Chapter 5

Magnetism And Matter

Magnet & Magnetic Field

16. Magnet:

16.1 Pole strength, magnetic dipole and magnetic dipole moment:
A magnet always has two poles `N' and `S' and like poles of two magnets repel other and the unlike poles of two magnets attract each other they form action reaction pair.

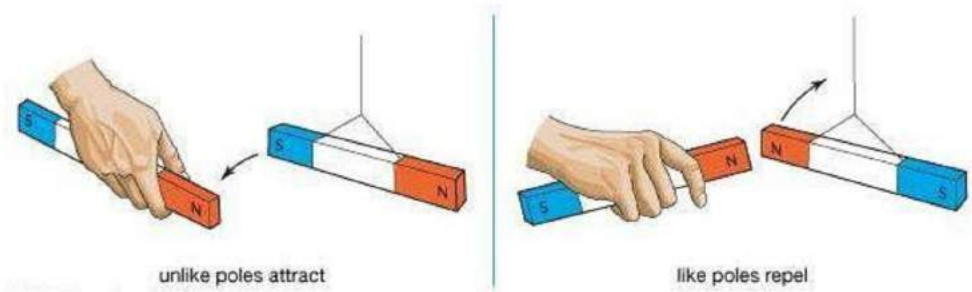
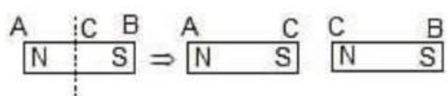
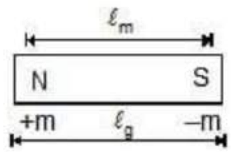


Fig: Magnetic poles

The poles of the same magnet do not comes to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. SO, `N' and `S' always exist together.
Therefore, they are:



Known as +ve and -ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as -ve pole (or -ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" + m and - m respectively (just like we have charge +q and -q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also). A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges -q and +q). It is called MAGNETIC DIPOLE and it has a direction is from -m to +m that means from `S' to `N').



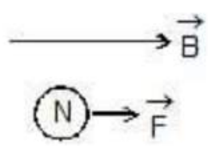
$M = m.l_m$ here l_m = magnetic length of the magnet. l_m is slightly less than l_g (it is geometrical length of the magnet = end to end distance). The `N' and `S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m / \ell_g \simeq$ 0.84].
The units of m and M will be mentioned afterwards where you can remember and understand.

16.2 Magnetic field and strength of magnetic field:

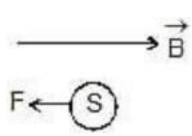
The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called "MAGNETIC FORCE". This field is quantitatively represented by "STRENGTH OF MAGNETIC FIELD" or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by. It is a vector quantity.

Definition of $\vec{B}$: The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \frac{\vec{F}}{m}$
Here = magnetic force on pole of pole strength m. m may be +ve or -ve and of any value. S.I. unit of is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).
We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on -ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



and

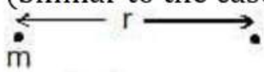


The field generated by sources does not depend on the test pole (for its any value and any sign).

(A) $\vec{B}$ due to various sources:

(i) Due to a single pole:

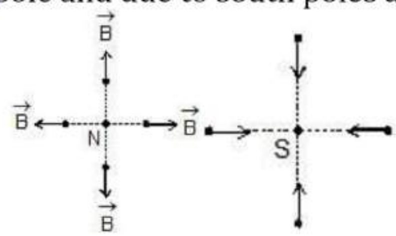
(Similar to the case of a point charge in electrostatics)



$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}$.(23)

This is magnitude

Direction of B due to north pole and due to south poles are as shown.

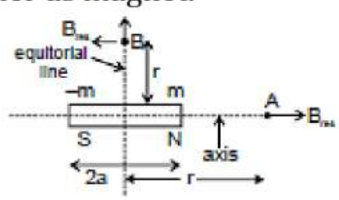


in vector from $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$.(24)

here m is with sign and = position vector of the test point with respect to the pole.

(ii) Due to a bar magnet:

(Same as the case of electric dipole in electrostatics) independent case never found. Always `N' and `S' exist together as magnet.



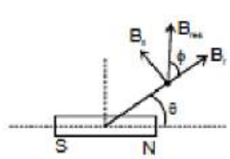
at A (on the axis) = $2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r.(25)

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r ...(26)

At General point:

$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$

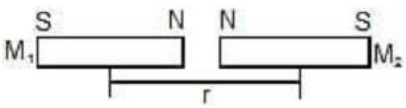
$B_n = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$



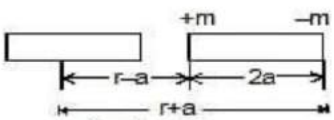
$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$ ...(27 (a))

$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$.(28 (b))

Ex. 36: Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Ans: To find the magnetic force we will use the formula of `B' due to a magnet. We will also assume m and -m as pole strengths of `N' and `S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and -m respectively. They experience magnetic forces F_1 and F_2 as shown.



$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$ and $F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$

Therefore, F_{res} F_1-F_2=2 $\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$

By using, Binomial expansion, and neglecting terms of high power we get

$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4} = 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$

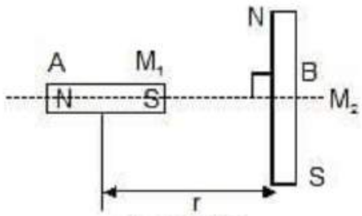
Direction of F_{res} is towards right.

Alternative Method:

$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \Rightarrow \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$

$F = -M_2 \times \frac{dB}{dr} \Rightarrow F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1M_2}{r^4}$

Ex. 37: Two short magnets A and B of magnetic dipole moments M_1 and M_2 respectively are placed as shown. The axis of `A' and the equatorial line of `B' are the same. Find the magnetic force on one magnet due to the other.



Ans: $F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2M_1}{r^4}$ upwards on M_1
down wards on M_2

Ex 38. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Ans: The pole strength is m = 120
CGS units = 12 A-m
Magnetic length is $2l = 10$ cm or $l = 0.05$ m
Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)]^2-(0.05m)^2]^2} = 3.4\times10^{-5}\ T.$$

Ex. 39: Find the magnetic field due to a dipole of magnetic moment 1.2 A-m² at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.

Ans: The magnitude of the field is

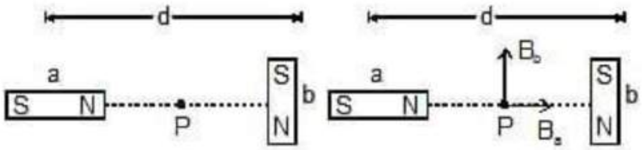
$$B = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{12A-m^2}{1m^2}\sqrt{1+3\cos^2 60^\circ}$$

The direction of the field makes an angle a with the radial line where

$\tan a = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$

Ex. 40: Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



(a) (b)

Ans: The point p is in end-on position for the dipole a and in broadside-on position for the dipole b'. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore

$B = \sqrt{B_a^2 + B_b^2}$

$= \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2 + 2^2}$

$= \frac{2\sqrt{5}\mu_0 M}{\pi d^2}$

The direction of this field makes an angle a with B_a such that $\tan a = B_b/B_a = 1/2$.

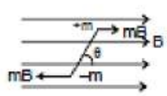
16.3 Magnet in an external uniform magnetic field:

(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

$t = MB\sin\theta$

* here θ is angle between $\vec{B}$ and $\vec{M}$



Note:

$\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$

$\vec{\tau}$ is same about every point of the dipole its potential energy is $U = -MB\cos\theta = -\vec{M}.\vec{B}$

$\theta = 0°$ is stable equilibrium

$\theta = p$ is unstable equilibrium

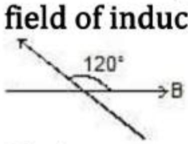
for small `q' the dipole performs SHM about $q = 0°$ position

$\tau = - MB \sin \theta$;
$I \alpha = - M B \sin \theta$
$\alpha = - \left(\frac{MB}{I}\right)\theta$
for small θ, $\sin \theta \approx \theta$
Angular frequency of SHM
$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \Rightarrow T = 2p\sqrt{\frac{I}{MB}}$
here $I = I_{cm}$ if the dipole is free to rotate
$= I_{hinge}$ if the dipole is hinged

***Ex.* 41: A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.**

Ans: The work done by the external agent = change in potential energy
$= (-MB \cos\theta_2) - (-MB \cos\theta_1)$
$= - MB (\cos 60° - \cos 0°)$
$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^4 \text{ J/T}) (4 \times 10^{-5} \text{ T}) = 0.2 \text{ J}$

Ex. 42: A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure.



Find:
(i) Its kinetic energy at $\theta = 90°$
(ii) its maximum kinetic energy during the motion.
(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Ans: (i) Apply energy conservation at $\theta = 120°$ and $\theta = 90°$
$= - MB \cos 120° + 0$
$= - MB \cos 90° + (K.E)$
$KE = \frac{MB}{2}$

Ans.
(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0º$.
Now apply energy conservation between $\theta = 120º$ and $\theta = 0º$.
$= -mB \cos 120º + 0$

$= - mB \cos 0^{\circ} + (KE)_{max}$

$(KE)_{max} = \frac{3}{2} MB$

Ans.

The K.E. is max at $\theta = 0°$ can also be proved by torque method. From $\theta = 120°$ to $\theta = 0°$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0°$. Beyond that t reverses its direction and then K.E. starts decreasing. Therefore, $\theta = 0°$ is the orientation of M to here the maximum K.E.

(iii) Since `q' is not small.

Therefore, the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120°.

Ex. 43: A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes p/2 seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Ans: (a) The moment of inertia of the magnet about the axis of rotation is

$I = \frac{m'}{12}(L^2 + b^2)$

$= \frac{100 \times 10^{-3}}{12} [(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]$ kg-m² $= \frac{25}{6} \times 10^{-5}$ kg $-$ m²

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$

or, $M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5} \text{kg}/\text{m}^2}{6 \times (25 \times 10^{-6}\text{T}) \times \frac{\pi^2}{4} \text{s}^2} = 27$ A-m²

(b) In this case the moment of inertia becomes

$I' = \frac{m'}{12}(L^2 + b^2)$ where b' = 0.5 cm.

The time period would be

$T' = \sqrt{\frac{I'}{MB}}$ ...(ii)

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7cm)^2 + (0.5\,cm)^2}}{\sqrt{(7\,cm)^2 + (1.0\,cm)^2}} = 0.992$$

$$\text{or, } T' = \frac{0.992 \times \pi}{2}s = 0.496\ \text{p s.}$$

16.4 Magnet in an External Non-uniform Magnetic field:

No special formula are applied is such problems. Instead see the force on individual poles and calculate the resultant force torque on the dipole.

Note:

- Force due to Non-uniform Magnetic field:

$$F = -M\frac{dB}{dr}$$

- If a source of Magnetic Moment $\vec{M}$ have dimension very less than the distance of point of application then we can replace it with magnet of magnetic moment equal to $\vec{M}$.

17. TERRESTRIAL MAGNETISM:

Earth is a natural source of magnetic field.

17.1 Elements of the Earth's Magnetic Field:

The earth's magnetic field at a point on its surface is usually characterised by three quantities:
(a) declination
(b) inclination or dip and
(c) horizontal component of the field. These are known as the elements of the earth's magnetic field.

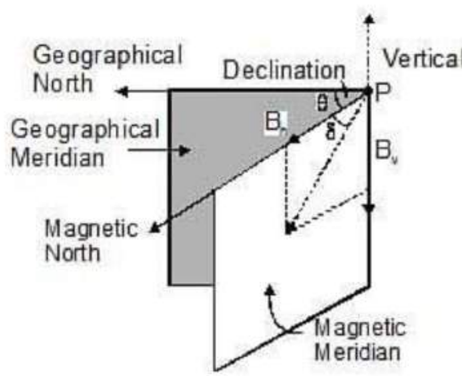
(a) Declination:

A plane passing through the geographical poles (that is, through the axis of rotation of the earth) and a given point P on the earth's surface is called the geographical meridian at the point P. Similarly, the plane passing through the geomagnetic poles (that is, through the dipole-axis of the earth) and the point P is called the magnetic meridian at the point P.

The angle made by the magnetic meridian at a point with the geographical meridian is called the declination at that point.

(b) Inclination or dip:
The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian, is called the inclination or dip at that point.

(c) Horizontal component of the earth's magnetic field:
As the name indicates, the horizontal component is component of the earth's magnetic field in the horizontal direction in the magnetic meridian. This direction is towards the magnetic north.
Figure shows the three elements. Starting from the geographical meridian we draw the magnetic meridian at an angle q (declination). In the magnetic meridian we draw the horizontal direction specifying magnetic north. The magnetic field is at an angle d (dip) from this direction. The horizontal component B_H and the total field B are related as
$B_H = B \cos \delta$
or, $B = B_H / \cos \delta$



Thus, from the knowledge of the three elements, both the magnitude and direction of the earth's magnetic field can be obtained.

Ex. 45 The horizontal component of the earth's magnetic field is 3.6×10^{-5} T where the dip is 60º. Find the magnitude of the earth's magnetic field.

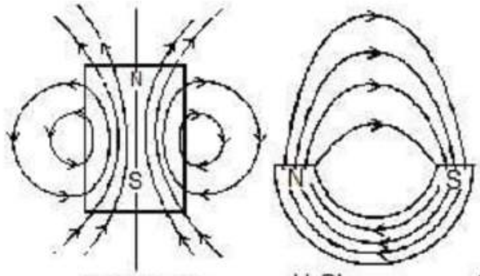
Ans: We have $B_H = B \cos \delta$

$$B = \frac{B_H}{\cos\delta} = \frac{3.6 \times 10^{-5} T}{\cos 60^\circ} = 7.2 \times 10^{-5} T$$

Magnetic Lines & Their Characteristics

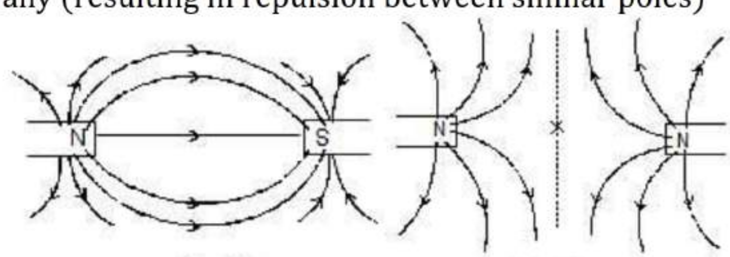
6. Magnetic Lines and Their Characteristics

The space surrounding a magnet or magnetic configuration in which its effects are perceptible is called the **magnetic field** of the given magnet or magnetic configuration.



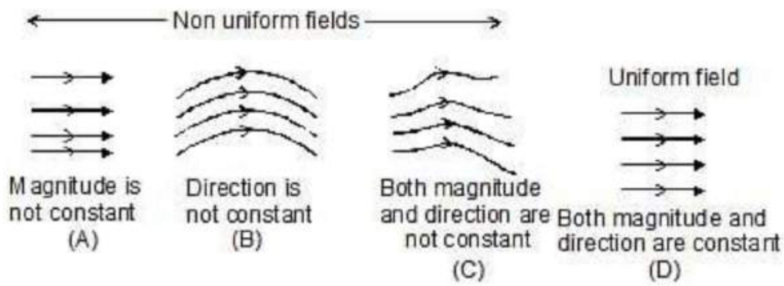
Bar-magnet (A)   U-Shape magnet (B)

In order to visualize a magnetic field graphically, **Michael Faraday** introduced the concept of lines.
According to him a line is an imaginary curve the tangent to which at a point gives the direction of the field at that point.
Regarding magnetic field it is worth noting that:
(i) Outside a magnet, field are from north to to south pole while inside from south to north, i.e., magnetic lines are closed curves i.e., they appear to converge or diverge at poles.
(ii) The number of magnetic lines of field originating or terminating on a pole is proportional to its strength. μ_0 lines are assumed to be associated with a unit pole. so if a body encloses a pole of strength m, total lines linked with the body (called **magnetic flux**) will be μ_0(m).
(iii) Magnetic lines of field can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is absurd.
(iv) Magnetic lines of field have a tendency to contract longitudinally like a stretched elastic string (producing attraction between opposite poles) and repel each other laterally (resulting in repulsion between similar poles)



Attraction (A)   Repulsion (B)

(v) Number of lines of field per unit area, normal to the area at a point, represents the magnitude of field at that point. so crowded lines represent a strong field while distant lines represent weak field. Further, if the lines of force are equidistant and straight the field is uniform otherwise not



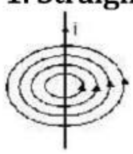
(vi) In a region of space where there is no magnetic field, there will be no lines of field. This is why, at a **neutral point** (where resultant field is zero) there cannot be any line of field.
(vii) Magnetic lines of field originate from or enter in the surface of a magnetic material at any angle.
(viii) Magnetic lines of field exist inside every magnetised material
(ix) As mono-poles do not exist, the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B}.\vec{ds} = \mu_0(0) = 0$$

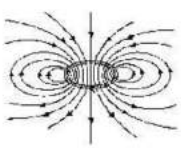
This law is called Gauss's law for magnetism.

Magnetic field line due to some important structure

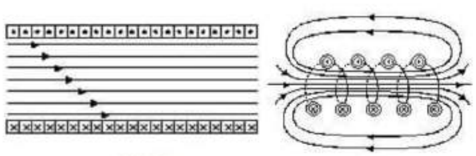
1. Straight current carrying wire



2. Circular coil



3. Solenoid



(ideal) (Real)

Magnetic Dipole

Magnetic Dipole

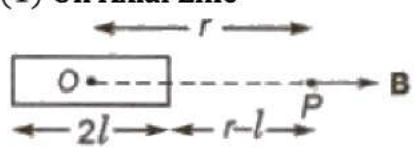
Magnetic dipole is an arrangement of two unlike magnetic poles of equal pole strength separated by a very small distance, e.g., a small bar magnet, a magnetic needle, a current carrying loop etc.

Magnetic Dipole Moment

The product of the distance (2 l) between the two poles and the pole strength of either pole is called magnetic dipole moment.
Magnetic dipole moment
$M = m\ (2\ l)$
Its SI unit is 'joule/tesla' or 'ampere-metre^2'.
Its direction is from south pole towards north pole.

Magnetic Field Due to a Magnetic Dipole

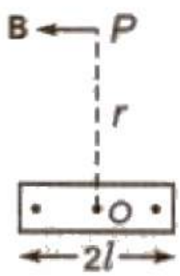
(1) **On Axial Line**



$$B = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}$$

If $r >> l$, then
$B = \mu_o / 4\pi\ 2M / r^3$

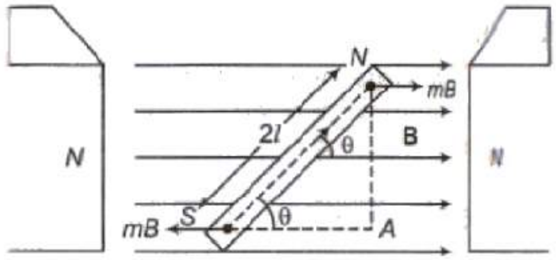
(ii) **On Equatorial Line**



$B = \mu_o / 4\pi\ M / (r^2 + l^2)^{3/2}$
If $r >> l$, then
$B = \mu_o / 4\pi\ 2M / r^3$

Torque Acting on a Magnetic Dipole

When a Magnetic Dipole (M) is placed in a uniform magnetic field (B), then a Torque acts on it, Which is given by



$\tau = M * B$
or $\tau = MB \sin \theta$
Where θ is angle between the dipole axis and magnetic field.

Potential Energy of a Magnetic Dipole in a Uniform Magnetic Field

The work done in rotating the dipole against the action of the torque is stored as potential energy of the dipole.
Potential Energy, $U = W = - MB \cos \theta = - M. B$

Earth's Magnetism & Magnetic Declination

Introduction to The Earth's Magnetism

The reason why, a bar magnet, when suspended freely, points in north - south direction is due to earth's giant magnetic field. It is believed that the electric currents circulating from earth's core to the space give rise to the earth's magnetic field. The earth's magnetic field is supposed to save earth from the solar wind which might cause the ozone layer of the earth to strip away. The SI unit of earth's magnetic field is given by tesla.

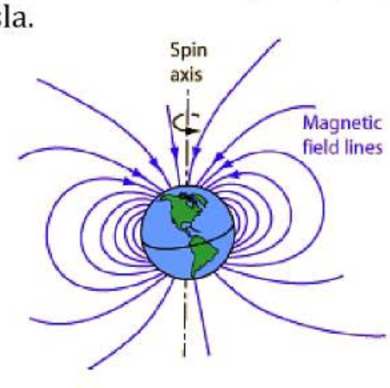


Fig: SI unit of earth's magnetic field is given by tesla

Theory of Earth's Magnetism

There is no valid reason for the cause of earth's magnetism or why earth has giant magnetic field but there are some theory related to earth's magnetic field which helps us to understand that why earth behaves as a giant magnet.

- It is believed that the magnetic field of earth is due to dynamo effect. Dynamo effect is caused by the motion of metallic fluids in the outer core of the earth which results in electric current. It is because of this electric current that the earth has its own magnetic field lines.
- Another theory suggests that the rotation of earth in its own axis produces strong electric current since the outer layers of earth is ionized. As a result of which when the earth rotates, there is a movement of charged ions, which in return produces electric current.

Distinctive Aspect of Earth's Magnetism

- A hypothetical giant magnetic dipole is supposed to be located at the centre of the earth. It does-not coincides with the axis of earth. The dipole is tilted by 11.3° with respect to earth's axis as shown in the diagram given below:

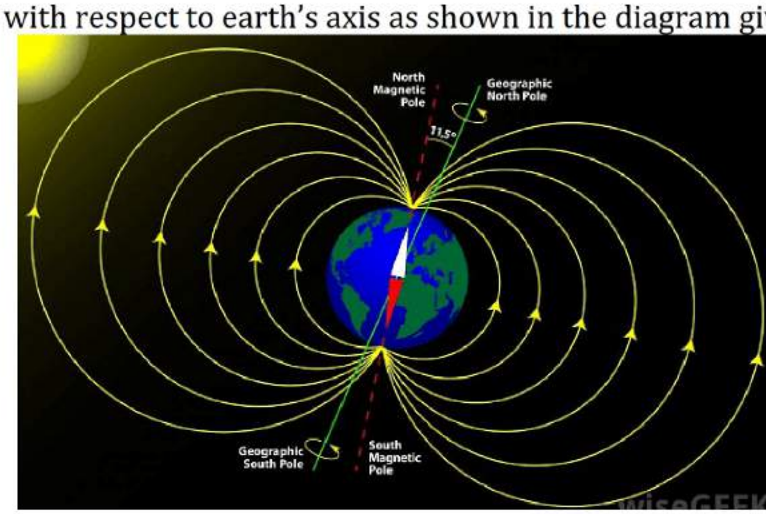


Fig: Pictorial representation of earth's magnetic field

- As it can be seen from the above diagram that there are two north (magnetic north and geographic north) poles and two south (magnetic south and geographic south) poles located on the poles of the earth. The magnetic north and magnetic South Pole is the result of the dipole. The magnetic north pole is located at 79.74° N (latitude) and 71.8° W (longitude). Similarly, the magnetic south pole is located at 79.74° S (latitude) and 108.22°E (longitude)
- If we observe carefully the magnetic field lines of the earth, we observe that the magnetic field lines enter the north pole and leaves the south pole

unlike Bar magnet, where the magnetic field lines enter the south pole and leaves the north pole. This is because the magnetic north pole actually behaves like the south pole of a bar magnets and vice versa. It was named as magnetic north because the magnetic needle (North Pole) of the bar magnet pointed in this direction.

Component of Earth's Magnetic Field

The components that are responsible for the magnitude as well as the direction of earth's magnetic field at a particular place is given by:

- Magnetic Declination
- Horizontal Component of Earth's Magnetic Field
- Angle of Dip or Magnetic Inclination

Magnetic Declination

When a magnetic needle is suspended freely in the air, it always points in the north-south direction. This direction in which a magnetic needle points freely in a particular direction free form all other attracting forces is known as **Magnetic Meridian**.

Magnetic Declination is defined as the angle made by the Magnetic meridian with the geographic meridian. Here geographic meridian is defined as the plane passing through the north and south poles of the earth.

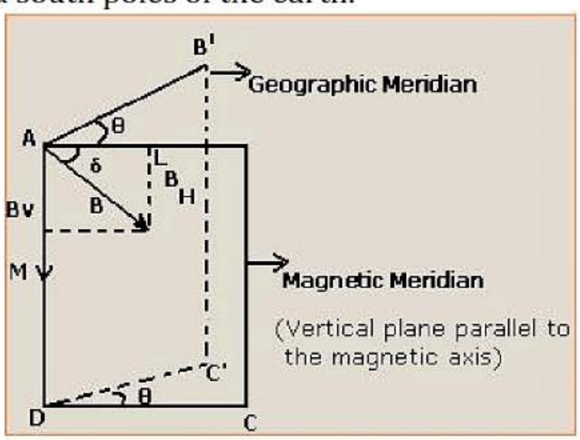


Fig: The angle θ is known as the magnetic declination

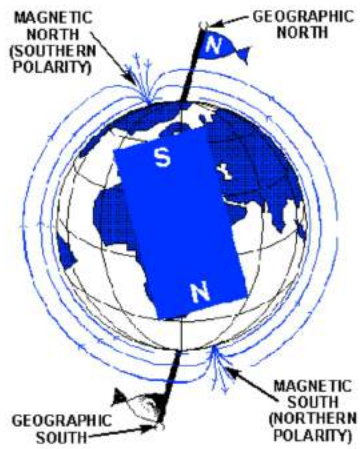


Fig: Pictorial representation of true geographic north and south-pole

Angle of Dip or Magnetic Inclination

Take a magnetic needle and suspend it freely so that it can rotate about a horizontal axis as shown in the diagram below:

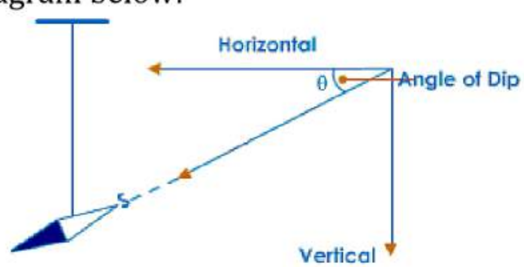


Fig: Angle of dip or Magnetic inclination

The angle that the north pole of the needle makes with the horizontal axis is known as the **Angle of Dip or Magnetic Inclination**.

Permanent Magnets & Electromagnets

Permanent Magnets

Commonly steel is used to make a permanent magnet because steel has high residual magnetism and high coercivity.

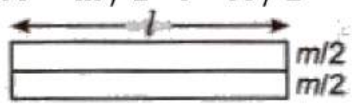
Electromagnets

Electromagnets are made of soft iron because area of hysteresis loop for soft iron is small. Therefore, energy loss is small for a cycle of magnetisation and demagnetisation.
(Permanent magnets are made by the materials such as steel, for which residual magnetism as well as coercivity should be high. Electromagnets are made by the materials such as soft iron for which residual magnetism is high, coercivity is low and hysteresis loss is low).

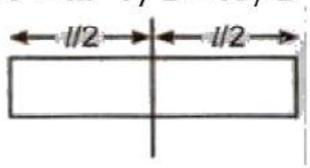
Important Points

- Magnetic length = 5 / 6 * geometric length of magnet.
- About 90% of magnetic moment is due to spin motion of electrons and remaining 10% of magnetic moment is due to the orbital motion of electrons.
- When a magnet having magnetic moment M is cut into two equal parts

(i) Parallel to its length
$M' = m / 2 * l = M / 2$

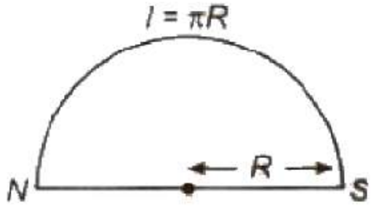


(ii) Perpendicular to its length
$M' = m * l / 2 = M / 2$



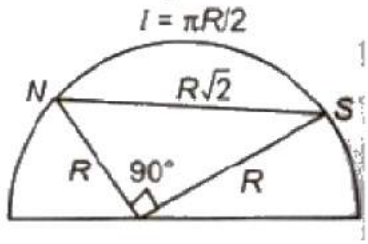
- When a magnet of length I, pole strength m and of magnetic moment M is turned into a semicircular arc then it new magnetic moment

$M' = m * 2R = m * 2 * 1 / \pi \ (\pi R = I)$
$= 2M / \pi \ (M = m * l)$



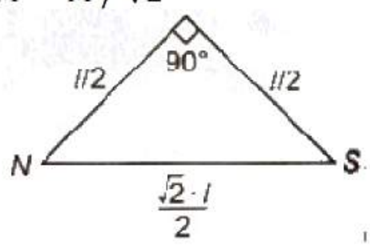
• A thin magnet of moment M is turned into an arc of 90°. Then new magnetic moment

M' = 2√2M / π



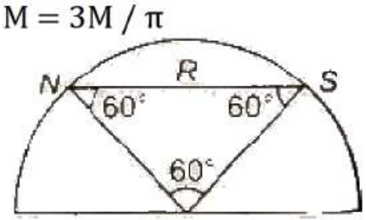
• A thin magnet of moment M is turned at mid point 90°.Then new magnet moment

M' = M / √2



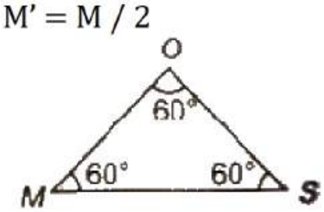
• A thin magnet of moment M is turned into an arc of 60°. Then new magnetic moment

M = 3M / π



• A thin magnet of moment M is bent at mid point at angle 60°. Then new magnetic moment.

M' = M / 2



- Original magnet MOS is bent at O, the mid point at 60°.All sidesare equal
- The mutual interaction force between two small magnets of moments M_1 and M_2 is given by

$F = K\,6M_1M_2 / d^4$ in end- on position.
Hered denotes the separation between magnets.

- Magnetic length = 5 / 6 * geometric length of magnet.
- Cause of diamagnetism is orbital motion and cause of paramagnetism is spin motion of electrons. Cause of ferromagnetism lies in formation of domains.
- The perpendicular bisector of magnetic axis is known as neutral axis of magnet. Magnetism at neutral axis is zero and at poles is maximum.
- For steel coercivity is large. However, retentivity is comparatively smaller in case of steel. So, steel is used to make permanent magnets.
- For soft iron, coercivity is very small and area of hysteresis loop is small. So, soft iron is an ideal material for making electromagnets.

Magnetisation & Magnetic Intensity

Magnetization, also termed as magnetic polarization, is a vector quantity that gives the measure of the density of permanent or induced dipole moment in a given magnetic material. As we know, magnetization results from the magnetic moment, which results from the motion of electrons in the atoms or the spin of electrons or the nuclei. The net magnetization results from the response of a material to the external magnetic field, together with any unbalanced magnetic dipole moment that is inherent in the material due to the motion in its electrons as mentioned earlier. The concept of magnetization helps us in classifying the materials on the basis of their magnetic property. In this section, we will learn more about magnetization and the concept of magnetic intensity.

What is Magnetization?

The magnetization of a given sample material M can be defined as the net magnetic moment for that material per unit volume.

Mathematically,

$$M = \frac{m_{net}}{V}$$

Let us now consider the case of a solenoid. Let us take a solenoid with n turns per unit length and the current passing through it be given by I, then the magnetic field in the interior of the solenoid can be given as,

$B_0 = \mu_0 nI$

Now, if we fill the interior with the solenoid with a material of non-zero magnetization, the field inside the solenoid must be greater than before. The net magnetic field B inside the solenoid can be given as,

$B = B_0 + B_m$

Where B_m gives the field contributed by the core material. Here, B_m is proportional to the magnetization of the material, M. Mathematically,

$B_m = \mu_0 M$

Here, $\mu 0$ is the constant of permeability of a vacuum.

Let us now discuss another concept here, the magnetic intensity of a material. The magnetic intensity of a material can be given as,

$$H = \frac{B}{\mu_0} - M$$

From this equation, we see that the total magnetic field can also be defined as,

$B = \mu_o(H+M)$

Here, the magnetic field due to the external factors such as the current in the solenoid is given as H and that due to the nature of the core is given by M. The latter quantity, that is M is dependent on external influences and is given by,

$M = \chi H$

Where χ is the magnetic susceptibility of the material. It gives the measure of the response of a material to an external field. The magnetic susceptibility of a material is small and positive for paramagnetic materials and is small and negative for diamagnetic materials.

$B=\mu_0(1+X)\ H=\mu_0\mu_r H= \mu H$

Here, the term μr is termed as the relative magnetic permeability of a material, which is analogous to the dielectric constants in the case of electrostatics. We define the magnetic permeability as,

$\mu=\mu_0\mu_r=\mu_0(1+X)$

Magnetic Properties of Materials

Paramagnetism

Paramagnetism is a kind of magnetism where several objects are attracted through an externally applied magnetic field. Whereas the diamagnetic materials

are repelled by magnetic fields and develop induced magnetic fields in the direction which is opposite to that of the applied magnetic fields.
These materials include most of the chemical elements and some compounds as they have a magnetic permeability greater than or equal to 1. The magnetic moment induced by the applied field is linear to the strength and weakness of the field. It usually needs a sensitive analytical balance to detect the effect and different modern measurements on paramagnetic materials that are often conducted with a SQUID magnetometer.
The paramagnetic material becomes good magnetic material when placed in the strong magnetic material. It acts as a magnet which attracts and repels other magnetic and ferromagnetic materials. When the magnetic field is removed, the total magnetic alignment of the magnetic dipoles is lost and the dipole returned to their normal random motion. This condition is known as Paramagnetism.
The Super Paramagnetism is described as the property as some materials magnetic moments change their direction at the nanoscale and behaves like a paramagnet event. According to Curie law, If there is no magnetic field applied, curie temperature is applied at the same time as they show high magnetic susceptibility.

Curie's Law

According to this law, the magnetization in the paramagnetic material is inversely proportional to the temperature, which means the more the temperature of the paramagnetic material increases, its magnetization decreases.

$$M = C(B/T)$$

Where,
C = Curie constant,
T = temperature in Kelvin and,
B = applied the magnetic field.

Magnetism & Gauss's Law

Introduction to Magnetism and Gauss's Law

The property of magnet to attract or repel other substance is known as Magnetism. We know that, there exist an imaginary magnetic field lines around a magnet which is the main source, responsible for the behaviour of the magnets. When these magnetic field lines penetrates through an area perpendicularly, then (The average of the magnetic field lines) X (the area through which it penetrates) is known as

Magnetic Flux. Gauss law in magnetism states that the magnetic flux through any closed surface is zero.

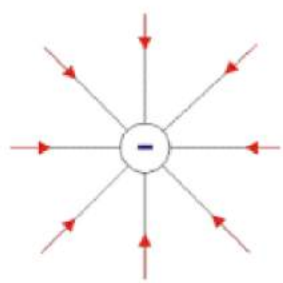
Electrostatics

In physics, electrostatics deals with the study of electric charges that are stationary ie. they are slow moving or have very less acceleration. With respect to the static charges

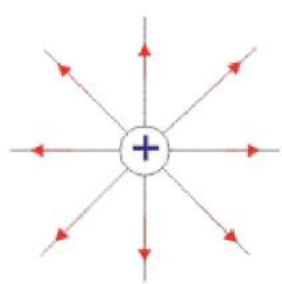
Coulomb stated that **"The magnitude of electrostatic force of interaction between twopoint charges is directly proportional to the scalar multiplication of the magnitude of charges and inversely proportional to the square of the distance between them".** This law is known as **Coulomb's Law**.

Electric Field

A charge particle is always surrounded by a field in which other charge particles can experience its force. In other words, the region where a charge particle can exert force on other charge particle is known as Electric Field. It is a vector quantity which means that it has both magnitude as well as direction. For a positive charge the line of force is directed outwards while for a negative charge the lines of force is directed inwards.



Lines of Force Due to and Isolated -ve charge



Lines of Force Due to and Isolated +e charge

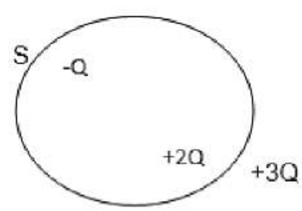
Gauss's Law in Electrostatics

Theorem: Gauss's Law states that "The net electric flux through any closed surface is equal to 1/ times the net electric charge within that closed surface (or imaginary Gaussian surface)".

$\int$ **E.ds = q/ϵ**

Explanation: In the fig 1.1 two charges +2Q and -Q is enclosed within a closed surface S, and a third charge +3Q is placed outside the closed surface. The net electric flux through the closed surface S will be given by,

$\Phi_S = \sum q/\epsilon$
that is, $= (+2Q + (-Q))/\epsilon$
$= Q/\epsilon$
Since the charge +3Q is placed outside the closed surface, therefore this charge will not be taken into consideration while applying gauss theorem



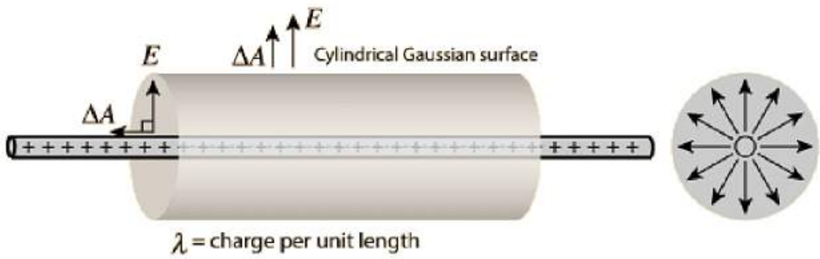
(Here the imaginary closed Gaussian surface is considered to be a circle

Application of Gauss's Law

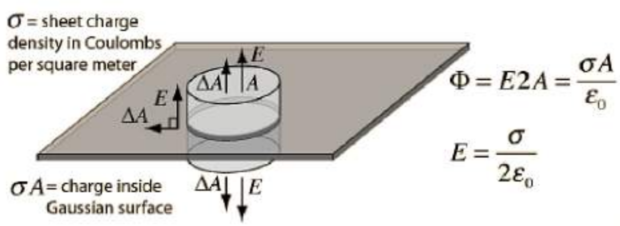
Three types of Application of Gauss's Law

- Electric Field intensity due to Infinitely long uniformly Charged Wire
- Electric Field due to Plane Sheet
- Electric Field due to Spherical shell

1. **Electric Field intensity due to infinitely long uniformly charged wire**
   Let us consider a long linear wire of length L with uniformly distributed positive charge λ.

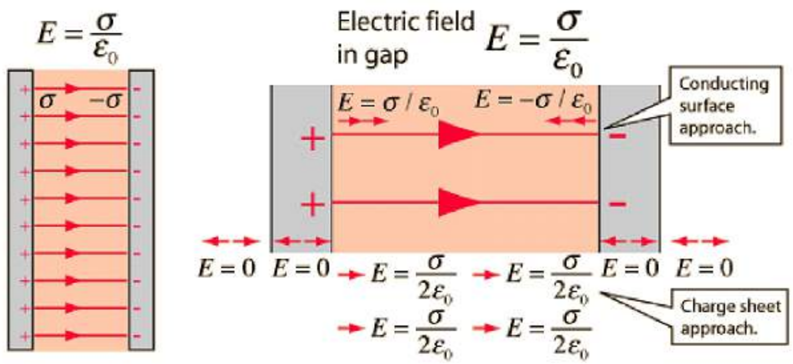


1. $\int E.ds = q/\epsilon$
   $= \lambda L/\epsilon$
   $\Phi = E2\pi rL$
   $E = \lambda/2\pi r\epsilon$
   Where r is the perpendicular distance from the charged wire.
2. **Electric Field due to Plane Sheet**
   - **Electric field due to thin infinite plane sheet with uniform surface charge density.**



n case of an infinite sheet charge, a cylindrical Gaussian surface is considered, such that the electric field is perpendicular to the surface. As a result of which only the ends of the Gaussian surface will constitute the magnetic flux.

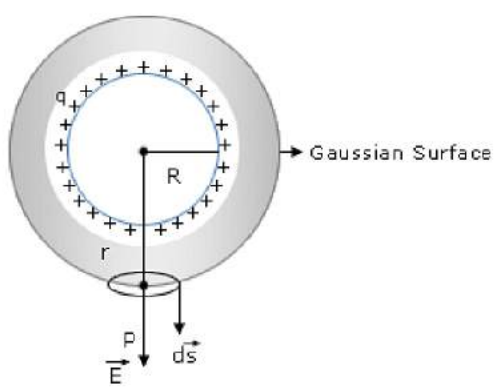
- **Electric field due to two parallel sheets of equal and opposite charge at any point.**
  Let us assume that the two parallel plates with equal and opposite charge, is at equilibrium which means that the field is perpendicular to the surface and have zero electric field inside the conductor. Then by applying gauss law we can find the electric field between the two plates, which is given in the below diagram.



Electric Field due to Spherical Shell

Let us consider a sphere of radius R with positive charge (q) distributed uniformly all over the surface. To calculate the electric field on the surface of the shell (inside and outside), a spherical Gaussian surface is taken into consideration.

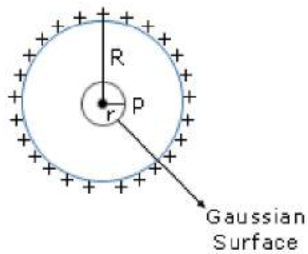
- **Electric field outside the shell:** To calculate the electric field outside the shell, we consider the radius of Gaussian surface r is greater than the radius of shell R that is ($r > R$)



Then by applying integral form of gauss law,
$\varphi E = \oint (E\cos\theta)\, ds$
$= E \oint ds$
$= E\,(4\pi^2)$

- **Electric field inside the charged shell:** Electric field inside the shell can be calculated by considering a cylindrical Gaussian surface of radius r. Now in case of electric field inside the shell, we assume that the radius of Gaussian surface r is smaller than the Radius of the shell R that is, (r <

R). Then the electric flux through the Gaussian surface is given by,
$\varphi E = \oint (E\cos\theta)\, ds = E\,(4\pi^2)$



- According to Gauss's law,

  $$\varphi_E = \frac{Q}{\varepsilon_0}$$

  Since all the charge lies outside the Gaussian surface therefore the net charge is zero.

  $$E\,(4\pi r^2) = \frac{Q}{\varepsilon_0} = 0$$

  Therefore the electric field is zero at all points inside the shell.
- **Electric field on the surface of shell:** In this case we consider that the radius of Gaussian surface is equal to the radius of shell that is, r =

$$\therefore \quad \text{Electric Field } E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

$$= \frac{1}{\varepsilon_0}\frac{q}{4\pi R^2}$$

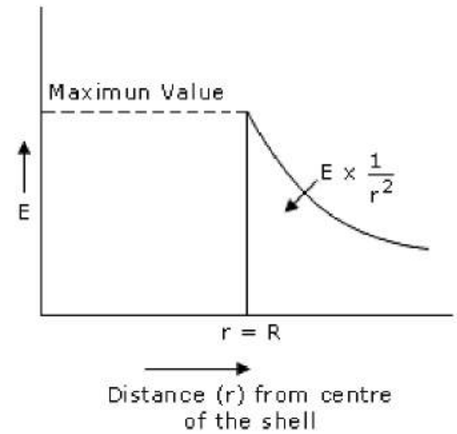
R

$$= \frac{\sigma}{\varepsilon_0}$$

Where

$$\sigma = \frac{q}{4\pi R^2}$$

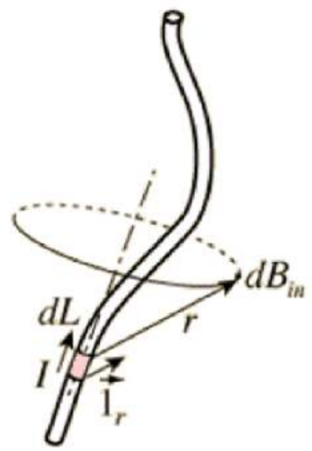
is the surface charge density on the shell.



From the above observation it is seen that the electric field inside the shell is zero and electric field is maximum at the surface of the shell.

Magnetostatics

Magnetostatics is the study of magnetic fields where the current is not changing with time that is, it is steady. The magnetic field produced by a steady current is given by



Magnetic field of a current element $\vec{dB} = \frac{\mu_0 I \vec{dL} \, x \, \vec{1}_r}{4\pi r^2}$

where

$\vec{dL}$ = infinitesmal length of conductor carrying electric current I

$\vec{1}_r$ = unit vector to specify the direction of the the vector distance r from the current to the field point.

This is known as **Biot- Savarts Law**.

Biot-savart law states that:

The magnetic field due to current carrying conductor is directly proportional to a) the magnitude of current b) length of element dL c) sine of angle between r and dL and inversely proportional to the square of the distance between source and the field point. Therefore the total magnetic field due to current carrying conductor is

$$B = \int dB = \int \frac{\mu_\circ}{4\pi}\frac{Idlsin\theta}{r^2} = \frac{\mu_\circ}{4\pi}\int \frac{Idlsin\theta}{r^2} Tesla$$

Gauss Law in Magnetism

Gauss law in magnetism states that the magnetic flux through a closed surface is zero. The integral form of gauss law is given by

$$\oiint_S B.\, dA = 0$$

Where S is any closed surface.

Ampere's Law

While the gauss theorem strictly deals with the electric field lines, Amperes law deals with the magnetic field lines. Ampere's law states that the line integral of magnetic field along a closed hypothetical loop (an amperian loop) is equal to μ times the current enclosed by loop where μ is the permeability of the free space.
Difference between Biot- Savart's Law and Ampere's Law
Both ampere's law and Biot-Savart's law help us to calculate the magnitude of magnetic field lines but the basic difference between the biot-savart law and amperes law is that in ampere's law a symmetric amperian loop is considered along a straightline charge. In other words amperes law is used for symmetrical distribution of currents, while Biotsavart's law is used for both symmetrical and asymmetrical distribution of currents.