\left(f^{-1}(a)\right)}$

$$\left(f^{-1}\right)'(2)=\frac{1}{f'\left(f^{-1}(2)\right)}$$
$$=\frac{1}{f'(0)}$$

Now find the value of $f'(0)$

$$f'(x)=3x^2+3\cos x-2\sin x$$
$$f'(0)=3(0)^2+3\cos 0-2\sin 0$$
$$=3$$

Therefore $f'(0)=3$

Now $\left(f^{-1}\right)'(2)=\dfrac{1}{f'(0)}$

$$=\frac{1}{3}$$

Thus, the solution is $\boxed{\left(f^{-1}\right)'(2)=\frac{1}{3}}$.

Answer 41E.

Since $f(x)=3+x^2+\tan\left(\dfrac{\pi x}{2}\right)$, $\quad -1<x<1, \quad a=3$

We see that $f'(x)=2x+\dfrac{\pi}{2}.\sec^2\left(\dfrac{\pi x}{2}\right)>0$, $\quad -1<x<1$

So $f(x)$ is an increasing function on (-1, 1), thus $f(x)$ is one to one

We have to find $f^{-1}(3)$, this we can find by inspection

$$f(0)=3+0+\tan 0=3$$

So $\quad f^{-1}(3)=0$

Therefore by theorem

$$g'(a)=\frac{1}{f'(g(a))}, \quad \text{Where } g(a)=f^{-1}(a)$$

So we have $\quad \left(f^{-1}\right)'(3)=\dfrac{1}{f'\left(f^{-1}(3)\right)}$

$$=\frac{1}{f'(0)}$$
$$=\frac{1}{2(0)+\frac{\pi}{2}.\sec^2(0)}=\frac{1}{\pi/2}$$

Or $\quad \boxed{\left(f^{-1}\right)'(3)=\frac{2}{\pi}}$

Answer 42E.

Since $f(x)=\sqrt{x^3+x^2+x+1}$, $a=2$

Differentiating by chain rule with respect to x

$$f'(x)=\frac{1}{2}\frac{(3x^2+2x+1)}{\sqrt{x^3+x^2+x+1}}>0$$

So $f(x)$ is an increasing function thus $f(x)$ is one to one function and inverse of $f(x)$ exists. We have to find $f^{-1}(2)$, this we can find by inspection

$$f(1)=\sqrt{1^3+1^2+1+1}=\sqrt{4}=2$$

So $f^{-1}(2)=1$

Therefore by theorem $g'(a)=\frac{1}{f'(g(a))}$ where $g(a)=f^{-1}(a)$

So we have $(f^{-1})'(2)=\frac{1}{f'(f^{-1}(2))}=\frac{1}{f'(1)}$

$$=\frac{1}{\frac{1}{2}\frac{(3+2+1)}{\sqrt{4}}}=\frac{4}{6}$$

Or $\boxed{(f^{-1})'(2)=\frac{2}{3}}$

Answer 43E.

Suppose f^{-1} is the inverse function of a differentiable function f and $f(4)=5,\ f'(4)=\frac{2}{3}$.

Need to find $(f^{-1})'(5)$.

If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a))\neq 0$, then the inverse function is differentiable at a is as follows:

$$(f^{-1})'(a)=\frac{1}{f'(f^{-1}(a))}$$

Use the above condition write $(f^{-1})'(5)$.

$$(f^{-1})'(5)=\frac{1}{f'(f^{-1}(5))}$$

Given $f(4)=5$ and the definition of the inverse function says that $f(4)=5$ is the same as $f^{-1}(5)=4$.

$$\frac{1}{f'(f^{-1}(5))}=\frac{1}{f'(4)}$$ Since $f^{-1}(5)=4$

$$=\frac{1}{\left(\frac{2}{3}\right)}$$ Since $f'(4)=\frac{2}{3}$

$$=\frac{3}{2}$$

Therefore, the inverse function $(f^{-1})'(5)$ is $\boxed{\frac{3}{2}}$.

Answer 44E.

Given that g is an increasing function

$g(2)=8$ and $g'(2)=5$

Then $(g^{-1})^1(8)=\frac{1}{g'(g^{-1}(8))}$

$$=\frac{1}{g'(2)}$$

$$=\frac{1}{5}$$

Therefore $\boxed{(g^{-1})^1(8)=\frac{1}{5}}$

Answer 45E.

Consider the following function:

$$f(x)=\int_3^x \sqrt{1+t^3}\,dt$$

The objective is to find the function $(f^{-1})'(0)$

The function *f* is one-to-one differentiable function, to find $f'(x)$.

The function $f(x)$ is differentiate with respect to *x*.

$$\frac{d}{dx}(f(x))=\frac{d}{dx}\left(\int_3^x \sqrt{1+t^3}\,dt\right)$$
$$f'(x)=\sqrt{1+x^3}$$
$$f'(x)>0$$

Now calculate $f^{-1}(0)$ from $f(x)$

$$f(x)=\int_3^x \sqrt{1+t^3}\,dt$$
$$f(3)=\int_3^3 \sqrt{1+t^3}\,dt$$
$$=0 \quad \left(\text{since} \int_a^a f(x)dx=f(a)-f(a)=0\right)$$

Therefore, $f(3)=0$
$$f^{-1}(0)=3$$

To find the function $(f^{-1})'(0)$

The theorem states that if *f* is one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a))\neq 0$, then the inverse function is differentiable at *a* and ,

$$(f^{-1})'(a)=\frac{1}{f'(f^{-1}(a))}$$

Substitute 0 for *a*

$$(f^{-1})'(0)=\frac{1}{f'(f^{-1}(0))}$$
$$=\frac{1}{f'(3)}$$
$$=\frac{1}{\sqrt{1+3^3}}$$
$$=\frac{1}{\sqrt{28}}$$

Therefore, the function $(f^{-1})'(0)=\boxed{\frac{1}{\sqrt{28}}}$

Answer 46E.

We have $f(3)=2$ then $f^{-1}(2)=3$

$f'(3)=1/9$

And $G(x)=1/f^{-1}(x)=\left[f^{-1}(x)\right]^{-1}$ …………(1)

Differentiating (1) with respect to x

$$G'(x)=-\left[f^{-1}(x)\right]^{-2}\cdot\left[\frac{d}{dx}f^{-1}(x)\right] \quad \text{By chain rule}$$

Now since

$$\frac{d}{dx}f^{-1}(x)=\frac{1}{f'\left(f^{-1}(x)\right)}$$

Then

$$G'(x)=-\frac{\left[f^{-1}(x)\right]^{-2}}{f'\left(f^{-1}(x)\right)}$$

Substituting x = 2

$$G'(2)=-\frac{\left[f^{-1}(2)\right]^{-2}}{f'\left(f^{-1}(2)\right)}$$

Since $f^{-1}(2)=3$

Then $G'(2)=-\dfrac{[3]^{-2}}{f'(3)}$

Or $G'(2)=-\dfrac{1}{9f'(3)}$

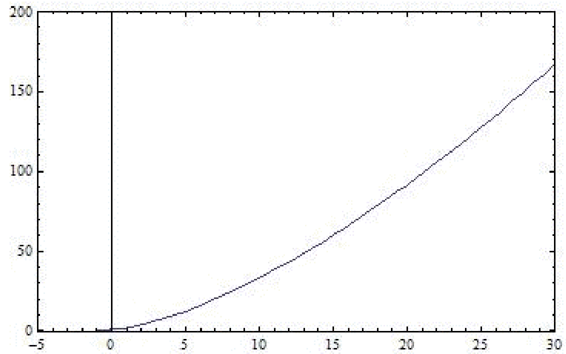
Or $G'(2)=-\dfrac{1}{9(1/9)}$

Therefore

$$\boxed{G'(2)=-1}$$

Answer 47E.

Graph the function $f(x)=\sqrt{x^3+x^2+x+1}$



We can see that the function is increasing, so $f(x)$ is one-to-one.

For the inverse we need the roots of $x=\sqrt{y^3+y^2+y+1}$ you can use any computer algebra system to find the explicit expression for $f^{-1}(x)$.

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y= 1/3 (Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1]/Power[2, (3)^-1]-1-(2 Power[2, (3)^-1])/Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1]

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y= -((1-\[ImaginaryI] Sqrt[3]) Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1])/(6 Power[2, (3)^-1])-1/3+(Power[2, (3)^-1] (1+\[ImaginaryI] Sqrt[3]))/(3 Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1])

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y= -((1+\[ImaginaryI] Sqrt[3]) Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1])/(6 Power[2, (3)^-1])-1/3+(Power[2, (3)^-1] (1-\[ImaginaryI] Sqrt[3]))/(3 Power[27 y^2+3 Sqrt[3] Sqrt[27 y^4-40 y^2+16]-20, (3)^-1])

We get two solutions involving imaginary expressions [irrelevant solution for real analysis].

The real solution give us the inverse:

f^{-1}(x)= 1/3 (Power[27 x^2+3 Sqrt[3] Sqrt[27 x^4-40 x^2+16]-20, (3)^-1]/Power[2, (3)^-1]-1-(2 Power[2, (3)^-1])/Power[27 x^2+3 Sqrt[3] Sqrt[27 x^4-40 y^2+16]-20, (3)^-1]

Answer 48E.

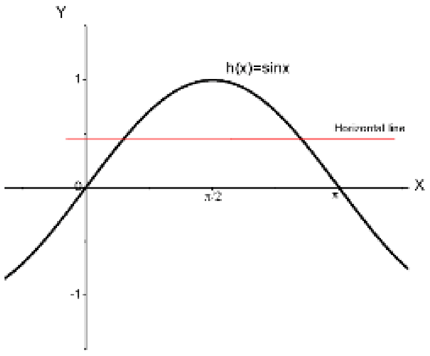


Fig.1

$f(x)=\sin x,\quad x\in R$ is not one to one [By the horizontal line test]

If we restrict it's domain as $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$ then we see that it is a one to one function and its inverse exists, $g=f^{-1}$

Since $f'(x)=\cos x$

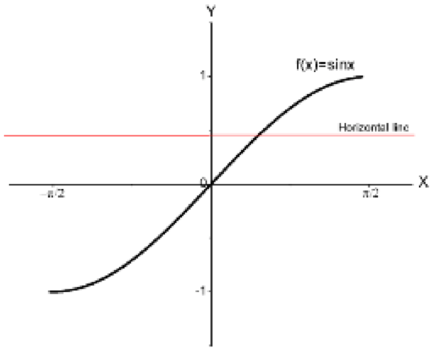


Fig.1

$f(x)$ has domain $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ and range [- 1, 1], since

$$g(x)=f^{-1}(x)$$
$$=\sin^{-1}x$$

So by the theorem $g'(a)=\dfrac{1}{\left[f'(g(a))\right]}$, where $g(a)=f^{-1}(a)$

We have $g'(x)=\dfrac{1}{\left[f'(g(x))\right]}$

Or $\quad g'(x)=\dfrac{1}{f'\left(\sin^{-1}x\right)}$

Or $\quad \boxed{g'(x)=\dfrac{1}{\cos\left(\sin^{-1}x\right)}}$

Answer 49E.

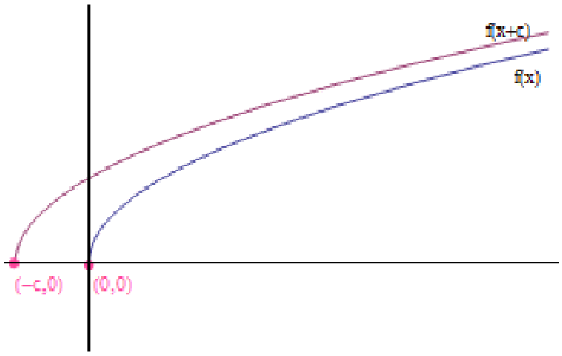
a) Let $f(x)$ the curve function.

$f(x)$ is one-to-one.

The graph of $f^{-1}(x)$ is obtained by reflecting the graph of $f(x)$ about the line $y=x$.

if we shift a curve c units to the left the new function is given by $g(x)=f(x+c)$ where $c>0$

If the pair (x,y) is on the graph of $f(x)$ then the point $(x-c,y)$ is on the graph of $g(x)=f(x+c)$.



Let $f(x)$ be a one-to-one function with domain A and range B. Then its inverse function $f^{-1}(x)$ has domain B and range A, therefore, if the pair (x,y) is on the graph of $f(x)$ then the point $(y,x-c)$ is on the graph of $f^{-1}(x)$ and if the point $(x-c,y)$ is on the graph of $g(x)=f(x+c)$ the point $(y,x-c)$ is on $f^{-1}(x+c)$. So we have that $f^{-1}(x+c)$, shift the graph of $g(x)=f(x+c)$ a distance c units downward, Using this geometric principle an expression for the inverse function is $g^{-1}(x)=f^{-1}(x)-c$, that is, the curve's reflection is shifted downward c units as the curve itself is shifted c units to the left.

b) $h(x)=f(cx)$, compress the graph of $f(x)$ horizontally by a factor of c, where $c>1$

$h(x)=f(cx)$, stretch the graph of $f(x)$ horizontally by a factor of c, where $0<c<1$

If the pair (x,y) is on the graph of $f(x)$ then the point (cx,y) is on the graph of $h(x)=f(cx)$ then the point (y,cx) is on the graph of $h^{-1}(x)$

Then if we compress or stretch the graph of $f(x)$ horizontally by a factor of c, the curve's reflection in the line $y=x$ is compressed or stretched vertically by the same factor. Using this geometric principle, an expression for the inverse function is $h^{-1}(x)=\frac{1}{c}f^{-1}(x)$

Answer 50E.

If *f* is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a))\neq 0$, then the inverse function is differentiable at *a* and

$$(f^{-1})'(a)=\frac{1}{f'(f^{-1}(a))}$$

Here f is one-to-one twice differentiable function with inverse function g.

That $f^{-1}=g$

From above definition, write $(g)'(x)=\frac{1}{f'(g(x))}$

Use the Chain Rule to evaluate this derivative of $(g)'(x)$

$$g''(x)=\frac{d}{dx}\left[f'(g(x))\right]^{-1}$$
$$=-\frac{\frac{d}{dx}\left[f'(g(x))\right]}{\left[f'(g(x))\right]^2}$$
$$=-\frac{f''(g(x))\cdot\frac{d}{dx}\left[g(x)\right]}{\left[f'(g(x))\right]^2}$$

Finally, use the formula for the derivative of $f^{-1}(a)$ to further simplify the expression.

$$g''(x) = -\frac{f''(g(x))\frac{1}{f'(g(x))}}{\left[f'(g(x))\right]^2} \qquad \left(\text{Since } (g)'(x) = \frac{1}{f'(g(x))}\right)$$

$$= -\frac{f''(g(x))}{\left[f'(g(x))\right]^3}$$

(b)

If f is increasing then $f'(g(x)) > 0$

Therefore $\left[f'(g(x))\right]^3 > 0$

If f is concave upward, then $f''(g(x)) > 0$.

Now the function $g''(x) = -\frac{f''(g(x))}{\left[f'(g(x))\right]^3}$ must be negative.

Since $f'(g(x)) > 0$ and $f''(g(x)) > 0$.

Therefore second derivative of g is negative.

By concavity principal g is concave downward.

Hence inverse function of f is concave downward.