Probability

Question1

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6 | X > 3). Then b + c/a is equal to___

[27-Jan-2024 Shift 1]

Answer: 12

Solution:

$$a = P(X = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$b = P(X \geq 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots\ldots$$

$$= \frac{\frac{25}{216}}{1 - \frac{5}{6}} = \frac{25}{216} \times \frac{6}{1} = \frac{25}{36}$$

$$P(X \geq 6) = \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \ldots\ldots\ldots$$

$$= \frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$\frac{b + c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}} = 12$$

Question2

An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

[27-Jan-2024 Shift 2]

Options:

A.

5/256

B.

5/715

C.

3/715

D.

3/256

Answer: C

Solution:

$$\frac{{}^{6}C_4}{{}^{15}C_4} \times \frac{{}^{9}C_4}{{}^{11}C_4} = \frac{3}{715}$$

Hence option (3) is correct.

Question3

A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

[29-Jan-2024 Shift 1]

Options:

A.

5/6

B.

1/6

C.

5/11

D.

6/11

Answer: C

Solution:

$z = \frac{1}{2} - 2i$

$|z+1| = \alpha z + \beta(1+i)$

$\left|\frac{3}{2} - 2i\right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$

$\left|\frac{3}{2} - 2i\right| = \left(\frac{\alpha}{2} + \beta\right) + (\beta - 2\alpha)i$

$\beta = 2\alpha$ and $\frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4}$

$\alpha + \beta = 3$

Question4

An integer is chosen at random from the integers 1, 2, 3,...,50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

[29-Jan-2024 Shift 2]

Options:

A.

8/25

B.

21/50

C.

9/50

D.

14/25

Answer: B

Solution:

Given set $= \{1, 2, 3, \ldots\ldots.50\}$

$P(A) =$ Probability that number is multiple of 4

$P(B) =$ Probability that number is multiple of 6

$P(C) =$ Probability that number is multiple of 7

Now,

$$P(A) = \frac{12}{50}, P(B) = \frac{8}{50}, P(C) = \frac{7}{50}$$

again

$$P(A \cap B) = \frac{4}{50}, P(B \cap C) = \frac{1}{50}, P(A \cap C) = \frac{1}{50}$$

$$P(A \cap B \cap C) = 0$$

Thus

$$P(A \cup B \cup C) = \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0$$

$$= \frac{21}{50}$$

Question5

Two integers x and y are chosen with replacement from the set {0, 1, 2, 3,....,10}. Then the probability that $|x - y| > 5$ is :

[30-Jan-2024 Shift 1]

Options:

A.

30/121

B.

62/121

C.

60/121

D.

31/121

Answer: A

Solution:

If $x = 0, y = 6, 7, 8, 9, 10$

If $x = 1, y = 7, 8, 9, 10$

If $x = 2, y = 8, 9, 10$

If $x = 3, y = 9, 10$

If $x = 4, y = 10$

If $x = 5, y =$ no possible value

Total possible ways $= (5 + 4 + 3 + 2 + 1) \times 2$ Required probability $= \dfrac{30}{11 \times 11} = \dfrac{30}{121}$

Question6

Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

[30-Jan-2024 Shift 2]

Options:

A.

1/3

B.

1/9

C.

1/3

D.

3/10

Answer: C

Solution:

A	B
3W	3W
7R	2R

E_1 : A is selected

E_2 : B is selected

E : white ball is drawn

$$P(E_1/E) = \frac{P(E)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)} = \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{5}}$$

$$= \frac{3}{3+6} = \frac{1}{3}$$

Question7

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

[31-Jan-2024 Shift 1]

Options:

A.

2/25

B.

4/25

C.

2/3

D.

4/75

Answer: D

Solution:

Probability of drawing first red and then white = 10/75 × 30/75 = 4/75

Question8

Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

[31-Jan-2024 Shift 1]

Options:

A.

37/153

B.

57/153

C.

47/153

D.

40/153

Answer: D

Solution:

3 bad apples, 15 good apples.

Let X be no of bad apples

Then $P(X=0)=\frac{^{15}C_2}{^{18}C_2}=\frac{105}{153}$

$P(X=1)=\frac{^{3}C_1\times{}^{15}C_1}{^{18}C_2}=\frac{45}{153}$

$P(X=2)=\frac{^{3}C_2}{^{18}C_2}=\frac{3}{153}$

$E(X)=0\times\frac{105}{153}+1\times\frac{45}{153}+2\times\frac{3}{153}=\frac{51}{153}$

$=\frac{1}{3}$

$\operatorname{Var}(X)=E(X^2)-(E(X))^2$

$=0\times\frac{105}{153}+1\times\frac{45}{153}+4\times\frac{3}{153}-\left(\frac{1}{3}\right)^2$

$=\frac{57}{153}-\frac{1}{9}=\frac{40}{153}$

Question9

A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is-

[31-Jan-2024 Shift 2]

Options:

A.

2/9

B.

1/9

C.

2/27

D.

1/27

Answer: A

Solution:

Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head $= \frac{2}{3}$

$\therefore$ Probability of getting 2 tails and 1 head

$= \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}\right) \times 3$

$= \frac{2}{27} \times 3$

$= \frac{2}{9}$

Question10

A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

[1-Feb-2024 Shift 1]

Options:

A.

2/5

B.

2/7

C.

1/7

D.

1/5

Answer: B

Solution:

$$P(4W4B/2W2B) = \frac{P(4W4B) \times P(2W2B/4W4B)}{P(2W6B) \times P(2W2B/2W6B) + P(3W5B) \times P(2W2B/3W5B) + \ldots\ldots\ldots + P(6W2B) \times P(2W2B/6W2B)}$$

$$= \frac{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{5} \times \frac{{}^2C_2 \times {}^6C_2}{{}^8C_4} + \frac{1}{5} \times \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} + \ldots + \frac{1}{5} \times \frac{{}^6C_2 \times {}^2C_2}{{}^8C_4}}$$

$$= \frac{2}{7}$$

Question11

Let Ajay will not appear in JEE exam with probability p = 2/7, while both Ajay and Vijay will appear in the exam with probability q = 1/5. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

[1-Feb-2024 Shift 2]

Options:

A.

9/35

B.

18/35

C.

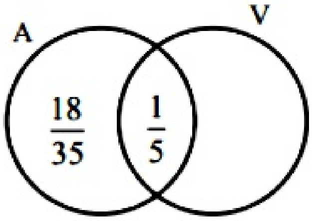
24/35

D.

3/35

Answer: B

Solution:



$P(\overline{A}) = \frac{2}{7} = p$

$P(A \cap V) = \frac{1}{5} = q$

$P(A) = \frac{5}{7}$

Ans. $P(A \cap \overline{V}) = \frac{18}{35}$

Question12

Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations
$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$
has unique solution is −, then the sum of value of k and all possible values of N is
[24-Jan-2023 Shift 1]

Options:

A. 18

B. 19

C. 20

D. 21

Answer: C

Solution:

Solution:
$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$= (N - 2)(N - 3)$
For unique solution $\Delta \neq 0$
So $N \neq 2, 3$
P(system has unique solution) = −
So $k = 4$
Therefore sum $= 4 + 1 + 4 + 5 + 6 = 20$

Question13

Let Ω be the sample space and A ⊆ Ω be an event. Given below are two

statements :
(S1) : If P(A) = 0, then A = φ
(S2) : If P(A) = 1, then A = Ω Then
[24-Jan-2023 Shift 1]

Options:

A. only (S1) is true

B. only (S2) is true

C. both (S1) and (S2) are true

D. both (S1) and (S2) are false

Answer: D

Solution:

Solution:
$\Omega =$ sample space
$A =$ be an event
$A = \left\{ \frac{1}{2} \right\}, \Omega = [0, 1]$
If $P(A) = 0 \;\; A \neq \varphi$
If $P(\overline{A}) = 1 \Rightarrow \overline{A} \neq \Omega$
Then both statement are false

Question14

Let M be the maximum value of the product of two positive integers when their sum is 66 . Let the sample space S = $\left\{ x \in Z : x(66 - x) \geq \frac{5}{9}M \right\}$ and the event A = { x ∈ S : x is a multiple of 3}. Then P(A) is equal to
[25-Jan-2023 Shift 1]

Options:

A. $\frac{15}{44}$

B. $\frac{1}{3}$

C. $\frac{1}{5}$

D. $\frac{7}{22}$

Answer: B

Solution:

Solution:
$M = 33 \times 33$
$x(66 - x) \geq \frac{5}{9} \times 33 \times 33$
$11 \leq x \leq 55$
$A : \{12, 15, 18, \ldots .54\}$
$P(A) = \frac{15}{45} = \frac{1}{3}$

Question15

Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5 , is _______.
[25-Jan-2023 Shift 1]

Options:

A.

Answer: 120

Solution:

Solution:
$x + y = 5\lambda$
Cases:
Total $= 120$

Question16

Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N - 2, \sqrt{3N}, N + 2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is
[25-Jan-2023 Shift 2]

Options:

A. 2

B. 4

C. 16

D. 8

Answer: B

Solution:

Solution:
$n(s) = 36$
Given : $N - 2, \sqrt{3N}, N + 2$ are in G.P.
$3N = (N - 2)(N + 2)$
$3N = N^2 - 4$
$\Rightarrow N^2 - 3N - 4 = 0$
$(N - 4)(N + 1) = 0 \Rightarrow N = 4$ or $N = -1$ rejected
$(\text{Sum} = 4) \equiv \{(1, 3), (3, 1), (2, 2)\}$
$n(A) = 3$
$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow k = 4$

Question17

25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer then a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is ________.

[25-Jan-2023 Shift 2]

Answer: 9

Solution:

Solution:

E_1 : Smokers

$P(E_1) = \frac{1}{4}$

E_2 : non-smokers

$P(E_2) = \frac{3}{4}$

E : diagnosed with lung cancer

$P(E / E_1) = \frac{27}{28}$

$P(E / E_2) = \frac{1}{28}$

$P(E_1 / E) = \frac{P(E_1)P(E / E_1)}{P(E)}$

$$= \frac{\frac{1}{4} \times \frac{27}{28}}{\frac{1}{4} \times \frac{27}{28} + \frac{3}{4} \times \frac{1}{28}} = \frac{27^{9}}{30_{10}} = \frac{9}{10}$$

K = 9

Question18

Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is [29-Jan-2023 Shift 1]

Options:

A. $\frac{5}{24}$

B. $\frac{2}{15}$

C. $\frac{1}{6}$

D. $\frac{5}{36}$

Answer: D

Solution:

Solution:

Required probability $= 1 - \frac{D_{(15)} + {}^{15}C_1 \cdot D_{(14)} + {}^{15}C_2 D_{(13)}}{15!}$

Taking $D_{(15)}$ as $\frac{15\ !}{e}$

$D_{(14)}$ as $\frac{14!}{e}$

$D_{(13)}$ as $\frac{13!}{e}$

We get, $1 - \left(\frac{\frac{15!}{e} + 15 \cdot \frac{14!}{e} + \frac{15 \times 14}{2} \times \frac{13!}{e}}{15!} \right)$

$= 1 - \left(\frac{1}{e} + \frac{1}{e} + \frac{1}{2e} \right) = 1 - \frac{5}{2e} \approx .08$

Question19

Let $S = \{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}$, $n \geq 2$. Let $A = \{2k + 3\ell; k, \ell \in \mathbb{N}\}$ and $B = \{w_n; n \in A\}$. Then $P(B)$ is equal to [29-Jan-2023 Shift 2]

Options:

A. $\frac{3}{32}$

B. $\frac{3}{64}$

C. $\frac{1}{16}$

D. $\frac{1}{32}$

Answer: B

Solution:

Solution:

Let $P(w_1) = \lambda$ then $P(w_2) = \frac{\lambda}{2} ... P(w_n) = \frac{\lambda}{2^{n-1}}$

As $\sum_{k=1}^{\infty} P(w_k) = 1 \Rightarrow \frac{\lambda}{1 - \frac{1}{2}} = 1 \Rightarrow \lambda = \frac{1}{2}$

So, $P(w_n) = \frac{1}{2^n}$

$A = \{2k + 3\ell; k, \ell \in N\} = \{5, 7, 8, 9, 10 ...\}$

$B = \{w_n : n \in A\}$

$B = \{w_5, w_7, w_8, w_9, w_{10}, w_{11}, ...\}$

$A = \mathbb{N} - \{1, 2, 3, 4, 6\}$

$\therefore P(B) = 1 - [P(w_1) + P(w_2) + P(w_3) + P(w_4) + P(w_6)]$

$= 1 - \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} \right]$

$= 1 - \frac{32 + 16 + 8 + 4 + 1}{64} = \frac{3}{64}$

Question20

If an unbiased die, marked with −2, −1, 0, 1, 2, 3 on its faces, is through five times, then the probability that the product of the outcomes is positive, is : [30-Jan-2023 Shift 1]

Options:

A. $\frac{881}{2592}$

B. $\frac{521}{2592}$

C. $\frac{440}{2592}$

D. $\frac{27}{288}$

Answer: B

Solution:

Solution:
Either all outcomes are positive or any two are negative.

$$\text{Now, } p = P(\text{ positive }) = \frac{3}{6} = \frac{1}{2}$$

$$q = p(\text{ negative }) = \frac{2}{6} = \frac{1}{3}$$

Required probability

$$= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$$

$$= \frac{521}{2592}$$

∴ Option (2) is correct.

Question21

A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If p : q = m :n, where m and n are coprime, then m + n is equal to _______.
[30-Jan-2023 Shift 2]

Answer: 14

Solution:

$$p = \frac{{}^6C_1}{6 \times 6} = \frac{1}{6}$$

$$q = \frac{{}^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$$

$$\therefore p : q = 9 : 5 \Rightarrow m + n = 14$$

Question22

A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is
[31-Jan-2023 Shift 1]

Options:

A. $\frac{5}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{5}{6}$

Answer: A

Solution:

Solution:

$$\frac{^5C_2 + {}^6C_2}{^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^8C_2} = \frac{10+15}{1+3+6+10+15}$$

$$= \frac{25}{35} = \frac{5}{7}$$

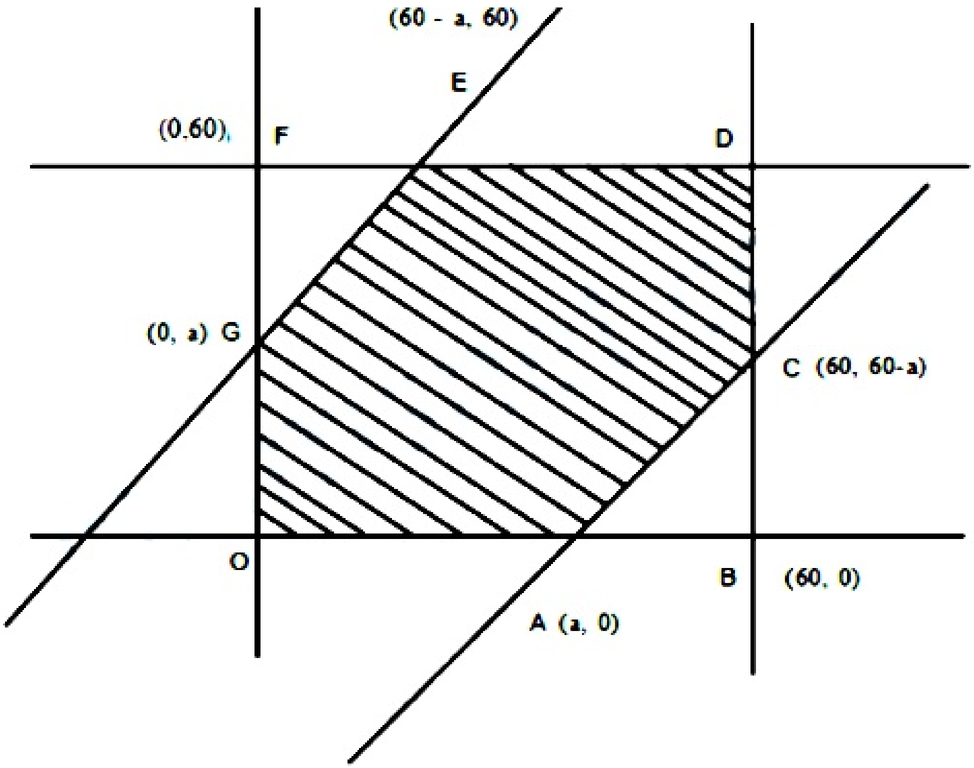
Question23

Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0, 60] is less than or equal to a If P(A) = $\frac{11}{36}$, then a is equal to _______.
[31-Jan-2023 Shift 2]

Answer: 10

Solution:

$|x-y| < a \Rightarrow -a < x-y < a$
$\Rightarrow x-y < a$ and $x-y > -a$



$$P(A) = \frac{ar(OACDEG)}{(OBDF)}$$
$$= \frac{ar(OBDF) - ar(ABC) - ar(EFG)}{ar(OBDF)}$$
$$\Rightarrow \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$$
$$\Rightarrow 1100 = 3600 - (60-a)^2$$
$$\Rightarrow (60-a)^2 = 2500 \Rightarrow 60 - a = 50$$
$$\Rightarrow a = 10$$

Question24

In a binomial distribution B(n, p), the sum and product of the mean & variance are 5 and 6 respectively, then find 6(n + p − q) is equal to :- [1-Feb-2023 Shift 1]

Options:

A. 51

B. 52

C. 53

D. 50

Answer: B

Solution:

$$np + npq = 5,\ np \cdot npq = 6$$
$$np(1+q) = 5,\ n^2p^2q = 6$$
$$n^2p^2(1+q)^2 = 25,\ n^2p^2q = 6$$
$$\frac{6}{q}(1+q)^2 = 25$$
$$6q^2 + 12q + 6 = 25q$$
$$6q^2 - 13q + 6 = 0$$
$$6q^2 - 9q - 4q + 6 = 0$$
$$(3q-2)(2q-3) = 0$$
$$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3} \text{ is accepted}$$
$$p = \frac{1}{3} \Rightarrow n \cdot \frac{1}{3} + n \cdot \frac{1}{3} \cdot \frac{2}{3} = 5$$
$$\frac{3n + 2n}{9} = 5$$
$$n = 9$$

So $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

Question25

Two dice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the 1^{st} die is even and that on the second die is odd, and C be the event that the number appeared on the 1^{st} die is odd and that on the 2^{nd} is even. Then
[1-Feb-2023 Shift 2]

Options:

A. the number of favourable cases of the event $(A \cup B) \cap C$ is 6

B. A and B are mutually exclusive

C. The number of favourable cases of the events A, B and C are 15,6 and 6 respectively

D. B and C are independent

Answer: A

Solution:

Solution:

A : no. on 1^{st} die $<$ no. on 2^{nd} die

A : no. on 1^{st} die $=$ even & no. of 2^{nd} die $=$ odd

C : no. on 1^{st} die $=$ odd & no. on 2^{nd} die $=$ even

$n(A) = 5 + 4 + 3 + 2 + 1 = 15$

$n(B) = 9$

$n(C) = 9$

$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$

$= (3 + 2 + 1) + 0 = 6.$

Question26

A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to :

[6-Apr-2023 shift 1]

Options:

A. 164

B. 123

C. 82

D. 75

Answer: B

Solution:

Solution:
$n(\text{ total } 5) = \{1, 4), (2, 3), (3, 2), (4, 1)\} = 4$
$P(\text{ success }) = \frac{4}{36} = \frac{1}{9}$
$P(\text{ at least } 4 \text{ success }) = P(4 \text{ success }) + P(5 \text{ success })$
$= {}^{5}C_{4} \cdot \left(\frac{1}{9}\right) \cdot \frac{8}{9} + {}^{5}C5 \left(\frac{1}{9}\right)^{5} = \frac{41}{9^{5}} = \frac{41}{3^{10}} = \frac{123}{3^{11}} = \frac{k}{3^{11}}$
$K = 123$

Question27

Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to : [6-Apr-2023 shift 2]

Options:

A. 1

B. 2

C. 4

D. 3

Answer: C

Solution:

Solution:
$F\text{ av.} = \frac{({}^{6}C_{3})(3!)}{6 \times 6 \times 6}$
$= \frac{(20)(6)}{6 \cdot 6 \cdot 6} = \frac{20}{36} = \frac{5}{9} = \frac{p}{q}$
$p = 5$
$q = 9] \Rightarrow q - p = 4$
Ans. Option 3

Question28

In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random form the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is. [8-Apr-2023 shift 1]

Options:

A. $\frac{5}{14}$

B. $\frac{3}{7}$

C. $\frac{9}{28}$

D. $\frac{2}{7}$

Answer: A

Solution:

Solution:

$P(A) = \frac{2}{10} P(B) = \frac{3}{10} P(C) = \frac{5}{10}$

$P(\text{ Defective } / A) = \frac{3}{100}, P(\text{ Defective } / B) = \frac{4}{100}, P(\text{ Defective } / C) = \frac{2}{100}$

$$P(E) = \frac{5/10 \times \frac{2}{100}}{\frac{2}{10} \times \frac{3}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{5}{10} \times \frac{2}{100}} = \frac{10}{6 + 12 + 10}$$

$= \frac{10}{28}$

$= \frac{5}{14}$

Question29

If the probability that the random variable X takes values x is given by $P(X = x) = k(x + 1)3^{-x}$, x = 0, 1, 2, 3, ..., where k is a constant, then $P(X \geq 2)$ is equal to
[8-Apr-2023 shift 2]

Options:

A. $\frac{7}{27}$

B. $\frac{11}{18}$

C. $\frac{7}{18}$

D. $\frac{20}{27}$

Answer: A

Solution:

Solution:

$\sum_{x=0}^{\infty} P(X = x) = 1$

$k(1 + 2 \cdot 3^{-1} + 3 \cdot 3^{-2} + 4 \cdot 3^{-3} + \ldots \infty) = 1$

Let $s = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots \infty$

$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^2} + \ldots \infty$

$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^2} + \ldots \infty$

$\frac{2s}{3} - \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$

$s = \frac{9}{4}$

so $k = \frac{4}{9}$
$P(X \geq 2) = 1 - P(x = 0) - P(x = 1)$
$= 1 - \frac{4}{9}\left(1 + \frac{2}{3}\right)$
$= \frac{7}{27}$(Option 1)

Question30

Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then $4m - 3n$ equal to :
[10-Apr-2023 shift 1]

Options:

A. 12

B. 8

C. 10

D. 6

Answer: B

Solution:

Solution:
$2^N < N!$ is satisfied for $N \geq 4$
Required probability $P(N \geq 4) = 1 - P(N < 4)$
N = 1 (Not possible)
$N = 2(1, 1)$
$\Rightarrow P(N = 2) = \frac{1}{36}$
$N = 3(1, 2), (2, 1)$
$\Rightarrow P(N = 3) = \frac{2}{36}$
$P(N < 4) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$
$\therefore P(N \geq 4) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12} = \frac{m}{n}$
$\Rightarrow m = 11, n = 12$
$\therefore 4m - 3n = 4(11) - 3(12) = 8$

Question31

Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

[10-Apr-2023 shift 2]

Options:

A. 60

B. 30

C. 90

D. 15

Answer: A

Solution:

Solution:
$P(\text{ odd number } 7 \text{ times }) = P(\text{ odd number9times })$
${}^{n}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{n-9}$
${}^{n}C_{7} = {}^{n}C_{9}$
$\Rightarrow n = 16$
Required
$P = {}^{16}C_{2} \times \left(\frac{1}{2}\right)^{16}$
$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$
$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$

Question32

Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \leq i, j \leq 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible }\}$ be an event. Then $P(A)$ is equal to : [11-Apr-2023 shift 1]

Options:

A. $\frac{16}{27}$

B. $\frac{50}{81}$

C. $\frac{47}{81}$

D. $\frac{49}{81}$

Answer: B

Solution:

Solution:
$M\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d, \in \{0, 1, 2\}$
$n(s) = 3^{4} = 81$
we first bound $p(\overline{A})$
$|m| = 0 \Rightarrow ad = bc$
$ad = bc = 0 \Rightarrow$ no. of $(a, b, c, d) = (3^{2} - 2^{2})^{2} = 25$
$ad = bc = 1 \Rightarrow$ no. of $(a, b, c, d) = 1^{2} = 1$
$ad = bc = 2 \Rightarrow$ no. of $(a, b, c, d) = 2^{2} = 4$
$ad = bc = 4 \Rightarrow$ no. of $(a, b, c, d) = 1^{2} = 1$
$: P(\overline{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$

Question33

Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^{2} + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to _______.

[11-Apr-2023 shift 2]

Answer: 27

Solution:

Solution:

$64x^2 + 5Nx + 1 = 0$

$D = 25N^2 - 256 < 0$

$\Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$

$\therefore N = 1, 2, 3$

$\therefore$ Probability $= \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64}$

$\therefore q - p = 27$

Question34

Two dice A and B are rolled. Let numbers obtained on A and B be α and β respectively. If the variance of α − β is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisor of p is equal to
[12-Apr-2023 shift 1]

Options:

A. 36

B. 31

C. 48

D. 72

Answer: C

Solution:

$\alpha-\beta$	Case	P
5	(6,1)	1/36
4	(6,2) (5,1)	2/36
3	(6,3) (5,2) (4,1)	3/36
2	(6,4) (5,3) (4,3) (3,1)	4/36
1	(6,5) (5,4) (4,3) (3,2) (2,1)	5/36
0	(6,6) (5,5) . . . . . . (1,1)	6/36
-1	- - - - -	5/36
-2	- - - - -	4/36
-3	- - - - -	3/36
-4	(2,6) (1,5)	2/36
-5	(1,6)	1/36

$\sum(x^2) = \sum x^2 P(x) = 2\left[\frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36}\right]$

$= \frac{105}{18} = \frac{35}{6}$

$\mu = \sum(x) = 0$ as data is symmetric

$\sigma^2 = \sum(x^2) = \sum x^2 P(x) = \frac{35}{6} P = 35 = 5 \times 7$

Sum of divisors $= (5^\circ + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$

Question35

A fair n(n > 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ________.

[12-Apr-2023 shift 1]

Answer: 10

Solution:

Solution:

$\text{Mean} = 1 \cdot \frac{n-1}{n} + 2\frac{1}{n}\left(\frac{n-1}{n}\right) + 3\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)\ldots$

$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 \ldots\ldots\right)$

$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 - \frac{1}{n}\right)^{-2} = \left(\frac{n-1}{n}\right) \cdot \frac{n^2}{(n-1)^2}$

$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$

Question36

A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is-

[13-Apr-2023 shift 1]

Options:

A. $\frac{21}{16}$

B. $\frac{15}{16}$

C. $\frac{81}{64}$

D. $\frac{37}{16}$

Answer: A

Solution:

Solution:

$P(H) = \frac{3}{4}$

$P(T) = \frac{1}{4}$

$\text{Mean } \overline{X} = \frac{3}{4} + \frac{3}{8} + 3\left(\frac{1}{64} + \frac{3}{64}\right)$

$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$

$= 3\left(\frac{7}{16}\right) = \frac{21}{16}$

Question37

The random variable X follows binomial distribution B(n, p), for which the difference of the mean and the variance is 1 . If 2P(x = 2) = 3P(x = 1), then $n^2P(X > 1)$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 16

B. 11

C. 12

D. 15

Answer: B

Solution:

$2P(x = 2) = 3P(x = 1)$

$2 \times {}^nc_2 P^2(1 - P)^{n-2} = 3\,{}^nc_1 P^1(1 - P)^{n-1}$

$\Rightarrow 2\,\frac{n(n-1)}{2} \cdot P = 3n(1 - P)$

$\Rightarrow (n - 1)P = 3(1 - P) \cdots \text{(i)}$

$nP - nPq = 1$

$\Rightarrow nP - nP(1 - p) = 1$

$\Rightarrow nP = 1 \Rightarrow n = \frac{1}{p^2}$

$\Rightarrow$ put in equ (i)

$\left(\frac{1}{p^2} - 1\right)P = 3(1 - p)$

$\frac{1}{p} - P = 3 - 3P$
$\Rightarrow 1 - P^2 = 3P - 3p^2$
$\Rightarrow 2P^2 - 3P + 1 = 0$
$\Rightarrow 2P^2 - 2P - P + 1 = 0$
$\Rightarrow 2P(P - 1) - 1(P - 1) = 0$
$\Rightarrow P = \frac{1}{2}, P = 1\{\text{Rejected}\}$
$n = \frac{1}{(1/2)^2} = 4$
$n^2P(x > 1) = n^2p(1 - P(x = 0) - P(x = 1))$
$\Rightarrow n^2P(1 - (1 - P)^n - nP(1 - P)^{n-1})$
$\Rightarrow (4)^2\left(1 - \left(\frac{1}{2}\right)^4 - 4\left(\frac{2}{2}\right)^4\right)$
$\Rightarrow 16 - 1 - 4 = 11$

Question38

A bag contains 6 white and 4 black balls. A die is rolled once and the number of ball equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is [15-Apr-2023 shift 1]

Options:

A. $\frac{1}{4}$

B. $\frac{9}{50}$

C. $\frac{11}{50}$

D. $\frac{1}{5}$

Answer: D

Solution:

Solution:

$\frac{1}{6} \times \left[\frac{{}^6C_1}{{}^{10}C_1} + \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_3}{{}^{10}C_3} + \frac{{}^6C_4}{{}^{10}C_4} + \frac{{}^6C_5}{{}^{10}C_5} + \frac{{}^6C_6}{{}^{10}C_6}\right]$
$= \frac{1}{6}\left(\frac{126 + 70 + 35 + 15 + 5 + 1}{210}\right) = \frac{42}{210} = \frac{1}{5}$

Question39

A random variable X has the following probability distribution:

X	0	1	2	3	4
P(X)	k	2k	4k	6k	8k

The value of P(1 < X < 4 | X < 2) is equal to: [24-Jun-2022-Shift-2]

Options:

A. $\frac{4}{7}$

B. $\frac{2}{3}$

C. $\frac{3}{7}$

D. $\frac{4}{5}$

Answer: A

Solution:

Solution:

$\because$x is a random variable

$\therefore k+2k+4k+6k+8k=1$

$\therefore k=\frac{1}{21}$

Now, $P(1<x<4 \mid x \leq 2)=\frac{4k}{7k}=\frac{4}{7}$

Question40

In an examination, there are 10 true-false type questions. Out of 10 , a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to __
[24-Jun-2022-Shift-2]

Answer: 479

Solution:

Solution:

Student guesses only two wrong. So there are three possibilities.

(i) Student guesses both wrong from 1^{st} section

(ii) Student guesses both wrong from 2^{nd} section

(iii) Student guesses two wrong one from each section

Required probabilities

$={}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2\left(\frac{1}{6}\right)^6+{}^6C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^4+{}^4C_1\cdot{}^6C_1\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^5$

$=\frac{1}{4^{10}}[6\times 9+15\times 9^4+24\times 9^2]$

$=\frac{27}{4^{10}}[2+27\times 15+72]$

$=\frac{27\times 479}{4^{10}}$

Question41

Let E_1 and E_2 be two events such that the conditional probabilities

$P(E_1 \mid E_2) = \frac{1}{2}$, $P(E_2 \mid E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then :
[25-Jun-2022-Shift-1]

Options:

A. $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

B. $P(E_1' \cap E_2') = P(E_1') \cdot P(E_2)$

C. $P(E_1 \cap E_2') = P(E_1) \cdot P(E_2)$

D. $P(E_1' \cap E_2) = P(E_1) \cdot P(E_2)$

Answer: C

Solution:

Solution:

$$P\left(\frac{E_1}{E_2}\right) = \frac{1}{2} \Rightarrow \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{2}$$

$$P\left(\frac{E_2}{E_1}\right) = \frac{3}{4} \Rightarrow \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{3}{4}$$

$$P(E_1 \cap E_2) = \frac{1}{8}$$

$$P(E_2) = \frac{1}{4}, P(E_1) = \frac{1}{6}$$

(A) $P(E_1 \cap E_2) = \frac{1}{8}$ and $P(E_1) \cdot P(E_2) = \frac{1}{24}$

$$\Rightarrow P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2)$$

(B) $P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$

$$= 1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{8}\right] = \frac{17}{24}$$

$$P(E_1') = \frac{3}{4} \Rightarrow P(E_1')P(E_2) = \frac{3}{24}$$

$$\Rightarrow P(E_1' \cap E_2') \neq P(E_1').P(E_2)$$

(C) $P(E_1 \cap E_2') = P(E_1) - P(E_1 \cap E_2)$

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$$

$$P(E_1) \cdot P(E_2) = \frac{1}{24}$$

$$\Rightarrow P(E_1 \cap E_2') = P(E_1) \cdot P(E_2)$$

(D) $P(E_1' \cap E_2) = P(E_2) - P(E_1 \cap E_2)$

$$= \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

$$P(E_1)P(E_2) = \frac{1}{24}$$

$$\Rightarrow P(E_1' \cap E_2) \neq P(E_1) \cdot P(E_2)$$

Question42

A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48 , is :
[25-Jun-2022-Shift-2]

Options:

A. $\frac{7}{2^{11}}$

B. $\frac{7}{2^{12}}$

C. $\frac{3}{2^{10}}$

D. $\frac{13}{2^{12}}$

Answer: D

Solution:

Solution:
There are only two ways to get sum 48 , which are (32, 8, 8) and (16, 16, 16)
So, required probability
$= 3\left(\frac{2}{32}\cdot\frac{1}{8}\cdot\frac{1}{8}\right) + \left(\frac{1}{16}\cdot\frac{1}{16}\cdot\frac{1}{16}\right)$
$= \frac{3}{2^{10}} + \frac{1}{2^{12}}$
$= \frac{13}{2^{12}}$

Question43

Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is :
[27-Jun-2022-Shift-1]

Options:

A. $\frac{1}{136}$

B. $\frac{1}{72}$

C. $\frac{1}{68}$

D. $\frac{1}{34}$

Answer: C

Solution:

Solution:
No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3.................. $n(s) = {}^{18}C_5$
$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$
$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$
$\frac{1}{17 \times 4} = \frac{1}{68}$

Question44

Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(x = 4), then the sum of the mean and the variance of X is : [27-Jun-2022-Shift-1]

Options:

A. $\frac{105}{16}$

B. $\frac{7}{16}$

C. $\frac{77}{36}$

D. $\frac{49}{16}$

Answer: C

Solution:

Solution:
B(7, p) n = 7 p = p
given
P(x = 3) = 5P(x = 4)
$^7C_3 \times p^3(1-p)^4 = 5\ .\ ^7C_4 p^4(1-p)^3$
$\frac{^7C_3}{5 \times {^7C_4}} = \frac{p}{1-p}$
1 − p = 5p
6p = 1
$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$
n = 7
Mean $= np = 7 \times \frac{1}{6} = \frac{7}{6}$
Var $= npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$
Sum
$= \frac{7}{6} + \frac{35}{36}$
$= \frac{42+35}{36}$
$= \frac{77}{36}$

Question45

If a point A(x, y) lies in the region bounded by the y-axis, straight lines 2y + x = 6 and 5x − 6y = 30, then the probability that y < 1 is [27-Jun-2022-Shift-2]

Options:

A. $\frac{1}{6}$

B. $\frac{5}{6}$

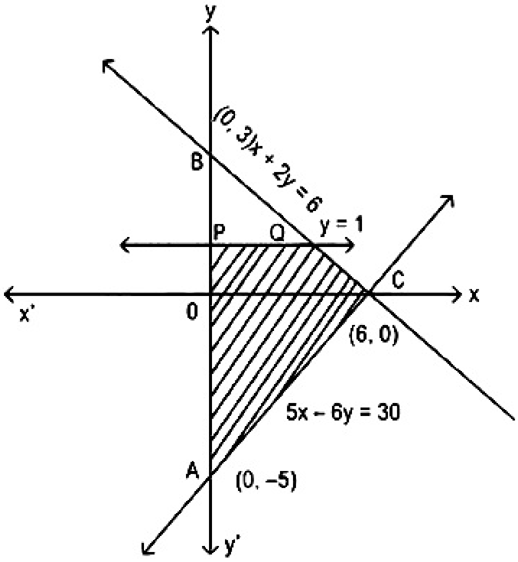
C. $\frac{2}{3}$

D. $\frac{6}{7}$

Answer: B

Solution:

Solution:
The required probability



$= \frac{\text{Area of Region PQCAP}}{\text{Area of Region ABCA}}$

$= \frac{\frac{1}{2} \times 8 \times 6 - \frac{1}{2} \times 2 \times 4}{\frac{1}{2} \times 8 \times 6}$

$= \frac{5}{6}$

Question46

Let S = {E_1, E_2, ..., E_8} be a sample space of a random experiment such that P(E_n) = $\frac{n}{36}$ for every n = 1, 2, ...

8. Then the number of elements in the set $\left\{ A \subseteq S : P(A) \geq \frac{4}{5} \right\}$ is___

[27-Jun-2022-Shift-2]

Answer: 19

Solution:

Here $P(E_n) = \frac{n}{36}$ for n = 1, 2, 3, ……, 8

Here $P(A) = \frac{\text{Any possible sum of } (1, 2, 3, \ldots, 8)(= a \text{ say })}{36}$

$\because \frac{a}{36} \geq \frac{4}{5}$

$\therefore a \geq 29$

If one of the number from {1, 2, ……, 8} is left then total $a \geq 29$ by 3 ways.

Similarly by leaving terms more 2 or 3 we get 16 more combinations.

∴ Total number of different set A possible is 16 + 3 = 19

Question47

The probability, that in a randomly selected 3-digit number at least two digits are odd, is
[28-Jun-2022-Shift-1]

Options:

A. $\frac{19}{36}$

B. $\frac{15}{36}$

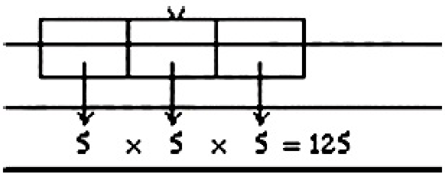
C. $\frac{13}{36}$

D. $\frac{23}{36}$

Answer: A

Solution:

Solution:



Atleast two digits are odd=Exactly 2 digits odd+exactly 3 digits odd
For exactly three digits are odd $= 5 \times 5 \times 5 = 125$ For exactly two digits odd :
If 0 is used then no of 3 digit numbers with 2 odd digits $= 2 \times 5 \times 5 = 50$
If 0 is not used then no of 3 digit numbers with 2 odd digits
$= {}^{3}C_{1} \times 4 \times 5 \times 5 = 300$
Required Probability $= \frac{125 + 50 + 300}{900} = \frac{475}{900} = \frac{19}{36}$

Question48

Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to
[24-Jun-2022-Shift-1]

Options:

A. 13

B. 6

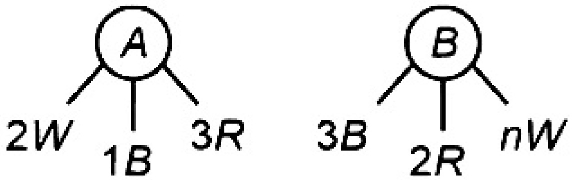
C. 4

D. 3

Answer: C

Solution:

Solution:



$P(1R \text{ and } 1B) = P(A) \cdot P\left(\frac{1R1B}{A}\right) + P(B) \cdot P\left(\frac{1R1B}{B}\right)$

$= \frac{1}{2} \cdot \frac{{}^3C_1 \cdot {}^1C_1}{{}^6C_2} + \frac{1}{2} \cdot \frac{{}^2C_1 \cdot {}^3C_1}{{}^{n+5}C_2}$

$P\left(\frac{1R1B}{A}\right) = \frac{\frac{1}{2} \cdot \frac{3}{15}}{\frac{1}{2} \cdot \frac{3}{15} + \frac{1}{2} \cdot \frac{6 \cdot 2}{(n+5)(n+4)}} = \frac{6}{11}$

$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$

$\Rightarrow \frac{11}{10} = \frac{6}{10} + \frac{36}{(n+5)(n+4)}$

$\Rightarrow \frac{5}{10 \times 36} = \frac{1}{(n+5)(n+4)}$

$\Rightarrow n^2 + 9n - 52 = 0$

$\Rightarrow n = 4$ is only possible value

Question49

If a random variable X follows the Binomial distribution B(33, p) such that 3P(X = 0) = P(X = 1), then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to: [24-Jun-2022-Shift-1]

Options:

A. 1320

B. 1088

C. $\frac{120}{1331}$

D. $\frac{1088}{1089}$

Answer: A

Solution:

$3P(X=0)=P(X=1)$

$3\cdot{}^{n}C_{0}P^{0}(1-P)^{n}={}^{n}C_{1}P^{1}(1-P)^{n-1}$

$\frac{3}{n}=\frac{P}{1-P}\Rightarrow\frac{1}{11}=\frac{P}{1-P}$

$\Rightarrow 1-P=11P$

$\Rightarrow P=\frac{1}{12}$

$\frac{P(X=15)}{P(X=18)}-\frac{P(X=16)}{P(X=17)}$

$\Rightarrow\frac{{}^{33}C_{15}P^{15}(1-P)^{18}}{{}^{33}C_{18}P^{18}(1-P)^{15}}-\frac{{}^{33}C_{16}P^{16}(1-P)^{17}}{{}^{33}C_{17}P^{17}(1-P)^{16}}$

$\Rightarrow\left(\frac{1-P}{P}\right)^{3}-\left(\frac{1-P}{P}\right)$

$\Rightarrow\ 11^{3}-11=1320$

Question50

If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is: [25-Jul-2022-Shift-1]

Options:

A. $\frac{33}{2^{32}}$

B. $\frac{33}{2^{29}}$

C. $\frac{33}{2^{28}}$

D. $\frac{33}{2^{27}}$

Answer: C

Solution:

Solution:
If n is number of trails, p is probability of success and q is probability of unsuccess then,
Mean $=np$ and variance $=npq$.
Here
$np+npq=24\ \ldots(i)$
$np\,.\,npq=128\ \ldots(ii)$
and $q=1-p\ \ldots(iii)$
from eq. (i), (ii) and (iii) : $p=q=\frac{1}{2}$ and $n=32$.
∴ Required probability $=p(X=1)+p(X=2)$

$={}^{32}C_{1}\cdot\left(\frac{1}{2}\right)^{32}+{}^{32}C_{2}\cdot\left(\frac{1}{2}\right)^{32}$

$=\left(32+\frac{32\times31}{2}\right)\cdot\frac{1}{2^{32}}$

$=\frac{33}{2^{28}}$

Question51

If A and B are two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$, then $P(A\mid B')+P(B\mid A')$ is equal to

[25-Jul-2022-Shift-2]

Options:

A. $\frac{3}{4}$

B. $\frac{5}{8}$

C. $\frac{5}{4}$

D. $\frac{7}{8}$

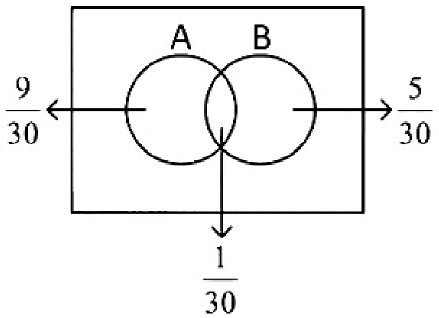
Answer: B

Solution:

Solution:

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

$\therefore P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}$



Now, $P(A \mid B') + P(B \mid A') = \frac{P(A \cap B')}{P(B')} + \frac{P(B \cap A')}{P(A')}$

$= \frac{\frac{9}{30}}{\frac{4}{5}} + \frac{\frac{5}{30}}{\frac{2}{3}} = \frac{5}{8}$

Question52

The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to :
[26-Jul-2022-Shift-1]

Options:

A. $\frac{5}{9}$

B. $\frac{64}{81}$

C. $\frac{16}{27}$

D. $\frac{145}{243}$

Answer: C

Solution:

Solution:

Given, mean $= np = \alpha$.
and variance $= npq = \frac{\alpha}{3}$
$\Rightarrow q = \frac{1}{3}$ and $p = \frac{2}{3}$
$P(X = 1) = n \cdot p^1 \cdot q^{n-1} = \frac{4}{243}$
$\Rightarrow n \cdot \frac{2}{3} \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$
$\Rightarrow n = 6$
$P(X = 4 \text{ or } 5) = {}^6C_4 \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^2 + {}^6C_5 \cdot \left(\frac{2}{5}\right)^5 \cdot \frac{1}{3}$
$= \frac{16}{27}$

Question53

Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to
[26-Jul-2022-Shift-1]

Options:

A. $\frac{2}{3}$

B. $\frac{5}{3}$

C. $\frac{5}{4}$

D. 1

Answer: B

Solution:

Solution:
$0 \le \frac{2+3P}{6} \le 1 \Rightarrow P \in \left[-\frac{2}{3}, \frac{4}{3}\right]$
$0 \le \frac{2-P}{8} \le 1 \Rightarrow P \in [-6, 2]$
$0 \le \frac{1-P}{2} \le 1 \Rightarrow P \in [-1, 1]$
$0 < P(E_1) + P(E_2) + P(E_3) \le 1$
$0 < \frac{13}{12} - \frac{P}{8} \le 1$
$P \in \left[\frac{2}{3}, \frac{26}{3}\right]$
Taking intersection of all
$P \in \left[\frac{2}{3}, 1\right)$
$P_1 + P_2 = \frac{5}{3}$

Question54

Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54P(X \le 2)$ is equal to
[26-Jul-2022-Shift-2]

Options:

A. $\frac{73}{27}$

B. $\frac{146}{27}$

C. $\frac{146}{81}$

D. $\frac{126}{81}$

Answer: B

Solution:

Solution:
Mean $= 4 = \mu = np$
Variance $= \sigma^2 = np(1 - P) = \frac{4}{3}$
$4(1 - P) = \frac{4}{3}$
$P = \frac{2}{3}$
$n \times \frac{2}{3} = 4$
$n = 6$
$P(X = k) = {}^nC_k P^k (1 - P)^{n-k}$
$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
$= {}^6C_0 P^0 (1 - P)^6 + {}^6C_1 P^1 (1 - P)^5 + {}^6C_2 P^2 (1 - P)^4$
$= {}^6C_0 \left(\frac{1}{3}\right)^6 + {}^6C_1 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + {}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4$
$= \left(\frac{1}{3}\right)^6 [1 + 12 + 60] = \frac{73}{3^6}$
$54P(X \leq 2) = \frac{73}{3^6} \times 54 = \frac{146}{27}$

Question55

Let X have a binomial distribution B(n, p) such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$, then k is equal to :

[27-Jul-2022-Shift-2]

Options:

A. 528

B. 529

C. 629

D. 630

Answer: B

Solution:

Solution:
Mean $= np = 16$
Variance $= npq = 8$
$\Rightarrow q = p = \frac{1}{2}$ and $n = 32$
$P(x > n - 3) = p(x = n - 2) + p(x = n - 1) + p(x = n)$
$= ({}^{32}C_2 + {}^{32}C_1 + {}^{32}C_0) \cdot \frac{1}{2^n}$

$= \frac{529}{2^n}$

Question56

A six faced die is biased such that
$3 \times P($ a prime number $) = 6 \times P($ a composite number $) = 2 \times P$.
Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :
[27-Jul-2022-Shift-2]

Options:

A. $\frac{3}{11}$

B. $\frac{5}{11}$

C. $\frac{7}{11}$

D. $\frac{8}{11}$

Answer: D

Solution:

Solution:
Let $P($ a prime number $) = \alpha$
$P($ a composite number $) = \beta$
and $P(1) = \gamma$
$\because 3\alpha = 6\beta = 2\gamma = k$ (say)
and $3\alpha + 2\beta + \gamma = 1$
$\Rightarrow k + \frac{k}{3} + \frac{k}{2} = 1 \Rightarrow k = \frac{6}{11}$
Mean $= np$ where $n = 2$
and $p =$ probability of getting perfect square
$= P(1) + P(4) = \frac{k}{2} + \frac{k}{6} = \frac{4}{11}$
So, mean $= 2 \cdot \left(\frac{4}{11}\right) = \frac{8}{11}$

Question57

Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is :
[28-Jul-2022-Shift-1]

Options:

A. $\frac{3}{4}$

B. $\frac{11}{16}$

C. $\frac{23}{32}$

D. $\frac{13}{16}$

Answer: A

Solution:

Solution:
P(Female $\big)$ = $\frac{60}{100}$ = $\frac{3}{5}$
P(Male $\big)$ = $\frac{2}{5}$
P(Female/Qualified $\big)$ = $\frac{40}{60}$ = $\frac{2}{3}$
P(Male/qualified $\big)$ = $\frac{20}{60}$ = $\frac{1}{3}$

Question58

Let A and B be two events such that $P(B \mid A) = \frac{2}{5}$, $P(A \mid B) = \frac{1}{7}$ and $P(A \cap B) = \frac{1}{9}$. Consider
(S1) $P(A' \cup B) = \frac{5}{6}$,
(S2) $P(A' \cap B') = \frac{1}{18}$
Then
[28-Jul-2022-Shift-2]

Options:

A. Both (S1) and (S2) are true

B. Both (S1) and (S2) are false

C. Only (S1) is true

D. Only (S2) is true

Answer: A

Solution:

Solution:
$P(A / B) = \frac{1}{7} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{7}$
$\Rightarrow P(B) = \frac{7}{9}$
$P(B / A) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{2}{5}$
$P(A) = \frac{5}{2} \cdot \frac{1}{9} = \frac{5}{18}$
$S2 : P(A' \cap B') = \frac{1}{18}$
S1 : and $P(A' \cup B) = \frac{1}{9} + \frac{6}{9} + \frac{1}{18} = \frac{5}{6}$.

Question59

A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\sigma^2$ is equal to_______.
[28-Jul-2022-Shift-2]

Answer: 56

Solution:

Solution:
X = Number of white ball drawn

$P(X = 0) = \frac{^6C_3}{^{10}C_3} = \frac{1}{6}$

$P(X = 1) = \frac{^6C_2 \times {}^4C_1}{^{10}C_3} = \frac{1}{2}$

$P(X = 2) = \frac{^6C_1 \times {}^4C_2}{^{10}C_3} = \frac{3}{10}$

and $P(X = 3) = \frac{^6C_0 \times {}^4C_3}{^{10}C_3} = \frac{1}{30}$

Variance $= \sigma^2 = \sum P_i X_i^2 - (\sum P_i X_i)^2$

$\sigma^2 = \frac{1}{2} + \frac{12}{10} + \frac{3}{10} - \left(\frac{1}{2} + \frac{6}{10} + \frac{1}{10}\right)^2$

$= \frac{56}{100}$

$100\sigma^2 = 56$

Question60

Let S = {1, 2, 3, ..., 2022}. Then the probability, that a randomly chosen number n from the set S such that HCF(n, 2022) = 1, is : [29-Jul-2022-Shift-1]

Options:

A. $\frac{128}{1011}$

B. $\frac{166}{1011}$

C. $\frac{127}{337}$

D. $\frac{112}{337}$

Answer: D

Solution:

Solution:
$S = \{1, 2, 3, \ldots\ldots\ldots .2022\}$
$HCF(n, 2022) = 1$
$\Rightarrow$n and 2022 have no common factor
Total elements $= 2022$
$2022 = 2 \times 3 \times 337$
M : numbers divisible by 2.
$\{2, 4, 6, \ldots\ldots, 2022\}$ $n(M) = 1011$
N : numbers divisible by 3.
$\{3, 6, 9, \ldots\ldots .2022\}$ $n(N) = 674$

L : numbers divisible by 6 .
$\{6, 12, 18, \ldots\ldots, 2022\}$ $n(L) = 337$
$n(M \cup N) = n(M) + n(N) - n(L)$
$= 1011 + 674 - 337$
$= 1348$
$0 =$ Number divisible by 337 but not in $M \cup N$
$\{337, 1685\}$

Number divisible by 2,3 or 337
$= 1348 + 2 = 1350$

Required probability $= \frac{2022 - 1350}{2022}$

$= \frac{672}{2022}$

$= \frac{112}{337}$

Question61

Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:
[29-Jul-2022-Shift-2]

Options:

A. $\frac{4}{9}$

B. $\frac{5}{18}$

C. $\frac{1}{6}$

D. $\frac{3}{10}$

Answer: B

Solution:

Solution:
A : Drown ball from boy II is black
B : Red ball transferred

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{9} \times \frac{5}{10}}{\frac{3}{9} \times \frac{5}{10} + \frac{4}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{5}{10}}$$

$$= \frac{15}{15 + 24 + 15} = \frac{15}{54} = \frac{5}{18}$$

Question62

The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is ______.
[29-Jul-2022-Shift-2]

Answer: 96

Solution:

Solution:
Let two roots of a quadratic equation are mean $= np$ and variance $= npq$.

Given np + npq = 82.5
and np(npq) = 1350
∴ Quadratic equation is
$x^2 - 82.5x + 1350 = 0$
$\Rightarrow x^2 - 22.5x - 60x + 1350 = 0$
$\Rightarrow x - (x - 22.5) - 60(x - 22.5) = 0$
Mean = 60 and Variance = 22.5
np = 60, npq = 22.5
$\Rightarrow q = \frac{9}{24} = \frac{3}{8}, p = \frac{5}{8}$
$\therefore\ n\frac{5}{8} = 60 \quad \Rightarrow n = 96$

Question63

A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2 , is [2021, 26 Feb. Shift-II]

Options:

A. $\frac{6}{7}$

B. $\frac{1}{7}$

C. $\frac{3}{7}$

D. $\frac{4}{7}$

Answer: C

Solution:

Solution:
Given, digits 3, 3, 4, 4, 4, 5, 5
Total 7 digit number from given digits will be
$\frac{7!}{3!2!2!} = \frac{7 \times 6 \times 5 \times 4}{2 \times 2}$
$= 210$
Now, if the 7 digit number is divisible by 2 , then last digit must be 4 .
Now, 7 digit number divisible by 2 will be
$= \frac{6!}{2!2!2!} = \frac{.6 \times 5 \times 4 \times 3}{2 \times 2} = 90$
Required probability $= \frac{90}{210} = \frac{9}{21} = \frac{3}{7}$

Question64

In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10%, respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is [2021, 25 Feb. Shift-II]

Options:

A. $\frac{7}{45}$

B. $\frac{8}{45}$

C. $\frac{28}{45}$

D. $\frac{14}{45}$

Answer: C

Solution:

Solution:
Let 'A' be the event of smokers and non-vegetarian.
Let ' B ' be the event of smokers and vegetarian.
Let ' C ' be the event of non-smokers and vegetarian.
Let ' E ' be the event of chest disorders.
According to question,

$P(A) = \frac{160}{400} = \frac{2}{5}, P(B) = \frac{100}{400} = \frac{1}{4}$

$P(C) = \frac{140}{400} = \frac{7}{20}\ P\left(\frac{E}{A}\right) = 35\% = \frac{35}{100},$

$P\left(\frac{E}{B}\right) = \frac{20}{100}, P\left(\frac{E}{C}\right) = \frac{10}{100}$

Thus,

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right) + P(C)P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{2}{5} \cdot \frac{35}{100}}{\frac{2}{5} \cdot \frac{35}{100} + \frac{1}{4} \cdot \frac{20}{100} + \frac{7}{20} \cdot \frac{10}{100}}$$

$$= = \frac{\frac{14}{100}}{\frac{14}{100} + \frac{5}{100} + \frac{\frac{7}{2}}{100}}$$

$$= \frac{14}{14 + 5 + \frac{7}{2}} = \frac{28}{45}$$

Question65

Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let P be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$(All the probabilities are assumed to lie in the interval $(0, 1)$). Then, $\frac{P(B_1)}{P(B_3)}$ is equal to ....
[2021, 24 Feb. Shift-l]

Answer: 6

Solution:

Given, B_1, B_2 and B_3 are three independent events.
Let x, y, z be the probability of B_1, B_2, B_3, respectively.

$P(\text{only } B_1 \text{ occur}) = \alpha$

$P(B_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = \alpha$
$\Rightarrow x \cdot (1-y) \cdot (1-z) = \alpha$
$\Rightarrow$ P(only B_2 occur) $= \beta$
$P(\overline{B}_1) \cdot P(B_2) \cdot P(\overline{B}_3) = \beta$
$\Rightarrow (1-x) \cdot y \cdot (1-z) = \beta$
P(none occur) $= P$
$P(\overline{B}_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = P$
$\Rightarrow (1-x) \cdot (1-y) \cdot (1-z) = P$
Now, we have given relations
$(\alpha - 2\beta)P = \alpha\beta$
$\Rightarrow [x(1-y)(1-z) - 2y(1-x)(1-z)]$
$(1-x)(1-y)(1-z)$
$= x \cdot (1-y)(1-z) \cdot y(1-x)(1-z)$
[putting the value of α, β, P]
$\Rightarrow (1-z)[x(1-y) - 2y(1-x)] = x \cdot y \cdot (1-z)$
$\Rightarrow x - xy - 2y + 2xy = xy$
$\Rightarrow x = 2y$ . . . (i)
Similarly, on solving the second relation,
$(\beta - 3\gamma)P = 2\beta\gamma$ by putting β, γ and P,
We get, $y = 3z$ . . . (ii)
From Eqs. (i) and (ii), we get
$x = 2 \times 3z \Rightarrow x = 6z$
$\Rightarrow \frac{x}{z} = 6$
Now, $\frac{P(B_1)}{P(B_3)} = \frac{x}{z} = 6$
$\therefore$ Required ratio is 6 .

Question66

When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is
[2021, 25 Feb. Shift-1]

Options:

A. $\frac{1}{27}$

B. $\frac{3}{4}$

C. $\frac{1}{8}$

D. $\frac{3}{8}$

Answer: C

Solution:

Solution:

Probability of missile to get intercepted $= \frac{1}{3}$

$\therefore$ Probability of missile to not get intercepted $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability of missile to hit the target $= \frac{3}{4}$

$\therefore$ Probability of three missiles to hit the target
$= \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{8}$

Question67

An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :
24 Feb 2021 Shift 1

Options:

A. $\frac{1}{32}$

B. $\frac{5}{16}$

C. $\frac{3}{16}$

D. $\frac{1}{2}$

Answer: D

Solution:

Solution:
${}^{n}C_2\left(\frac{1}{2}\right)^n = {}^{n}C_3\left(\frac{1}{2}\right)^n \Rightarrow {}^{n}C_2 = {}^{n}C_3$
$\Rightarrow n = 5$
Probability of getting an odd number for odd number of times is
${}^{5}C_1\left(\frac{1}{2}\right)^5 + {}^{5}C_2\left(\frac{1}{2}\right)^5 + {}^{5}C_3\left(\frac{1}{2}\right)^5$
$= \frac{1}{2^5}(5 + 10 + 1) = \frac{1}{2}$

Question68

Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α. Only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to
24 Feb 2021 Shift 1

Answer: 6

Solution:

Let $P(B_1) = p_1$, $P(B_2) = p_2$, $P(B_3) = p_3$
given that $p_1(1 - p_2)(1 - p_3) = \alpha$
$p_2(1 - p_1)(1 - p_3) = \beta$ (i)
$p_3(1 - p_1)(1 - p_2) = \gamma$
and $(1 - p_1)(1 - p_2)(1 - p_3) = p$ (ii)
$\Rightarrow \frac{p_1}{1 - p_1} = \frac{\alpha}{p}, \frac{p_2}{1 - p_2} = \frac{\beta}{p} \& \frac{p_3}{1 - p_3} = \frac{\gamma}{p}$
Also $\beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$
$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$

$\Rightarrow \alpha p - 6p\gamma - 5\alpha\gamma$

$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)}$

$\Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \frac{p_1}{p_3} = 6$

Question69

Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then, the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is [2021, 25 Feb. Shift-11]

Options:

A. $\frac{1}{5}$

B. $\frac{2}{9}$

C. $\frac{97}{297}$

D. $\frac{122}{297}$

Answer: C

Solution:

Solution:
Here, we have four digit natural numbers, then total cases will exclude those number which contain zero at thousands place.
Hence, total cases will be
$= ({}^4C_1 \times 9 \times 9 \times 9) - ({}^3C_1 \times 9 \times 9) = 2673$
Again, only those numbers will have remainder 2 when divided by 5 either they have 2 at its unit place or 7 at its unit place.
When unit digit is 2, then total number of four digit numbers will be
$= ({}^3C_1 \times 9 \times 9) - ({}^2C_1 \times 9) = 225$
When unit digit is 7 , then total number of four digit numbers will be $= 8 \times 9 \times 9 = 648$
Now, total favourable cases
$= 225 + 648 = 873$
Required probability
$= \frac{\text{Total favourable cases}}{\text{Total number of cases}}$
$= \frac{873}{2673} = \frac{97}{297}$

Question70

The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is [2021, 25 Feb. Shift-1]

Options:

A. $\frac{1}{72}$

B. $\frac{5}{216}$

C. $\frac{1}{36}$

D. $\frac{1}{54}$

Answer: B

Solution:

Solution:
Given, $ax^2 + bx + c = 0$
According to the question,
$a, b, c \in \{1, 2, 3, 4, 5, 6\}$
$\therefore n(s) = 6 \times 6 \times 6 = 216$
$\because D = 0$
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow b^2 = 4ac$
$\Rightarrow ac = \frac{b^2}{4}$
If $b = 2, ac = 1 \Rightarrow (a = 1, c = 1)$
If $b = 4, ac = 4 \Rightarrow (a = 1, c = 4)$
or $(a = 4, c = 1)$
or $(a = 2, c = 2)$
If $b = 6, ac = 9 \Rightarrow (a = 3, c = 3)$
$\therefore$ Required probability $= 5 / 216$

Question71

The probability that two randomly selected subsets of the set { 1, 2, 3, 4, 5 } have exactly two elements in their intersection, is [2021, 24 Feb. Shift-II]

Options:

A. $\frac{65}{2^7}$

B. $\frac{65}{2^8}$

C. $\frac{135}{2^9}$

D. $\frac{35}{2^7}$

Answer: C

Solution:

Solution:
Given, set $= \{1, 2, 3, 4, 5\}$
Let the two subsets be A and B.
Then, $n(A \cap B) = 2$ (as given in question)
$\therefore$ Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5} = \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

Question72

Two dices are rolle(d) If both dices have six faces numbered 1, 2, 3, 5, 7 and 11 , then the probability that the sum of the numbers on the top faces is less than or equal to 8 is

[2021, 17 March Shift-1]

Options:

A. $\frac{4}{9}$

B. $\frac{17}{36}$

C. $\frac{5}{12}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:
Six faces have numbers $\{1, 2, 3, 5, 7, 11\}$.
Sum of the numbers on top faces of both the sides is less than or equal to 8
$= (1, 1), (1, 2), (1, 3), (1, 5), (1, 7) \rightarrow 5$
$(2, 1), (2, 2), (2, 3), (2, 5) \rightarrow 4$
$(3, 1), (3, 2), (3, 3), (3, 5) \rightarrow 4$
$(5, 1), (5, 2), (5, 3) \rightarrow 3$
$(7, 1) \rightarrow 1$
$n(S) = 5 + 4 + 4 + 3 + 1 = 17$
$n(T) = 6 \times 6 = 36$
$\therefore$ Probability $= \frac{17}{36}$

Question73

Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then, the probability that '10' is followed by ' $01'$ is equal to
[2021, 17 March Shift-II]

Options:

A. $\frac{1}{18}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: D

Solution:

Solution:
Probability of 0 at even place =
$P(\text{O at even place}) = \frac{1}{2}$ (given)
and $P(\text{0 at odd places}) = \frac{1}{3}$ (given)
$\therefore P(\text{1 at even place}) = \frac{1}{2}$

and $P(1 \text{ at odd place}) = 1 - \frac{1}{3} = \frac{2}{3}$
$\therefore P(10 \text{ is followed by } 01)$ = Concepts
$\left(\frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} \times \frac{2}{3}\right)$
$= \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$

Question74

Let A denote the event that a 6 -digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3 . Then, probability of event A is equal to [2021, 16 March Shift-II]

Options:

A. $\frac{9}{56}$

B. $\frac{4}{9}$

C. $\frac{3}{7}$

D. $\frac{11}{27}$

Answer: B

Solution:

Solution:
Given, digits $= \{0, 1, 2, 3, 4, 5, 6\}$
Number of ways in which 6 digit number can be formed using these 7 digits $= 6 \cdot 6!$
Number of ways (0 can't be filled here)
If the required number is divisible by 3 , then the sum of the digits must be divisible by 3 .
Sum of all 7 digits
$= 0 + 1 + 2 + 3 + 4 + 5 + 6$
$= \frac{6 \cdot 7}{2} = 21$
Now, this further implies we have to remove anyone digit from the given digits. We can remove only the multiple of 3 as the sum of all 7 digits is already 21. So, removing any other digit will lead to a number of 6 digit which will not be a multiple of 3.
Possible digits $\Rightarrow \{1, 2, 3, 4, 5, 6\}$
$\{0, 1, 2, 4, 5, 6\}$
$\{0, 1, 2, 3, 4, 5\}$
Case | $\{1, 2, 3, 4, 5, 6\}$
Number of 6 digit numbers $= 6!$
Case II $\{0, 1, 2, 4, 5, 6\}$
Number of 6 digit numbers $= 5.5!$
Case III $\{0, 1, 2, 3, 4, 5\}$
Number of 6 digit numbers $= 5 \cdot 5!$
$\therefore \text{ Probability } = \left(\frac{6! + 5 \cdot 5! + 5 \cdot 5!}{6 \cdot 6!}\right)$
$= \left(\frac{1}{6} + \frac{5}{36} + \frac{5}{36}\right) = \frac{16}{36} = \frac{4}{9}$

Question75

A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is [2021, 16 March Shift-1]

Options:

A. $\frac{3}{4}$

B. $\frac{52}{867}$

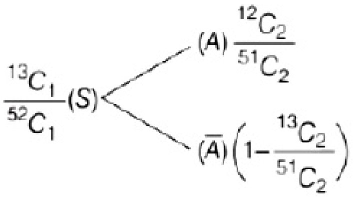
C. $\frac{39}{50}$

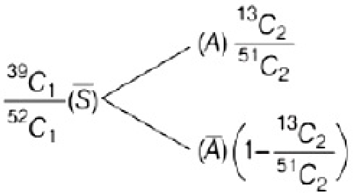
D. $\frac{22}{425}$

Answer: C

Solution:

Solution:
Let S and NS be the respective events of choosing a spade card and a card which is not a spade.
$P(S) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{1}{4}$
$P(N\,S) = \frac{{}^{39}C_1}{{}^{52}C_1} = \frac{3}{4}$
Let $\underline{A}$ be the event when both the chosen cards are spade.
Let $\overline{A}$ be the event when both the chosen cards are not spade.





We have to find the probability of missing card not being spade when both the chosen cards are spade $= P(\overline{S} \mid A)$ By using Bayes theorem,

$$P(\overline{S} \mid A) = \frac{P(\overline{S} \cap A)}{P(A)}$$
$$= \frac{P(A \mid \overline{S})P(\overline{S})}{P(A \mid S)P(S) + P(A \mid \overline{S})P(\overline{S})}$$
$$= \frac{\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)\left(\frac{{}^{39}C_1}{{}^{52}C_1}\right)}{\left(\frac{{}^{13}C_1}{{}^{52}C_1}\right)\left(\frac{{}^{12}C_2}{{}^{15}C_2}\right) + \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)\left(\frac{{}^{39}C_1}{{}^{52}C_1}\right)}$$
$$= \frac{\left(\frac{13 \cdot 12}{51 \cdot 50}\right)\left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right)\left(\frac{12 \cdot 11}{51 \cdot 50}\right) + \left(\frac{3}{4}\right)\left(\frac{13 \cdot 12}{51 \cdot 50}\right)}$$
$$= \frac{13 \times 12 \times 3}{(12 \times 11) + (3 \times 12 \times 13)}$$
$$= \frac{39}{50}$$

Question76

Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let p denote the probability of none of events occur that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). Then, probability of occurrence of E_1 probability of occurrence of E_3 is equal to ....
[2021, 17 March Shift-1]

Answer: 6

Solution:

Solution:
Let $P(E_1) = x$, $P(E_2) = y$, $P(E_3) = z$
Then, $(1 - x)(1 - y)(1 - z) = P \ldots$ (A)
Probability of only $E_1 = P(E_1 \cap \overline{E}_2 \cap \overline{E}_3)$
$\alpha = x(1 - y)(1 - z) \ldots$ (B)
Similarly, $\beta = (1 - x)y(1 - z) \ldots$ (C)
$\gamma = (1 - x)(1 - y)z \ldots$ (D)
On dividing Eq. (A) ÷(B),
$$\frac{P}{\alpha} = \frac{1 - x}{x}$$
$\Rightarrow xP = \alpha - \alpha x \Rightarrow x(P + \alpha) = \alpha$
$$\Rightarrow x = \frac{\alpha}{\alpha + P}$$
Similarly, $y = \frac{\beta}{\beta + P}$
$$z = \frac{\gamma}{\gamma + P}$$
$$\frac{P(E_1)}{P(E_3)} = \frac{x}{z} = \left(\frac{\alpha}{\alpha + P}\right) \times \left(\frac{\gamma + P}{\gamma}\right)$$
$$\frac{x}{z} = \left(\frac{\alpha}{\gamma}\right)\left(\frac{\gamma + P}{\alpha + P}\right) = \left(\frac{1 + P / \gamma}{1 + P / \alpha}\right)$$
Also given, $P = \frac{\alpha\beta}{\alpha - 2\beta} = \frac{2\beta\gamma}{\beta - 3\gamma}$
$\alpha P - 2\beta P = \alpha\beta$
$$\Rightarrow \frac{\alpha P}{\alpha + 2P} = \beta$$
$\beta P - 3P\gamma = 2\beta\gamma$
$\beta(P - 2Y) = 3PY$
$$\Rightarrow \beta = \frac{3P\gamma}{P - 2\gamma}$$
From Eqs. (i)and(ii),
$$\Rightarrow \frac{\alpha P}{\alpha + 2P} = \frac{3P\gamma}{P - 2\gamma}$$
$$\Rightarrow \frac{1}{1 + 2\left(\frac{P}{\alpha}\right)} = \frac{3}{\left(\frac{P}{\gamma}\right) - 2}$$
$\Rightarrow (P / \gamma) - 2 = 3 + 6(P / \alpha)$
$$\frac{P}{\gamma} = 5 + 6\left(\frac{P}{\alpha}\right)$$
$$\frac{x}{z} = \frac{1 + P / \gamma}{1 + P / \alpha} = \frac{1 + 5 + 6(P / \alpha)}{1 + P / \alpha}$$
$$= 6\left(\frac{1 + P / \alpha}{1 + P / \alpha}\right) = 6$$

Question77

Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$, then k lies in the set [2021, 25 July Shift-1]

Options:

A. $\{x \in R: |x - 3| < 1\}$

B. $\{x \in R: |x - 2| \leq 1\}$

C. $\{x \in R: |x - 13| < 1\}$

D. $\{x \in R: |x - 5| \leq 1\}$

Answer: A

Solution:

Solution:
B_3 contains exactly 3 balls among 9 balls.
So, number of ways $= {}^9C_3$
And the rest of the 6 balls can be placed in 3^6 ways.
If E is the event and S is the sample,
$n(E) = {}^9C_3 \times 3^6$
And $n(s) = 4^6$
Probability $= \frac{{}^9C_3 \times 3^6}{4^9}$
According to the question,
$\frac{{}^9C_3 \times 3^6}{4^9} = k\frac{3^9}{4^9}$
$k = \frac{{}^9C_3}{3^3} = \frac{9 \times 8 \times 7}{1 \times 2 \times 3 \times 27} = \frac{28}{9}$
Which satisfies $|x - 3| < 1$.

Question78

The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of $3\}$ is equal to [2021, 27 July Shift-1]

Options:

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: C

Solution:

Solution:
Sample set $S = \{10, 11, 12, \ldots, 99\}$
$n(S) = 99 - 10 + 1 = 90$

n	2^n	$2^n - 2$	Divisible by 3
1	2	0	Yes
2	4	2	No
3	8	6	Yes
4	16	14	No
5	32	30	Yes
6	64	62	No
7	128	126	Yes
8	256	254	No
9	512	510	Yes
10	1024	1022	No
11	2048	2046	Yes
12	4096	4094	No

As we can see, for all the odd values of n, $2^n - 2$ is divisible by 3 .
So, the event set will be all the two digit odd numbers.

$E = \{11, 13, 15, \ldots, 99\}$

$n(E) = \left(\frac{99-11}{2} + 1 \right) = 45$

$\therefore$ Probability $= \frac{n(E)}{n(S)} = \frac{45}{90} = \frac{1}{2}$

Question79

The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is
[2021, 20 July Shift-1]

Options:

A. $\frac{7}{36}$

B. $\frac{2}{9}$

C. $\frac{1}{6}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:

$f(x) = x^2 + 2(a+4)x - 5a + 64$

$f(x) > 0$, When $D < 0$ as coefficient of $x^2 > 0$

$4(a+4)^2 - 4(64 - 5a) < 0$

$\Rightarrow a^2 + 16 + 8a - 64 + 5a < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a+16)(a-3) < 0$

$\Rightarrow$ n

According to the question, $a \in [-5, 30]$

$n(S) = 30 + 5 + 1 = 36$

$n(E) = 2 + 5 + 1 = 8$

$\Rightarrow$ 16, 3)$\Rightarrow$

$\Rightarrow$ Probability $= \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$

Question80

Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non singular, is
[2021, 22 July Shift-II]

Options:

A. $\frac{45}{162}$

B. $\frac{23}{81}$

C. $\frac{22}{81}$

D. $\frac{43}{162}$

Answer: D

Solution:

Solution:

Let $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, \left|A\right| = ad - bc$

$n(s) = 6 \times 6 \times 6 \times 6 = 6^4$

For non-singular matrix $|A| \neq 0$

$\Rightarrow\ ad \neq bc$

Now, a, b, c and d should be different.

If $ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

If $a = 1$, $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$

$\{a, b, c, d\} = \{1, 2, 3, 6\}$ or $\{1, 3, 2, 6\}$

If $a = 2$

$\frac{2}{1} = \frac{6}{3}, \frac{2}{3} = \frac{4}{6}, \frac{2}{4} = \frac{3}{6} = \frac{1}{2}, \frac{2}{6} = \frac{1}{3}$

So, $\{a, b, c, d\} = \{2, 1, 6, 3\}, \{2, 3, 4, 6\}, \{2, 4, 3, 6\}, \{2, 4, 1, 2\}\{2, 6, 1, 3\}$

If $a = 3$, $\frac{3}{1} = \frac{6}{2}, \frac{3}{2} = \frac{6}{4}$

$\{a, b, c, d\} = \{3, 1, 6, 2\}, \{3, 2, 6, 4\}\{4, 6, 2, 3\}$

If $a = 4$, $\frac{4}{2} = \frac{6}{3}, \frac{4}{6} = \frac{2}{3}, \{a, b, c, d\} = \{4, 2, 6, 3\}$

If $a = 5 \Rightarrow$ Not possible

If $a = 6$, $\frac{6}{2} = \frac{3}{1}, \frac{6}{3} = \frac{2}{1}, \frac{6}{4} = \frac{3}{2}$

$\{a, b, c, d\} = \{6, 2, 3, 1\}, \{6, 3, 2, 1\}, \{6, 4, 3, 2\}$

So, $n(\overline{E}) = 2 + 5 + 2 + 2 + 0 + 3 = 16$

So, $n(E) = {}^6C_4 - 16$

Probability $= \frac{{}^6C_4 - 16}{6^4} = \frac{43}{162}$

Question81

Let A, B and C be three events such that the probability that exactly one of A and B occurs is $(1 - k)$, the probability that exactly one of B and C occurs is $(1 - 2k)$, the probability that exactly one of C and A occurs is $(1 - k)$ and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is [2021, 20 July Shift-II]

Options:

A. greater than 1 / 8 but less than 1 / 4

B. greater than 1 / 2

C. greater than 1 / 4 but less than 2 / 2

D. exactly equal to $\frac{1}{2}$

Answer: B

Solution:

Solution:

There are three events A, B, C.

Probability that exactly one of A and B occur is

$P(\overline{A} \cap B) + P(A \cap \overline{B})$

$= 1 - k \quad \ldots(i)$

Probability that exactly one of B and C occur is

$P(\overline{B} \cap C) + P(B \cap \overline{C}) = 1 - 2k \quad \ldots(ii)$

Probability that exactly one of C and A occur is $P(\overline{A} \cap C) + P(A \cap \overline{C}) = 1 - k$. (iii)
Probability of all A, B, C occur simultaneously, is
$P(A \cap B \cap C) = k^2$
From Eqs. (i), (ii) and (iii),
$P(A) + P(B) - 2P(A \cap B) = 1 - k$ . . . (v)
$P(B) + P(C) - 2P(B \cap C) = 1 - 2k$ ... (vi)
$P(A) + P(C) - 2P(A \cap C) = 1 - k$ ...(vii)
Adding Eqs. (v), (vi) and (vii),
$2[P(A) + P(B) + P(C) - P(A \cap B)$
$-P(B \cap C) - P(C \cap A)]$
$= (1 - k) + (1 - 2k) + (1 - k) = 3 - 4k$
$\therefore P(A) + P(B) + P(C) - P(A \cap B)$
$-P(B \cap C) - P(A \cap C) = \frac{3-4k}{2}$
$\Rightarrow P(A \cup B \cup C) = \frac{3-4k}{2} + P(A \cap B \cap C)$
$= \frac{3-4k}{2} + k^2 = \frac{2k^2 - 4k + 3}{2}$
$\Rightarrow P(A \cup B \cup C) = \frac{2(k-1)^2 + 1}{2}$
$= (k-1)^2 + \frac{1}{2} > \frac{1}{2}$
$\therefore P(A \cup B \cup C) > \frac{1}{2}$

Question82

Let S = {1, 2, 3, 4, 5, 6}. Then, the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is
[2021, 31 Aug. Shift-II]

Options:

A. $\frac{1}{10}$

B. $\frac{1}{15}$

C. $\frac{1}{5}$

D. $\frac{1}{30}$

Answer: A

Solution:

Solution:
$\because g : S \rightarrow S$
where $S = \{1, 2, 3, 4, 5, 6\}$
Total number of onto function $= 6!$
Given condition $g(3) = 2g(1)$
If $g(1) = 1$, then $g(3) = 2$
$g(1) = 2$, then $g(3) = 4$
and $g(1) = 3$, then $g(3) = 6$
There are only 3 possible cases
Number onto functions $= 3 \times 4!$
Required probability $= \frac{3 \times 4!}{6!} = \frac{1}{10}$

Question83

When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$.
All other faces occur with probability 1 / 6. Note that opposite faces sum to 7

in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice is 13 / 96, then the value of x is
[2021, 27 Aug. Shift-1]

Options:

A. 1 / 16

B. 1 / 8

C. 1 / 9

D. $\frac{1}{12}$

Answer: B

Solution:

Solution:

Let $P_1 = \frac{1}{6} - x$

$P_6 = \frac{1}{6} + x$[sum of opposite faces is 7]

$P_2 = P_3 = P_4 = P_5 = \frac{1}{6}$ Psum $= 7\big)$

$= 2(P_6 \times P_1 + P_5 \times P_2 + P_4 \times P_5)$

$\Rightarrow \frac{13}{96} = 2\left(\frac{1}{6} + x\right)\left(\frac{1}{6} - x\right) + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2\Big]$

$\Rightarrow \frac{13}{96} = 2\left[\frac{1}{36} - x^2 + \frac{1}{36} + \frac{1}{36}\right]$

$\Rightarrow \frac{1}{6} - 2x^2 = \frac{13}{96}$

$\Rightarrow x^2 = \frac{1}{64} \Rightarrow x = \frac{1}{8}$

Question84

An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is P, then 98P is equal to
[2021, 31 Aug. Shift-1]

Answer: 28

Solution:

Solution:
$P(A) =$ Probability of first unit to function
$P(B) =$ Probability of second unit to function
$P(A) = 0.9$, $P(\overline{A}) = 0.1$
$P(B) = 0.8$, $P(\overline{B}) = 0.2$
$AB \rightarrow$ Operate
$A\overline{B} \rightarrow$ Not operate
$\overline{A}B \rightarrow$ Not operate
$\overline{A}\overline{B} \rightarrow$ Not operate
According to question, first failed and second functioning, given that instrument has failed, So according to Bayes theorem

$$\therefore P = \frac{P(\overline{A}) \cdot P(B)}{P(A)P(\overline{B}) + P(\overline{A})P(B) + P(\overline{A})P(\overline{B})}$$

$= \frac{(0.1 \times 0.8)}{(0.9 \times 0.2) + (0.1 \times 0.8) + (0.1 \times 0.2)} = \frac{8}{28}$

$98P = \frac{8}{28} \times 98 = 28$

Question85

Let A and B be independent events such that P(A) = p and P(B) = 2p. The largest value of p, for which P (exactly one of A, B occurs) $= \frac{5}{9}$, is [2021, 26 Aug. Shift-1]

Options:

A. $\frac{1}{3}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{5}{12}$

Answer: D

Solution:

Solution:

P(Exactly one out of A or B occurs $\Big) = \frac{5}{9}$

$p(A \cap \overline{B}) + p(\overline{A} \cap B) = \frac{5}{9}$

$p(A)p(\overline{B}) + p(\overline{A})p(B) = \frac{5}{9}$

$\therefore\ p(1-2p) + (1-p)2p = \frac{5}{9}$

$\Rightarrow\ 9p(1-2p) + 18p(1-p) = 5$

$\Rightarrow\ 9p - 18p^2 + 18p - 18p^2 = 5$

$\Rightarrow\ 36^2p - 27p + 5 = 0$

$\Rightarrow\ (3p-1)(12p-5) = 0$

$p = \frac{1}{3}$ or $p = \frac{5}{12}$

$p_{max} = \frac{5}{12}$

Question86

A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability P(X ≥ 5 | X > 2) is [2021, 26 Aug. Shift-II]

Options:

A. $\frac{125}{216}$

B. $\frac{11}{36}$

C. $\frac{5}{6}$

D. $\frac{25}{36}$

Answer: D

Solution:

Solution:

$P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5 \cap x > 2)}{P(x > 2)} = \frac{P(x \geq 5)}{P(x > 2)}$

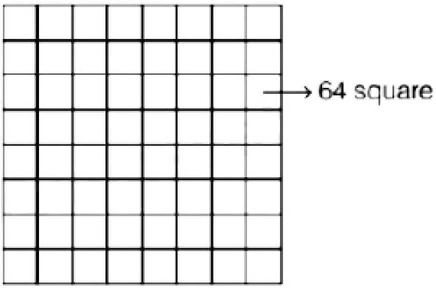
$= \frac{\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \ldots \infty}{\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \ldots \infty}$

$= \frac{\left(\frac{5}{6}\right)^4 \cdot \left(\frac{1}{6}\right)}{\left(\frac{5}{6}\right)^2 \cdot \left(\frac{1}{6}\right)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$\frac{1 - \frac{5}{6}}{1 - \frac{5}{6}}$

Question87

Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is



[2021, 01 Sep. Shift-II]

Options:

A. $\frac{2}{7}$

B. $\frac{1}{18}$

C. $\frac{1}{7}$

D. $\frac{1}{9}$

Answer: B

Solution:

Solution:

Total ways $= {}^{64}C_2 = 32 \times 63$

Favourable ways $= 2 \times 7 \times 8$

Required probability $= \frac{2 \times 7 \times 8}{32 \times 63} = \frac{1}{18}$

Question88

In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability

that the second A-card appears before the third B-card is:
[Jan. 9,2020 (I)]

Options:

A. $\frac{9}{16}$

B. $\frac{11}{16}$

C. $\frac{13}{16}$

D. $\frac{15}{16}$

Answer: B

Solution:

Solution:
P(second A – card appears before the third B– card) = P (AA) + P (ABA) + P (BAA) + P (ABBA) + P (BBAA) + P (BABA)
$\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{11}{16}$

Question89

Let A and B be two independent events such that$P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE?
[Jan. 8, 2020 (I)]

Options:

A. $P(A / B)=\frac{2}{3}$

B. $P(A / B')=\frac{1}{3}$

C. $P(A' / B')=\frac{1}{3}$

D. $P(A / (A \cup B))=\frac{1}{4}$

Answer: B

Solution:

Solution:
A and B are independent events.

So, $P\left(\frac{A}{B'}\right)=\frac{P(A \cap B')}{P(B')}=\frac{\frac{1}{3}-\frac{1}{3}\cdot\frac{1}{6}}{\frac{1}{6}}=\frac{1}{3}$

Question90

An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value -1 . Then the expected value of X, is:
[Jan. 7, 2020 (I)]

Options:

A. $\frac{3}{16}$

B. $\frac{1}{8}$

C. $-\frac{3}{16}$

D. $-\frac{1}{8}$

Answer: B

Solution:

Solution:

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = No. of times head occur consecutively
Now expectation

$= \sum xP(k) = (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32}$

$+3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32} = \frac{1}{8}$

Question91

In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to: [Jan. 7,2020 (II)]

Options:

A. $\frac{17}{8}$

B. $\frac{17}{4}$

C. $\frac{17}{2}$

D. 4

Answer: A

Solution:

Solution:
Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27\times 17}{64\times 8}$

$= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$

$\therefore k = \frac{17}{8}$

Question92

Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability thatA or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is:
[Jan. 8, 2020 (II)]

Options:

A. 0.02

B. 0.20

C. 0.01

D. 0.10

Answer: D

Solution:

Solution:
P(exactly one) = $\frac{2}{5}$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

P(A or B) = $P(A \cup B) = \frac{1}{2}$

$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$\therefore P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10} = 0.10$

Question93

If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :
[Jan. 9, 2020 (II)]

Options:

A. $\frac{965}{2^{11}}$

B. $\frac{965}{2^{10}}$

C. $\frac{945}{2^{10}}$

D. (Bonus)

Answer: D

Solution:

Solution:
Total number of ways placing 10 different balls in 4 distinct boxes $= 4^{10}$
Since, two of the 4 distinct boxes contains exactly 2 and 3 balls.
Then, there are three cases to place exactly 2 and 3 balls in 2 of the 4 boxes.
Case-1: When boxes contains balls in order 2,3,0,5

Then, number of ways of placing the balls $= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4!$

Case-2: When boxes contains ball in order 2, 3, 1, 4.

Then, number of ways of placing the balls $= \dfrac{10!}{2! \times 3! \times 1! \times 4!} \times 4!$

Case-3: When boxes contains ball in order 2, 3, 2, 3

Then, number of ways of placing the balls $= \dfrac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$

Therefore, number of ways of placing the balls that contains exactly 2 and 3 balls. $= \dfrac{10!}{2! \times 3! \times 0! \times 5!} \times 4! +$

$\dfrac{10!}{2! \times 3! \times 1! \times 4!} \times 4! + \dfrac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$

$= 2^5 \times 17 \times 945$

Hence, the required probability

$= \dfrac{2^5 \times 17 \times 945}{4^{10}} = \dfrac{17 \times 945}{2^{15}}$

Question94

A random variable X has the following probability distribution:

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then, P(X > 2) is equal to:
[Jan. 9, 2020 (II)]

Options:

A. $\frac{7}{12}$

B. $\frac{1}{36}$

C. $\frac{1}{6}$

D. $\frac{23}{36}$

Answer: D

Solution:

Solution:

$\sum P(K) = 1 \Rightarrow 6K^2 + 5K = 1$

$6K^2 + 5K - 1 = 0$

$6K^2 + 6K - K - 1 = 0$

$\Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = \frac{1}{6}$ ($K = -1$ rejected)

$P(X > 2) = K + 2K + 5K^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6 + 12 + 5}{36} = \frac{23}{36}$

Question95

In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins.
The probability of A winning the game is :
[Sep. 04, 2020 (II)]

Options:

A. $\frac{5}{31}$

B. $\frac{31}{61}$

C. $\frac{5}{6}$

D. $\frac{30}{61}$

Answer: D

Solution:

Solution:

Probability of sum getting 6, $P(A) = \frac{5}{36}$

Probability of sum getting 7, $P(B) = \frac{6}{36} = \frac{1}{6}$

$P(A wins) = P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A}) . P(\overline{B})P(\overline{A})P(\overline{B})P(A) + ...$

$\Rightarrow \frac{5}{36} + \left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right) + ... \infty$

$\Rightarrow \frac{5}{36}\left(1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + ... \infty\right)$

$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}} = \frac{30}{61} \left(\because S_\infty = \frac{a}{1-r}\right)$

Question96

A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is : [Sep. 03, 2020 (I)]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{8}$

D. $\frac{1}{9}$

Answer: D

Solution:

Solution:

E_1 [the event for getting score a multiple of 4] = (1, 3), (3, 1), (2, 2), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4) & (6, 6)

E_2[4 has appeared atleast once] = (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3)(4,5) & (4, 6)

$E_1 \cap E_2 = (4, 4)$

$P\left(\frac{E_2}{E_1}\right) = \frac{1}{9}$

Question97

The probability that a randomly chosen 5 -digit number is made from exactly two digits is:
[Sep. 03, 2020 (II)]

Options:

A. $\frac{135}{10^4}$

B. $\frac{121}{10^4}$

C. $\frac{150}{10^4}$

D. $\frac{134}{10^4}$

Answer: A

Solution:

Solution:
Total outcomes $= 9(10^4)$
Favourable outcomes $= {}^9C_2(2^5-2) + {}^9C_1(2^4-1) = 36(30) + 9(15)$
Probability $= \frac{36 \times 30 + 9 \times 15}{9 \times 10^4} = \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$

Question98

Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :
[Sep. 02, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{8}{17}$

C. $\frac{4}{17}$

D. $\frac{2}{5}$

Answer: B

Solution:

Solution:
Let B_1 and B_2 be the boxes and N be the number of non-prime number.
$\because P(B_1) = P(B_2) = \frac{1}{2}$
and P (non-prime number)
$= P(B_1) \times P\left(\frac{N}{B_1}\right) + P(B_2) \times P\left(\frac{N}{B_2}\right)$
$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$

So,

$$P\left(\frac{B_1}{N}\right) = \frac{P(B_1) \times P\left(\frac{N}{B_1}\right)}{P(B_1) \times P\left(\frac{N}{B_1}\right) + P(B_2) \times P\left(\frac{N}{B_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{15}{40}} = \frac{8}{17}$$

Question99

Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$
and $P(E_1 \cap E_2 \cap E_3) = 0$
Then $P(E_2^C \cap E_3^C / E_1)$ is equal to:
[Sep. 02, 2020 (II)]

Options:

A. $P(E_2^C) + P(E_3)$

B. $P(E_3^C) - P(E_2^C)$

C. $P(E_3) - P(E_2^C)$

D. $P(E_3^C) - P(E_2)$

Answer: D

Solution:

Solution:

$$P\left(\frac{E_2^C \cap E_3^C}{E_1}\right) = \frac{P[E_1 \cap E_2^C \cap E_3^C)]}{P(E_1)}$$

$$= \frac{P(E_1) - P[E_1 \cap (E_2 \cup E_3)]}{P(E_1)} = [\because P(A \cap B^C) = P(A) - P(A \cap B)]$$

$$= \frac{P(E_1) - P[(E_1 \cap E_2) \cup (E_1 \cap E_3)]}{P(E_1)}$$

$$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$ $[\because P(A \cap B) = P(A) . P(B)]$

$= P(E_2^C) - P(E_3)$ or $P(E_3^C) - P(E_2)$

Question100

Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ______.
[NA Sep. 05, 2020 (I)]

Answer: 11

Solution:

Probability of getting at least two 3's or 5's in one trial

$= {}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$

$= \frac{33}{3^4} = \frac{11}{27}$

$E(x) = np = 27\left(\frac{11}{27}\right) = 11$

Question101

In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is ________.
[NA Sep. 05, 2020 (II)]

Answer: 11

Solution:

Solution:
Let 'n' bombs are required, then

$1 - {}^nC_1 \cdot \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} - {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$\Rightarrow \frac{1}{100} \geq \frac{n+1}{2^n} \Rightarrow 2^n \geq 100(n+1) \Rightarrow n \geq 11$

Question102

The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.
[NA Sep. 04, 2020 (I)]

Answer: 3

Solution:

$p = \frac{1}{10}, q = \frac{9}{10}$

P (not hitting target in n trials) $= \left(\frac{9}{10}\right)^n$

P(at least one hit $) = 1 - \left(\frac{9}{10}\right)^n$

$\because 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4} \Rightarrow (0.9)^n < 0.75$

$\therefore n_{minimum} = 3$

Question103

Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:
[Sep. 06, 2020 (I)]

Options:

A. $\frac{15}{101}$

B. $\frac{5}{101}$

C. $\frac{5}{33}$

D. $\frac{10}{99}$

Answer: C

Solution:

Solution:
For an A.P. $2b = a + c$ (even), so both a and c even numbers or odd numbers from given numbers and b number will be fixed automatically.

Required probability $= \frac{{}^{6}C_{2} + {}^{5}C_{2}}{{}^{11}C_{3}} = \frac{25}{165} = \frac{5}{33}$

Question104

The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:
[Sep. 06, 2020 (II)]

Options:

A. [0.35, 0.36]

B. [0.25, 0.35]

C. [0.20, 0.25]

D. [0.36, 0.40]

Answer: B

Solution:

Solution:
$\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 1 - 0.8 = 0.2$
Now,
$\because P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
$\Rightarrow \alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$
$\Rightarrow \beta = 1.2 - \alpha$
$\because \alpha \in [0.85, 0.95]$ then $\beta \in [0.25, 0.35]$

Question105

Let S = {1, 2, ..., 20}. A subset B of S is said to be “nice”, if the sum of the elements of B is 203. Than the probability that a randomly chosen subset of S is “nice” is :
[Jan. 11, 2019 (II)]

Options:

A. $\frac{7}{2^{20}}$

B. $\frac{5}{2^{20}}$

C. $\frac{4}{2^{20}}$

D. $\frac{6}{2^{20}}$

Answer: B

Solution:

Solution:
Since total number of subsets of the set S = 2^{20}
Now, the sum of all number from 1 to 20 = $\frac{20 \times 21}{2} = 210$
Then, find the sets which has sum 7 .
(1) $\{7\}$
(2) $\{1, 6\}$
(3) $\{2, 5\}$
(4) $\{3, 4\}$
(5) $\{1, 2, 4\}$
Then, there is only 5 sets which has sum 203
Hence required probability $= \frac{5}{2^{20}}$

Question106

In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :
[Jan. 12, 2019 (II)]

Options:

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{5}{6}$

Answer: A

Solution:

Solution:
$P =$ Set of students who opted for NCC
$Q =$ Set of Students who opted for NSS
$n(P) = 40, n(Q) = 30, n(P \cap Q) = 20$

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
$= 40 + 30 - 20$
$= 50$

$\therefore$ Hence, required probability $= 1 - \frac{50}{60}$

$= \frac{1}{6}$

Question107

In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to:
[Jan. 12, 2019 (I)]

Options:

A. $\frac{200}{6^5}$

B. $\frac{150}{6^5}$

C. $\frac{225}{6^5}$

D. $\frac{175}{6^5}$

Answer: D

Solution:

Solution:
(d) Since, the experiment will end in the fifth throw.
Hence, the possibilities are $4**44$, $*4*44$, $***44$ (where * is any number except 4)

Required Probability $= \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$

$= \frac{25 + 25 + 125}{6^5} = \frac{175}{6^5}$

Question108

In a game, a man wins Rs.100 if the gets 5 or 6 on a throw of a fair die and loses Rs.50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is:
[Jan. 12, 2019 (II)]

Options:

A. $\frac{400}{9}$ loss

B. 0

C. $\frac{400}{3}$ gain

D. $\frac{400}{3}$ loss

Answer: B

Solution:

Solution:

Probability of getting 5 or 6 = $P(E) = \frac{2}{6} = \frac{1}{3}$

Probability of not getting 5 or 6 = $P(E) = 1 - \frac{1}{3} = \frac{2}{3}$

E will consider as success.

Event	Success in Ist attempt	Success in IInd attempt	Success in IIIrd attempt	No success in IIIrd attempt
Probability	$\frac{1}{3}$	$\frac{2}{3} \times \frac{1}{3}$	$\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}$	$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
Gain/loss	100	50	0	-150

His expected gain/loss = $\frac{1}{3} \times 100 + \frac{2}{9} \times 50 + \frac{8}{27} \times (-150)$

$= \frac{900 + 300 - 1200}{27} = 0$

Question109

Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:
[Jan. 11, 2019 (I)]

Options:

A. $\frac{7}{10}$

B. $\frac{1}{2}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

Answer: C

Solution:

Solution:

Probability of getting sum of selected two numbers is even $= P(E_1) = \frac{{}^5C_2 + {}^5C_2}{{}^{11}C_2}$

Probability of getting sum is even and selected numbers are also even $P(E_2) = \frac{{}^5C_2}{{}^{11}C_2}$

Hence, $P\left(\frac{E_2}{E_1}\right) = \frac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \frac{10}{15 + 10} = \frac{2}{5}$.

Question110

An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is:

[Jan 10, 2019 (I)]

Options:

A. $\frac{13}{36}$

B. $\frac{15}{72}$

C. $\frac{19}{72}$

D. $\frac{19}{36}$

Answer: C

Solution:

Solution:

P(Outcome is head) $= \frac{1}{2}$

P(Outcome is tail) $= \frac{1}{2}$

P(7 or 8 is the sum of two dice) $= \frac{6}{36} + \frac{5}{36} = \frac{11}{36}$

P(7 or 8 is the number of card) $= \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

Required probability $= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9}$

$= \frac{1}{2}\left(\frac{11+8}{36}\right) = \frac{19}{72}$

Question111

If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is:
[Jan. 10, 2019 (II)]

Options:

A. 3

B. 6

C. 5

D. 4

Answer: C

Solution:

Solution:

Let the number of independent shots required to hit the target at least once be n, then

$1 - \left(\frac{2}{3}\right)^{n} > \frac{5}{6}\left(\frac{2}{3}\right)^{n} < \frac{1}{6}$

Hence, the above inequality holds when least value of n is 5

Question112

Two cards are drawn successively with replacement from a well-shuffled deck

of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals:
[Jan 09, 2019 (I)]

Options:

A. 49/169

B. 52/169

C. 24/169

D. 25/169

Answer: D

Solution:

Solution:
X = number of aces drawn
$\therefore P(X = 1) + P(X = 2)$
$= \left\{ \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} \right\} + \left\{ \frac{4}{52} \times \frac{4}{52} \right\}$
$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$

Question113

An urn contains 5 red and 2 green balls. Aball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red is:
[Jan. 09, 2019 (II)]

Options:

A. $\frac{21}{49}$

B. $\frac{27}{49}$

C. $\frac{26}{49}$

D. $\frac{32}{49}$

Answer: D

Solution:

Solution:
Let G represents drawing a green ball and R represents drawing a red ball
So, the probability that second drawn ball is red $= P(G) . P\left(\frac{R}{G}\right) + P(R)P\left(\frac{R}{R}\right)$
$= \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$
$= \frac{12 + 20}{49}$
$= \frac{32}{49}$

Question114

A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to:
[Jan. 11, 2019 (II)]

Options:

A. 4

B. $4\sqrt{3}$

C. $3\sqrt{2}$

D. $\frac{4\sqrt{3}}{3}$

Answer: B

Solution:

Solution:

$P(\text{ white ball }) = \frac{30}{40} = \frac{3}{4}, Q(\text{ red ball }) = \frac{10}{40} = \frac{1}{4}, n = 16$

$\frac{\text{Mean of X}}{\text{standard deviation ofX}} = \frac{nP}{\sqrt{nPQ}} = \frac{\sqrt{nP}}{\sqrt{Q}}$

$= \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = \sqrt{48} = 4\sqrt{3}$

Question115

Let A and B be two non-null events such that A ⊂ B. Then, which of the following statements is always correct?
[April 08, 2019 (I)]

Options:

A. $P(A \mid B) = P(B) - P(A)$

B. $P(A \mid B) \geq P(A)$

C. $P(A \mid B) \leq P(A)$

D. $P(A \mid B) = 1$

Answer: B

Solution:

Solution:

$\because A \subset B$; so $A \cap B = A$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)}$

$\because P(B) \leq 1$

$\Rightarrow P\left(\frac{A}{B}\right) \geq P(A)$

Question116

The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is : [April. 08, 2019 (II)]

Options:

A. 5

B. 3

C. 4

D. 2

Answer: C

Solution:

Solution:
Let, p is probability for getting head and is probability for getting tail.

$p = P(H) = \frac{1}{2}, q = 1 - p = \frac{1}{2}$

$P(x \geq 1) \geq \frac{9}{10} \Rightarrow 1 - P(x = 0) \geq \frac{9}{10}$

$1 - {}^{n}C_{0}\left(\frac{1}{2}\right)^{n} \geq \frac{9}{10} \Rightarrow \frac{1}{2^{n}} \leq 1 - \frac{9}{10} \Rightarrow \frac{1}{2^{n}} \leq \frac{1}{10}$

$2^{n} \geq 10 \Rightarrow n \geq 4 \Rightarrow n_{min} = 4$

Question117

Four persons can hit a target correctly with probabilities $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is: [April 09, 2019 (I)]

Options:

A. $\frac{25}{192}$

B. $\frac{7}{32}$

C. $\frac{1}{192}$

D. $\frac{25}{32}$

Answer: D

Solution:

Solution:
P (at least one hits the target) = 1 – P (none of them hits the target)

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{7}{32} = \frac{25}{32}$

Question118

Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is:
[April 10, 2019 (I)]

Options:

A. $\frac{1}{11}$

B. $\frac{1}{10}$

C. $\frac{1}{12}$

D. $\frac{1}{17}$

Answer: A

Solution:

Solution:
Let, A = At least two girls
B = All girls

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)}$$

$$= \frac{\left(\frac{1}{4}\right)^4}{1 - {}^4C_0\left(\frac{1}{2}\right)^4 - {}^4C_1\left(\frac{1}{2}\right)^4} = \frac{1}{16-1-4} = \frac{1}{11}$$

Question119

Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :
[April 10, 2019 (II)]

Options:

A. 5

B. 6

C. 8

D. 7

Answer: D

Solution:

Solution:
According to the question,

$$1 - \left(\frac{1}{2}\right)^n > \frac{99}{100} \Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100} \Rightarrow n \geq 7$$

Hence, minimum value is 7 .

Question120

If three of the six vertices of a regular hexazon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :
[April 12, 2019 (I)]

Options:

A. $\frac{1}{10}$

B. $\frac{1}{5}$

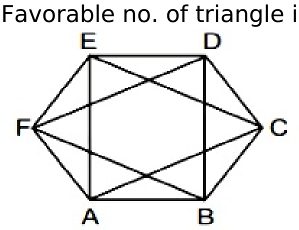
C. $\frac{3}{10}$

D. $\frac{3}{20}$

Answer: A

Solution:

Solution:
Total no. of triangles $= {}^6C_3$
Favorable no. of triangle i.e, equilateral triangles (ΔAEC and ΔBDF) = 2



Hence, required probability $= \frac{2}{{}^6C_3} = \frac{1}{10}$

Question121

For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is:
[April 12, 2019(II)]

Options:

A. $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

B. $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

C. $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

D. $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

Answer: C

Solution:

Solution:
Let p is the probability that candidate can solve a problem and q is the probability that candidate can not not solve a problem.

$p = \frac{4}{5}$ and $q = \frac{1}{5}$($\because p + q = 1$)
Probability of solving either 50 or 49 problem by the candidate
$= {}^{50}C_{50} . p^{50} . q^0 + {}^{50}C_{49} . p^{49} . q^1 = p^{49}[p + 50q]$
$= \left(\frac{4}{5}\right)^{49} . \left(\frac{4}{5} + \frac{50}{5}\right) = \frac{54}{5} . \left(\frac{4}{5}\right)^{49}$

Question122

Let a random variable X have a binomial distribution with mean 8 and variance 4. If P(X d "2) = $\frac{k}{2^{16}}$, then k is equal to:

[April 12, 2019 (I)]

Options:

A. 17

B. 121

C. 1

D. 137

Answer: D

Solution:

Solution:
Given mean $\mu = 8$ and variance $\sigma^2 = 4$
$\Rightarrow \mu = np = 8$ and $\sigma^2 = npq = 4$
$p + q = 1 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}$ and $n = 16$
$\because P(X \le 2) = \frac{k}{2^{16}}$
$\therefore {}^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$
$\Rightarrow k = (1 + 16 + 120) = 137$

Question123

A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

[April 12, 2019 (II)]

Options:

A. $\frac{1}{2}$ gain

B. $\frac{1}{4}$ loss

C. $\frac{1}{2}$ loss

D. 2 gain

Answer: C

Solution:

Solution:
Let X be the random variable which denotes the Rs gained by the person.
Total cases $= 6 \times 6 = 36$
Favorable cases for the person on winning $Rs.15$ are (1,1),(2,2),(3,3),(4,4), (5,5),(6,6) i.e., 6 cases.
$\therefore P(X = 15) = \frac{6}{36} = \frac{1}{6}$
Favorable cases for the person on winning $Rs.12$ are (6,3) , (5,4),(4,5), (3,6) i.e., 4
$\therefore P(X = 12) = \frac{4}{36} = \frac{1}{9}$
Remaining cases $= 36 - 6 - 4 = 26$
$\therefore P(X = -6) = \frac{26}{36} = \frac{13}{18}$

X	15	12	-6
P(X)	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
X.P(X)	$\frac{5}{2}$	$\frac{4}{3}$	$-\frac{13}{3}$

Hence, $E(X) = \sum X.P(X) = \frac{5}{2} + \frac{4}{3} - \frac{13}{3} = -\frac{1}{2}$

Question124

Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children ofthe family B is $\frac{1}{12}$, then the number of children in each family is? [Online April 16, 2018]

Options:

A. 4

B. 6

C. 3

D. 5

Answer: D

Solution:

Solution:
Let the number of children in each family be x.
Thus the total number of children in both the families are 2x
Now, it is given that 3 tickets are distributed amongst the children of these two families.
Thus, the probability that all the three tickets go to the children in family B
$= \frac{{}^{x}C_3}{{}^{2x}C_3} = \frac{1}{12}$
$\Rightarrow \frac{x(x-1)(x-2)}{2x(2x-1)(2x-2)} = \frac{1}{12}$
$\Rightarrow \frac{(x-2)}{(2x-1)} = \frac{1}{6}$
$\Rightarrow x = 5$
Thus, the number of children in each family is 5.

Question125

A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without

replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is
[Online April 15, 2018]

Options:

A. $\frac{7}{16}$

B. $\frac{9}{32}$

C. $\frac{7}{8}$

D. $\frac{9}{16}$

Answer: A

Solution:

Solution:

Probability of drawing a white ball and then a red ballfrom bag B is given by $\frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{2}{9}$

Probability of drawing a white ball and then a red ball frombag A is given by $\frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \frac{2}{7}$

Hence, the probability of drawing a white ball and then ared ball from bag B = $\frac{\frac{2}{9}}{\frac{2}{7}+\frac{2}{9}} = \frac{2 \times 7}{18+14} = \frac{7}{16}$

Question126

A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :
[2018]

Options:

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{3}{4}$

D. $\frac{3}{10}$

Answer: A

Solution:

Solution:
Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.
Now, in the given bag there are 4 red and 6 black balls.
$\therefore P(R_1) = \frac{4}{10}$ and $P(B_1) = \frac{6}{10}$

And, $P\left(\frac{R_2}{R_1}\right) = \frac{6}{12}$ and $P\left(\frac{R_2}{B_1}\right) = \frac{4}{12}$

Now, required probability

$= P(R_1) \times P\left(\frac{R_2}{R_1}\right) + P(B_1) \times P\left(\frac{R_2}{B_1}\right)$

$= \left(\frac{4}{10} \times \frac{6}{12}\right) + \left(\frac{6}{10} \times \frac{4}{12}\right) = \frac{2}{5}$

Question127

Let A, B and C be three events, which are pair-wise independence and $\overline{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\overline{A} \cap \overline{B}) \mid C]$ is equal to.
[Online April 16, 2018]

Options:

A. $P(A) + P(\overline{B})$

B. $P(\overline{A}) - P(\overline{B})$

C. $P(\overline{A}) - P(B)$

D. $P(\overline{A}) + P(\overline{B})$

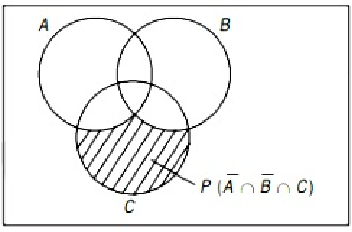
Answer: C

Solution:

Solution:

Here, $P(\overline{A} \cap \overline{B} \mid C) = \frac{P(\overline{A} \cap \overline{B} \cap C)}{P(C)}$.

$= \frac{P(C) - P(A \cap C - P(B \cap C) + P(A \cap B \cap C))}{P(C)}$



$= 1 - \left[\frac{P(A) . P(C) + P(B) . P(C)}{P(C)}\right]$

$= 1 - P(A) - P(B) = P(\overline{A}) - P(B)$ ($\because P(A \cap B \cap C) = 0$)

Question128

A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of ' p ' is
[Online April 15, 2018]

Options:

A. $\frac{1}{3}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Answer: A

Solution:

Solution:
If the outcome is one of the following:
H, TTH, TTTTH, ..., then X wins.
As subsequent tosses are independent, so the probability that X wins is
$p + \frac{p}{4} + \frac{p}{16} + \ldots. = \frac{4p}{3}$
Similarly Y wins if the outcome is one of the following: TH, TTTH, TTTTTH, ...
Therefore, the probability that Y wins is
$\frac{1-p}{2} + \frac{1-p}{8} + \frac{1-p}{32} = \frac{2(1-p)}{3}$
Since, the probability of winning the game by both the players is equal then, we have
$\frac{4p}{3} = \frac{2(1-p)}{3} \Rightarrow p = \frac{1}{3}$

Question129

For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is : [2017]

Options:

A. $\frac{3}{16}$

B. $\frac{7}{32}$

C. $\frac{7}{16}$

D. $\frac{7}{64}$

Answer: C

Solution:

Solution:
P (exactly one of A or B occurs) $= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ......(1)

$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ......(2)
P (Exactly one of C or A occurs)
$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ....(3)
Adding (1),(2) and (3), we get
$2\sum P(A) - 2\sum P(A \cap B) = \frac{3}{4}$

$\therefore \sum P(A) - \sum P(A \cap B) = \frac{3}{8}$

Now, $P(A \cap B \cap C) = \frac{1}{16}$
$\therefore P(A \cup B \cup C)$

$= \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$
$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

Question130

From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is : [Online April 9, 2017]

Options:

A. $\frac{21}{220}$

B. $\frac{3}{11}$

C. $\frac{1}{11}$

D. $\frac{2}{23}$

Answer: C

Solution:

Solution:
Probability of 4 member committee which contain atleast one woman.
⇒ P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)
$\Rightarrow \frac{{}^{10}C_3\,{}^5C_1}{{}^{15}C_4} + \frac{{}^{10}C_2\,{}^5C_2}{{}^{15}C_4} + \frac{{}^{10}C_1\,{}^5C_3}{{}^{15}C_4} + \frac{{}^{10}C_0\,{}^5C_4}{{}^{15}C_4}$
$\Rightarrow \frac{600}{1365} + \frac{450}{1365} + \frac{100}{1365} + \frac{5}{1365}$
$\Rightarrow \frac{1155}{1365}$
∴ Probability of committees to have more women than men.
$= \frac{P(1M, 3W) + P(0M, 4W)}{P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)}$
$= \frac{\frac{105}{1365}}{\frac{1155}{1365}} = \frac{1}{11}$

Question131

If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is : [2017]

Options:

A. $\frac{7}{55}$

B. $\frac{6}{55}$

C. $\frac{12}{55}$

D. $\frac{14}{55}$

Answer: B

Solution:

Solution:
Let $A \equiv \{0, 1, 2, 3, 4, \ldots\ldots, 10\}$
$n(S) = {}^{11}C_2 = 55$ where 'S' denotes sample
space Let E be the given event
$\therefore E \equiv \{(0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)\}$
$\Rightarrow n(E) = 6$
$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{55}$

Question132

Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is
[Online April 9, 2017]

Options:

A. $\frac{4}{3}$

B. $\frac{3}{2}$

C. $\frac{1}{3}$

D. $\frac{5}{12}$

Answer: A

Solution:

Solution:
$P(E \cap F) = P(E) . P(F) = \frac{1}{12}$

$P(\overline{E} \cap \overline{F}) = P(\overline{E}) . P(\overline{F}) = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \frac{1}{2}$

Let $P(E) = x$
$P(F) = y$
$\Rightarrow 1 - x - y + xy = \frac{1}{2}$

$\Rightarrow 1 - x - y = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$

$\Rightarrow x + y = \frac{7}{12}$

$\Rightarrow x + \frac{1}{12x} = \frac{7}{12} \left[\because x . y = \frac{1}{12} \right]$
$\Rightarrow 12x^2 - 7x + 1 = 0$
$\Rightarrow 12x^2 - 4x - 3x + 1 = 0$
$\Rightarrow (4x - 1)(3x - 1) = 0$
$\Rightarrow x = \frac{1}{3}, x = \frac{1}{4}$

$\Rightarrow$ and $y = \frac{1}{4}, y = \frac{1}{3}$

$\therefore \frac{x}{y} = \frac{1/3}{1/4} = \frac{4}{3}$ or $\frac{1/4}{1/3} = \frac{3}{4}$

Question133

Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is :
[Online April 8, 2017]

Options:

A. $\frac{21}{64}$

B. $\frac{9}{64}$

C. $\frac{15}{64}$

D. $\frac{39}{64}$

Answer: A

Solution:

Solution:
Required probability = $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)+\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)$

$= \frac{12+9}{64} = \frac{21}{64}$

Question134

An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is:
[Online April 8, 2017]

Options:

A. $\frac{255}{256}$

B. $\frac{127}{128}$

C. $\frac{63}{64}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:
Required probability = 1 – {P (All Head) + P (All Tail)}

$= 1 - \left\{\frac{1}{2^8}+\frac{1}{2^8}\right\}$

$= 1 - \left\{\frac{1}{2^7}\right\}$

$= 1 - \left\{\frac{1}{128}\right\} = \frac{127}{128}$

Question135

A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:
[2017]

Options:

A. $\frac{6}{25}$

B. $\frac{12}{5}$

C. 6

D. 4

Answer: B

Solution:

Solution:
We can apply binomial probability distribution
We have n = 10
p = Probability of drawing a green ball $= \frac{15}{25} = \frac{3}{5}$
Also $q = 1 - \frac{3}{5} = \frac{2}{5}$
Variance = $npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

Question136

Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?
[2016]

Options:

A. E_1 and E_3 are independent.

B. E_1, E_2 and E_3 are independent.

C. E_1 and E_2 are independent.

D. E_2 and E_3 are independent.

Answer: B

Solution:

Solution:
$P(E_1) = \frac{1}{6}; P(E_2) = \frac{1}{6}; P(E_3) = \frac{1}{2}$
$P(E_1 \cap E_2) = \frac{1}{36}, P(E_2 \cap E_3) = \frac{1}{12}, P(E_1 \cap E_3) = \frac{1}{12}$
And $P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1) . P(E_2) . P(E_3)$
$\Rightarrow E_1, E_2, E_3$ are not independent.

Question137

If A and B are any two events such that $P(A) = \frac{2}{5}$ and$P(A \cap B) = \frac{3}{20}$, then the conditional probability,$P(A \mid A' \cup B')$), where A' denotes the complement of A, is equal to :
[Online April 9, 2016]

Options:

A. $\frac{11}{20}$

B. $\frac{5}{17}$

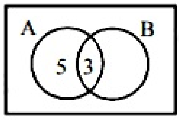
C. $\frac{8}{17}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:



$P(A) = \frac{2}{5} = \frac{8}{20}; \; P(A \cap B) = \frac{3}{20}$

$P(\overline{A \cap B}) = 1 - \frac{3}{20}$

$\Rightarrow P(\overline{A} \cup \overline{B}) = \frac{17}{20}$

$A \cap (A' \cup B') \; = A - (A \cap B)$

$\therefore P(A - (A \cap B)) = \frac{5}{20}$

$\therefore P(\, A \,/\, (A' \cap B') \; = \frac{P(A - (A \cap B))}{P(\overline{A} \cup \overline{B})} \; = \frac{5}{17}$

Question138

An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is:
[Online April 10, 2016]

Options:

A. $\frac{496}{729}$

B. $\frac{192}{729}$

C. $\frac{240}{729}$

D. $\frac{256}{729}$

Answer: D

Solution:

Solution:
Let $p(F) = p \Rightarrow p(S) = 2p$
$\because p + 2p = 1 \Rightarrow p = \frac{1}{3}$
$p(x \geq 5) = p(x = 5) + p(x = 6)$
$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_5\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$
$= \left(\frac{2}{3}\right)^5\left(6 \times \frac{1}{3} + \frac{2}{3}\right) = \frac{256}{729}$

Question139

If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is : [Online April 11, 2015]

Options:

A. $\frac{1}{21}$

B. $\frac{1}{27}$

C. $\frac{1}{15}$

D. $\frac{1}{26}$

Answer: B

Solution:

Solution:
Favourable case = (6, 6, 6)
Total case = {(1, 1, 1) (2, 2, 1), (2, 2, 2), (2, 2, 3), (3, 3, 1) ..... (3, 3, 5) (4, 4, 1).... (4, 4, 6) (5, 5, 1).... (5, 5, 6) (6, 6, 1).... (6, 6, 6)}
which satisfies condition $a + b > c$
Number of total case = 27
Probability = $\frac{1}{27}$

Question140

If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : (2015)

Options:

A. $220\left(\frac{1}{3}\right)^{12}$

B. $22\left(\frac{1}{3}\right)^{11}$

C. $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

D. $55\left(\frac{2}{3}\right)^{10}$

Answer: C

Solution:

Solution:
Note:- The question should state ‘3 different’ boxes instead of ‘3 identical boxes’ and one particular box has 3 balls. Then the solution would be:

Required probability $= \frac{{}^{12}C_3 \times 2^9}{3^{12}}$

$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Question141

Let X be a set containing 10 elements and $P(X)$ be its power set. If A and B are picked up at random from $P(X)$ with replacement, then the probability that A and B have equal number elements, is:
[Online April 10, 2015]

Options:

A. $\frac{(2^{10}-1)}{2^{10}}$

B. $\frac{{}^{20}C_{10}}{2^{10}}$

C. $\frac{(2^{10}-1)}{2^{20}}$

D. $\frac{{}^{20}C_{10}}{2^{20}}$

Answer: D

Solution:

Solution:
Required probability is

$$\frac{({}^{10}C_0)^2 + ({}^{10}C_1)^2 + ({}^{10}C_2)^2 + \ldots\ldots + ({}^{10}C_{10})^2}{2^{10}}$$

$= \frac{{}^{20}C_{10}}{2^{20}}$

Question142

If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is:
[Online April 11, 2015]

Options:

A. $\frac{9}{16}$

B. $\frac{3}{4}$

C. $\frac{1}{16}$

D. $\frac{15}{16}$

Answer: D

Solution:

Solution:
Let mean = np = 2 ...(1)
and variance = npq = 1 ...(2)
On solving eqn (1) and (2), we get
$q = \frac{1}{2}$ and $p = \frac{1}{2}$
From eqn (1), we have
$n = 4$
$P(x \geq 1) = {}^4C_1 p^1 q^3 + {}^4C_2 p^2 q^2 + {}^4C_3 p^3 q + {}^4C_4 p^4$
$= 1 - {}^4C_0 p^0 q^4 = 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$

Question143

A number x is chosen at random from the set { 1, 2, 3, 4, ... 100\} . Define the event: A = the chosen number x satisfies
$\frac{(x-10)(x-50)}{(x-30)} \geq 0$
Then P (A) is:
[Online April 12, 2014]

Options:

A. 0.71

B. 0.70

C. 0.51

D. 0.20

Answer: A

Solution:

Solution:
Given $\frac{(x-10)(x-50)}{(x-30)} \geq 0$
Let $x \geq 10$, $x \geq 50$ equation will be true $\forall x \geq 50$
as $\left(\frac{x-50}{x-30}\right) \geq 0, \quad \forall x \in [10, 30)$
$\frac{(x-10)(x-50)}{x-30} \geq 0 \ \ \forall x \in [10, 30)$
Total value of x between 10 to 30 is 20 .
Total values of x between 50 to 100 including 50 and 100 is 51
Total values of $x = 51 + 20 = 71$
$P(A) = \frac{71}{100} = 0.71$

Question144

A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that x ∈ A is:
[Online April 11, 2014]

Options:

A. $\frac{1}{2}$

B. $\frac{64}{127}$

C. $\frac{63}{128}$

D. $\frac{31}{128}$

Answer: B

Solution:

Solution:
Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
Let the chosen element be x_i
Total number of subsets of S = $2^7 = 128$
No. of non-empty subsets of S = $128 - 1 = 127$
We need to find number of those subsets that contains x_i.

2	2	2	2	1	2	2

$x_1 x_2 - - - - - - - x_i - - - - x_7$
For those subsets containing x_i , each element has 2 choices.
i.e., (included or not included) in subset, However as the subset must contain x_i, x_i has only one choice. (included one)
So, total no. of subsets containing
$x_i = 2 \times 2 \times 2 \times 2 \times 1 \times 2 \times 2 = 64$

$$\text{Required prob} = \frac{\text{No. of subsets containing } x_i}{\text{Total no. of non-empty subsets}}$$

$$= \frac{64}{127}$$

Question145

If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the incorrect statement amongst the following statements is: [Online April 9, 2014]

Options:

A. A and B are equally likely

B. $P(A \cap B') = 0$

C. $P(A' \cap B) = 0$

D. $P(A) + P(B) = 1$

Answer: D

Solution:

Solution:
Let A and B be two events such that
$P(A \cup B) = P(A \cap B)$
and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
option (a): since $P(A \cup B) = P(A \cap B)$ (given)
therefore A and B are equally likely
Suppose option (b) and option (c) are correct.
$\therefore P(A \cap B') = 0$ and $P(A' \cap B) = 0$
$\Rightarrow P(A) - P(A \cap B) = 0$ and $P(B) - P(A \cap B) = 0$
$\Rightarrow P(A) = P(A \cap B)$ and $P(B) = P(A \cap B)$
Thus $P(A) = P(B) = P(A \cap B) = P(A \cup B)$ [$\because$ Given $P(A \cap B) = P(A \cup B)$]

Also, we know
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A \cap B) + P(A \cap B) - P(A \cap B)$
$= P(A \cap B)$
which is true from given condition
Hence, option (a), (b) and (c) are correct.

Question146

Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(\overline{A \cap B}) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are
[2014]

Options:

A. independent but not equally likely.

B. independent and equally likely.

C. mutually exclusive and independent.

D. equally likely but not independent.

Answer: A

Solution:

Solution:
Given, $P(\overline{A \cup B}) = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$
$P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$
We know,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$
$\left(\because P(A \cap B) = \frac{1}{4} \right)$
$\Rightarrow P(B) = \frac{1}{3}$
$\because P(A) \neq P(B)$ so they are not equally likely.
Also $P(A) \times P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A \cap B)$
So A & B are independent.

Question147

Let A and E be any two events with positive probabilities:
Statement -1: $P(E / A) \geq P(A / E)P(E)$
Statement -2: $P(A / E) \geq P(A \cap E)$
[Online April 19, 2014]

Options:

A. Both the statements are true

B. Both the statements are false

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: A

Solution:

Solution:
Let A and E be any two events with positive probabilities.
Consider statement-1 :
$P(E / A) \geq P(A / E)P(E)$
LHS : $P(E / A) = \frac{P(E \cap A)}{P(A)}$ ....(1)
RHS : $P(A / E) . P(E) = \frac{P(E \cap A)}{P(E)} . P(E)$
$= P(A \cap E)$ ....(2)
Clearly, from (1) and (2), we have
$P(A / E) \geq P(A \cap E)$
Thus, statement-1 is true.
Similarly, statement-2 is also true.

Question148

If X has a binomial distribution, B(n, p) with parameters n and p such that $P(X = 2) = P(X = 3)$, then $E(X)$, the mean of variable X, is
[Online April 11, 2014]

Options:

A. $2 - p$

B. $3 - p$

C. $\frac{p}{2}$

D. $\frac{p}{3}$

Answer: B

Solution:

Solution:
Since X has a binomial distribution, B(n, p)
$\therefore P(X = 2) = {}^nC_2(p)^2(1 - p)^{n-2}$
and $P(X = 3) = {}^nC_3(p)^3(1 - p)^{n-3}$
Given $P(X = 2) = P(X = 3)$
$\Rightarrow {}^nC_2p^2(1 - p)^{n-2} = {}^nC_3(p)^3(1 - p)^{n-3}$
$\Rightarrow \frac{n!}{2!(n - 2)!} \cdot \frac{p^2(1 - p)^n}{(1 - p)^2} = \frac{n!}{3!(n - 3)!} \cdot \frac{p^3(1 - p)^n}{(1 - p)^3}$
$\Rightarrow \frac{1}{n - 2} = \frac{1}{3} \cdot \frac{p}{1 - p} \Rightarrow 3(1 - p) = p(n - 2)$
$\Rightarrow 3 - 3p = np - 2p$
$\Rightarrow np = 3 - p$
$\Rightarrow E(X) =$ mean $= 3 - p$ ($\because$ mean of B (n, p) = np)

Question149

If the events A and B are mutually exclusive events such that $P(A) = \frac{3x + 1}{3}$ and $P(B) = \frac{1 - x}{4}$, then the set of possible values of x lies in the interval :
[Online April 25, 2013]

Options:

A. [0,1]

B. $\left[\frac{1}{3}, \frac{2}{3}\right]$

C. $\left[-\frac{1}{3}, \frac{5}{9}\right]$

D. $\left[-\frac{7}{9}, \frac{4}{9}\right]$

Answer: C

Solution:

Solution:
Since events A and B are mutually exclusive
$\therefore P(A) + P(B) = 1$
$\Rightarrow \frac{3x+1}{3} + \frac{1-x}{4} = 1$
$\Rightarrow 12x + 4 + 3 - 3x = 12$
$\Rightarrow 9x = 5 \Rightarrow x = \frac{5}{9}$
$\therefore x \in \left[-\frac{1}{3}, \frac{5}{9}\right]$

Question150

A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is :
[Online April 23, 2013]

Options:

A. 21/64

B. 7/8

C. 7/32

D. 9/64

Answer: A

Solution:

Solution:
P(A or B but not byC) = $P((A \cup B) \cup \overline{C})$
$= P(A \cup B) \times P(\overline{C})$
$= [P(A) + P(B) - P(A \cap B)] \times P(\overline{C})$
$= \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right] \times \frac{3}{8} = \left(\frac{6+4-3}{8}\right) \times \frac{3}{8} = \frac{21}{64}$

Question151

Given two independent events, if the probability that exactly one of them occurs is $\frac{26}{49}$ and the probability that none of them occurs is $\frac{15}{49}$, then the probability of more probable of the two events is:
[Online April 22, 2013]

Options:

A. 4/7

B. 6/7

C. 3/7

D. 5/7

Answer: A

Solution:

Solution:
Let the probability of occurrence of first event A, be ‘a’
i.e., P(A) = a
∴ P(not A) = 1 – a
And also suppose that probability of occurrence of second event B, P(B) = b,
∴ P(not B) = 1 – b
Now, $\mathrm{P}(\mathrm{A}\text{ and }\mathrm{not}\,\mathrm{B}) + \mathrm{P}(\mathrm{not}\,\mathrm{A}\text{ and }\mathrm{B}) = \frac{26}{49}$

$\Rightarrow \mathrm{P(A)} \times \mathrm{P(not\,B)} + \mathrm{P(not\,A)} \times \mathrm{P(B)} = \frac{26}{49}$

$\Rightarrow \mathrm{a} \times (1 - \mathrm{b}) + (1 - \mathrm{a})\mathrm{b} = \frac{26}{49}$

$\Rightarrow \mathrm{a} + \mathrm{b} - 2\mathrm{ab} = \frac{26}{49}$ ......(i)

And A and $\mathrm{not}\,\mathrm{B}\,) = \frac{15}{49}$

$\Rightarrow \mathrm{P(not\,A)} \times \mathrm{P(not\,B)} = \frac{15}{49}$

$\Rightarrow (1 - \mathrm{a}) \times (1 - \mathrm{b}) = \frac{15}{49}$

$\Rightarrow 1 - \mathrm{b} - \mathrm{a} + \mathrm{ab} = \frac{15}{49}$

$\Rightarrow \mathrm{a} + \mathrm{b} - \mathrm{ab} = \frac{34}{49}$ ......(ii)
From (i) and (ii),
$\mathrm{a} + \mathrm{b} = \frac{42}{49}$ ....(iii)

and $\mathrm{ab} = \frac{8}{49}$

$(\mathrm{a} - \mathrm{b})^2 = (\mathrm{a} + \mathrm{b})^2 - 4\mathrm{ab} = \frac{42}{49} \times \frac{42}{49} - \frac{4 \times 8}{49} = \frac{196}{2401}$

$\therefore \mathrm{a} - \mathrm{b} = \frac{14}{49}$ ....(iv)
From (iii) and (iv),
$\mathrm{a} = \frac{4}{7}, \mathrm{b} = \frac{2}{7}$

Hence probability of more probable of the two events = $\frac{4}{7}$

Question152

The probability of a man hitting a target is $\frac{2}{5}$. He fires at the target k times (k, a given number). Then the minimum k, so that the probability of hitting the target at least once is more than $\frac{7}{10}$, is :
[Online April 9, 2013]

Options:

A. 3

B. 5

C. 2

D. 4

Answer: A

Solution:

Solution:

$\frac{2}{5} + \frac{3}{5} \times \frac{2}{5} + \left(\frac{3}{5}\right)^2 \times \frac{2}{5} + \ldots\ldots + \left(\frac{3}{5}\right)^k . \frac{2}{5} > \frac{7}{10}$

$\Rightarrow \frac{2}{5}\left[1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \ldots\ldots. + \left(\frac{3}{5}\right)^k\right] > \frac{7}{10}$

$\Rightarrow \frac{2}{5} \times \frac{1 - \left(\frac{3}{5}\right)^k}{1 - \frac{3}{5}} > \frac{7}{10} \Rightarrow 1 - \left(\frac{3}{5}\right)^k > \frac{7}{10}$

$\Rightarrow \left(\frac{3}{5}\right)^k < \frac{3}{10} \Rightarrow k \geq 3$

Hence minimum value of k = 3

Question153

A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: [2013]

Options:

A. $\frac{17}{3^5}$

B. $\frac{13}{3^5}$

C. $\frac{11}{3^5}$

D. $\frac{10}{3^5}$

Answer: C

Solution:

Solution:

p = p (correct answer), q = p (wrong answer)

$\Rightarrow P = \frac{1}{3}, q = \frac{2}{3}, n = 5$

By using Binomial distribution

Required probability

$P(x \geq 4) = {}^5C_4\left(\frac{1}{3}\right)^4 . \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5$

$= 5 . \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

Question154

There are two balls in an urn. Each ball can be either white or black. If a white ball is put into the urn and there after a ball is drawn at random from the urn, then the probability that it is white is [Online May 26, 2012]

Options:

A. $\frac{1}{4}$

B. $\frac{2}{3}$

C. $\frac{1}{5}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
Total possible event when one ball is taken out $= {}^3C_1$
Let E : The event of 1 white ball coming out
No. of ways to 1 white ball coming out $= {}^2C_1$
$\therefore P(E) = \frac{{}^2C_1}{{}^3C_1} = \frac{2}{3}$

Question155

If six students, including two particular students A and B, stand in a row, then the probability that A and B are separated with one student in between them is
[Online May 19, 2012]

Options:

A. $\frac{8}{15}$

B. $\frac{4}{15}$

C. $\frac{2}{15}$

D. $\frac{1}{15}$

Answer: B

Solution:

Solution:
Consider a group of three students A, B and an other student in between A and B. Choice for a student between A and B is 4. A and B can interchange their places in the group in 2 ways.
Now the group of three students (student A, student B and a student in between A and B) and the remaining 3 students can be stand in a row in 4! ways.
Hence total number of ways to stand in a row such that A and B are separated with one student in between them = 4 × 2 × 4!
Now total number of ways to stand 6 student stand in a row without any restriction = 6!
Hence required probability $= \frac{4 \times 2 \times 4!}{6!} = \frac{4 \times 2}{6 \times 5} = \frac{4}{15}$

Question156

A number n is randomly selected from the set {1, 2, 3, ..., 1000}. The probability that $\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i}$ is an integer is
[Online May 12, 2012]

Options:

A. 0.331

B. 0.333

C. 0.334

D. 0.332

Answer: C

Solution:

Solution:

$$\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$$

For $n = 1, 2, 3, \ldots\ldots, 1000$

Value of $\frac{2n+1}{3} = \frac{3}{3}, \frac{5}{3}, \frac{7}{3}, \ldots\ldots, \frac{2001}{3}$ respectively. Out of $\frac{3}{3}, \frac{5}{3}, \frac{7}{3}, \ldots\ldots, \frac{2001}{3}$ only first term $\left(\frac{3}{3} = 1\right)$, fourth term $\left(\frac{9}{3} = 3\right)$, 667^{th} term $\left(\frac{2001}{3} = 667\right)$ are integers.

Hence, out of 1000 values of $\frac{2n+1}{3}$,

total number of integral values of $\frac{2n+1}{3} = 333 + 1 = 334$

$\therefore$ Required probability $= \frac{334}{1000} = 0.334$

Question157

Let X and Y are two events such that $P(X \cup Y) = P(X \cap Y)$
Statement 1: $P(X \cap Y') = P(X' \cap Y) = 0$
Statement 2: $P(X) + P(Y) = 2P(X \cap Y)$
[Online May 7, 2012]

Options:

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

Answer: B

Solution:

Solution:
Let X and Y be two events such that
$P(X \cup Y) = P(X \cap Y)$ ......(1)
We know
$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
$P(X \cap Y) = P(X) + P(Y) - P(X \cap Y)$(from (1)
$\Rightarrow P(X) + P(Y) = 2P(X \cap Y)$
Hence, Statement -2 is true.
Now, $P(X \cap Y') = P(X) - P(X \cap Y)$
and $P(X' \cap Y) = P(Y) - P(X \cap Y)$
This implies statement- 1 is also true.

Question158

Three numbers are chosen at random without replacement from {1, 2, 3, ..8}. The probability that their minimum is 3 , given that their maximum is 6 , is :
[2012]

Options:

A. $\frac{3}{8}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Answer: B

Solution:

Solution:
Given three numbers are chosen without replacement from = {1,2,3,.....,8}
Let Event
F : Maximum of three numbers is 6.
E : Minimum of three numbers is 3.
This is the case of conditional probability
We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

$$\therefore P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{{}^{2}C_{1}}{{}^{5}C_{2}} = \frac{2}{10} = \frac{1}{5}$$

Question159

Let A, B, C, be pairwise independent events with P(C) > 0and $P(A \cap B \cap C) = 0$. Then $P(A^c \cap B^c / C)$.
[2011RS]

Options:

A. $P(B^c) - P(B)$

B. $P(A^c) + P(B^c)$

C. $P(A^c) - P(B^c)$

D. $P(A^c) - P(B)$

Answer: D

Solution:

Solution:
$P(A^c \cap B^c / C) =$

$$\frac{P((A^c \cap B^c) \cap C)}{P(C)} = \frac{P((A \cup B)^c \cap C)}{P(C)}$$

$$= \frac{P((S - A \cup B) \cap C)}{P(C)}$$

$$= \frac{P((S - A - B + A \cap B) \cap C)}{P(C)}$$

$$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$

$= \frac{P(C) - P(A) . P(C) - P(B)P(C) + 0}{P(C)}$
$= 1 - P(A) - P(B)$ $[\because P(A^C) = 1 - P(A)]$
$= P(A^c) - P(B)$

Question160

If C and D are two events such that $C \subset D$ and $P(D) \neq 0$ then the correct statement among the following is [2011]

Options:

A. $P(C \mid D) \geq P(C)$

B. $P(C \mid D) < P(C)$

C. $P(C \mid D) = \frac{P(D)}{P(C)}$

D. $P(C \mid D) = P(C)$

Answer: A

Solution:

Solution:
We know,
$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$ $[\because C \subset D]$
Where, $0 \leq P(D) \leq 1$, hence
$P\left(\frac{C}{D}\right) \geq P(C)$

Question161

Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval [2011]

Options:

A. $\left(\frac{3}{4}, \frac{11}{12}\right]$

B. $\left[0, \frac{1}{2}\right]$

C. $\left(\frac{11}{12}, 1\right]$

D. $\left(\frac{1}{2}, \frac{3}{4}\right]$

Answer: B

Solution:

Solution:
Given that p (at least one failure) $\geq \frac{31}{32}$

$\Rightarrow 1 - p(\text{ no failure}) \geq \frac{31}{32}$

$\Rightarrow 1 - p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \frac{1}{32} \Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

Hence p lies in the interval $\left[0, \frac{1}{2}\right]$.

Question162

Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ...20}.
Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.
Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is (±1, ±2, ±3, ±4, ±5) .
[2010]

Options:

A. Statement -1 is true, Statement - 2 is true; Statement -2 is not a correct explanation for Statement -1

B. Statement -1 is true, Statment -2 is false

C. Statement -1 is false, Statment -2 is true.

D. Statement -1 is true, Statement -2 is true ; Statement - 2 is a correct explanation for Statement -1.

Answer: B

Solution:

Solution:
Four numbers are chosen from $\{1, 2, 3.....20\}$
$n(S) = {}^{20}C_4$
Statement-1:
Common difference is 1; total number of ways = 17
common difference is 2; total number of ways = 14
common difference is 3; total number of ways = 11
common difference is 4; total number of ways = 8
common difference is 5; total number of ways = 5
common difference is 6; total number of ways = 2
Prob. $= \frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}$
Statement -2 is false, because common difference can be 6 also.

Question163

An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is
[2010]

Options:

A. $\frac{2}{7}$

B. $\frac{1}{21}$

C. $\frac{2}{23}$

D. $\frac{1}{3}$

Answer: A

Solution:

Solution:
$n(S) = {}^{9}C_{3}$
$n(E) = {}^{3}C_{1} \times {}^{4}C_{1} \times {}^{2}C_{1}$
Probability $= 3 \times 4 \times 2 {}^{9}C_{3} = \frac{24 \times 3!}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$

Question164

One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8 , given that the product of these digits is zero, equals: [2009]

Options:

A. $\frac{1}{7}$

B. $\frac{5}{14}$

C. $\frac{1}{50}$

D. $\frac{1}{14}$

Answer: D

Solution:

Solution:
A = Events that sum of the digits on selected ticket is 8
$= \{08, 17, 26, 35, 44\}$
Therefore, total 5 cases when sum of digits is 8
$\Rightarrow n(A) = 5$
Event that product of digits is zero
$= \{00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40\}$
$\Rightarrow n(A \cap B) = 1$ as only 08 is the number with sum 8 and product zero
$= n(B) = 14$
$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{14}$

Question165

In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: [2009]

Options:

A. $\frac{1}{\log_{10}4 + \log_{10}3}$

B. $\frac{9}{\log_{10}4 - \log_{10}3}$

C. $\frac{4}{\log_{10}4 - \log_{10}3}$

D. $\frac{1}{\log_{10}4 - \log_{10}3}$

Answer: D

Solution:

Solution:
Given that
$P = \frac{1}{4} \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$
and $P(x \geq 1) \geq \frac{9}{10}$
$\Rightarrow 1 - P(x = 0) \geq \frac{9}{10}$
$\Rightarrow 1 - {}^{n}C_{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{n} \geq \frac{9}{10}$
$\Rightarrow 1 - \frac{9}{10} \geq \left(\frac{3}{4}\right)^{n}$
$\Rightarrow \left(\frac{3}{4}\right)^{n} \leq \left(\frac{1}{10}\right)$
Taking log at the base 3 / 4, on both sides, we get
$n\log_{3/4}\left(\frac{3}{4}\right) \leq \log_{3/4}\left(\frac{1}{10}\right)$
$\Rightarrow n \geq -\log_{3/4}10 = \frac{-\log_{10}10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10}3 - \log_{10}4}$
$\Rightarrow n \geq \frac{1}{\log_{10}4 - \log_{10}3}$

Question166

A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P(A ∪ B) is
[2008]

Options:

A. $\frac{3}{5}$

B. 0

C. 1

D. $\frac{2}{5}$

Answer: C

Solution:

Solution:
A (number is greater than 3) = {4, 5, 6}
$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$
B(number is less than 5) = {1, 2, 3, 4} $\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$\therefore A \cap B = \{4\}$
$\Rightarrow P(A \cap B) = \frac{1}{6}$
$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3 + 4 - 1}{6} = 1$

Question167

It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A \mid B) = \frac{1}{2}$ and $P(B \mid A) = \frac{2}{3}$. Then P(B) is
[2008]

Options:

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:
Given that $P(A) = 1 / 4$, $P(A / B) = \frac{1}{2}$, $P(B / A) = 2 / 3$
By conditional probability,
$P(A \cap B) = P(A)P(B / A) = P(B)P(A / B)$
$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

Question168

Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is
[2007]

Options:

A. 0.2

B. 0.7

C. 0.06

D. 0.14

Answer: D

Solution:

Solution:
Given that $P(I) = 0.3$ and $P(II) = 0.2$

$\therefore P(\overline{I}) = 1 - 0.3 = 0.7$
$\therefore$ The required probability $= P(\overline{I} \cap II) = P(\overline{I}) \cdot P(II) = 0.7 \times 0.2 = 0.14$

Question169

A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is
[2007]

Options:

A. 8/729

B. 8/243

C. 1/729

D. 8/9.

Answer: B

Solution:

Solution:
The sample space of pair of fair dice is thrown,
S = (1, 1), (1, 2)(1, 3).... = 36
Sum 9 are (5,4),(4,5),(6,3),(3,6)
$P(\text{score } 9) = \frac{4}{36} = \frac{1}{9}$
Number of trial = 3
therefore Probability of getting score 9 exactly twice $= {}^3C_2 \times \left(\frac{1}{9}\right)^2 \cdot \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$
$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

Question170

At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is
[2006]

Options:

A. $\frac{6}{5^e}$

B. $\frac{5}{6}$

C. $\frac{6}{55}$

D. $\frac{6}{e^5}$

Answer: D

Solution:

Solution:
From poission distribution

$P(X = r) = \frac{e^{-m} m^r}{r!}$
Given mean (m) = 5
P (at most 1 phone call)
$= P(X \le 1) = P(X = 0) + P(X = 1)$
$= e^{-5} + 5 \times e^{-5} = \frac{6}{e^5}$

Question171

Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is [2005]

Options:

A. $\frac{2}{9}$

B. $\frac{1}{9}$

C. $\frac{8}{9}$

D. $\frac{7}{9}$

Answer: B

Solution:

Solution:
Probability of particular house being selected $= \frac{1}{3}$
P(all the persons apply for the same house)
$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) 3 = \frac{1}{9}.$

Question172

Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are [2005]

Options:

A. equally likely and mutually exclusive

B. equally likely but not independent

C. independent but not equally likely

D. mutually exclusive and independent

Answer: C

Solution:

Solution:
$P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = \frac{5}{6}, P(A) = \frac{3}{4}$
Also $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$
$\Rightarrow P(A)P(B) = \frac{3}{4} - \frac{1}{3} \ = \frac{1}{4} = P(A \cap B)$
Hence A and B are independent but not equally likely.

Question173

A random variable X has Poisson distribution with mean 2 . Then $P(X > 1.5)$ equals [2005]

Options:

A. $\frac{2}{e^2}$

B. 0

C. $1 - \frac{3}{e^2}$

D. $\frac{3}{e^2}$

Answer: C

Solution:

Solution:
From poission distribution, probability of getting k successes is
$P(x = k) = e^{-\lambda}\frac{\lambda^k}{k!}$
Given mean $(\lambda) = 2$
$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$
$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right) = 1 - \frac{3}{e^2}$

Question174

The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is [2004]

Options:

A. $\frac{4}{5}$

B. $\frac{1}{5}$

C. $\frac{7}{20}$

D. $\frac{3}{20}$

Answer: C

Solution:

Solution:
A and B will contradict each other if one speaks truth and other false . So, the required probability

$P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{4}{5}\left(1 - \frac{3}{4}\right) + \left(1 - \frac{4}{5}\right)\frac{3}{4}$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$

Question175

The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is [2004]

Options:

A. $\frac{28}{256}$

B. $\frac{219}{256}$

C. $\frac{128}{256}$

D. $\frac{37}{256}$

Answer: A

Solution:

Solution:
Given that mean $= np = 4$ and variance $= npq = 2$

$\Rightarrow p = q = \frac{1}{2}$ and $n = 8$

$\therefore P(\text{ 2 success }) = {}^8C_2\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2$

$= \frac{28}{2^8} = \frac{28}{256}$

Question176

A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.1

For the events $E = \{X$ is a prime number $\}$ and $F = \{X < 4\}$, the $P(E \cup F)$ is [2004]

Options:

A. 0.50

B. 0.77

C. 0.35

D. 0.87

Answer: B

Solution:

Solution:
P(E) = P(2 or 3 or 5 or 7)
= 0.23 + 0.12 + 0.20 + 0.07 = 0.62
P(F) = P(1 or 2 or 3) = 0.15 + 0.23 + 0.12 = 0.50
P(E ∩ F) = P(2 or 3) = 0.23 + 0.12 = 0.35
We know that
P(E ∪ F) = P(E) + P(F) − P(E ∩ F)
= 0.62 + 0.50 − 0.35 = 0.77

Question177

Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is [2003]

Options:

A. $\frac{5}{2}$

B. $\frac{5}{4}$

C. $\frac{5}{3}$

D. $\frac{1}{5}$

Answer: A

Solution:

Solution:
Let 5 horses are H_1, H_2, H_3, H_4 and H_5.
Total ways of selecting pair of horses be = ${}^5C_2 = 10$[i . e . H_1H_2, H_1H_3, H_1H_4, H_1H_5 H_2H_3, H_2H_4, H_2H_5, H_3H_4, H_3H_5, H_4H_5]
Any horse can win the race in 4 ways
(e.g. for H_1 : H_1H_2, H_1H_3, H_1H_4, H_1H_5)
Hence required probability $= \frac{4}{10} = 25$

Question178

Events A, B, C are mutually exclusive events such that P(A) = $\frac{3x+1}{3}$, P(B) = $\frac{1-x}{4}$ and P(C) = $\frac{1-2x}{2}$ The setof possible values of x are in the interval. [2003]

Options:

A. [0,1]

B. $\left[\frac{1}{3}, \frac{1}{2}\right]$

C. $\left[\frac{1}{3}, \frac{2}{3}\right]$

D. $\left[\frac{1}{3}, \frac{13}{3}\right]$

Answer: B

Solution:

Solution:
Given that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$
We know that $0 \le P(E) \le 1$
$\Rightarrow 0 \le \frac{3x+1}{3} \le 1, \ge -1 \ \le 3x \le 2$
$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$ ......(i)
$0 \le \frac{1-x}{4} \le 1 \Rightarrow -3 \le x \le 1$ ......(ii)
and $0 \le \frac{1-2x}{2} \le 1 \Rightarrow -1 \le 2x \le 1$
$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$ ......(iii)
Also for mutually exclusive events A, B, C
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$
$\therefore 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$
$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$
$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$ ......(iv)
From (i), (ii), (iii) and (iv), we get
$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

Question179

The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then P (X = 1) is
[2003]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{32}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

Answer: B

Solution:

Solution:
Given that $np = 4$ and
$npq = 2 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$
$P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8 \cdot \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

Question180

A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\overline{A}) = \frac{2}{3}$ then $P(\overline{A} \cap B)$ is
[2002]

Options:

A. 5/12

B. 3/8

C. 5/8

D. 1/4

Answer: A

Solution:

Solution:
We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$ $[\because P(A) = 1 - P(\overline{A})]$
$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3}$
Now, $P(\overline{A} \cap B) = P(B) - P(A \cap B)$
$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

Question181

A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is [2002]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: A

Solution:

Solution:
Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and$P(C) = \frac{1}{4}$; $P(A \cup B \cup C) = 1 - P(\overline{A})P(\overline{B})P(\overline{C})$
$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$

Question182

A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is [2002]

Options:

A. 8/3

B. 3/8

C. 4/5

D. 5/4

Answer: D

Solution:

The experiment follows binomial distribution with
$n = 5$, $p = 3/6 = 1/2$.
$q = 1 – p = 1/2$.;
∴ Variance = $npq = 5/4$.
