that, the height of student B is double to the height of student A or the height of student A is half of the height of student B.

Note : To compare two quantities, the units must be the same.

RATIO AND PROPORTION

Ratio : The ratio of two quantities of the same kind and in the same unit is the fraction that one quantitiy is of the other.

The ratio of a to b is the fraction $\frac{a}{b}$ and it is written as $a : b$

Where a is called the first term and b is called the second term.

A ratio has no units.

Example-1 : Find the ratio of $4km$ to $300m$

Sol. To find the ratio of $4km$ to $300m$.

Firstly, convert both the quantities into same unit.

$\because \quad 1km = 1000m \Rightarrow 4km = 4 \times 1000 = 4000m$

So, ratio of $4000m$ to $300m = \frac{4000}{300} = 40 : 3$

Hence, required ratio is 40 : 3

SIMPLEST FORM

The ratio $(a : b)$ is in the simplest form, if H.C.F of a and b is 1, *i.e.* there is no common factor between a and b other than 1.

EQUIVALENT RATIO

To compare different ratios, first write the given ratios in the form of fraction. Convert fractions into like fractions. If the resulting fractions are equal, then the given ratios are said to be equivalent.

A ratio remains unchanged, if both of its terms are multiplied or divided by the same non-zero quantity.

Note : The ratio of two numbers is usually expressed in its simplest form.

For example if we have 1 : 3 and 2 : 9, we can compare these as follows :

Fraction of 1 : 3 is $\frac{1}{3}$, fraction of 2 : 9 is $\frac{2}{9}$

Now, change these fractions into like fractions $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$ and $\frac{2}{9} \times \frac{1}{1} = \frac{2}{9}$

Like fraction are $\frac{3}{9}$ and $\frac{2}{9}$, where $3 > 2$, so $\frac{3}{9} > \frac{2}{9}$.

Hence 1 : 3 is greater than 2 : 9.

Example-2 : Are the ratios 1 : 5 and 2 : 15 equivalent ?

Sol. To check this, we need to know whether 1 : 5 and 2 : 15 are equal.

First convert the ratios into fractions. 1 : 5 is written as $\frac{1}{5}$ 2 : 15 is written as $\frac{2}{15}$. For converting these into like fraction, make denominator of both the fractions same.

$$\frac{1}{5} \times \frac{3}{3} = \frac{3}{15} \text{ and } \frac{2}{15} \times \frac{1}{1} = \frac{2}{15}$$

so $$\frac{3}{15} > \frac{2}{15}$$

Hence 1 : 5 and 2 : 15 are not equivalent.

Example-3 : Following is the performance of a circket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This Year	4	2

in which year, was the record better ?

Sol. Last year, wins : losses = 8 : 2 = 4 : 1

This year, wins : losses = 4 : 2 = 2 : 1

Obviously, 4 : 1 > 2 : 1 (in fractional form, $\frac{4}{1} > \frac{2}{1}$)

Hence, we can say that the team performed better, last year

Example-4 : There are 10 soffa sets, 8 double beds and 16 dining tables in a furniture showroom. Find the ratio of :

(*i*) Number of dining tables to double beds

(*ii*) Number of double beds to sofa sets.

Sol. (*i*) Number of dining tables = 16

Number of double beds = 8

$\therefore$ Ratio of dining tables to double beds = 16 : 8 $= \frac{16}{8} = \frac{2}{1} = 2 : 1$

(*ii*) Number of double beds = 8

Number of sofa sets = 10

Ratio of double beds to sofa sets = 8 : 10 $= \frac{8}{10} = \frac{4}{5} = 4 : 5$

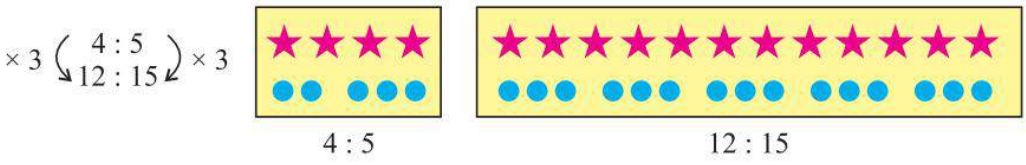
PROPORTION

We know that proportion is the equivalence of two ratios, therefore, a proportion involves four quantities. For example, when we say 4, 5, 12 and 15 are in proportion, we mean

4 : 5 : : 12 : 15 (: : denotes the symbol for proportion)

i.e. $$\frac{4}{5} = \frac{12}{15}$$

The first and the last terms (4 and 15) are called extremes, the second and third terms (5 and 12) are called middle terms.



Product of middle terms = product of exterme terms.

Given three terms of a proportion, it is easy to find the missing term. Let's see how this helps us solve problems.

USE OF PROPORTION IN SOLVING PROBLEMS

Before you start solving a problem using proportion, you need to determine whether the type of proportion is direct or indirect Let us analyse some problems to identify the type of proportion and solve them.

Proportion can be direct or indirect

Direct Proportion : The given quantities are said to be in direct proportion if with increase or decrease in one quantity leads to increase or decrease respectively in the other quantity.

Indirect Proportion : The given quantities are said to be in indirect proportion if with increase in one quantity the other quantity decreases and vice versa.

Example-5 : If 2 pens cost Rs 15, how many pens can you purchase with Rs 90 ?

Sol. The more the number of pens one purchases, the more is the amout to be paid.

More Pens → More is the amount to be paid therefore, there is a direct proportion between the number of pens and the amount to be paid. Let x be number of pens to be purchased.

$2 : 15 : : x : 90$

$$\frac{2}{15} = \frac{x}{90}$$

So, $$x = \frac{2\times 90}{15} = 12$$

Thus, we can buy 12 pens for Rs 90.

Example-6 : If it takes 6 days for 4 men to repair a road, how long will it take for 7 men to do the same job if they work at the same rate ?

Sol. The more the number of men, the lesser the number of days they will take to complete a job.

More Men → Less days to complete a job

Therefore, there is an indirect proportion between the number of men and number of days they take to complete a job. Let the number of days required be x. using the formula for indirect proportion, we get.

$4 : 7 : : x : 6$ or

$$4 \times 6 = 7 \times x$$

So, $$x = \frac{4\times 6}{7} = \frac{24}{7} = 3\frac{3}{7}$$

Therefore, it would take $3\frac{3}{7}$ days for 7 men to repair the road.

EXERCISE - 8.1

1. Find the ratio of

(*i*) ₹ 5 to 50 paise

(*ii*) 15kg to 210g

(*iii*) 4*m* to 400*cm*

(*iv*) 30 days to 36 hours

2. Are the ratios 1 : 2 and 2 : 3 equivalent ?
3. If the cost of 6 toys is ₹ 240, find the cost of 21 toys.
4. The car that I own can go 150 km with 25 litres of petrol. How far can it go with 30 litres of petrol ?
5. In a computer lab, there are 3 computers for every 6 students. How many computers will be needed for 24 students ?

PERCENTAGE – ANOTHER WAY OF COMPARING QUANTITIES

Do you remember percentage ?

- A fraction with denominator 100 is called percent.
- Symbol used for percent is %

Example of percent are $\frac{19}{100} = 19\%$, $\frac{7}{100} = 7\%$

Remember : Percent can be

(*i*) Converted into fraction.
(*ii*) Expressed as a ratio.
(*iii*) Converted into a decimal.

Do you know ?

The word 'percent' is an abbreviation of the latin word percentum which means per hundred or hundredths.

For example

Aman got 88 marks out of hundred marks; it means she got 88 percent marks. conversely, when a student got 65 percent marks; it means that the student scored 65 marks out of hundred marks.

Thus 25% means 25 out of hundred $= \frac{25}{100}$,

62% means 62 out of hundred $= \frac{62}{100}$

So, the symbol % stands for one hundredth *i.e* $\frac{1}{100}$

To understand percentage, consider the following example.

Suman made table top of 100 different coloured tiles. She counted blue, red, yellow and green tiles separately and filled the table given below.

Colour	Number of tiles	Fraction	Percentage	Written as
Blue	16	$\frac{16}{100}$	16	16%
Red	33	$\frac{33}{100}$	33	33%
Yellow	23	$\frac{23}{100}$	23	23%
Green	28	$\frac{28}{100}$	28	28%
Total	**100**			

This leads to :

In the fraction $\frac{r}{100}$, percentage = r it is written as r %. The numerator of the fraction i.e. r is also called rate percent. Thus, percentage = rate percent.

PERCENTAGE WHEN THE TOTAL IS NOT HUNDRED

If the total is not hundred, then convert it into an equivalent fraction with denominator 100. Consider the following example :

Rina has a necklace with 20 beads in two different colours.

Colour	Number of beads	fraction	Fraction with denominator 100	Percentage
Red	12	$\frac{12}{20}$	$\frac{12}{20} \times \frac{5}{5} = \frac{60}{100}$	60%
Green	8	$\frac{8}{20}$	$\frac{8}{20} \times \frac{5}{5} = \frac{40}{100}$	40%
Total	**20**			

Note : In practice, the words percent and percentage both are used synonymously.

Example-1 : Out of 25 students of a class, 16 are girls. What is the percentage of girls ?

Sol. Out of 25 students, there are 16 girls.

$$\therefore \quad \text{Percentage of girls} = \left(\frac{16}{25} \times 100\right)\% = 64\%$$

Example-2 : Teena scored 320 marks out of 400 marks and Reena scored 300 marks out of 360 marks. Whose performance is better ?

Sol. Teena scored 320 marks out of 400 marks.

$$\therefore \quad \text{Percentage of marks scored by Teena} = \left(\frac{320}{400} \times 100\right)\% = 80\%$$

Reena scored 300 marks out of 360 marks

$\therefore$ Percentage of marks scored by Reena

$$= \left(\frac{300}{360} \times 100\right)\%$$

$$= \frac{250}{3}\% = 83\frac{1}{3}\%$$

As $83\frac{1}{3} > 80$, therefore, performance of Reena is better than that of Teena.

Example-3 : Radhika spends Rs 350 every month. If this is 70% of her pocket money find her pocket money.

Sol. Let Radhika's pocket money = Rs x

Money spent = Rs 350

Also money spent is = 70% of x

$\therefore$ 70% of x = 350

$\Rightarrow \quad \frac{70}{100} \times x = 350$

$\Rightarrow \quad x = \frac{350 \times 100}{70} = 500, \; x = 500$

Hence Radhika's pocket money is Rs 500.

CONVERTING A PERCENTAGE INTO A FRACTION

Rule : To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and reduce the fraction to simplest form.

Example-4 : Express the following percentages as fractions :

(*i*) 20% **(*ii*) 6.5%** **(*iii*) $3\frac{1}{8}$ %** **(*iv*) 135%**

Sol. (*i*) $20\% = \frac{200}{100} = \frac{1}{5}$

(*ii*) $6.5\% = \frac{65}{1000} = \frac{13}{200}$

(*iii*) $3\frac{1}{8}\% = \frac{25}{8}\% = \frac{\frac{25}{8}}{100} = \frac{25}{8} \times \frac{1}{100} = \frac{1}{32}$

(*iv*) $135\% = \frac{135}{100} = \frac{27}{20} = 1\frac{7}{20}$

(Replace % sign with $\frac{1}{100}$ and simplify)

CONVERTING A FRACTION INTO PERCENTAGE

Rule : To convert a fraction into percentage, multiply the given fraction by 100, put the sign of %.

Example-5 : Converting the following fractions into percentages.

(*i*) $\frac{1}{2}$ **(*ii*) $\frac{2}{3}$** **(*iii*) $1\frac{5}{8}$**

Sol. (*i*) $\frac{1}{2} = \frac{1}{2} \times 100 = 50$

Thus, $\frac{1}{2} = 50\%$

(*ii*) $\frac{2}{3} \times 100 = 66.67\%$

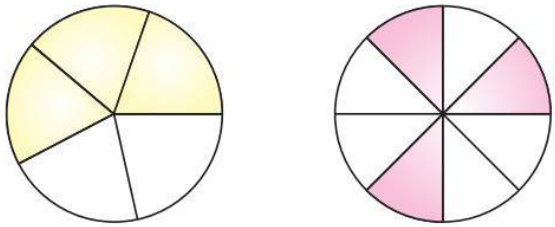
Thus, $\frac{2}{3} = 66.67\%$

(Multiply by 100 and put % sign)

(*iii*) $1\frac{5}{8} = \frac{13}{8} \times 100 = 13 \times \frac{25}{2} = \frac{325}{2} = 162.5$

Thus, $1\frac{5}{8} = 162.5\%$

Example-6 : Write the part of the circle which is shaded and hence find percentage of part which is shaded.



(*i*) (*ii*)

Sol.

(*i*) Shaded part $= \frac{3}{5}$

Percentage of shaded part $= \left(\frac{3}{5} \times 100\right)\% = 60\%$

(*ii*) Shaded part $= \frac{3}{8}$

Percentage of shaded part $= \left(\frac{3}{8} \times 100\right)\%$

$= \frac{75}{2}\% = 37.5\%$

CONVERTING A PERCENTAGE INTO A RATIO

Rule : To convert a percentage into a ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.

Example-7 : Convert the following percentages as ratios in simplest form.

(*i*) 28% **(*ii*) 17.5%** **(*iii*) $66\frac{2}{3}\%$**

Sol. (*i*) $28\% = \frac{28}{100} = \frac{7}{25} = 7 : 25$

(*ii*) $17.5\% = \frac{17.5}{100} = \frac{175}{1000} = \frac{7}{40} = 7 : 40$

(*iii*) $66\frac{2}{3}\% = \frac{200}{3}\% = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3} = 2 : 3$

(First convert a fraction in simplest form)

CONVERTING A RATIO INTO A PERCENTAGE

To convert a ratio into percentage, first convert the given ratio into a fraction and then to a percentage.

Example-8 : Express the following ratios as percentages :

(*i*) 1 : 2 (*ii*) 7 : 6

Sol. (*i*) $1 : 2 = \frac{1}{2} = \left(\frac{1}{2} \times 100\right)\% = 50\%$

(ii) $7 : 6 = \frac{7}{6} = \left(\frac{7}{6} \times 100\right)\% = \frac{350}{3}\%$

$= 116\frac{2}{3}\%$

(First convert ratio into a fraction)

Example-9 : Ritu's mother said, to make idlis, you must take 5 parts rice and 3 parts urad dal. What percentage of such a mixture would be rice and what percent would be urad dal ?

Sol. In terms of ratio we write this as rice : urad dal = 5 : 3

Total number of parts = 5 + 3 = 8

This means $\frac{5}{8}$ part in rice and $\frac{3}{8}$ part in urad dal.

Then, Percentage of rice = $\left(\frac{5}{8} \times 100\right)\% = \frac{125}{2}\% = 62.5\%$

Percentage of urad dal = $\left(\frac{3}{8} \times 100\right)\% = \frac{75}{2}\% = 37.5\%$

CONVERTING A PERCENTAGE INTO A DECIMAL

Rule : To convert a percentage into a decimal, first convert the percentage into a fraction by replacing the % with $\frac{1}{100}$. Then convert the fraction to decimal.

Example-10 : Convert the following percentages into decimals.

(*i*) 25% (*ii*) 78.5% (*iii*) 150%

Sol. (*i*) $25\% = 25 \times \frac{1}{100} = \frac{25}{100} = 0.25$

(*ii*) $78.5\% = 78.5 \times \frac{1}{100} = \frac{78.5}{100} = 0.785$

(*iii*) $150\% = 150 \times \frac{1}{100} = \frac{15}{10} = 1.5$

Note : From the above example, we observe that to convert a percentage into a decimal, remove the sign % and move the decimal point two places to the left.

CONVERTING A DECIMAL INTO A PERCENTAGE

Rule : To convert a decimal into a percentage, multiply the decimal by 100 and put the sign %

Example-11 : Convert the following decimals to percent

(*i*) 0.75 (*ii*) 0.025 (*iii*) 0.4

Sol. (*i*) $0.75 = (0.75 \times 100)\% = 75\%$

(*ii*) $0.025 = (0.025 \times 100)\% = 2.5\%$

(*iii*) $0.4 = (0.4 \times 100)\% = 40\%$

Note : From the above example, we observe that to convert a decimal into a percentage, move the decimal point two places to the right (adding zeros if necessary) and put the % sign.

FIND A PERCENTAGE OF A GIVEN QUANTITY

Rule : To find a percentage of a given quantity, change the percentage into fraction and multiply by the given quantity.

Example-12 : Find the value of

(*i*) 75% of 12 (*ii*) $12\frac{1}{2}$ % of 64

Sol. (*i*) $75\% \text{ of } 12 = \frac{75}{100} \times 12 = \frac{3}{4} \times 12 = 9$

(*ii*) $12\frac{1}{2}\% \text{ of } 64 = \frac{\frac{25}{2}}{100} \times 64 = \frac{25}{2} \times \frac{16}{25} = 8$

Example-13 : A survey of 50 children showed that 20% like playing cricket. How many children liked playing cricket ?

Sol. Total number of children = 50

Out of these, 20% liked playing cricket.

$\therefore$ Number of children who liked playing circket

$$= 20\% \text{ of } 50$$

$$= \frac{20}{100} \times 50$$

$$= \frac{1}{5} \times 50 = 10$$

EXPRESSING ONE QUANTITY AS PERCENTAGE OF ANOTHER QUANTITY

Rule : To express one quantity as a percentage of another quantity,

$$\text{Percentage} = \left(\frac{\text{one quantity}}{\text{other quantity}} \times 100 \right)\%$$

Note that both quantities must be of same kind (in same units)

Example-14 : A person ate 3 icecream cups out of 5 kept in the fridge what percent did he eat ?

Sol. Required percentage $= \left(\frac{3}{5}\times 100\right)\% = 60\%$

Example-15 : Express

(*i*) 15 as a percentage of 45.

(*ii*) 20 paise as a percentage of ₹ 5.

Sol. (*i*) Required percentage $= \left(\frac{15}{45}\times 100\right)\% = \frac{100}{3}\% = 33\frac{1}{3}\%$

(*ii*) ₹ 5 = 500 paise

Required percentage $= \left(\frac{20}{500}\times 100\right)\% = 4\%$

FINDING INCREASE / DECREASE PERCENTAGE

Rule : Percentage increase $= \left(\frac{\text{increase in value}}{\text{original value}}\times 100\right)\%$

Percentage decrease $= \left(\frac{\text{decrease in value}}{\text{orignal value}}\times 100\right)\%$

Example-16 : Apples were selling at ₹ 50 per kg last season. This season they are selling at ₹ 55 per kg. Find the percentage increase or decrease in price.

Sol. Obviously the price has increased from ₹ 50 to ₹ 55.

Original price = ₹ 50

Increase in Price = ₹ 55 – ₹ 50

= ₹ 5

$\therefore$ Percentage increase $= \left(\frac{\text{increase in value}}{\text{original value}}\times 100\right)\%$

$= \left(\frac{5}{50}\times 100\right)\%$

Hence, the price of apples has increased by 10%.

Example-17 : A computer costing ₹ 60000 one year ago now costs ₹ 40000. Find the percentage increase or decrease in the price.

Sol. The price has decreased from ₹ 60000 to ₹ 40000.

Original price = ₹ 60000,

Decrease in price = ₹ 60000 – ₹40000

= ₹ 20000

$$\therefore \quad \text{Percentage decrease} = \left(\frac{\text{decrease in value}}{\text{original value}} \times 100\right)\%$$

$$= \left(\frac{20000}{60,000} \times 100\right)\% = \frac{100}{3}\% = 33\frac{1}{3}\%$$

Hence the price has decreased by $33\frac{1}{3}\%$

USE OF PERCENTAGE

We shall now solve some real life problems on percentages.

Example-18 : In a class of 50 students, 20% students wear spectacles. How many students do not wear spectacles ?

Sol. Since 20% wear spectacles.

$\therefore$ Percentage of students who do not wear spectacles.

$$= (100 - 20)\% = 80\%$$

Hence, the number of students who do not wear spectacles

$$= 80\% \text{ of } 50 = \frac{80}{100} \times 50 = 40$$

Example-19 : On a rainy day, only 36 students out of 48 came to the class. What percentage were absent ?

Sol. Total number of students in class = 48

Number of students absent = 48 – 36 = 12

$\therefore$ Percentage of absent students $= \left(\frac{12}{48} \times 100\right)\% = 25\%$

EXERCISE - 8.2

1. Convert the following fractions into percents

(*i*) $\frac{1}{8}$ (*ii*) $\frac{49}{50}$ (*iii*) $\frac{5}{4}$ (*iv*) $1\frac{3}{8}$

2. Convert the following percents into fractions in simplest form :

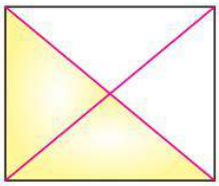
(*i*) 25% (*ii*) 150% (*iii*) $7\frac{1}{2}\%$

3. (*i*) Anita secured 324 marks out of 400 marks. Find the percentage of marks secured by Anita.

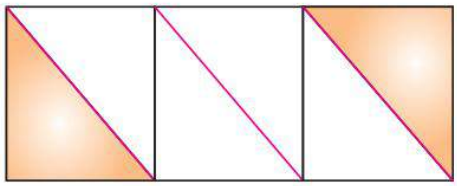
(*ii*) Out of 32 students, 8 are absent from the class. What is the percentage of students who are absent.

(*iii*) There are 120 voters, 90 out of them voted. What percent did not vote ?

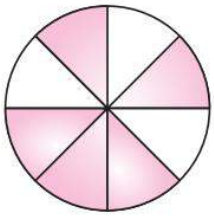
4. Estimate the part of figure which is shaded and hence find the percentage of the part which is shaded.



(*i*)



(*ii*)



(*iii*)

5. Convert the following percentages into ratios in simplest form :

(*i*) 14% (*ii*) $1\frac{3}{4}\%$ (*iii*) $33\frac{1}{3}\%$

6. Express the following ratios as percentages :

(*i*) 5 : 4 (*ii*) 1 : 1 (*iii*) 2 : 3 (*iv*) 9 : 16

7. Chalk contains calcium, carbon and sand in the ratio 12 : 3 : 10. Find the percentage of carbon in the chalk.

8. Convert each part of the following ratios into percentage :

(*i*) 3 : 1 (*ii*) 1 : 4 (*iii*) 4 : 5 : 6

9. Convert the following percentages to decimals :

(*i*) 28% (*ii*) 3% (*iii*) $37\frac{1}{2}\%$

10. Convert the following decimals to percentage :

(*i*) 0.65 (*ii*) 0.9

(*iii*) 2.1

11. (*i*) If 65% of students in a class have a bicycle, then what percent of the students do not have a bicycle ?

(*ii*) We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what percent are mangoes ?

12. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

13. Arun bought a plot for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the percentage of price increase ?

14. Find :

(*i*) 15% of 250 (*ii*) 25% of 120 litres

(*iii*) 4% of 12.5 (*iv*) 12% of ₹ 250

15. Multiple Choice Questions :

(*i*) The ratio 2 : 3 expressed as percentage is

(*a*) 40% (*b*) 60% (*c*) $66\frac{2}{3}\%$ (*d*) $33\frac{1}{3}\%$

(*ii*) If 30% of x is 72, then x is equal to

(*a*) 120 (*b*) 240 (*c*) 360 (*d*) 480

(*iii*) 0.025 when expressed as a percent is

(*a*) 250% (*b*) 25% (*c*) 4% (*d*) 2.5%

(*iv*) In a class, 45% of students are girls. If there are 22 boys in the class, then the total number of students in the class is

(*a*) 30 (*b*) 36 (*c*) 40 (*d*) 44

(*v*) What percent of $\frac{1}{7}$ is $\frac{2}{35}$?

(*a*) 20% (*b*) 25% (*c*) 30% (*d*) 40%

PROFIT AND LOSS

A shopkeeper (dealer or retailer) buys his goods from a manufacturer or a wholesale dealer and then he sells them to a customer. If he sells his goods at a higher price than he paid for them, he makes a profit (gain). If for some reasons, he sells his goods at a lower price than he paid for them, then he suffers a loss.

Cost price → The price at which an article is purchased, is called its cost price (abbreviated C.P)

Selling price → The price at which an article is sold, is called its selling price (abbreviated S.P)

Profit → If the selling price of an article is more than its cost price, then there is a profit.

Profit = selling price – cost price

If SP > CP, then there is gain or profit

Loss → If the selling price of an article is less than its cost price, then there is a loss.

Loss = Cost price – Selling price

If SP < CP, then there is a loss

Also if SP = CP, then there is no profit no loss

For Example :

If a dealer buys a T.V for ₹ 11, 000 and sells it at ₹ 12,100, then he makes a profit, and,

Profit = ₹ 12100 – ₹11000 = ₹1100

If a dealer sells it at ₹10,000, then he suffers a loss and

Loss = ₹11000 – ₹10000 = ₹1000

PROFIT OR LOSS PERCENTAGE

Often in bussiness, instead of talking about what actual profit or loss is, we have to find out the profit or loss percentage.

Profit or loss percentage is always calculated on the cost price.

$$\text{Profit percentage} = \left[\frac{\text{Profit}}{\text{cost price}} \times 100\right]\%$$

$$\text{Loss percentage} = \left[\frac{\text{Loss}}{\text{cost price}} \times 100\right]\%$$

Example-1 : A dealer buys a watch for ₹ 580 and sells at ₹667. Find his profit and profit percentage.

Sol. C.P of the watch = ₹580

S.P of watch = ₹667

$\therefore$ Profit = SP – CP = ₹667 – ₹580 = ₹87

$$\text{Profit percentage} = \left[\frac{\text{Profit}}{\text{cost price}} \times 100\right]\% = \left[\frac{87}{580} \times 100\right]\% = 15\%$$

Example-2 : Sakshi bought a gold ring for ₹5,500 and two years later sold it for ₹4,000. What was her profit or loss ? Also, find her profit or loss percent.

Sol. The selling price of the ring is less than its cost price. Thus, Sakshi suffers a loss on the ring.

$$\text{Loss} = \text{CP} - \text{SP} = ₹\,(5{,}500 - 4{,}000)$$
$$= ₹1{,}500$$
$$\text{Loss percent} = \frac{\text{Loss}}{\text{cost price}} \times 100$$
$$= \frac{1{,}500}{5{,}500} \times 100 = 27.27\%$$

Example-3 : A shopkeeper buys an article at ₹150 and sells it at a profit of 12%. Find the selling price

Sol. Cost price = ₹150

$$\text{Profit} = 12\% \text{ of cost price} = \frac{12}{100} \text{ of } ₹150$$
$$= ₹\left[\frac{12}{100} \times 150\right] = ₹\,18$$
$$\text{Selling price} = \text{Cost price} + \text{profit} = ₹150 + ₹18$$
$$= ₹168$$

Example-4 : Find the selling price of an article which is purchased for ₹1240 and sold at a loss of 7%

Sol. Cost price = ₹12400

$$\text{Loss} = 7\% \text{ of cost price} = \frac{7}{100} \times ₹12400 = ₹868$$
$$\text{Selling price} = \text{Cost price} - \text{Loss}$$

$\therefore$ = ₹12400 – ₹868 = ₹11532.

Note : We can also calculate S.P. by using formula

$$\text{S.P.} = \text{C.P.} \times \left[\frac{100 - \text{Loss }\%}{100}\right]$$

Example-5 : By selling an article for ₹ 475 Rahul lost 5%. Find C.P of the article.

Sol.

$$\begin{aligned}
\text{Let C.P of article} &= ₹100\\
\text{Loss} &= 5\% \text{ of } ₹100\\
&= ₹5\\
\text{S.P. of article} &= ₹(100-5)\\
&= ₹95\\
\text{If S.P. of article is ₹95, then C.P} &= ₹100\\
\text{If S.P of article is ₹1, then C.P} &= ₹\frac{100}{95}\\
\text{If S.P of article is ₹475, then C.P} &= ₹\left[\frac{100}{95}\times 475\right]\\
&= ₹500
\end{aligned}$$

Note → We can also calculate C.P using formula.

$$\text{C.P.} = \text{S.P.} \times \left[\frac{100}{100-\text{Loss}\%}\right]$$

SIMPLE INTEREST

When you borrow money from a bank or a money lender, you need to pay the money back after a period along with some extra money. This extra money is called the Interest

The amount of interest you pay depends on :

- The money you borrow, called the Principal (P).
- The rate of interest per annum, R (in percent).
- The time for which the money is borrowed, T (in years).

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} \quad i.e. \quad I = \frac{P \times R \times T}{100}$$

Note → Given any three quantities out of P, R, T and I;

We can calculate the fourth quantity by using the above formula.

- **Amount :** The total money paid by the borrower to the money lender is called amount.

  Thus, Amount = Principal + Interest

If P denotes the principal, I is interest paid and A the amount, then

$$A = P + I$$

Example-1 : Find the simple interest on ₹1500 at 6% per annum for 3 years. Also find the amount.

Sol. Here, Principal P = ₹1500, rate of interest R = 6% per annum and Time T = 3 years

$$\therefore \quad \text{Simple Interest I} = \frac{P \times R \times T}{100} = ₹\frac{1500 \times 6 \times 3}{100}$$

$$= ₹270$$

$$\text{Amount A} = P + I = ₹1500 + ₹270 = ₹1770$$

Example-2 : On a certain sum the interest paid after 3 years is ₹450 at 5% rate of interest per annum. Find the sum.

Sol. Here I = ₹450, R = 5% p.a. and T = 3 years Let the sum i.e. principal be ₹ P, we want to find P

$$\text{Simple Interest I} = \frac{P \times R \times T}{100}, \text{ we get}$$

$$450 = \frac{P \times 5 \times 3}{100} \Rightarrow P = 450 \times \frac{100}{5 \times 3}$$

$$P = 3000$$

Hence, the required sum = ₹ 3000.

Example-3 : Jyoti take a loan of ₹6000 and pays back 7,080 at the end of three years. What is the rate of interest that she paid ?

Sol. Jyoti paid ₹(7,080 – 6,000) = 1,080 as interest, we know that

SI = 1,080, T = 3 years and P = ₹ 6000

We need to find R

The formula for S.I is

$$SI = \frac{P \times R \times T}{100}$$

So,
$$R = \frac{SI \times 100}{P \times T}$$

Substituting the values in the formula

$$R = \frac{1,080 \times 100}{6,000 \times 3} = 6$$

Jyoti paid interest at the rate of 6% per annum

Example-4 : Tanveer lends ₹7,000 to a shopkeeper and charges an interest of 7%. If he gets back ₹8,470 find the time for which tanveer lent the money.

Sol. The interest Tanveer earned = (Amount – Principal)

$$= ₹(8,470 - 7,000)$$

$$= ₹(1,470)$$

P = ₹7,000, S.I = ₹1,470 and R = 7% we need to find T

The formula for S.I. is

$$S.I = \frac{P \times R \times T}{100}$$

So,
$$T = \frac{S.I \times 100}{P \times R}$$

Substituting the values in the formula :

$$T = \frac{1,470 \times 100}{7,000 \times 7} = 3$$

So, Tanveer gave the loan for 3 years.

EXERCISE - 8.3

1. Find what is the profit or loss in the following transactions. Also find profit percent or loss percent in each case.

(*i*) Gardening shears bought for ₹250 and sold for ₹325

(*ii*) A refrigerater bought for ₹12,000 and sold at ₹13,500

(*iii*) A cupboard bought for ₹2,500 and sold at ₹3,000.

(*iv*) A shirt bought for ₹250 and sold at ₹150

2. A shopkeeper buys an article for ₹735 and sold it for ₹850. Find his profit or loss.

3. Kirti bought a saree for ₹ 2500 and sold it for ₹ 2300. Find her loss and loss percent.

4. An article was sold for ₹252 with a profit of 5%. What was its cost price.

5. Amrit buys a book for ₹275 and sells it at a loss of 15%. For how much does she sell it ?

6. Juhi sells a washing machine for ₹13500. She losses 20% in the bargain. What was the price at which she bought it ?

7. Anita takes a loan of ₹ 5000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

8. Find the amount to be paid at the end of 3 years in each case :

(*i*) Principal = ₹1200 at 12% p.a.

(*ii*) Principal = ₹7500 at 5% p.a.

9. Find the time when simple interest on ₹2500 at 6% p.a. is ₹450

10. Find the rate of interest when simple interest on ₹1560 in 3 years is ₹585.

11. If Nakul gives an interest of ₹45 for one year at 9% rate p.a. what is the sum he borrowed?

12. If ₹14,000 is invested at 4% per annum simple interest, how long will it take for the amount to reach ₹16240 ?

13. Multiple Choice Questions :

(*i*) If a man buys an article for ₹ 80 and sells it for ₹100, then gain percentage is

(*a*) 20% (*b*) 25%

(*c*) 40% (*d*) 125%

(*ii*) If a man buys an article for ₹120 and sells it for ₹100, then his loss percentage is

(*a*) 10% (*b*) 20% (*c*) 25%(*d*) $16\frac{2}{3}\%$

(*iii*) The salary of a man is ₹24000 per month. If he gets an increase of 25% in the salary, then the new salary per month is

(*a*) ₹2,500 (*b*) ₹28,000 (*c*) 30,000 (*d*) 36,000

(*iv*) On selling an article for ₹100, Renu gains ₹20 Her gain percintage is

(*a*) 25% (*b*) 20% (*c*) 15% (*d*) 40%

(*v*) The simple interest on ₹6000 at 8% *p.a.* for one year is

(*a*) ₹600 (*b*) ₹480 (*c*) ₹400 (*d*) ₹240

(*vi*) If Rohini borrows ₹4800 at 5% p.a. simple interest, then the amount she has to return at the end of 2 years is.

(*a*) ₹480 (*b*) ₹5040 (*c*) ₹5280 (*d*) ₹5600

WHAT HAVE WE DISCUSSED ?

1. We are often required to compare two quantities in our daily life. They may be heights, weights, salaries, marks, etc.
2. Two ratios can be compared by converting them to like fractions. If two fractions are equal, we say the two given ratios are equivalent.
3. If two ratios are equivalent then the four quantities are said to be in proportion. For example, the ratios 8 : 2 and 16 : 4 are equivalent there fore 8, 2, 16 and 4 are in proportion.
4. A proportion is the equivalence of two ratios.
5. In a proportion, product of middle terms is equal to product of extreme terms.
6. Percent means per hundred or out of hundred. The symbol % stands for percent i.e. $\frac{1}{100}$
7. To convert the percentage into a fraction, replace the % sign with $\frac{1}{100}$ and simplify.
8. To convert a fraction into percentage, multiply the fraction by 100 and put % sign.
9. To convert the percentage into decimal, first convert the percentage into a fraction by replacing the sign % with $\frac{1}{100}$ and then convert fraction to decimal.
10. To convert a decimal into percentage, multiply the decimal by 100 and put the sign %
11. To convert percentage into ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.
12. To convert ratio into percentage, first convert the given ratio into a fraction and then to percentage.
13. To find the percentage of a given quantity, change the percentage into fraction and multiply by the given quantity.
14. Percentage increase / decrease in a quantity. $= \left[\frac{\text{Change in quantity}}{\text{Original quantity}} \times 100\right]\%$

15. The price at which an article is bought by a dealer is called its cost price (C.P.)

16. The price at which the article is sold by a dealer is called its selling price (S.P)

17. If selling price is more than the cost price, then the dealer makes a profit, and Profit = Selling price – Cost price.

18. If the selling price is less than the cost price, then the dealer suffers a Loss and,
Loss = Cost Price – Selling Price.

19. Profit or Loss percentage is calculated on the cost price.

$$\text{Profit percentage} = \left[\frac{\text{profit}}{\text{cost price}} \times 100\right]\%$$

$$\text{Loss percentage} = \left[\frac{\text{loss}}{\text{cost price}} \times 100\right]\%$$

20. $\text{Simple Interest (S.I.)} = \dfrac{P \times R \times T}{100}$

Where P = Principal
R = Rate of interest per annum
T = Time (in years)

21. Amount = Principal + Interest

22.
- $P = \dfrac{S.I \times 100}{R \times T}$
- $R = \dfrac{S.I \times 100}{P \times T}$
- $T = \dfrac{S.I \times 100}{P \times R}$

LEARNING OUTCOMES

After completion of the chapter, the students are now able to

1. Compare the two quantities in their daily life.
2. Convert two ratios in to like fractions.
3. Understand equivalent ratios.
4. Distinguish quantities that are in proportion.
5. Find the percentage of given problems.
6. Convert percentage into fraction, fraction into percentage, percentage into decimal, decimal into percentage, percentage into ratio and ratio into percentage.
7. Solve the problems related to profit, loss, profit% or loss%.
8. Find simple interest, principal, rate of interest or time in the given problem.
9. Differentiate between simple interest and amount.

ANSWERS

EXERCISE 8.1

1. (*i*) 10 : 1 (*ii*) 500 : 7
(*iii*) 1 : 1 (*iv*) 20 : 1

2. No **3.** 840

4. 180*km* **5.** 12

EXERCISE 8.2

1. (*i*) 12.5% (*ii*) 98%
(*iii*) 125% (*iv*) $137\frac{1}{2}\%$

2. (*i*) $\frac{1}{4}$ (*ii*) $\frac{3}{2}$ (*iii*) $\frac{3}{40}$

3. (*i*) 81% (*ii*) 25% (*iii*) 25%

4. (*i*) $\frac{1}{2}$; 50% (*ii*) $\frac{1}{3}$; $33\frac{1}{3}\%$ (*iii*) $\frac{5}{8}$; 62.5%

5. (*i*) 7 : 50 (*ii*) 7 : 400
(*iii*) 1 : 3

6. (*i*) 125% (*ii*) 100%
(*iii*) $66\frac{2}{3}\%$ (*iv*) $56\frac{1}{4}\%$

7. 12%

8. (*i*) 75%, 25% (*ii*) 20%, 80% (*iii*) $26\frac{2}{3}\%$, $33\frac{1}{3}$, 40%

9. (*i*) 0.28 (*ii*) 0.03 (*iii*) 0.375

10. (*i*) 65% (*ii*) 90% (*iii*) 210%

11. (*i*) 35% (*ii*) 20%

12. 2% **13.** $5\frac{5}{7}\%$

14. (*i*) 37.5 (*ii*) 30 litres
(*iii*) 0.5 (*iv*) ₹300

15. (*i*) (*c*) (*ii*) (*b*) (*iii*) (*d*)
(*iv*) (*c*) (*v*) (*d*)

EXERCISE 8.3

1. (*i*) Profit = ₹75 ; Profit % = 30 (*ii*) Profit = ₹1500 ; profit % = 12.5
(*iii*) Profit = ₹500 ; profit% = 20 (*iv*) Loss = ₹100 ; Loss % = 40

2. Profit = ₹115

3. ₹ 200 ; 8%

4. ₹ 240

5. ₹ 233.75

6. ₹ 16875

7. ₹ 750

8. (*i*) ₹1632 (*ii*) ₹ 8625

9. 3 years

10. 12.5% *p.a.*

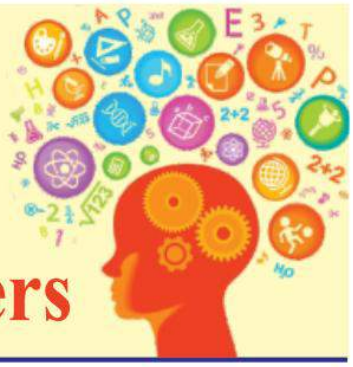
11. ₹ 500

12. 4 years

13. (*i*) (*b*) (*ii*) (*d*)
(*iii*) (*c*) (*iv*) (*a*)
(*v*) (*b*) (*vi*) (*c*)

CHAPTER 9

H4I4L7



Rational Numbers

Learning Objectives :-

In this chapter, you will learn :-

1. To define rational numbers and reduce them to their standard form.
2. The concept of equivalent rational numbers.
3. To represent the rational number on a number line.
4. To find more rational numbers between given rational numbers.
5. To compare rational numbers and perform basic mathematical operations on them.
6. To use rational numbers in solving your daily life problems.

3. OUR NATION'S PRIDE

Aryabhata : Aryabhata was a great Indian mathematican born in 476CE at Kusumpura (Patliputra) presently Patna, India and died in 550CE. His research work include place value, number system and many more concepts in the field of mathematics. Great mathematician Lapluce (1749-1829) said that India has offered us a system for expressing all the numbers with the help of only ten symbols. This concept picks up its new height when we recall that the world known mathematicians like Appolonius and Archimedes failed to discover such essential system.

INTRODUCTION

It took a long time to discover the numbers. Earlier, man could not write the numbers but could only express them with the help of fingers or counting objects.

Natural numbers : The numbers which are used for counting are called natural numbers.

For example 1, 2, 3, 4, 5, 6, 7, ...

Whole Numbers : All natural numbers along with zero (0) are called whole numbers.

For example 0, 1, 2, 3, 4, 5, 6, ...

Integers : All the whole numbers and negative of natural numbers are called integers.

For example .....–6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ...

In the above 0 is an integer which is neither negative nor positive.

Fractions : The numbers of the form $\frac{a}{b}$ are known as fractions, where a is called numerator and b ($\neq 0$) is called the denominator.

Need of Rational number : To convert time, length and distance from one unit into another fractions are used e.g. If we convert 20 minutes into hours it will be $\frac{20}{60} = \frac{1}{3}$ hours. You can repersent a height of 500m above sea level as $\frac{1}{2}$ km. Can this height be repersented below sea level ? Can we denote $\frac{1}{2}$ km below sea level as $\frac{-1}{2}$? We see $\frac{-1}{2}$ is neither an integer nor a fractional number. We need to extend our number system to include such type of numbers.

What are rational numbers

The word 'rational' arises from the term 'ratio' and a ratio of 5 : 6 is written as $\frac{5}{6}$ where 5 is numerator and 6 is denominator.

The numbers of the form $\frac{a}{b}$

Where a and b are Integers and $b \neq 0$ are called rational number. e.g $\frac{5}{6}$, $\frac{-7}{8}$ and $\frac{21}{-9}$ rational numbers.

Equivalent of rational numbers : If we multiply or divide both the numerator and denominator of a rational number by a non zero integer then we get a rational number equivalent to the given rational number.

Example-1 : Write two equivalent rational numbers for the following :

(*i*) $\frac{-3}{5}$ (*ii*) $\frac{-3}{40}$

Sol. (*i*)
$$\frac{-3}{5} = \frac{-3}{5} \times \frac{2}{2} = \frac{-6}{10}$$
$$\frac{-3}{5} = \frac{-3}{5} \times \frac{3}{3} = \frac{-9}{15}$$

$\therefore$ Equivalent rational numbers of $\frac{-3}{5}$ are $\frac{-6}{10}$ and $\frac{-9}{15}$

(*ii*)
$$\frac{-8}{40} = \frac{-8 \div 4}{40 \div 4} = \frac{-2}{10}$$
$$\frac{-8}{40} = \frac{-8 \div (-8)}{40 \div (-8)} = \frac{1}{-5}$$

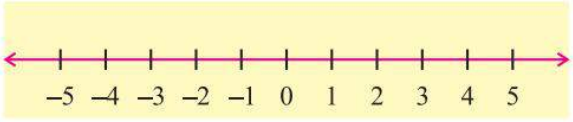
$\therefore$ Thus equivalent rational numbers of $\frac{-8}{40}$ are $\frac{-2}{10}$ and $\frac{1}{-5}$

This way we can write as many equivalent fractions as we want.

Positive rational numbers : A rational number is said to be positive if both the numerator and denominator are either positive or negative. For example : $\frac{3}{7}, \frac{5}{8}, \frac{-15}{-18}, \frac{-25}{-9}$ are positive rational numbers.

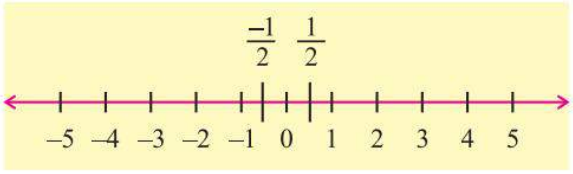
Negative rational numbers : A rational number is said to be negative if either numerator or denominator is negative. For example : $\frac{-6}{8}, \frac{5}{-9}, \frac{-15}{8}, \frac{8}{-17}$ are negative rational numbers.

Rational numbers on a number line : We have already learnt how to repersent integers on a number line.



Now let us repersent rational number $\frac{1}{2}$ and $\frac{-1}{2}$ on a number line. Half the distance between 0 and 1 which will be repersented as $\frac{1}{2}$.

Half the distance between 0 and –1 which will be repersented as $\frac{-1}{2}$.



Rational numbers in the standard form : A rational number is said to be in standard form if its denominator is positive and the highest common factor (HCF) of numerator and denominator is 1.

For example : $\frac{5}{7}, \frac{-4}{9}, \frac{2}{9}$

Example-2 : Find the standard form of

(*i*) $\frac{-21}{48}$ **(*ii*)** $\frac{42}{-28}$

Sol. (*i*) $\frac{-21}{48}$

$\because$ H.C.F of 21 and 48 is 3

So dividing both the numerator and denominator by 3 we get.

$\therefore \quad \frac{-21}{48} = \frac{-21 \div 3}{48 \div 3}$

$= \frac{-7}{16}$

Standard form of $\frac{-21}{48}$ is $\frac{-7}{16}$.

(*ii*) $\frac{42}{-28}$

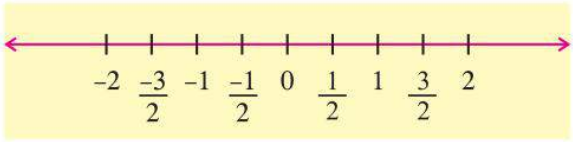
$\because$ HCF of 42 and 28 is 14.

$\therefore$ So dividing both the numerator and denominator by (–14) we get

$$\frac{42}{-28} = \frac{42 \div (-14)}{-28 \div (-14)} = \frac{-3}{2}$$

$\therefore$ Standard form of $\frac{42}{-28}$ is $\frac{-3}{2}$.

Comparison of two rational numbers : From a number line it is clear that :-



1. A positive rational number is always greater than zero.

2. A negative rational number is always less than zero.

3. If both the rational number are either positive or negative then they are compared as follows.

(*i*) Express each of the rational number with a positive denominator.

(*ii*) Take L.C.M of the denominators to make the denominators same.

(*iii*) The rational number having the greater numerator is greater.

Example-3 : Which is greater in each of the following :

(*i*) $\frac{4}{9}$ **and** $\frac{3}{6}$ **(*ii*)** $\frac{-5}{7}$ **and** $\frac{-4}{9}$

Sol. (*i*) Given rational numbers are $\frac{4}{9}$ and $\frac{3}{6}$.

$\because$ L.C.M of 9 and 6 is 18.

$\therefore$
$$\frac{4}{9} = \frac{4\times 2}{9\times 2} = \frac{8}{18}$$

$$\frac{3}{6} = \frac{3\times 3}{6\times 3} = \frac{9}{18}$$

$\because$ Numerator of second rational number is greater than first

i.e., $9 > 8$

so $\frac{3}{6} > \frac{4}{9}$

(*ii*) Given rational numbers are $\frac{-5}{7}$ and $\frac{-4}{9}$.

$\because$ LCM of 7 and 9 is 63.

$$\therefore \quad \frac{-5}{7} = \frac{-5}{7} \times \frac{9}{9} = \frac{-45}{63}$$

$$\frac{-4}{9} = \frac{-4}{9} \times \frac{7}{7} = \frac{-28}{63}$$

$$\because \quad -28 > -45$$

$$\text{So,} \quad \frac{-4}{9} > \frac{-5}{7}$$

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Integers between –4 and 3 are –3, –2, –1, 0, 1, 2. There are exactly six integers between –4 and 3 which are finite. In example 3 part (*ii*) the rational numbers between $\frac{-5}{7}$ and $\frac{-4}{9}$ are

$$\frac{-44}{63} < \frac{-43}{63} < \frac{-42}{63} < \frac{-41}{63} < \ldots\ldots\ldots < \frac{-29}{63}$$

You can insert as many as rational numbers as you want. To insert 'n' rational numbers, between two rational numbers, we shall multiply both the numerator and the denominator of the given rational number by '$n + 1$'.

So, to find 4 rational numbers between two rational numbers $\frac{2}{5}$ and $\frac{4}{5}$, we shall multiply the numerators and denominators of both the rational numbers by 4 + 1 = 5.

Example-4 : Find three rational numbers between –1 and 0.

Sol. Let's write –1 and 0 as rational numbers with 3 + 1 i.e 4 as denominator

we have $$-1 = -1 \times \frac{4}{4} = \frac{-4}{4}$$

$$0 = 0 \times \frac{4}{4} = \frac{0}{4}$$

$$\frac{-4}{4} < \frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < \frac{0}{4}$$

Hence rational number between –1 and 0 are $\frac{-3}{4}, \frac{-2}{4}, \frac{-1}{4}$.

Example-5 : Find five rational number between $\frac{-5}{7}$ and $\frac{-1}{3}$.

Sol. Give rational numbers are $\frac{-5}{7}$ and $\frac{-1}{3}$

Here the denominators are not same

$$\therefore \quad \frac{-5}{7} = \frac{-5}{7} \times \frac{3}{3} = \frac{-15}{21}$$

$$\frac{-1}{3} = \frac{-1}{3} \times \frac{7}{7} = \frac{-7}{21}$$

$$\frac{-15}{21} < \frac{-14}{21} < \frac{-13}{21} < \frac{-12}{21} < \frac{-11}{21} < \frac{-10}{21} < \frac{-7}{21}$$

or $\quad \frac{-5}{7} < \frac{-2}{3} < \frac{-13}{21} < \frac{-12}{21} < \frac{-11}{21} < \frac{-10}{21} < \frac{-1}{3}$

Hence five rational number between $\frac{-5}{7}$ and $\frac{-1}{3}$ are :

$$\frac{-2}{3}, \frac{-13}{21}, \frac{-4}{7}, \frac{-11}{21}, \frac{-10}{21}$$

EXERCISE - 9.1

1. Write two equivalent rational numbers of the following :-

(*i*) $\frac{4}{5}$ (*ii*) $\frac{-5}{9}$

(*iii*) $\frac{3}{-11}$

2. Find the standard form of the following rational numbers :-

(*i*) $\frac{35}{49}$ (*ii*) $\frac{-42}{56}$

(*iii*) $\frac{19}{-57}$ (*iv*) $\frac{-12}{-36}$

3. Which of the following pairs represent same rational number ?

(*i*) $\frac{-15}{25}$ and $\frac{18}{-30}$ (*ii*) $\frac{2}{3}$ and $\frac{-4}{6}$

(*iii*) $\frac{-3}{4}$ and $\frac{-12}{16}$ (*iv*) $\frac{-3}{-7}$ and $\frac{3}{7}$

4. Which is greater in each of the following ?

(*i*) $\frac{3}{7}, \frac{4}{5}$ (*ii*) $\frac{-4}{12}, \frac{-8}{12}$

(*iii*) $\frac{-3}{9}, \frac{4}{-18}$ (*iv*) $-2\frac{3}{5}, -3\frac{5}{8}$

5. Write the following rational numbers in ascending order.

(*i*) $\frac{-5}{7}, \frac{-3}{7}, \frac{-1}{7}$ (*ii*) $\frac{-1}{5}, \frac{-2}{15}, \frac{-4}{5}$

(*iii*) $\frac{-3}{8}, \frac{-2}{4}, \frac{-3}{2}$

6. Write five rational numbers between following rational numbers.

(*i*) -2 and -1 (*ii*) $\frac{-4}{5}$ and $\frac{-2}{3}$ (*iii*) $\frac{1}{3}$ and $\frac{5}{7}$

7. Write four more rational numbers in each of the following.

(*i*) $\frac{-1}{5}, \frac{-2}{10}, \frac{-3}{15}, \frac{-4}{20}, \ldots\ldots\ldots$ (*ii*) $\frac{-1}{7}, \frac{2}{-14}, \frac{3}{-21}, \frac{4}{-28}, \ldots\ldots\ldots$

8. Draw a number line and represent the following rational number on it.

(*i*) $\frac{2}{4}$ (*ii*) $\frac{-3}{4}$ (*iii*) $\frac{5}{8}$ (*iv*) $\frac{-6}{4}$

9. **Multiple choice questions :-**

(*i*) $\frac{3}{4} = \frac{?}{12}$, then ? =

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 12

(*ii*) $\frac{-4}{7} = \frac{?}{14}$, then ? =

(*a*) -4 (*b*) -8 (*c*) 4 (*d*) 8

(*iii*) The standard form of rational number $\frac{-21}{28}$ is

(*a*) $\frac{-3}{4}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{-3}{7}$

(*iv*) Which of the following rational number is not equal to $\frac{7}{-4}$?

(*a*) $\frac{14}{-8}$ (*b*) $\frac{21}{-12}$ (*c*) $\frac{28}{-16}$ (*d*) $\frac{7}{-8}$

(*v*) Which of the following is correct ?

(*a*) $0 > \frac{-4}{9}$ (*b*) $0 < \frac{-4}{9}$ (*c*) $0 = \frac{4}{9}$ (*d*) None

(*vi*) Which of the following is correct ?

(*a*) $\frac{-4}{5} < \frac{-3}{10}$ (*b*) $\frac{-4}{5} > \frac{3}{-10}$ (*c*) $\frac{-4}{5} = \frac{3}{-10}$ (*d*) None

OPERATIONS ON RATIONAL NUMBERS

Addition of Rational Numbers : To add two or more rational numbers, their denominators have to be positive and same. In case, the denominators are not same, we will make them same by taking their L.C.M., as we do for comparing the rational numbers and then we shall simply add their numerators.

Example-1 : Add $\frac{5}{9}$ and $\frac{-8}{9}$.

Sol. We have $\frac{5}{9} + \frac{-8}{9}$

$$= \frac{5+(-8)}{9}$$

$$= \frac{5-8}{9}$$

$$= \frac{-3}{9}$$

$$= \frac{-1}{3}$$

Example-2 : Add $\frac{9}{-17}$ and $\frac{-5}{17}$.

Sol. We have $\frac{9}{-17} + \frac{-5}{17}$

$$\frac{9}{-17} = \frac{9}{-17} \times \frac{-1}{-1} = \frac{-9}{17}$$

Now, $$\frac{9}{-17} + \frac{-5}{17} = \frac{-9}{17} + \frac{-5}{17}$$

$$= \frac{-14}{17}$$

Example-3 : Find the sum of $\frac{-4}{6}$ and $\frac{5}{9}$.

Sol. The rational number are $\frac{-4}{6}$ and $\frac{5}{9}$.

Here denominators are not same
so L.C.M of 6 and 9 $= 2 \times 3 \times 3 = 18$

2	6, 9
3	3, 9
3	1, 3
	1, 1

Now $$\frac{-4}{6} = \frac{-4}{6} \times \frac{3}{3} = \frac{-12}{18}$$

$$\frac{5}{9} = \frac{5}{9} \times \frac{2}{2} = \frac{10}{18}$$

Thus $$\frac{-4}{6} + \frac{5}{9} = \frac{-12}{18} + \frac{10}{18}$$

$$= \frac{-12+10}{18}$$

$$= \frac{-2}{18} = \frac{-1}{9}$$

Example-4 : Add $\frac{5}{-27}$ and $\frac{13}{36}$.

Sol. LCM of denominators $= (3 \times 3 \times 3 \times 4)$

$= 108$

3	27, 36
3	9, 12
	3, 4

Now $\frac{5}{-27} = \frac{5\times-4}{-27\times-4} = \frac{-20}{108}$

$\frac{13}{36} = \frac{13\times3}{36\times3} = \frac{39}{108}$

Thus $\frac{5}{-27} + \frac{13}{36} = \frac{-20}{108} + \frac{39}{108}$

$= \frac{-20+39}{108}$

$= \frac{19}{108}$

Additive Inverse : The additive Inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$ which is again a rational number.

- Sum of a rational number and its additive inverse is zero $\frac{a}{b} + \left(\frac{-a}{b}\right) = 0$
- 0 is only rational number which is additive inverse of itself.

Subtraction of a rational number : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then

$\left(\frac{a}{b} - \frac{c}{d}\right) = \frac{a}{b} + \left(\frac{-c}{d}\right)$

$= \frac{a}{b}$ + additive inverse of $\left(\frac{c}{d}\right)$

From above we conclude that while subtracting two rational numbers we add the additive inverse of the rational number that is being subtracted.

Example-5 : Find

(*i*) $\frac{3}{9} - \left(\frac{-4}{9}\right)$ (*ii*) $\frac{5}{12} - \frac{7}{24}$

Sol. (*i*) $\frac{3}{9} - \left(\frac{-4}{9}\right) = \frac{3}{9}$ + (additive inverse of $-\frac{4}{9}$)

$= \frac{3}{9} + \frac{4}{9}$

$= \frac{3+4}{9}$

$= \frac{7}{9}$

(*ii*) $\frac{5}{12} - \frac{7}{24} = \frac{5}{12} + (\text{additive inverse of } \frac{7}{24})$

$= \frac{5}{12} + \left(\frac{-7}{24}\right)$

L.C.M of 12 and 24 = 24

Now $\frac{5}{12} = \frac{5\times 2}{12\times 2} = \frac{10}{24}$

$\therefore \quad \frac{5}{12} - \frac{-7}{24} = \frac{10}{24} + \left(\frac{-7}{24}\right)$

$= \frac{10-7}{24}$

$= \frac{3}{24}$

$= \frac{1}{8}$

Multiplication of Rational numbers : Product of two rational numbers is defined as follows

$$\text{Product of two rational numbers} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

For any two rational number $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} \times \frac{c}{d} = \frac{(a\times c)}{(b\times d)}$$

Example-6 : Find the product of

(*i*) $\frac{9}{5} \times \frac{3}{7}$ (*ii*) $\frac{3}{7} \times \frac{-7}{3}$

Sol (*i*) $\frac{9}{5} \times \frac{3}{7} = \frac{9\times 3}{5\times 7}$

$= \frac{27}{35}$

(*ii*) $\frac{3}{-7} \times \frac{-7}{3} = \frac{3\times -7}{-7\times 3}$

$= 1$

Reciprocal of a rational number : Reciprocal of a rational number $\frac{a}{b}$ is $\frac{b}{a}$.

- Product of a rational number and its reciprocal is always one. $\left(\frac{a}{b}\times\frac{b}{a}=1\right)$
- Reciprocal of 1 is 1.
- Reciprocal of 0 does not exist.

Division of Rational numbers : If $\frac{a}{b}$ and $\frac{c}{d}$ be two rational number such that $\frac{c}{d}\neq 0$ then

$$\frac{a}{b}\div\frac{c}{d} = \frac{a}{b}\times(\text{Reciprocal of }\frac{c}{d})$$
$$= \frac{a}{b}\times\frac{d}{c}$$

Example-7 : Divide

(*i*) $\frac{9}{21}$ by $\frac{3}{7}$ **(*ii*) $\frac{-5}{9}$ by $\frac{7}{27}$**

Sol. (*i*) Given numbers are $\frac{9}{21}$ and $\frac{3}{7}$

Now $\frac{9}{21}\div\frac{3}{7} = \frac{9}{21}\times(\text{Reciprocal of }\frac{3}{7})$

$= \frac{9}{21}\times\frac{7}{3}$

$= 1$

(*ii*) Given numbers are $\frac{-5}{9}$ and $\frac{7}{27}$

Now $\frac{-5}{9}\div\frac{7}{27} = \frac{-5}{9}\times(\text{Reciprocal of }\frac{7}{27})$

$= \frac{-5}{9}\times\frac{27}{7}$

$= \frac{-15}{7}$

Example-8 : What number should be added to $\frac{-7}{12}$ to get $\frac{5}{9}$?

Sol. Let the required number to be added be x,

then $\frac{-7}{12}+x = \frac{5}{9}$

$\Rightarrow x = \frac{5}{9}-\left(\frac{-7}{12}\right)$

$\Rightarrow x = \frac{5}{9}+\frac{7}{12}=\frac{5\times4+7\times3}{36}$

$$= \frac{20+21}{36} = \frac{41}{36} = 1\frac{5}{36}$$

Hence the required number to be added is $1\frac{5}{36}$.

Example-9 : What number should be subtracted from $\frac{-3}{4}$ to get $\frac{-11}{4}$?

Sol. Let the required number to be subtracted be x, then

$$\frac{-3}{4} - x = \frac{-11}{4}$$

$$\Rightarrow \quad \frac{-3}{4} - \left(\frac{-11}{4}\right) = x$$

$$\Rightarrow \quad x = \frac{-3}{4} - \left(\frac{-11}{4}\right) = \frac{-3}{4} + \frac{11}{4} = \frac{-3+11}{4} = \frac{8}{4}$$

$$x = 2$$

Hence the required number to be subtracted is 2.

Example-10 : The product of two rational numbers is $\frac{-9}{16}$. If one of the number is $\frac{3}{14}$, find the other number.

Sol. Let the required number be x, then

$$\frac{3}{14} \times x = \frac{-9}{16}$$

$$\Rightarrow \quad x = \frac{-9}{16} \div \frac{3}{14}$$

$$x = \frac{-9}{16} \times \frac{14}{3} = \frac{(-9)\times 14}{16\times 3} = \frac{-126}{48} = \frac{-21}{8}$$

$$x = -2\frac{5}{8}$$

EXERCISE - 9.2

1. Find the sum

(*i*) $\frac{6}{9} + \frac{2}{9}$ (*ii*) $\frac{-15}{7} + \frac{9}{7}$ (*iii*) $\frac{17}{11} + \left(\frac{-9}{11}\right)$

(*iv*) $\frac{-5}{6} + \frac{3}{18}$ (*v*) $\frac{-7}{19} + \frac{-3}{38}$ (*vi*) $-3\frac{4}{7} + 2\frac{3}{7}$

(*vii*) $\frac{-5}{14} + \frac{8}{21}$ (*viii*) $-4\frac{1}{15} + 3\frac{2}{20}$

2. Find

(*i*) $\frac{7}{12} - \frac{11}{36}$ (*ii*) $\frac{-5}{9} - \frac{3}{5}$ (*iii*) $\frac{-7}{13} - \left(\frac{-5}{91}\right)$

(*iv*) $\frac{6}{11} - \frac{-3}{4}$ (*v*) $3\frac{4}{9} - \frac{28}{63}$

3. Find the product of

(*i*) $\frac{5}{9} \times \frac{-3}{8}$ (*ii*) $\frac{-3}{7} \times \frac{7}{-3}$ (*iii*) $\frac{3}{13} \times \frac{5}{8}$

(*iv*) $\frac{3}{10} \times (-18)$

4. Find the value of

(*i*) $-9 \div \frac{3}{5}$ (*ii*) $\frac{-4}{7} \div 4$ (*iii*) $\frac{7}{18} \div \frac{5}{6}$

(*iv*) $\frac{-8}{35} \div \left(\frac{-2}{7}\right)$ (*v*) $\frac{-9}{15} \div -18$

5. What rational number should be added to $\frac{-5}{12}$ to get $\frac{-7}{8}$?

6. What number should be subtracted from $\frac{-2}{3}$ to get $\frac{-5}{6}$?

7. The product of two rational numbers is $\frac{-11}{2}$. If one of them is $\frac{33}{8}$, find the other number.

8. Multiple choice questions

(*i*) The sum of $\frac{5}{4} + \left(\frac{25}{-4}\right) =$

(*a*) –5 (*b*) 5 (*c*) 4 (*d*) –4

(*ii*) $\frac{17}{11} - \frac{6}{11} =$

(*a*) 1 (*b*) –1 (*c*) 6 (*d*) 3

(*iii*) $\frac{2}{-5} \times \frac{-5}{2} =$

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –5

(*iv*) $\frac{7}{12} \div \left(\frac{-7}{12}\right) =$

(*a*) 1 (*b*) –1 (*c*) 7 (*d*) –7

(*v*) Which of the following is value of $(-4) \times [(-5) + (-3)]$

(*a*) –32 (*b*) 120 (*c*) 32 (*d*) –23

WHAT WE HAVE DISCUSSED ?

1. The number of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called rational numbers.
2. If we multiply or divide both the numerator and the denominator of a rational number by a non-zero integer, we get an equivalent ralional number.
3. A rational number is said to be positive if both the numerator and the denominator are either positive or negative.
4. A rational number is said to be negative if either numerator or denominator is negative.
5. The number zero (0) is neither positive nor negative rational number.
6. A rational number $\frac{a}{b}$ is said to be in standard form, if b is positive, a and b have no common divisor, other than 1.
7. Additive inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$.
8. Multiplicative inverse (Reciprocal) of a non-zero rational number $\frac{a}{b}$ is $\frac{b}{a}$.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Define rational numbers and reduce the given rational numbers to the standard form.
2. Write equivalent rational numbers.
3. Represent the given rational numbers on a number line.
4. Find more rational numbers between given rational numbers.
5. Add, subtract, multiply and divide two or more rational numbers.
6. Solve problems related to daily life situations involving rational numbers.

ANSWERS

EXERCISE 9.1

1. (*i*) $\frac{8}{10}, \frac{12}{15}$ (*ii*) $\frac{-10}{18}, \frac{15}{27}$

(*iii*) $\frac{6}{-22}, \frac{9}{-33}$

2. (*i*) $\frac{5}{9}$ (*ii*) $\frac{-3}{4}$

(*iii*) $\frac{-1}{3}$ (*iv*) $\frac{1}{3}$

3. (*i*), (*iii*), (*iv*)

4. (*i*) $\frac{4}{5} > \frac{3}{7}$ (*ii*) $\frac{-4}{12} > \frac{-8}{12}$

(*iii*) $\frac{4}{-18} > \frac{-3}{9}$ (*iv*) $-2\frac{3}{5} > -3\frac{5}{8}$

5. (*i*) $\frac{-5}{7}, \frac{-3}{7}, \frac{-1}{7}$ (*ii*) $\frac{-4}{5}, \frac{-1}{5}, \frac{-2}{15}$

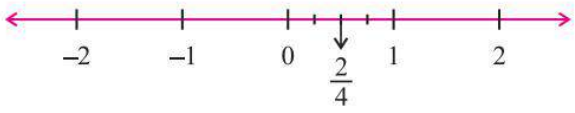
(*iii*) $\frac{-3}{2}, \frac{-2}{4}, \frac{-3}{8}$

6. (*i*) $\frac{-11}{6}, \frac{-5}{3}, \frac{-3}{2}, \frac{-4}{3}$ and $\frac{-7}{6}$ (*ii*) $\frac{-7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}$ and $\frac{-31}{45}$

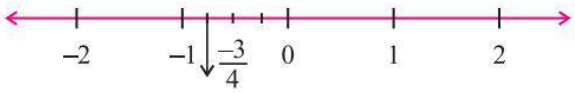
(*iii*) $\frac{8}{21}, \frac{3}{7}, \frac{10}{21}, \frac{11}{21}$ and $\frac{4}{7}$

7. (*i*) $\frac{-5}{25}, \frac{-6}{30}, \frac{-7}{35}, \frac{-8}{40}$ (*ii*) $\frac{5}{-35}, \frac{6}{-42}, \frac{7}{-49}, \frac{8}{-56}$

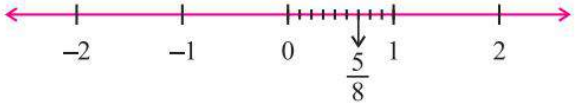
8. (*i*)



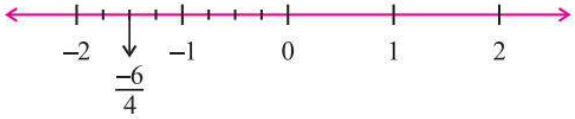
(*ii*)



(*iii*)



(*iv*)



9. (*i*) *c* (*ii*) *b*
(*iii*) *a* (*iv*) *d*
(*v*) *a* (*vi*) *a*

EXERCISE 9.2

1. (*i*) $\frac{8}{9}$ (*ii*) $\frac{-6}{7}$

(*iii*) $\frac{8}{11}$ (*iv*) $\frac{-2}{3}$

(*v*) $\frac{-17}{38}$ (*vi*) $\frac{-8}{7}$

(*vii*) $\frac{1}{42}$ (*viii*) $\frac{-29}{30}$

2. (*i*) $\frac{5}{18}$ (*ii*) $\frac{-52}{45}$

(*iii*) $\frac{-44}{91}$ (*iv*) $\frac{57}{44}$

(*v*) 3

3. (*i*) $\frac{-5}{24}$ (*ii*) 1

(*iii*) $\frac{15}{104}$ (*iv*) $\frac{-27}{5}$

4. (*i*) –15 (*ii*) $\frac{-1}{7}$

(*iii*) $\frac{7}{15}$ (*iv*) $\frac{4}{5}$

(*v*) $\frac{1}{30}$

5. $\frac{-11}{24}$ **6.** $\frac{1}{6}$ **7.** $\frac{-4}{3}$

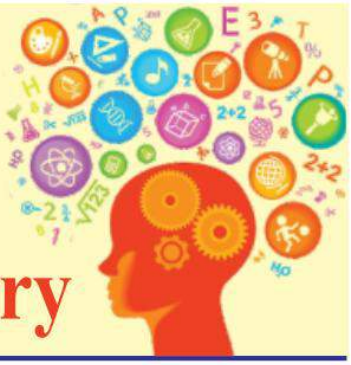
8. (*i*) *a* (*ii*) *a*

(*iii*) *a* (*iv*) *b*

(*v*) *c*

CHAPTER 10

M2T1Y7



Practical Geometry

Learning Objectives :-

In this chapter you will learn :-

1. To draw a line parallel to a given line.
2. To construct triangles using different construction criterias.
3. To determine whether the construction of a triangle with given measurements is possible or not.

INTRODUCTION

We have already learnt how to draw a line segment of given length, a line prependicular to a given line segment, an angle, an angle bisector and a circle etc previously. Now in this chapter. We will learn, how to

- Construct a line parallel to a given line.
- Construct triangles.

CONSTRUCTION OF PARALLEL LINE TO A GIVEN LINE

Construction of a line parallel to a given line 'l' through a point 'A', That does not lie on the given line. We shall draw it either.

(i) By using ruler and set square Or (ii) By using ruler and compasses

(i) Construction of a parallel line using ruler and set square

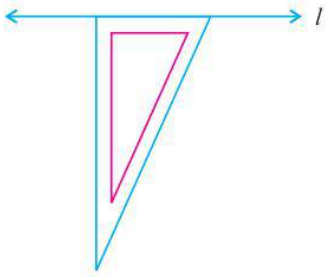
Step 1 : Draw a line 'l' and take a point 'A' that does not lie on the given line.

• A

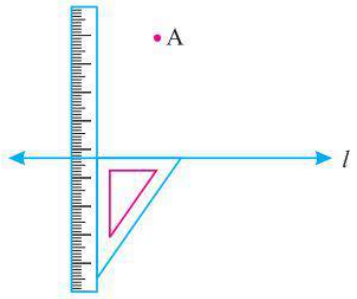
l

Step 2 : Place a set square such that one of its shorter edge lies along line 'l' as shown.

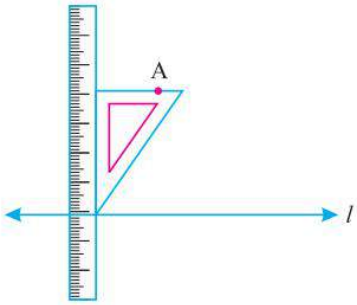
• A



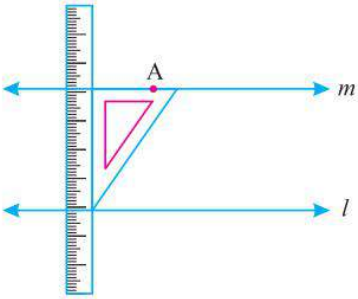
Step 3 : Now place a ruler along the other side of the set square so that it touches the standing edge of the set square.



Step 4 : Hold the ruler firmly and slide the set square along the ruler unitl the edge of the set square passes through 'A'.



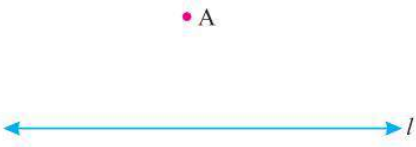
Step 5 : Draw a straight line m along the horizontal edge of the set square passing through point 'A'.



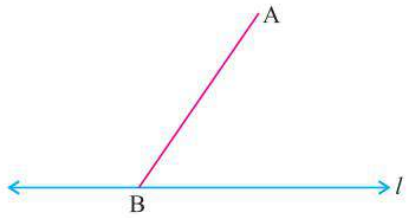
Step 6 : Line 'm' is the required line parallel to 'l'

(*ii*) Construction of a parallel line by using ruler and compass.

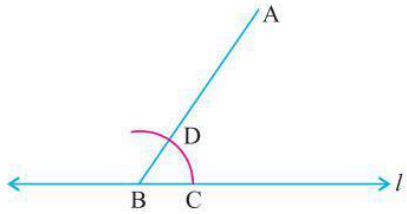
Step 1 : Draw a line 'l' and a point 'A' not lying on 'l'



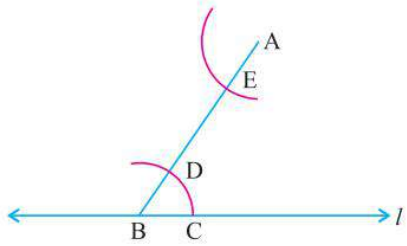
Step 2 : Take any point B on l and join B to A.



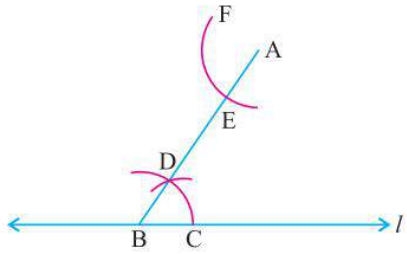
Step 3 : With B as centre draw an arc of any radius intersecting l at C and AB at D.



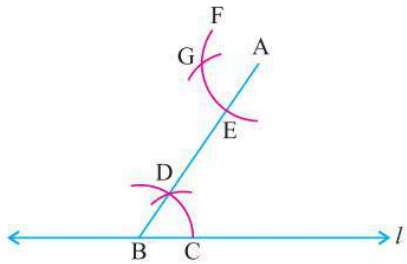
Step 4 : Now with A as centre and the same radius, draw another arc intersecting AB at a point E.



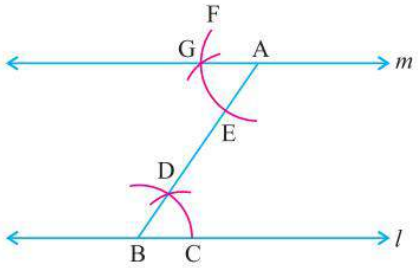
Step 5 : Measure the arc CD with compass.



Setp 6 : With E as centre cut an arc EG = arc CD



Step 7 : Join A to G and extend the line segment to both sides.



Note that $\angle EAG = \angle DBC$ are the alternate angles therefore $m \parallel l$.

EXERCISE - 10.1

1. Draw a line l, take a point p outside it, Through p draw a line parallel to l using ruler and compass only.
2. Draw a line parallel to a line l at a distance of $3.5cm$ from it.
3. Let l be a line and P be a point not on l. Through P draw a line m paralel to l. Now, join P to any point Q on l. Choose any other point R on m. Through R draw a line parallel to PQ. Let this meets l at S. What shape do the two sets of parallel lines enclose.
4. (*i*) How many parallel lines can be drawn, passing through a point not lying on the given line ?
   (*a*) 0 (*b*) 2
   (*c*) 1 (*d*) 3
   (*ii*) Which of the following is used to draw a line parallel to a given line ?
   (*a*) A protractor (*b*) A ruler
   (*c*) A compasses (*d*) A ruler and compasses.

CONSTRUCTION OF TRIANGLES

Let us recall some important propoerties of triangle.

1. Sum of lengths of any two sides of a triangle is greater than the length of the third side.
2. Sum of measures of the three angles of a triangle is 180°.
3. Exterior angle of a triangle is equal to the sum of opposite interior angles.
4. Pythagoras property i.e. In right angled triangle
   $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

A triangle can be drawn, if any one of the following set of measurements are given in the questions.

1. Three sides (SSS)
2. Two sides and angle between them (SAS)
3. Two angles and the side between them (ASA)
4. The length of Hypotenuse and a side in case of a right angled triangle.

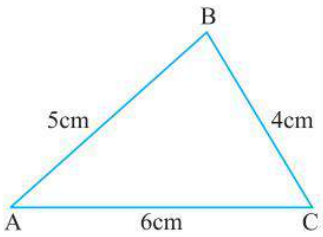
- Note : To construct a triangle with a given measure we should first draw a rough sketch to indicate the given measure.

CONSTRUCTION OF A TRIANGLE USING SSS CRITERION

SSS stands for side-side-side. In this section we would construct triangle when all its sides are known. To understand the criteria see the following example.

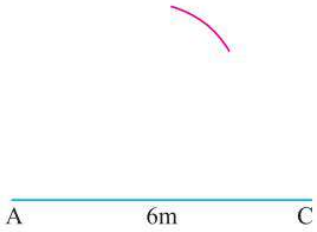
Example-1 : Construct a triangle ABC, given that AB = 5*cm*, BC = 4*cm*, AC = 6*cm*

Step 1 : Draw a rough sketch of ΔABC with given measures.

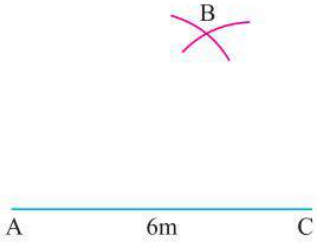


Step 2 : Draw a line segment AC of length 6cm (Note : Take longest side as a base it is optional but not compulsory)

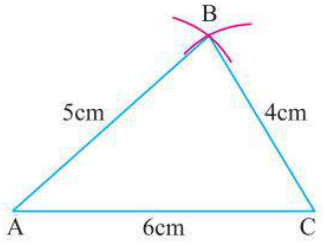
Step 3 : With A as centre and radius 5*cm* (∵ AB = 5*cm*) draw on arc.



Step 4 : With C as centre and radius 4cm (∵ BC = 4*cm*) draw another arc intersecting the previous arc at B.



Step 5 : Join AB and CB. ΔABCis the required triangle.



EXERCISE - 10.2

1. Construct a ΔABC in which AB = 3.5*cm*, BC = 5*cm* and CA = 7*cm*.
2. Construct a triangle ABC in which AB = BC = 6.5*cm* and CA = 4*cm*. Also name the kind of triangle drawn.
3. Construct a triangle XYZ such that length of each side is 5*cm*. Also name the kind of triangle drawn.
4. Construct a triangle PQR such that PQ = 2.5, QR = 6cm and RP = 6.5cm. Measure $\angle PQR$ and also name the kind of triangle drawn.
5. Construct a triangle ABC, in which AB = 6*cm*, BC = 2*cm*, CA = 3*cm*. (If possible). If not possible give the reason.
6. (*i*) Which of the following can be used to construct a triangle ?
   - (*a*) The lengths of the three sides
   - (*b*) The perimeter of the triangle
   - (*c*) The measures of three angles
   - (*d*) The name of three vertices

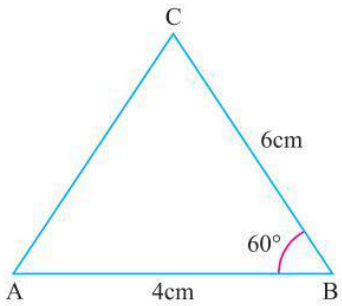
   (*ii*) A triangle can be constructed by taking its sides as
   - (*a*) 1.8*cm*, 2.6*cm*, 4.4*cm*
   - (*b*) 3*cm*, 4*cm*, 8*cm*
   - (*c*) 4*cm*, 7*cm*, 2*cm*.
   - (*d*) 5*cm*, 4*cm*, 4*cm*

CONSTRUCTION OF A TRIANGLE USING SAS CRITERION

SAS stands for side-angle-side. Here, we have two given sides and the one angle between them. We first draw a rough sketch. Follow example (1) to understand. the concept of construction of triangle using SAS.

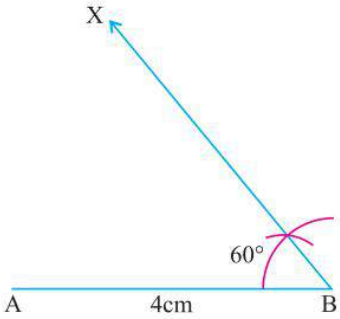
Example-1 : Construct a triangle ABC, Such that AB = 4*cm*, BC = 6*cm* and $\angle ABC = 60°$

Step 1 : Draw a rough sketch of ΔABC with given measures.

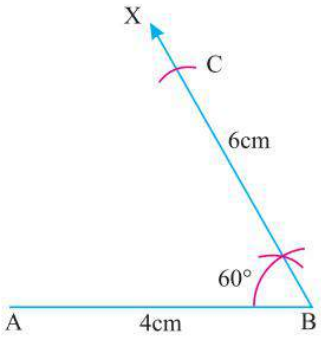


Step 2 : Draw a line segment AB of length 4*cm*.

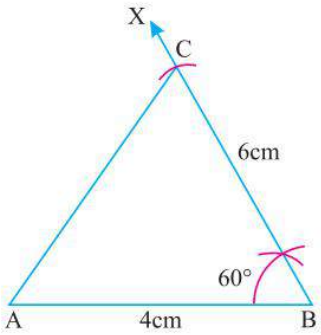
Step 3 : With the help of compass, at B, draw a ray BX making an angle 60° with AB.



Step 4 : With B as centre and radius 6cm. draw an arc intersecting the ray BX at point C.



Step 5 : Join AC. ΔABC is required triangle.



EXERCISE - 10.3

1. Construct ΔPQR such that AB = 4*cm*, ∠B = 30°, BC = 4*cm*. Also name the type of this triangle on the basis of sides.
2. Construct ΔABC with AB = 7.5*cm*, BC = 5*cm* and ∠B = 30°.
3. Construct a triangle XYZ such that XY = 6*cm*, YZ = 6*cm* and ∠Y = 60°. Also name the type of this triangle.

4. (*i*) Which of the following triangle can be constructed using SAS criterion.

(*a*) AB = 5*cm*, BC = 5*cm*, CA = 6*cm*

(*b*) AB = 5*cm*, BC = 5*cm*, $\angle B = 40°$

(*c*) $\angle A = 60°$, $\angle B = 60°$, $\angle C = 60°$

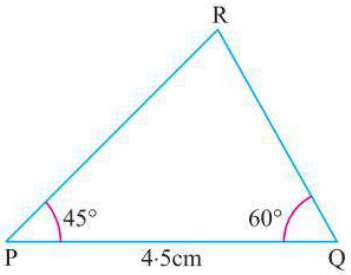
(*d*) BC = 5*cm*, $\angle B = \angle C = 45°$

CONSTRUCTION OF A TRIANGLE USING ASA CRITERION

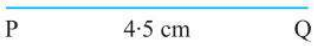
ASA stands for Angle-Side-Angle. First draw rough sketch of given measures, then draw the given line segment. Make angles on the both ends of line of given measures as shown in the following example.

Example-1 : Construct a triangle PQR such that PQ = 4.5*cm*, $\angle P = 45°$, $\angle Q = 60°$

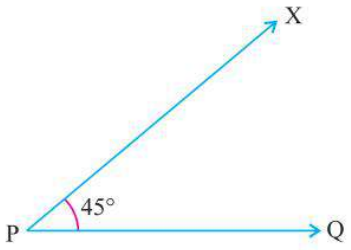
Step 1 : First we draw rough sketch of triangle PQR with given measures.



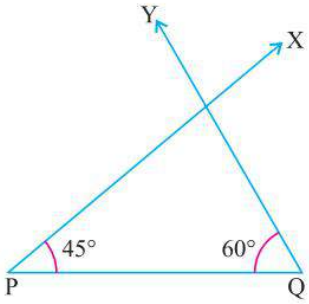
Step 2 : Draw a line segment PQ = 4.5*cm*.



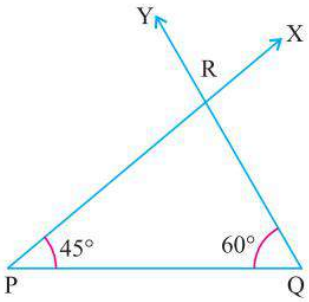
Step 3 : At P, Draw a ray PX making an angle 45° with PQ (with the help of compass as discussed in earlier classes)



Step 4 : With the help of compass, At Q draw a ray QY making an angle 60° with the line segment PQ.

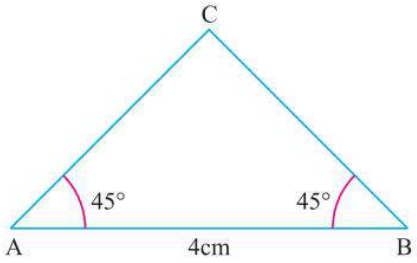


Step 5 : Rays PX and QY intersect at a point say R. then ΔPQR is the required triangle.



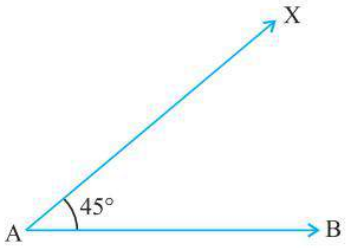
Example-2 : Construct an isosceles ΔABC with Base AB = 4*cm* and each base angle measuring 45°.

Step 1 : Draw a rough sketch of ΔABC with given measures.

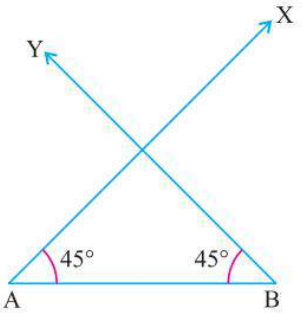


Step 2 : Draw a line segment AB = 4*cm*.

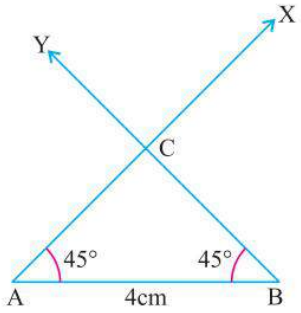
Step 3 : Taking A as centre with the help of compass, Draw a ray AX making an angle 45° with AB.



Step 4 : With the help of compass at taking B as a centre. Draw a ray BY making an angle 45° with the line segment AB.



Step 5 : Rays AX and BY intersect, at a point, say C, then ΔABC is the required triangle.



EXERCISE - 10.4

1. Construct ΔABC, given AB = 6*cm*, ∠A = 30° and ∠B = 75°
2. Construct an isosceles ΔABC such that base AB = 5.3*cm* and each base angle = 45°
3. Construct ΔXYZ if XY = 4*cm*, ∠X = 45° and ∠Z = 60°

   (Hint : ∠Y = 180° – 45° – 60° = 75°)
4. Examine whether you can construct ΔPQR such that ∠P = 100°, ∠Q = 90° and PQ = 4.3 cm. If not possible give reason.
5. (*i*) In which of the following cases a unique triangle can be drawn ?
   - (*a*) BC = 5*cm*, ∠B = 90° and ∠C = 100°
   - (*b*) AB = 4*cm*, BC = 7*cm* and CA = 2*cm*
   - (*c*) XY = 5*cm*, ∠X = 45°, ∠Y = 60°
   - (*d*) An isosceles triangle with length of each equal side equal to 5 cm.

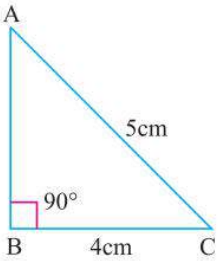
   (*ii*) A triangle can be constructed by taking two of its angles as.
   - (*a*) 110°, 40°
   - (*b*) 70°, 115°
   - (*c*) 135°, 45°
   - (*d*) 90°, 90°

CONSTRUCTION OF A TRIANGLE USING RHS CRITERION

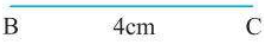
RHS stands for right angle, hypotenuse and side of a right angled triangle. First draw a rough sketch of given measure. Draw a line segment of given measure. Construct a right angle. Now mark length of side of hypotenuse of triangle. See example 1.

Example-1 : Construct a ΔABC such that ∠B = 90° BC = 4*cm* and AC = 5*cm*

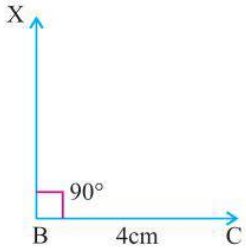
Step 1 : Draw a rough sketch of given measures.



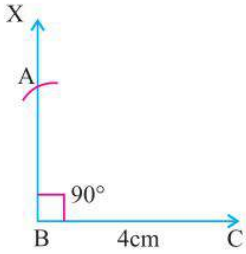
Step 2 : Draw a line segment BC = 4*cm*.



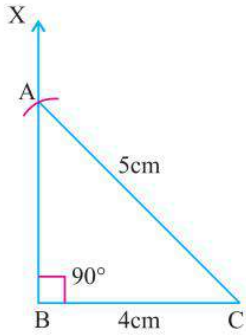
Step 3 : With the help of a compass taking B as centre, draw a ray BX making an angle 90° with BC.



Step 4 : With C as centre and radius 5cm (= AC) draw an arc interseting ray BX at a point A.



Step 5 : Join A and C therefore ΔABC is required triangle.



EXERCISE - 10.5

1. Construct a right angled triangle ABC with ∠C = 90°, AB = 5*cm* and BC = 3*cm*.
2. Construct an isosceles right angled triangle DEP where ∠E = 90° and EF = 6*cm*.
3. Construct a right angled triangle PQR in which ∠Q = 90°, PQ = 3.6*cm* and PR = 8.5*cm*.
4. (*i*) Which of the following is a pythagorian triplet ?

   (*a*) 1, 2, 3 (*b*) 2, 3, 4

   (*c*) 4, 5, 6 (*d*) 12, 13, 5

(*ii*) Construction of unique triangle is not possible when
- (*a*) Three sides are given.
- (*b*) Two sides and an included angle are given.
- (*c*) Three angles are given.
- (*d*) Two angles & included side are given.

WHAT HAVE WE DISCUSSED ?

1. A line can be drawn parallel to a given line through a point not lying on it. With compass and ruler by alternate angle or by corresponding angle method.
2. A triangle can uniquely be constructed by using indirectly the concept of congurence of triangles.
3. **ASA :** Measure of two angles and length of the side included between them is given.
4. **SSS :** Length of three sides of a triangle are given.
5. **SAS :** Measure of two sides and angle between the two sides are given.
6. **RHS :** Length of hypotenuse and one of the other two sides of a triangles are given.

LEARNING OUTCOMES

After completion of the chapter, students are now able to :

1. Handle geometrical instruments like scale, compass, protractor etc.
2. Construct a line parallel to the given line from a point outside it.
3. Construct a triangle with given measurements.
4. Check whether a triangle is possible with given measurements.

ANSWERS

EXERCISE 10.1

4. (*i*) *c* (*ii*) *d*

EXERCISE 10.2

6. (*i*) *a* (*ii*) *d*

EXERCISE 10.3

4. (*i*) *b*

EXERCISE 10.4

5. (*i*) *c* (*ii*) *a*

EXERCISE 10.5

4. (*i*) *d* (*ii*) *c*

CHAPTER

11

G5A9R7

Perimeter And Area

Learning Objectives :-

In this chapter you will learn :-

1. About measurements.
2. To convert the units of length and area.
3. The difference between perimeter and area of different plane figures.
4. To compute perimeter and area of square, rectangle, triangle and parallelogram using formulae.
5. To compute the circumference and area of a circle.
6. To apply your knowledge of perimeter and area in real life situations.

OUR NATIONS' PRIDE

History : Indian mathematicians played a crucial role in finding the area of plane figures. Aryabhata (476–550 AD) gave the formula for area of a triangle. He worked on the approximation for *pi* (π). In the second part of Aryabhatiya, he wrote that the ratio of the circumference of a circle to its diameter is 3.1416. Another mathematician Brahmagupt (598 – 668 AD) gave the formula for the area of a cyclic quadrilateral.

INTRODUCTION

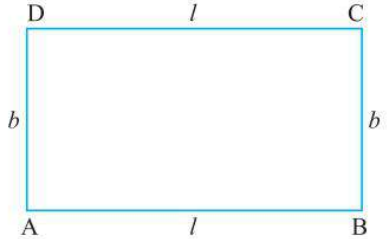
We have already learnt about the perimeter of plane figures and the area of a square and rectangle in class VI.

Perimeter : Perimeter of a simple closed figure is the length of its boundary. Its units are same as the units of length *i.e*, *cm* and *m* etc.

Area : Area of a simple closed figure is the measure of the surface enclosed in it. Units of area are cm^2 and m^2 etc.

PERIMETER AND AREA OF A RECTANGLE AND A SQUARE

Rectangle : Let us consider a rectangle ABCD with length = l units and breadth = b units

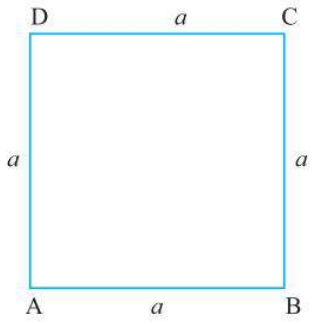


Then, Perimeter of rectangle = AB + BC + CD + DA
$= l + b + l + b$
$= 2l + 2b$
$= 2\,(l + b)$ units
Area of rectangle = (Length × Breadth) sq. units

We can find length and breadth by using the formula

$$\text{length} = \frac{\text{Area}}{\text{Breadth}} \text{ units and Breadth} = \frac{\text{Area}}{\text{Length}} \text{ units}$$

Square : Let us consider a square ABCD having each side equal to a units then,



Perimeter of square = AB + BC + CD + DA
$= a + a + a + a$
= 4a units
= (4 × side) units
Area of square = Side × side
A = a × a
$A = a^2$ sq units

Example-1 : Find the perimeter and area of a rectangle whose length is 18cm and breadth 9*cm*

Sol. Given length of rectangle = 18*cm*
Breadth of rectangle = 9*cm*
Perimeter of rectangle = 2 (Length + Breadth)
= 2 (18 + 9)
= 2 (27)
= 54*cm*
Area of rectangle = Length × Breadth
= 18 × 9
$= 162cm^2$

Example-2 : Find the perimeter and area of a square with side 3.5*cm*

Sol. Side of square = 3.5 c*m*
Perimeter of square = 4 × side
= 4 × 3.5
= 14.0 *cm*
Area of square = $(\text{Side})^2$
$= (3.5)^2$
= 3.5 × 3.5
$= 12.25\ cm^2$

Example-3 : Area of Rectangular park is 1386 m^2. If length of park is 42 *m*, find the breadth and the perimeter of the park.

Sol. Area of rectangular park = $1386m^2$
Length = 42*m*

$$\text{Area} = \text{Length} \times \text{Breadth}$$

$$\therefore \quad \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{1386}{42}$$

$$= 33m$$

$$\text{Perimeter of rectangular park} = 2\,(\text{Length} + \text{Breadth})$$

$$\text{Perimeter of rectangular park} = 2\,(42 + 33)$$

$$= 2\,(75)$$

$$= 150m$$

Example-4 : The area of square park is same as that of a rectangular park. If side of a square park is 36*m* and length of rectangular park is 54*m*. Find the breadth of rectangular park.

Sol.

$$\text{Side of a square park} = 36m$$

$$\text{Area of the square park} = (\text{side})^2$$

$$= 36 \times 36$$

$$= 1296m^2$$

$$\text{Length of a rectangular park} = 54m$$

$$\text{Let breadth of the rectangular park} = b$$

According to question,

$$\text{Area of rectangular park} = \text{Area of square park}$$

$$54 \times b = 1296$$

$$b = \frac{1296}{54}$$

$$b = 24m$$

$$\text{Hence breadth of rectangular park} = 24\text{m}$$

Example-5 : A wire is in the shape of a square of side 15 cm. If the wire is rebent into a rectangle of length 16cm. Find its breadth. Which encloses more area, square or rectangle ?

Sol.

$$\text{Side of square} = 15\ cm$$

$$\text{Perimeter of square} = 4 \times \text{side}$$

$$= 4 \times 15$$

$$= 60\ cm$$

$$\text{Length of rectangle} = 16\ cm$$

$$\text{Let breadth of rectangle} = \text{b}\ cm$$

$$\text{Perimeter of rectangle} = 2\,(l + b)$$

$$= 2\,(16 + b)\ cm$$

According to question,

$$\text{Perimeter of square} = \text{Perimeter of rectangle}$$

$$60 = 2\,(16 + b)$$

$$\frac{60}{2} = 16 + b$$
$$16 + b = 30$$
$$b = 30 - 16$$
$$b = 14cm$$

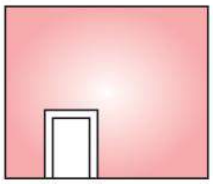
$\therefore$ Breadth of rectangle $= 14cm$

Area of square $= (\text{side})^2$
$= 15 \times 15$
$= 225cm^2$

Area of Rectangle $=$ Length $\times$ Breadth
$= 16 \times 14$
$= 224\ cm^2$

Square encloses more area

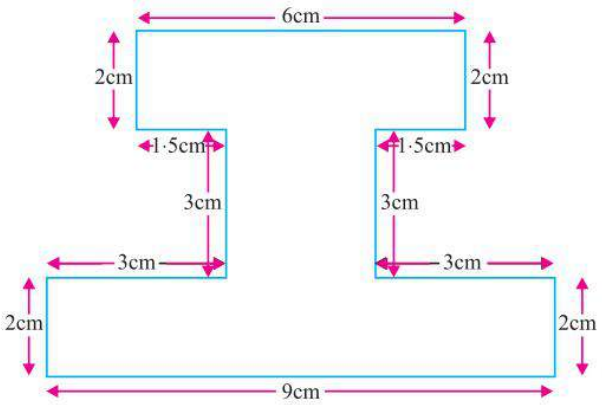
Example-6 : A door of dimensions $3m \times 2m$ is fitted in a wall. The length of the wall is $8m$ and the breadth is $5m$. Find the cost of painting the wall, if the rate of painting is ₹25 per m^2.



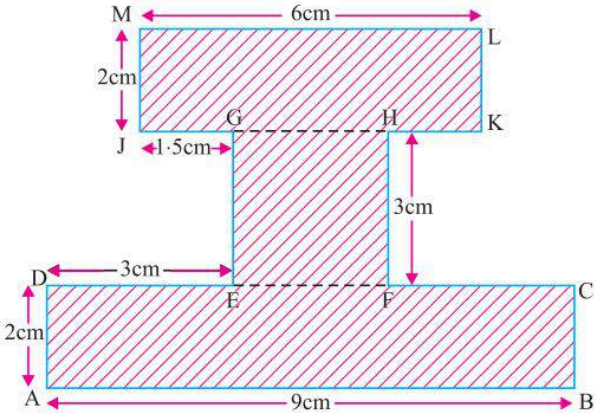
Sol.

Length of wall $= 8m$
Breadth of wall $= 5m$
Area of wall $=$ Length $\times$ Breadth
$= 8 \times 5$
$= 40m^2$
Area of door $= 3m \times 2m$
$= 6m^2$
Now area of wall for painting $=$ Area of wall including door $-$ Area of door
$= 40 - 6$
$= 34m^2$
Cost of painting $1m^2$ of wall $=$ ₹ 25
Cost of painting $34m^2$ of wall $= 34 \times 25$
$=$ ₹ 850

Example-7 : Find the perimeter and area of the given figure.



Sol.



$$AB = DC$$
$$AB = DE + EF + FC$$
$$9 = 3 + EF + 3$$
$$9 = 6 + EF$$
$$EF = 3cm$$

We obtain the rectangles ABCD, JKLM and a square EFHE

$$\text{Area of the figure} = \text{Area of rectangle ABCD} + \text{Area of rectangle JKLM} + \text{Area of square EFHG}$$
$$= (9 \times 2)cm^2 + (6 \times 2)cm^2 + (3 \times 3)cm^2$$
$$= (18 + 12 + 9)cm^2$$
$$= 39cm^2$$

$$\text{Perimeter of the given figure} = MJ + JG + GE + DE + DA + AB + BC + CF + FH + HK + KL + ML$$
$$= 2 + 1.5 + 3 + 3 + 2 + 9 + 2 + 3 + 3 + 1.5 + 2 + 6$$
$$= 38cm$$

EXERCISE - 11.1

1. Find the perimeter and the area of a rectangle having
   (*i*) Length = 28*cm*, Breadth = 15*cm*
   (*ii*) Length = 9.4*cm* Breadth = 2.5*cm*

2. Find the perimeter and the area of a square whose side measures
   (*i*) 29*cm*
   (*ii*) 8.3*cm*

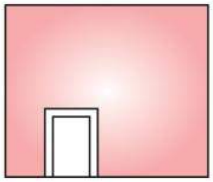
3. The perimeter of a square park is 148*m*. Find its area.

4. The area of a rectangle is 580cm^2. Its length is 29*cm*. Find its breadth and also, the perimeter.

5. A wire is in the shape of a rectangle. Its length is 48cm and breadth is 32*cm*. If the same wire is rebent into the shape of a square, what will be the measure of each side. Also, find which shape encloses more area and by how much ?

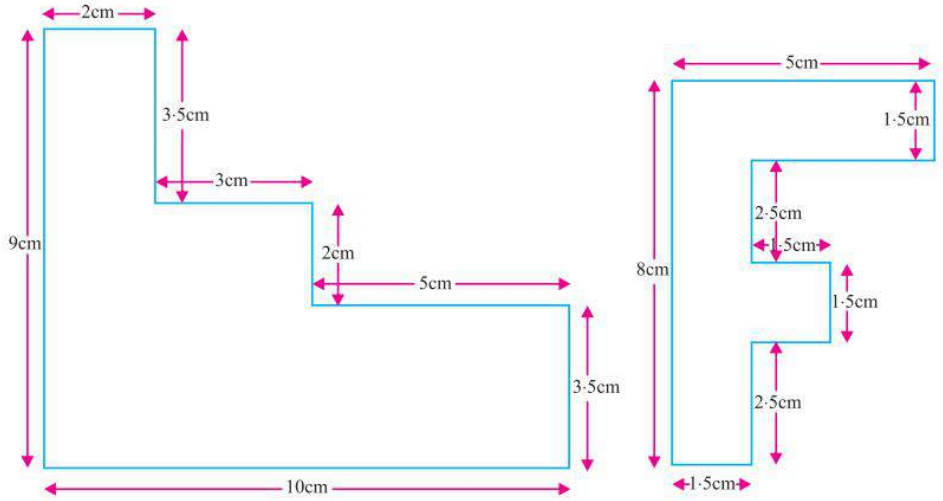
6. The area of a square park is the same as that of a rectangular park. If the side of the square park is 75m and the length of the rectangular park is 125m, find the breadth of the rectangular park. Also, find the perimeter of rectangular park.

7. A door of length 2.5m and breadth 1.5m is fitted in a wall. The length of wall is 9m and breadth is 6m. Find the cost of painting the wall, if the rate of painting the wall is ₹ 30 per m^2.



8. A door of dimensions $3m \times 2m$ and a window of dimensions $2.5m \times 1.5m$ is fitted in a wall. The length of the wall is 7.8m and breadth is 3.9m. Find the cost of painting the wall, if the rate of painting the wall is ₹ 25 per m^2.

9. Find the area and the perimeter of the following figures.



10. Multiple choice questions

(i) What is the area of a rectangle of dimensions $12cm \times 10cm$?

(a) 44cm^2 (b) 120cm^2

(c) 1200cm^2 (d) 1440cm^2

(ii) Find the breadth of a rectangle whose length is 12cm and perimeter is 36cm.

(a) 6cm (b) 3cm

(c) 9cm (d) 12cm

(iii) If each side of a square is 1m then its area is ?

(a) 10cm^2 (b) 100cm^2

(c) 1000cm^2 (d) 10000cm^2

(iv) Find the area of a square whose perimeter is 96cm.

(a) 576cm^2 (b) 626cm^2

(c) 726cm^2 (d) 748cm^2

(*v*) The area of a rectangular sheet is $500cm^2$. If the length of the sheet is $25cm$, what is its breadth ?

(*a*) $30cm$ (*b*) $40cm$

(*c*) $20cm$ (*d*) $25cm$

(*vi*) What happens to the area of a square, if its side is doubled ?

(*a*) The area becomes 4 times, the area of original square.

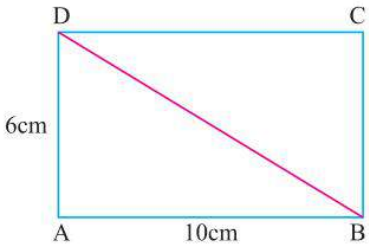
(*b*) The area becomes $\frac{1}{4}$ times, the area of original square.

(*c*) The are becomes 16 times, the area of orignal square.

(*d*) The area becomes $\frac{1}{6}$ times, the area of original square.

TRIANGLES AS PARTS OF A RECTANGLE

Consider a rectangle ABCD of length $10cm$ and breadth $6cm$. Now, if we draw a diagonal of the rectangle ; It divides the rectangle into two triangles and sum of the area of two triangles is equal to the area of rectangle.



i.e; Area of ΔABD + Area of ΔBCD = Area of Rectangle ABCD

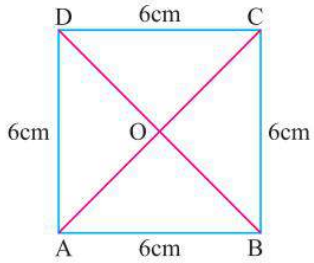
$$\therefore \quad \text{Area of each triangle} = \frac{1}{2} \times \text{area of rectangle}$$

$$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$$

$$= \frac{1}{2} \times 10 \times 6$$

$$= 30cm^2$$

On the other hand if we divide the rectangle into 4 triangles, then also the result is same i.e. sum of area of triangles is equal to the area of rectangle.



Let a Square ABCD of side $6cm$ is divided into four triangles as shown in the figure. Then

Area of ΔAOB + Area of ΔBOC + Area of ΔCOD + area of ΔDOA = Area of square ABCD

$$\text{Area of each triangle} = \frac{1}{4} \times \text{Area of square}$$

$$= \frac{1}{4} \times (\text{Side})^2$$

$$= \frac{1}{4} \times 6 \times 6 = 9cm^2$$

Example-1 : Estimate the area of the following figures by counting unit squares.

(*i*)

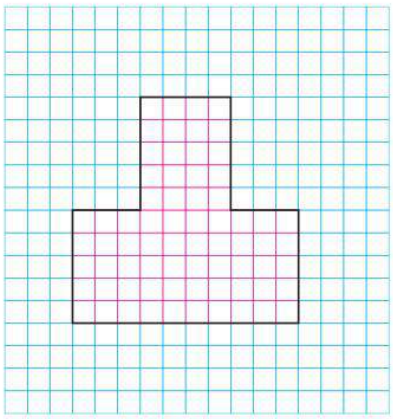


Figure 1

(*ii*)

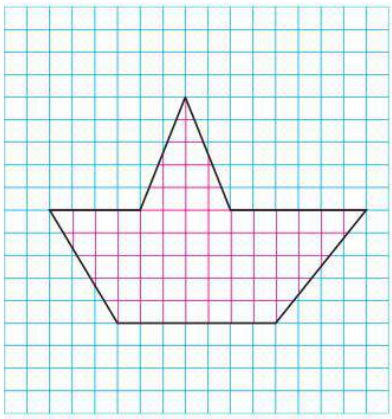


Figure 2

Sol. (*i*) In figure 1 no. of squares covered completely = 70

Area of 1 square = 1 sq. unit

Area of the figure = 70 sq. units.

(*ii*) No. of squares covered completely = 51

No. of squares covered half = 6

No. of squares to be considered in lieu of half squares = $\frac{1}{2} \times 6 = 3$

No. of squares covered more than half = 8

No. of squares to be considered in lieu of more than half = 8

No. of squares covered less than half = 9

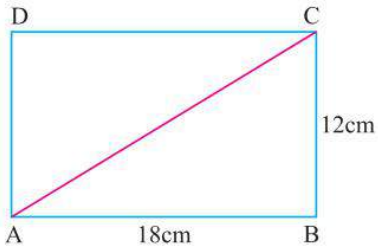
No. of squares to be considered in lieu of less than half = 0

Total no. of squares to be considered = 51 + 3 + 8 + 0 = 62

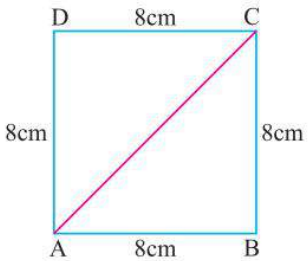
$\therefore$ Area of figure = 62 square units approx.

Example-2 : In- the following figures find the area of ΔABC

(*i*)



(*ii*)



Sol. (*i*) Given, Length of rectangle = $18cm$

Breadth of rectangle = $12cm$

AC the diagonal of rectangle ABCD, divides it into two equal triangles, ΔABC and ΔCDA

So, $$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Area of Rectangle ABCD}$$

$$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$$

$$= \frac{1}{2} \times 18 \times 12$$

$$= 108 cm^2$$

(*ii*) Given, side of a square = 8*cm*.

AC, the diagonal of square ABCD, divides it into two equal triangles, ΔABC and ΔCDA

So, $$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Area of square ABCD}$$

$$= \frac{1}{2} \times (\text{Side})^2$$

$$= \frac{1}{2} \times 8 \times 8$$

$$= 32\ cm^2$$

GENERALISING FOR OTHER CONGRUENT PARTS OF RECTANGLES

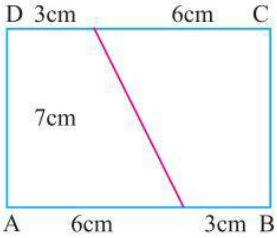
A rectangle of length 9*cm* and breadth 7*cm* is divided into two parts, congruent to each other.

$\therefore$ $$\text{Area of each congruent part} = \frac{1}{2} \times \text{Area of Rectangle ABCD}$$

$$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$$

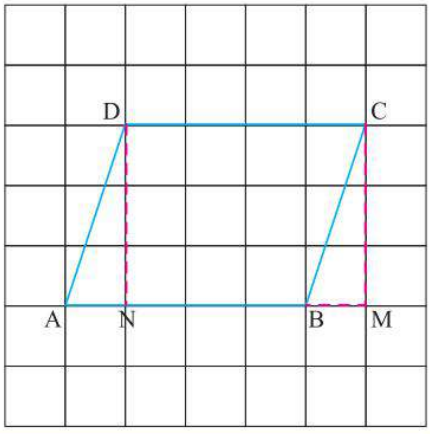
$$= \frac{1}{2} \times 9 \times 7$$

$$= 31.5 cm^2$$



AREA OF PARALLELOGRAM

A quadrilateral in which each pair of opposite sides is parallel and equal is called a parallelogram. In the figure, we see that the length of the rectangle DNMC is equal to the base of parallelogram ABCD and the breadth of the rectangle is equal to the height of parallelogram.



Area of parallelogram = Area of Rectangle

Area of parallelogram = Length × Breadth

Area of parallelogram = Base × height

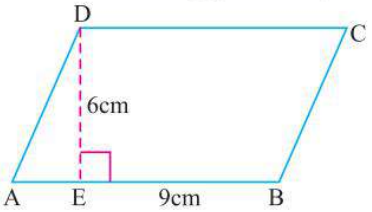
From above, we also have the following relatioship

$$\text{Base} = \frac{\text{Area of parallelogram}}{\text{Height}}$$

or $$\text{Height} = \frac{\text{Area of parallelogram}}{\text{Base}}$$

[Length of Rectangle = Base of Parallelogram
Breadth of Rectangle = height of Parallelogram]

Example-3 : Find the area of the following parallelogram.



Sol. Given, Base of parallelogram $= 9cm$

Height of parallelogram $= 6cm$

$\therefore$ Area of parallelogram $=$ Base $\times$ height

$= 9 \times 6$

$= 54cm^2$

Example-4 : Find the height of parallelogram, if the area of parallelogram is $42cm^2$ and base is $6cm$.

Sol. Given, Base of parallelogram $= 6cm$

Area of Parallelogram $= 42cm^2$

Base $\times$ height $= 42$

$6 \times$ height $= 42$

$$\text{height} = \frac{42}{6}$$

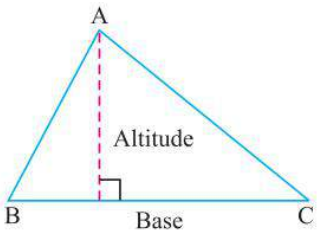
$$\text{height} = 7cm$$

$\therefore$ Height of parallelogram $= 7cm$

AREA OF A TRIANGLE

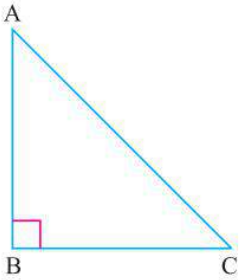
A closed plane figure made by joining three line segments is called a triangle.

(*i*) In a scalene triangle all the three sides have different lengths. If base and corresponding altitude are given.



Then area of $\Delta = \frac{1}{2} \times$ (base $\times$ height) sq. units

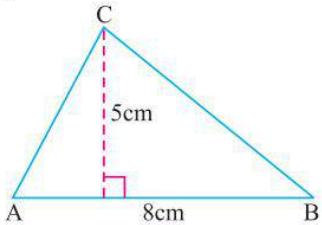
(*ii*) In a right angled triangle, the side opposite to the right angle is called the hypotenuse, the adjacent sides to right angle are called legs.



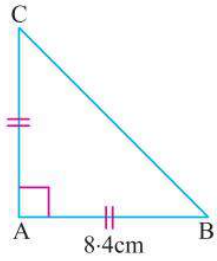
The area of right angled triangle $= \frac{1}{2} \times$ (base $\times$ height) sq unit.

Example-5 : Find the area of following triangles.

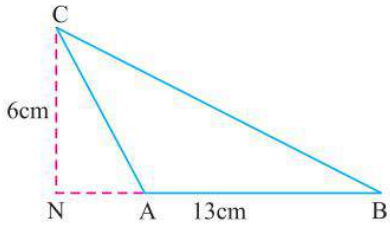
(*i*)



(*ii*)



(*iii*)



Sol. (*i*) Given, Base of triangle = $8cm$

Height of triangle = 5cm

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 8 \times 5$$

$$= 20cm^2$$

(*ii*) In ΔCAB, $AB = AC$

$AB = 8.4cm$

$\therefore$ $AC = 8.4cm$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 8.4 \times 8.4$$

$$= 35.28cm^2.$$

(*iii*) Base of triangle = 13 cm

Height of triangle = 6 cm

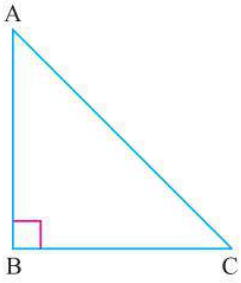
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 13 \times 6$$

$$= 39\ cm^2.$$

Example-6 : Area of a right angled triangle is $108cm^2$, if length of one leg is $9cm$. Find the length of other leg.

Sol. Let ABC be the triangle right angled at B



Let BC = $9cm$

AB = ?

Area of Triangle = $108cm^2$

$$\frac{1}{2} \times \text{Base} \times \text{height} = 108$$

$$\frac{1}{2} \times 9 \times \text{height} = 108$$

$$\text{height} = \frac{108 \times 2}{9}$$

$$\text{height} = 24cm$$

Example-7 : In ΔABC, BC = 5*cm* AN = 6*cm* and AB = 8*cm* find

(*i*) Area of ΔABC (*ii*) Length of CM

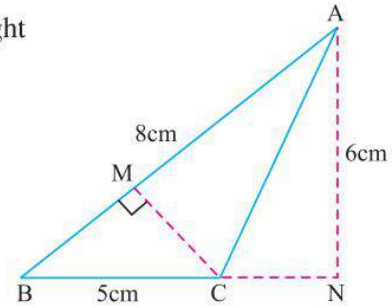
Sol. In ΔABC, BC = 5*cm* AN = 6*cm*

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AN$$

$$= \frac{1}{2} \times 5 \times 6$$

$$= 15cm^2$$



(*ii*) In ΔABC, AB = 8*cm*

$$CM = ?$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$15 = \frac{1}{2} \times AB \times CM$$

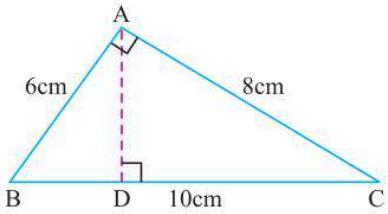
$$15 = \frac{1}{2} \times 8 \times CM$$

$$15 = 4 \times CM$$

$$CM = \frac{15}{4}$$

$$CM = 3.75cm$$

Example-8 : ΔABC is right angled at A as shown in figure. AD is prependicular to BC. If AB = 6*cm*, BC = 10*cm* and AC = 8*cm*. Find area of ΔABC. Also, find the length of AD.



Sol. Given ΔABC is right angled at A, AB = 6*cm*, BC = 10*cm* and AC = 8*cm*.

On taking AC as a base and AB as height we get

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 8 \times 6$$

$$= 24cm^2$$

Also, AD is prependicular to BC

Now, on taking BC as base and AD as height, We get

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$$

$$24 = \frac{1}{2} \times 10 \times AD$$

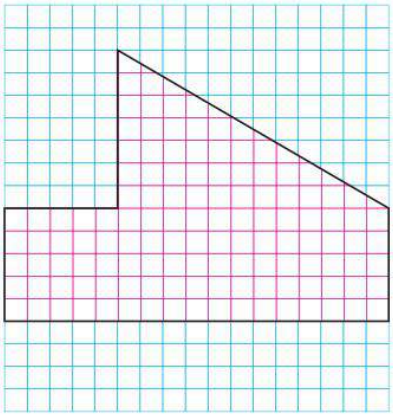
$$24 = 5 \times AD$$

$$AD = \frac{24}{5} = 4.8cm.$$

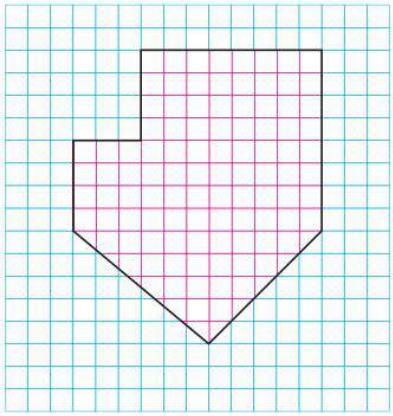
EXERCISE - 11.2

1. Estimate the area of the following figures by counting unit squares.

(*i*)

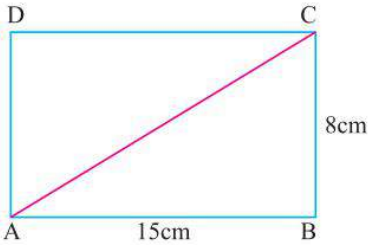


(*ii*)

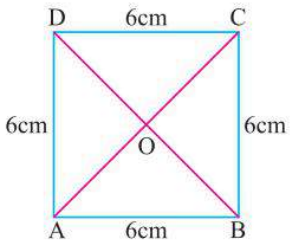


2. In the following figures find the area of

(*i*) ΔABC

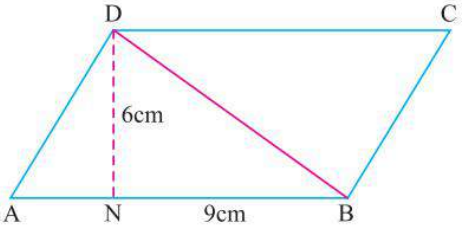


(*ii*) ΔCOD

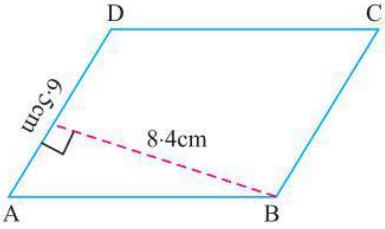


3. Find the area of following parallelograms.

(*i*)

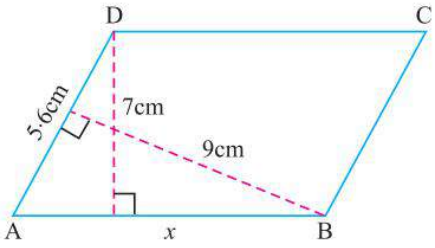


(*ii*)

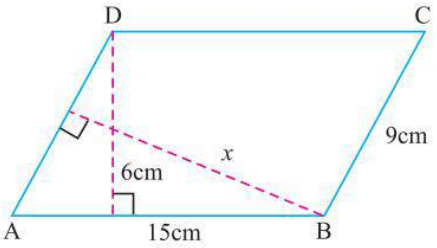


4. Find the value of x in the following parallelograms

(*i*)

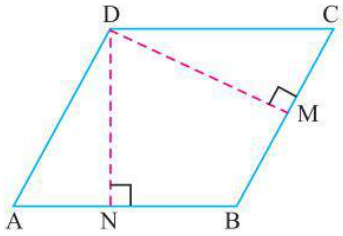


(*ii*)



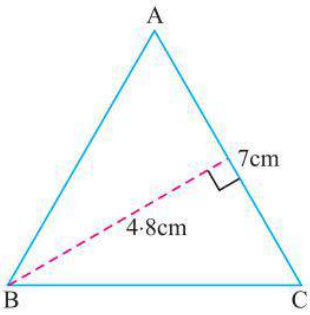
5. The adjacent sides of a parallelogram are $28cm$ and $45cm$ and the altitude on longer side is $18cm$. Find the area of parallelogram.

6. ABCD is a parallelogram given in figure. DN and DM are the altitudes on side AB and CB respectively. If area of the the parallelogram is $1225cm^2$, AB = $35cm$ and CB = $25cm$, find DN and DM.

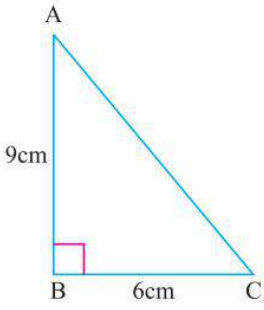


7. Find the area of the following triangles

(*i*)

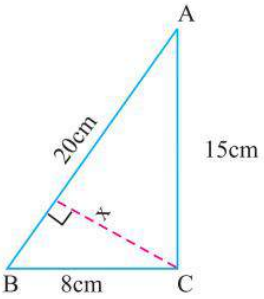


(*ii*)

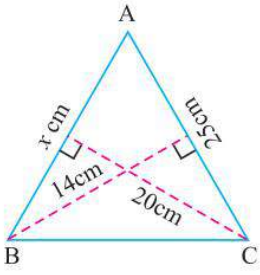


8. Find the value of x in the following triangles.

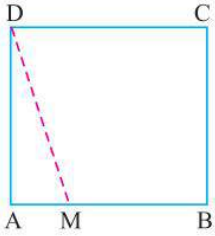
(*i*)



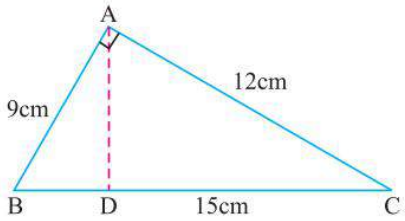
(*ii*)



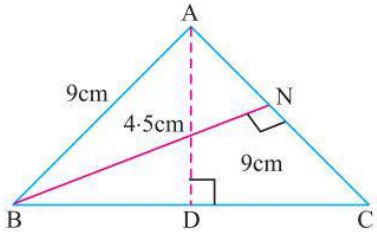
9. ABCD is a square, M is a point on AB such that AM = 9*cm* and area of ΔDAM is 171cm^2. What is the area of the square ?



10. ΔABC is right angled at A as shown in figure. AD is prependicular to BC, if AB = 9*cm*, BC = 15*cm* and AC = 12*cm*. Find the area of ΔABC, also find the length of AD.



11. ΔABC is isosceles with AB = AC = 9*cm*, BC = 12*cm* and the height AD from A to BC is 4.5*cm*. Find the area of ΔABC. What will be the height from B to AC i.e BN ?



12. Multiple choice questions :-

(*i*) Find the height of a parallelogram whose area is 246cm^2 and base is 20*cm*.

(*a*) 1.23cm^2 (*b*) 13.2cm^2

(*c*) 12.3cm^2 (*d*) 1.32cm^2

(*ii*) One of the side and the corresponding height of *a* parallelogram are 7*cm* and 3.5*cm* respectively. Find the area of the parallelogram.

(*a*) 21cm^2 (*b*) 24.5cm^2

(*c*) 21.5cm^2 (*d*) 24cm^2

(*iii*) The height of a triangle whose base is 13*cm* and area is 65cm^2 is.

(*a*) 12*cm* (*b*) 15*cm*

(*c*) 10*cm* (*d*) 20*cm*

(*iv*) Find the area of an isosceles right angled triangle, whose equal sides are of length 40*cm* each.

(*a*) 400cm^2 (*b*) 200cm^2

(*c*) 600cm^2 (*d*) 800cm^2

(*v*) If the sides of a parallelogram are increased to twice of its original length, how much will be the perimeter of the new parallelogram.

(*a*) 1.5 times (*b*) 2 times

(*c*) 3 times (*d*) 4 times

(*vi*) In a right angled triangle one leg is double the other and area is $64cm^2$ find the smaller leg.

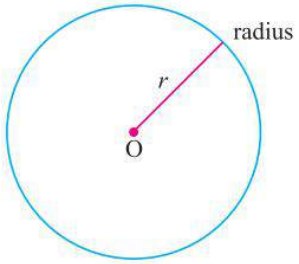
(*a*) $8cm$ (*b*) $16cm$

(*c*) $24cm$ (*d*) $32cm$

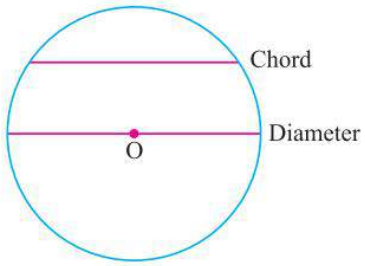
CIRCLE

A circle is a simple closed curve consisting of all those points in a plane each of which is at a constant distance from a fixed point O inside it. Let this constant distance be *r*.

- The fixed point O is called the **centre of the circle.**
- Line segment joining any point on the circle to its centre is called the **radius of the Circle.** It is denoted by *r*.



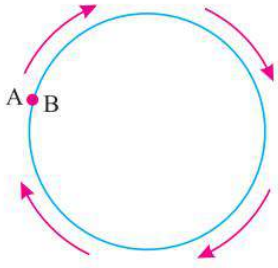
- A line segment joining any two points of a circle is called the **chord** of the circle.
- A chord passing through the centre of the circle is known as the **diameter of circle.** It is denoted by *d*.



Circumference of circle : The perimeter of a circle is called its circumference.

Circumference of a circle = Measure of the boundary of a circle.

To measure the boundary of a circle, we can put a piece of thread along its boundary and then straighten it to measure its length as shown in figure.



LAB ACTIVITY TO FIND THE VALUE OF Pi (π) AND FORMULA OF CIRCUMFERENCE

Aim : To find the value of Pi (π)

Material required : (*i*) Paper (*ii*) Thread (*iii*) Scissors (*iv*) Geometry box

Method : Draw six circles of different radii say 1cm, 2cm, 3cm, 4cm, 5cm and 6cm. Measure their circumference using thread. Now straighten the thread and measure its length.

Sr. No	Radius (in cm)	Diameter (in cm)	Circumference in cm (i.e., length of thread)	Value of $\pi = \frac{\text{Circumference}}{\text{Diameter}}$
1	1	2	6.3	3.15
2	2	4	12.5	3.125
3	3	6	18.8	3.133
4	4	8	25.1	3.14
5	5	10	31.4	3.14
6	6	12	37.6	3.133

From the above table we observe that the ratio of circumference to its diameter is constant and denoted by *pi* (π). The approximated value π is 3.14

From the above activity, we conclude that

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$$\pi = \frac{C}{d}$$

$$C = \pi d$$

$$C = \pi (2r) \quad [\because \text{ Diameter is double the radius of circle}]$$

$$C = 2\pi r$$

$\therefore$ circumference of circle $= 2\pi r$

$$\text{circumference of semi circle} = \frac{1}{2} \times 2\pi r = \pi r$$

Example-1: Find the circumference of circle whose diameter is 12cm (Take π = 3.14)

Sol.

Diameter of circle $d = 12cm$

circumference of circle $= \pi d$

$= 3.14 \times 12$

$= 37.68cm$

Example-2 : Circumference of circle is 88 *cm* find the radius of circle (Take $\pi = \frac{22}{7}$)

Sol.

Circumference of circle $= 88\ cm$

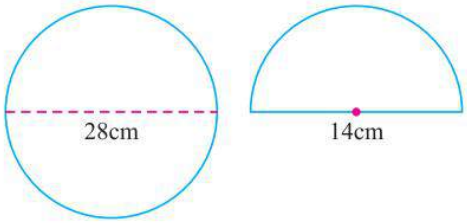
$2\pi r = 88$

$2 \times \frac{22}{7} \times r = 88$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14cm$$

Example-3 : A circular disc of diameter 28cm is divided into two parts. What is the perimeter of each semi circular disc ?



Sol.

Diameter of circular disc = $28cm$

Radius of circular disc = $14cm$

Circumference of semicircle = πr

$$= \frac{22}{7} \times 14$$

$$= 44cm$$

Perimeter of semi circular disc = Circumference of semi circle + length of diameter

$$= 44 + 28$$

$$= 72cm$$

Example-4 : A gardener wants to fence a circular garden of diameter 28*m*. Find the length of barbed wire if he makes 2 rounds of fence.

Sol.

Diameter of circular garden = $28m$

Circumference of garden = πd

$$= \frac{22}{7} \times 28$$

$$= 88m$$

Length of barbed wire needed to make one round of fence = 88m

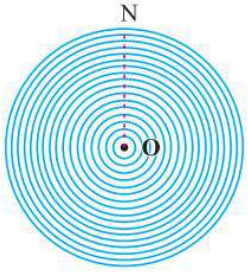
Length of barbed wire needed to make two round of fence = $88 \times 2 = 176m$

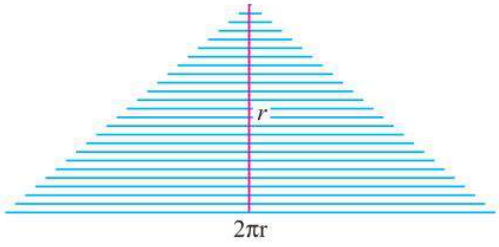
LAB ACTIVITY TO FIND AREA OF CIRCLE

Aim : To find the area of circle.

Material required : (*i*) wool of different colours (*ii*) scissor (*iii*) Fevicol (*iv*) coloured pens (*v*) geometery box

Method : Draw a circle of any radius. Fill up this circle with concentric circles with the help of wool. Without leaving any gap from the centre O cut off all the pieces of wool along ON and now arrange them starting from the outermost circular piece (as shown in figure). It will take the shape of a triangle whose base is equal to the circumference of the outermost circle and height is equal to the radius of the circle.





$$\text{Base of Triangle} = 2\pi r$$
$$\text{height of triangle} = r$$
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 2\pi r \times r$$
$$\text{Area of triangle} = \pi r^2 \text{ sq. unit}$$
$$\text{Hence area of circle} = \pi r^2 \text{ sq. unit}$$

Example-5 : Find the area of circle whose radius is 21cm (Take $\pi = \frac{22}{7}$)

Sol.

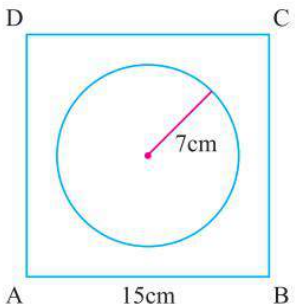
$$\text{Radius of circle} = 21cm$$
$$\text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 21 \times 21$$
$$= 1386\ cm^2$$

Example-6 : Find the area of circle whose circumference is 88*cm*.

Sol.

$$\text{Circumference of circle} = 88cm$$
$$2\pi r = 88$$
$$\pi r = \frac{88}{2}$$
$$\frac{22}{7} \times r = 44$$
$$r = \frac{44 \times 7}{22}$$
$$r = 14cm$$
$$\text{Then Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 14 \times 14$$
$$= 616\ \text{cm}^2$$

Example-7 : From a square sheet of side 15 cm *a* circular sheet of radius 7*cm* is removed. Find the area of remaining sheet (Take $\pi = \frac{22}{7}$)



Sol.

$$\text{Side of square sheet} = 15cm$$
$$\text{Area of square sheet} = (\text{side})^2$$
$$= 15 \times 15$$
$$= 225\ \text{cm}^2$$
$$\text{Radius of removed circle} = 7cm$$
$$\text{Area of removed circle} = \pi r^2$$
$$= \frac{22}{7} \times 7 \times 7$$
$$= 154cm^2$$

Area of Remaining sheet = Area of square sheet – Area of removed circular sheet

$= 225 - 154$

$= 71 cm^2$

EXERCISE - 11.3

1. Find the circumference of circle whose

(*i*) Radius $(r) = 21cm$ (*ii*) Radius $(r) = 3.5cm$

(*iii*) Diameter $= 84cm$

2. If the circumference of a circular sheet is $176m$, find its radius.

3. A circular disc of diameter $8.4cm$ is divided into two parts what is the perimeter of each semicircular part ?

4. Find the area of the circle having

(*i*) Radius $r = 49cm$ (*ii*) Radius $r = 2.8cm$

(*iii*) Diameter $= 4.2cm$

5. A gardener wants to fence a circular garden of radius $15m$. Find the length of wire, if he makes three rounds of fence. Also, find the cost of wire if it costs ₹ 5 per meter (Take $\pi = 3.14$)

6. Which of the following has larger area and by how much ?

(*a*) Rectangle with length $15cm$ and breadth $5.4cm$

(*b*) Circle of diameter $5.6cm$.

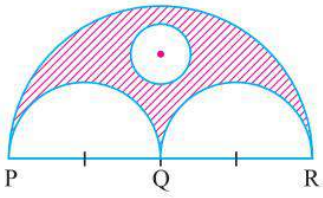
7. From a rectangular sheet of length $15cm$ and breadth $12cm$ a circle of radius $3.5cm$ is removed. Find the area of remaining sheet.

8. From a circular sheet of radius $7cm$, a circle of radius $2.1cm$ is removed, find the area of remaining sheet.

9. Smeep took a wire of length $88cm$ and bent it into the shape of a circle, find the radius and area of the circle. If the same wire is bent into a square, what will be the side of the square? Which figure encloses more area ?

10. A garden is $120m$ long and $85m$ broad. Inside the garden, there is a circular pit of diameter $14m$. Find the cost of planting the remaining part of the garden at the rate of ₹5.50 per square meter.

11. In the figure PQ = QR and PR = $56cm$. The radius of inscribed circle is $7cm$. Q is centre of semi circle. What is the area of shaded region ?



12. The minute hand of a circular clock is $18cm$ long. How far does the tip of minute hand move in one hour ?

13. Multiple choice questions :-

(*i*) The circumference of a circle of diameter 10*cm* is

(*a*) 31.4*cm* (*b*) 3.14*cm*

(*c*) 314*cm* (*d*) 35.4*cm*

(*ii*) The circumference of a circle with radius 14*cm* is

(*a*) 88*cm* (*b*) 44*cm*

(*c*) 22*cm* (*d*) 85*cm*

(*iii*) What is the area of the circle of radius 7*cm* ?

(*a*) 49*cm* (*b*) $22cm^2$

(*c*) $154cm^2$ (*d*) $308cm^2$

(*iv*) Find the diameter of a circle whose area is $154cm^2$.

(*a*) 4*cm* (*b*) 6*cm*

(*c*) 14*cm* (*d*) 12*cm*

(*v*) A circle has area 100 times the area of another circle. What is the ratio of their cirumferences ?

(*a*) 10 : 1 (*b*) 1 : 10

(*c*) 1 : 1 (*d*) 2 : 1

(*vi*) Diameter of a circular garden is 9.8*cm*. Which of the following is its area ?

(*a*) $75.46cm^2$ (*b*) $76.46cm^2$

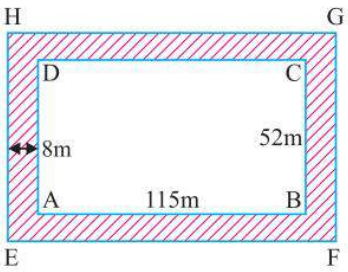
(*c*) $74.4cm^2$ (*d*) $76.4cm^2$

Conversion of units :-

Lenght units	Area Units
1*cm* = 10*mm*	$1\ cm^2 = (10 \times 10)\ mm^2 = 100\ mm^2$
1dm = 10cm	$1\ dm^2 = (10 \times 10)\ cm^2 = 100\ cm^2$
1m = 10dm	$1\ m^2 = (10 \times 10)\ dm^2 = 100\ dm^2$
1m = 100cm	$1\ m^2 = (100 \times 100)\ cm^2 = 10000\ cm^2$
1 dam = 10m	$1\ dam^2 = (10 \times 10)\ m^2 = 100\ m^2$
1 hm = 100m	$1\ hm^2 = (100 \times 100)\ m^2 = 10000\ m^2$
1km = 1000m	$1\ km^2 = (1000 \times 1000)\ m^2 = 1000000\ m^2$
	1 are = $100\ m^2$
	1 hectare = $10000\ m^2$

AREA OF PATHS, CROSS ROADS AND BORDERS

Example-1: A retangular garden is 115*m* long and 52*m* broad. A path of uniform width of 8*m* has to be constructed around it, on its outside. Find the cost of gravelling the path at 10.50 per m^2.



Sol. Let ABCD represents the rectangular garden and the shaded region represents the path of width 8*m* around the garden.

Length of rectangular garden $l = 115m$

Breadth of rectangular garden $b = 52m$

Area of rectangular garden ABCD $= (115 \times 52)m^2$

$= 5980m^2$

Length of rectangular garden including path $= 115m + (8m + 8m) = 131m$

Breadth of rectangular garden including path $= 52m + (8m + 8m) = 68m$

Area of garden including path $= (131 \times 68)\ m^2 = 8908\ m^2$

Area of path = Area of garden including path – Area of garden

Area of path $= (8908 - 5980)m^2$

$= 2928m^2$

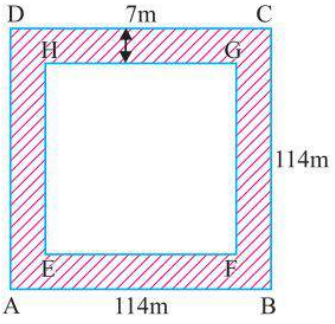
Cost of gravelling $1m^2$ of path = ₹ 10.50

Cost of gravelling $2928m^2$ of path $= 2928 \times 10.50$

= ₹ 30744

Example-2 : A path of $7m$ wide runs along inside a square park of side $114m$. Find the area of the path. Also, find the cost of cementing it at the rate of ₹225 per $15m^2$

Sol. Let ABCD be the square park of side $114m$ and the shaded region represents the path $7m$ wide



$EF = 114m - (7 + 7)m$

$= 100m$

Area of square park ABCD $= (\text{Side})^2$

$= 114 \times 114$

$= 12996m^2$

Area of EFGH $= (\text{Side})^2$

$= 100 \times 100$

$= 10000m^2$

Area of path = Area of square park ABCD – Area of EFGH

$= (12996 - 10000)\ m^2$

$= 2996m^2$

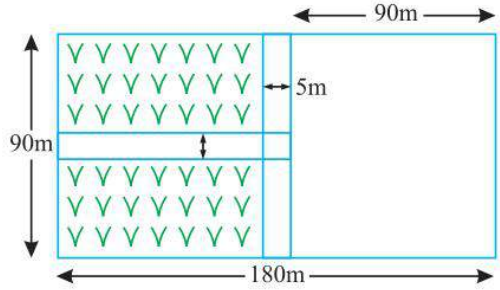
Cost of cementing $15m^2$ = ₹225

Cost of cementing $1m^2 = \frac{225}{15}$

Cost of cementing $2996m^2 = \frac{225}{15} \times 2996$

= ₹44940

Example-3 : A School ground is $180m$ long and $90m$ wide. An area of $90m \times 90m$ is kept for morning assembly. In the remaining portion there is $5m$ wide path parallel to its width and parallel to its remaining length (as shown in figure). The remaining area is covered by grass. Find the area covered by grass.



Sol. Area of school ground = $180m \times 90m$

$= 16200\ m^2$

Area kept for morning assembly $= 90 \times 90$

$= 8100\ m^2$

Area of path parallel to width of ground $= 90 \times 5$

$= 450\ m^2$

Area of path parallel to remaining length of ground $= 90 \times 5 = 450\ m^2$

Area common to both parths $= 5 \times 5$

$= 25m^2$

Total area covered by path $= (450 + 450 - 25)$

$= 875m^2$

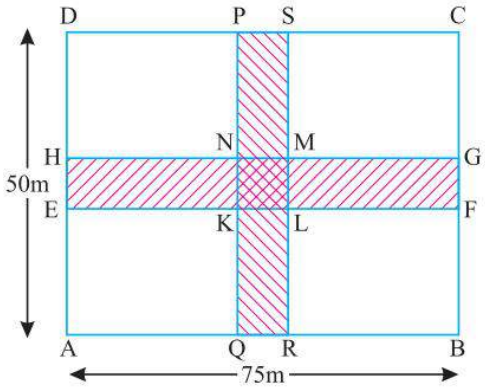
Area covered by grass = Area of ground – (Area kept for morning assembly + area covered by paths)

$= 16200 - (8100 + 875)$

$= 16200 - 8975$

$= 7225m^2$

Example-4 : Two crossroads each of width 6m, runs at right angle through the centre of rectangular park of length 75m and breadth 50*m* and parallel to its sides. Find the area of roads. Also find the cost of constructing the roads at the rate of ₹120 per m^2.



Sol. ABCD represent the rectangular park of length AB = 75m and breadth BC = 50m. Area of shaded portion i.e, area of rectangle EFGH and PQRS represent the area of cross roads, but the area of square KLMN is taken twice, So it will be subtracted

Now EF = 75*m*, FG = 6*m*, PQ = 50*m*, QR = 6*m*, KL = 6m. Area covered by road = Area of rectangle EFGH + area of rectangle PQRS – Area of square KLMN

$= (EF \times FG) + (PQ \times QR) - (KL)^2$

$= (75 \times 6) + (50 \times 6) - (6 \times 6)$

$= 450 + 300 - 36$

$= 714m^2$

Cost of constructing $1m^2$ of road=₹ 120

Cost of constructing $714m^2$ of road=714×120

= ₹ 85680

Hence, the cost of constructing the road is ₹ 85680

EXERCISE - 11.4

1. A rectangular park is 80*m* long and 65*m* wide. A path of 5*m* width is constructed outside the park. Find the area of path.

2. A rectangular garden is 110*m* long and 72*m* broad. A path of uniform width 8*m* has to be constructed around it. Find the cost of gravelling the path at ₹11.50 per m^2.

3. A room is 12*m* long and 8*m* broad. It is surrounded by a verandah, which is 3*m* wide all around it. Find the cost of flooring the verandah with marble at ₹275 per m^2.

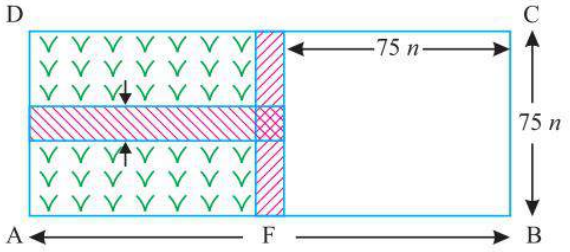
4. A sheet of paper measures 30*cm* × 24*cm*. A strip of 4*cm* width is cut from it, all around. Find the area of remaining sheet and also the area of cut out strip.

5. A path of 2*m* wide is built along the border inside a square garden of side 40*m*. Find

(*i*) The Area of path.

(*ii*) The cost of planting grass in the remaining portion of the garden at the rate of ₹50 per m^2.

6. A nursery school play ground is 150*m* long and 75*m* wide. A portion of 75*m* × 75*m* is kept for see-saw slides and other park equipments. In the remaining portion 3*m* wide path parallel to its width and parallel to remaing length (as shown in fig). The remaining area is covered by grass. Find the area covered by grass.



7. Two cross roads each of width 8*m* cut at right angle through the centre of a rectangular park of length 480m and breadth 250*m* and parallel to its sides. Find the area of roads. Also, find the area of park excluding cross roads.

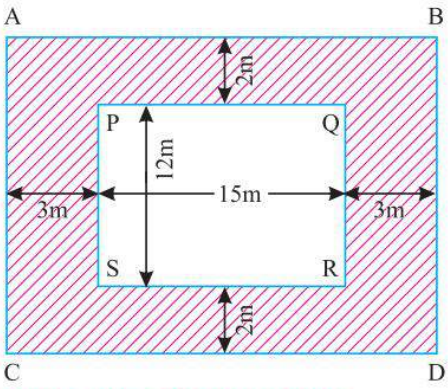
8. In a rectangular field of length 92*m* and breadth 70*m*, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of field. If the width of each road is 4*m*, find.

(*i*) The area covered by roads.

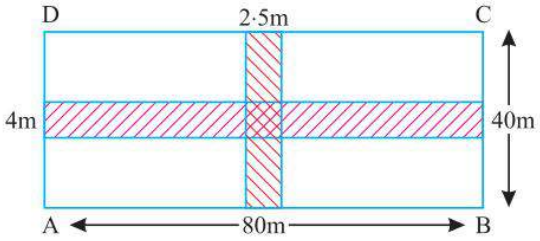
(*ii*) The cost of constructing the roads at the rate of ₹150 per m^2.

9. Find the area of shaded region in each of the following figures.

(*i*)



(*ii*)



WHAT HAVE WE DISCUSSED ?

1. For rectangle and square

(*i*) Perimeter of a rectangle = 2 (Length + Breadth) units

(*ii*) Area of a rectangle = (Length × Breadth) sq. units

(*iii*) $\text{Length} = \dfrac{\text{Area}}{\text{Breadth}}$ and $\text{Breadth} = \dfrac{\text{Area}}{\text{Length}}$

(*iv*) Perimeter of a square = (4 × side) units

(*v*) Area of a square $= (\text{side})^2$ sq units

(*vi*) Side of a square = $\sqrt{\text{Area}}$ units

2. For a parallelogram

(*i*) Area of a parallelogram = (Base × Height) sq. units

(*ii*) $\text{Base} = \dfrac{\text{Area of parallelogram}}{\text{Height}}$ and $\text{Height} = \dfrac{\text{Area of parallelogram}}{\text{Base}}$

3. For a triangle

(*i*) Area of a triangle = $\dfrac{1}{2}$ × (base × height) sq. units

4. For a circle

(*i*) Circumference of a circle = πd unit or $2\pi r$ units

(*ii*) Area of a circle = πr^2 sq. units.

LEARNING OUTCOMES

After completion of the chapter the students are now be able to :

1. Differentiate between perimeter and area of plane figures.
2. Find out approximate area of closed shapes by using unit square grid/graph sheet.
3. Calculate the perimeter and area of plane figures viz. square, rectangle, triangle, parallelogram.
4. Find the circumference and area of circle using formulae.
5. Convert various units of area and perimeter, wherever necessary.
6. Apply the formulae learnt to solve the problems related to their day to day life.

ANSWERS

EXERCISE 11.1

1. (*i*) 86*cm*; 420cm^2 (*ii*) 23.8*cm* ; 23.5cm^2

2. (*i*) 116*cm* ; 841cm^2 (*ii*) 33.2*cm* ; 68.89cm^2

3. 1369m^2 **4.** 20*cm* ; 98*cm*

5. 40*cm* ; Square encloses more area ; 64cm^2

6. 45*m* ; 340*m* **7.** ₹1507.50 **8.** ₹516.75

9. (*i*) 38*cm* ; 52cm^2 (*ii*) 29*cm* ; 19.5cm^2

10. (*i*) *b* (*ii*) *a* (*iii*) *d*
(*iv*) *a* (*v*) *c* (*vi*) *a*

EXERCISE 11.2

2. (*i*) 135 sq. units approx. (*ii*) 114 sq. units approx.

2. (*i*) 60m^2 (*ii*) 9cm^2

3. (*i*) 54cm^2 (*ii*) 54.6cm^2

4. (*i*) 7.2*cm* (*ii*) 10*cm*

5. 810cm^2 **6.** 35*cm* ; 49*cm*

7. (*i*) 16.8cm^2 (*iii*) 27cm^2

8. (*i*) 6*cm* (*ii*) 17.5*cm*

9. 324cm^2 **10.** 54cm^2 ; 7.2*cm*

11. 27cm^2 ; 6*cm*

12. (*i*) *c* (*ii*) *b* (*iii*) *c*
(*iv*) *d* (*v*) *b* (*vi*) *a*

EXERCISE 11.3

1. (*i*) 132*cm* (*ii*) 22*cm* (*iii*) 264*cm*

2. 28*m* **3.** 21.6*cm*

4. (*i*) 7546cm^2 (*ii*) 24.64cm^2 (*iii*) 13.86cm^2

5. 282.6 *m*; ₹1413 **6.** Rectangle has more area ; 56.36cm^2

7. 141.5cm^2 **8.** 140.14cm^2

9. 14*cm* ; 616cm^2 ; 22*cm* ; circle encloses more area **10.** ₹55243

11. 462cm^2 **12.** 113.04*cm*

13. (*i*) *a* (*ii*) *a* (*iii*) *c*
(*iv*) C (*v*) *a* (*vi*) *a*

EXERCISE 11.4

1. $1550 m^2$

2. ₹36432

3. ₹42900

4. $352 cm^2$; $368 cm^2$

5. (*i*) $304 m^2$ (*ii*) 64800

6. $5184 m^2$

7. $5776 m^2$; $114224 m^2$

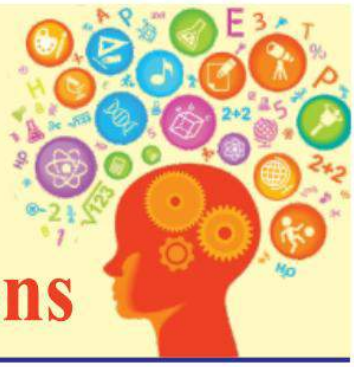
8. (*i*) $632 m^2$ (*ii*) ₹94800

9. (*i*) $156 m^2$ (*ii*) $410 m^2$

CHAPTER

12

T1W4G3



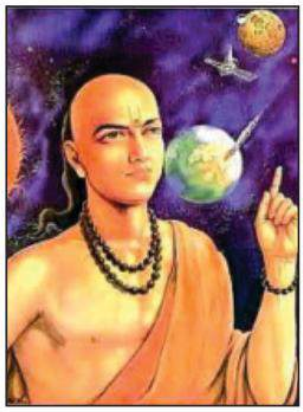
Algebraic Expressions

Learning Objectives :-

In this chapter, you will learn :-

1. To identify terms related to algebra like constant, variable, terms, coefficient of terms.
2. To generate algebraic expression with one or two variables.
3. About addition and subtraction of algebraic expressions.
4. To find the value of an expression for a given value of variable.
5. To apply your knowledge of algebraic expressions in your daily life.

OUR NATIONS' PRIDE



Bhaskaracharya was an Indian mathematician and astronomer. He was born in Bijapur in Karnatka. His works represent a significant contribution to mathematical and astronomical knowledge in the 12th century. He has been called the great mathematican of medieval India. His main work is divided into four parts called Lilavati, Bijaganita, Grahganita and Goladhyaya. These four sections deal with Arithmetic, Algebra, mathematics of the planets and sphere respectively. On 20 Nov. 1981, the Indian space research organisation launched the Bhaskara II satellite honouring the great mathematican and astronomer.

INTRODUCTION

In class VI, you have already learnt about simple algebraic expressions like $x + 5$, $2x - y$, $3x + y$, $2y - 7$ etc and we have seen the use of such expressions in forming words problems and simple equations.

In this chapter, we shall learn more about algebraic expressions. We shall study "How are Algebraic expressions formed" Factors of a term, coefficient of a term, like and unlike terms, types of polynomials. We shall learn to find the value of an expression for a particular value of the variable.

Algebraic Expression : Before proceeding further let us revise some definition related to algebraic expressions.

1. **Constant :** Constant is a term that has a fixed value. Some examples of a constant are $3, 5, 0, -7, \frac{-2}{3}, \sqrt{3}$ etc.
   - Infact every number is a constant.

2. **Variable :** Variable is a term that does not have a fixed value. We use letters of english alphabets for variables. For example x, y, z, s, t etc.

Let us assume any number less than 3. It may be –10, –7, –6, –3, –1, 0, 1, 2 and many more. Therefore, when we think any number less than 3. We observe that we do not have single fixed number less than 3 we will write $x < 3$.

Where x may have varying value. Which is less than 3. $\therefore$ x is variable.

3. **Term :** A term is a number (constant), a variable or a combination (Product or quotient) of numbers and variables. For example

$$7, y, 5b, xy, \frac{-3x}{2y}, \frac{7m}{8}, \frac{5}{t} \text{ etc.}$$

Algebraic expressions : A combination of one or more terms, which are seperated by addition, subtraction is called on Algebraic expression. For example. $4 + 10x$, $5x - 7y$, $3a + 7b$, $ax + by - cz$ etc.

- Only (–) minus and (+) plus signs seperate the terms. Where as the division and product do not seperate the terms.

Factors : The terms are made of the product of factors. For example the term $2xy$ of expression $2xy + 7z$ has three factors 2, x and y and the term 7z has 7 and 'z' two factors and expression $2xy + 7z$ has two terms.

Coefficient : Any of the factors of a term is called the coefficient of the product of all the remaining factors. In particular, the constant part is called the "numerical coefficient" and the remaining part is called the 'Literal coefficient" of term.

For example : Consider the expression.

$3x^2y + 7xy - 8$

In the term $3x^2y$ Numerical coefficient $= 3$

Coefficient of $y = 3x^2$

Coefficient of $x^2 = 3y$

Coefficient of $x = 3xy$

Simlarly : In the term $7xy$

Numerical coefficient $= 7$

Coefficient of $x = 7y$

Coefficient of $y = 7x$

Example-1 : Write the terms, factors and numerical coefficient for the following expressions.

(a) $xy - x$ **(b)** $17xy + 3$

(c) $30x^2yz + 70x$ **(d)** $10m^2n + 3pq + 17z$

Sol.

	Algebraic Expressiosn	Terms	Factors	Numerical Coefficient
(a)	$xy - x$	xy $-x$	x, y x	1 –1
(b)	$17xy + 3$	$17xy$ 3	17, x, y 3	17 3

(*c*)	$30\,x^2yz + 70x$	$30\,x^2yz$ $70\,x$	30, *x*, *x*, *y* ,*z* 70, *x*	30 70
(*d*)	$10\,m^2n + 3pq + 17z$	$10m^2n$ $3pq$ $17z$	10, *m*, *m*, *n* 3, *p*, *q* 17, *z*	10 3 17

Like Terms : The terms having same variable factors are called like terms. For example $3x^2y$ and $-7x^2y$, $2xyz$ and $7xyz$, $-3x^2yz^2$ and $2x^2yz^2$ etc.

Note : Like term may have different numerical coefficient, but same literal coefficient.

Unlike terms : The terms having different variable factors are called unlike terms. For example xy^2 and xyz, x^2y^2z and xyz^2, $3x^2$ and $3y^2$ etc.

Example-2 : Group the like terms.

(*a*) $2xy, 3x^2, -7x^2, 3xyz$ and $7xy$

(*b*) $7x^2yz, 3x^2y^2, 2xy^2, -8x^2y^2$

Sol. (*a*) Terms having xy as variable factor $2xy$, $7xy$.

$\therefore$ $2xy$, $7xy$ are like terms.

Terms having x^2 as varibale factor are

$3x^2, -7x^2$

$\therefore$ $3x^2, -7x^2$ are like terms

(*b*) Terms having x^2y^2 as variable factor are

$3x^2y^2, -8x^2y^2$

$\therefore$ $3x^2y^2$ and $-8x^2y^2$ are like terms.

Example-3 : State whether the given pair of terms is of like or unlike term.

(*a*) $10\,p^2q$ and $10\,pq^2$

(*b*) $7xy^2$ and $-3xy^2$

Sol. (*a*) $10p^2q$ and $10pq^2$

Variable coefficient in $10\,p^2q = p^2q$

Variable coefficient in $10\,pq^2 = pq^2$

$\therefore$ $10p^2q$ and $10pq^2$ are unlike terms.

(*b*) $7xy^2$ and $-3xy^2$

Variable coefficient in $7xy^2 = xy^2$

Variable coefficient in $-3xy^2 = xy^2$

$\therefore$ $7xy^2$ and $-3xy^2$ are like terms.

TYPES OF ALGEBRAIC EXPRESSIONS

Number of Terms	Name of Expression	Examples
One	Monomial	$x, 2y, \frac{5z}{3}, -\frac{7x^2}{9}$ etc
Two	Binomial	$x + 9, 3x - 2y$ $3x^2 - z^2$
Three	Trinomial	$x + y + z, p^2 + q^2 + r^2,$ $pq + r + t^2$
Two or more than two terms	Polynomial	$x, 3x + 2y, p + q + z,$ $x^2 + y^2 + zx$

- Every binomial, every trinomial, is a polynomial.

Example-4 : Classify the following algebraic expression as monomial, binomial or trinomial

(a) $xy + x - z$ **(b)** $16pqr$ **(c)** $m^2 + n^2$ **(d)** $\frac{7x}{2} + \frac{3y}{5}$

Sol. (a) Algebraic expression is $xy + x - z$

Number of terms = 3

$\therefore$ It is a trinomial

(b) Algebraic expression is $16\,pqr$

Number of terms = 1

$\therefore$ It is a monomial

(c) Algebraic expression is $m^2 + n^2$

Number of terms = 2

$\therefore$ It is a binomial

(d) Algebraic expresions is $\frac{7x}{2} + \frac{3y}{5}$

Number of terms = 2

$\therefore$ It is a binomial

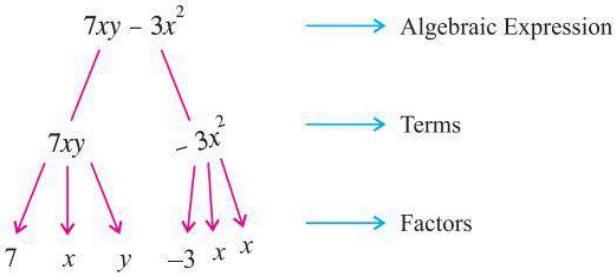
Tree diagram : It is a diagrammatic way of representing terms and factors of an algebraic expression. The terms and the factors of each terms of an algebraic expression can be shown by a tree diagram as shown in the following examples.

Example-5 : Identify the terms and their factors in the following expressions by tree diagrams.

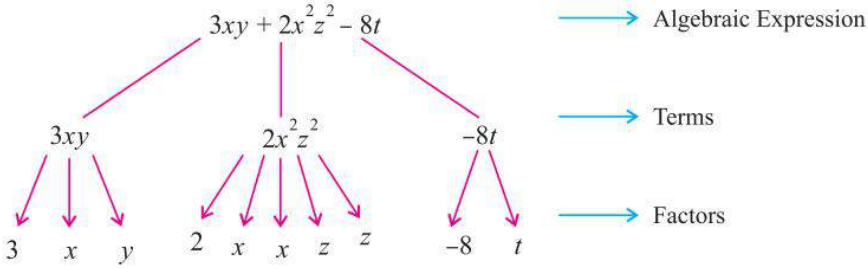
(a) $7xy - 3x^2$

(b) $3xy + 2x^2 z^2 - 8t$

Sol. (*a*) Given expression is $7xy - 3x^2$
Tree diagram.



(*b*) Given expression is $3xy + 2x^2 z^2 - 8t$
Tree diagram



EXERCISE - 12.1

1. **Generate algebraic expressions for the following :**

(*i*) The sum of a and b

(*ii*) The number z multiplied by itself.

(*iii*) The product of x and y added to the product of m and n.

(*iv*) The quotient of p by 5 is multiplied by q.

(*v*) One half of z added to twice the number t.

(*vi*) Sum of squares of the numbers x and z.

(*vii*) Sum of the numbers x and z is subtracted from their product.

2. Separate constant terms and variable terms from the following.

$$7, xy, \frac{3x^2}{2}, \frac{72z}{3}, \frac{-8z}{3x^2}$$

3. Write the terms and factors for each of the following algebraic expression.

(*a*) $2x^2 + 3yz$ (b) $15x^2y + 3xy^2$

(*c*) $-7xyz^2$ (*d*) $100pq + 10\,p^2q^2$

(*e*) $xy + 3x^2\,y^2$ (*f*) $-7x^2yz + 3xy^2\,z + 2xyz^2$

4. Classify the following algebraic expression into monomial, binomial and trinomial.

(*a*) $7x + 3y$ (*b*) $5 + 2x^2y^2z$

(c) $ax + by^2 + cz^2$ (d) $3x^2 y^2$

(e) $1 + x$ (f) 10

(g) $\frac{3}{2}p + \frac{7}{6}q$

5. Write numerical coefficient of each of the following algebraic expression.

(a) $2x$ (b) $\frac{-3}{2}xyz$

(c) $\frac{7}{2}x^2 p$ (d) $-p^2q^2$

(e) $-5mn^2$

6. State whether the given pairs of terms is of like or unlike terms.

(a) $-3y, \frac{7}{8}y$ (b) $-32, -32x$

(c) $3x^2y, 3xy^2$ (d) $14mn^2, 14mn^2q$

(e) $8pq, 32pq^2$ (f) 10, 15

7. In the following algebraic expressions write the coefficient of

(a) x in x^2y (b) xyz in $15x^2yz$

(c) $3pq^2$ in $3p^2q^2r^2$ (d) m^2 in $m^2 + n^2$

(e) xy in $x^2 y^2 + 2x + 3$

8. Identify the terms and their factors in the following algebraic expressions by tree diagrams

(a) $12xy + 7x^2$ (b) $p^2q^2 + 3mn^2 - pqr$

(c) $2x^2 y^2 + xyz^2 + zy$ (d) $\frac{3}{2}x^3 + 2x^2y^2 - 7y^3$

9. Multiple Choice Questions :

(*i*) An expression with only one term is called a

(a) Monomial (b) Binomial

(c) Trinomial (d) None of these

(*ii*) The coefficient of x in $8 - x + y$ is

(a) -1 (b) 1

(c) 8 (d) 0

(*iii*) Which of the following are like terms ?

(a) $7x, 12y$ (b) $15x, 12x$

(c) $3xy, 3x$ (d) $2y, -2yx$

(*iv*) Terms are added to form

(a) Expressions (b) Variables

(c) Constants (d) Factors

ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Suppose, you have 15 apples and your brother has 12 apples, then how many Apples do you both have together ? The answer is simple 15 + 12 = 27 Apples.

If we denote an apple by x then you have $15x$ and your brother has $12x$ which can be added as $15x + 12x = 27x$.

Now again suppose you have 12 pens and your brother has 8 pencils can you add 12 pens and 8 pencils. The answer is no. We can only say that we have 12 pens and 8 pencils.

Addition of like terms : The sum of two or more like terms is again the like terms whose numerical coefficient is the sum of the numerical coefficient of the given terms.

For example :

$$2y + 3y = (2 + 3)\, y = 5y$$
$$3x + 7x + 8x = (3 + 7 + 8)x = 18\, x$$
$$2ab + 5ab + 7ab = (2 + 5 + 7)ab = 14ab$$

Example-1 : Add $3xy^2, 7xy^2, -2xy^2$

Sol. Given terms are like terms.

Their coefficients are 3, 7 and –2 respectively.

$$3xy^2 + 7xy^2 + (-2xy^2)$$
$$= (3 + 7 - 2)xy^2$$
$$= (10 - 2)xy^2$$
$$= 8xy^2$$

Example-2 : Add $9xy, -3xy, -8xy, 5xy$

Sol.

$$\text{Required sum} = 9xy + (-3xy) + (-8xy) + (5xy)$$
$$= (9 - 3 - 8 + 5)xy$$
$$= (9 + 5 - 3 - 8)xy$$
$$= (14 - 11)xy$$
$$= 3xy$$

Addition of Algebraic expressions : To add algebraic expressions, we have to group like terms and then carry out addition on them. This can be done by two methods.

(*i*) **Horizontal Method :** In this method, All expressions are written in a horizontal line and then the terms are arranged in the group of like terms and then like terms are added.

(*ii*) **Column method :** In this method each expression is written in a seperate row such that there like terms are arranged one below the other in a column. Then the addition of terms is done column wise.

Example-3 : (*i*) Add the algebraic expressions $2x + 3y - 7z$ and $3x + 4y + 8z$

Sol. Horizontal Method

$$(2x + 3y - 7z) + (3x + 4y + 8z)$$
$$= 2x + 3y - 7z + 3x + 4y + 8z$$
$$= 2x + 3x + 3y + 4y - 7z + 8z$$
$$= (2 + 3)x + (3 + 4)y + (-7 + 8)z$$
$$= 5x + 7y + z$$

Column Method :

$$\begin{array}{r} 2x + 3y - 7z \\ 3x + 4y + 8z \\ \hline 5x + 7y + z \\ \hline \end{array}$$

(*ii*) Add the algebraic expressions $5x + 7y - 2z$, $3x + 3y + 8z$ and $7x + 2y - 3z$

Sol. Horizontal method

$(5x + 7y - 2z) + (3x + 3y + 8z) + (7x + 2y - 3z)$

$= 5x + 3x + 7x + 7y + 3y + 2y - 2z + 8z - 3z$

$= (5 + 3 + 7)x\ (7 + 3 + 2)y + (-2 + 8 - 3)z$

$= 15x + 12y + 3z$

Column Method :

$$\begin{array}{r} 5x + 7y - 2z \\ 3x + 3y + 8z \\ 7x + 2y - 3z \\ \hline 15x + 12y + 3z \end{array}$$

(*iii*) Add the algebraic expressions $3x^3 + 7xy - 8z^2x$, $2x^3 - 3xy + 3z^2x$, $x^3 - 2xy + 5z^2x$

Sol. Horizontal method

$(3x^3 + 7xy - 8z^2x) + (2x^3 - 3xy + 3z^2x) + (x^3 - 2xy + 5z^2x)$

$= 3x^3 + 2x^3 + x^3 + 7xy - 3xy - 2xy - 8z^2\,x + 3z^2\,x + 5z^2x$

$= (3 + 2 + 1)x^3 + (7 - 3 - 2)xy + (-8 + 3 + 5)\,z^2\,x$

$= 6x^3 + 2xy + 0z^2\,x$

$= 6x^3 + 2xy$

Column Method

$$\begin{array}{r} 3x^3 + 7xy - 8z^2x \\ 2x^3 - 3xy + 3z^2x \\ +\ x^3 - 2xy + 5z^2x \\ \hline 6x^3 + 2xy + 0z^2x \end{array}$$

Subtraction of like terms : Subtraction of like terms can be done in a manner exactly similar to that of integers. In other words change the sign of each term to be subtracted and then add.

Example-4 : Subtract (*a*) $3x^2$ from $7x^2$ (*b*) $-3xy^2$ from $2xy^2$

Sol. (*a*) $7x^2 - 3x^2 = (7 - 3)x^2 = 4x^2$

(*b*) $2xy^2 - (-3xy^2) = 2xy^2 + 3xy^2$

$= (2 + 3)xy^2 = 5xy^2.$

Subtraction of algebraic expression : Subtraction of algebraic expressions can be done by two methods.

(*i*) Horizontal method : Change the sign of each term of the expression to be subtracted and then add.

(*ii*) Column Method : Write both expressions one below the other such that the expression to be subtracted comes in the second row and the like terms come one below the other. Change the sign of every term of the expression in the second row and then add.

Example-5 : Subtract $15x^2 + 3xy$ from $20x^2 - 2xy$

Sol. Horizontal method

$20x^2 - 2xy - (15x^2 + 3xy)$

$= 20x^2 - 2xy - 15x^2 - 3xy$

$= 20x^2 - 15x^2 - 2xy - 3xy$

$= (20 - 15)x^2 + (-2 - 3)xy$

$= 5x^2 - xy$

Column Method

$$\begin{array}{r} 20x^2 - 2xy \\ 15x^2 + 3xy \\ - \quad - \\ \hline 5x^2 - 5xy \\ \hline \end{array}$$

Example-6 : Subtract $3a^2 - b^3 + 5c - 1$ from $2a^2 + 3b^3 - 7c + 2$

Sol. Horizontal method

$2a^2 + 3b^3 - 7c + 2 - (3a^2 - b^3 + 5c - 1)$

$= 2a^2 + 3b^3 - 7c + 2 - 3a^2 + b^3 - 5c + 1$

$= 2a^2 - 3a^2 + 3b^3 + b^3 - 7c - 5c + 2 + 1$

$= (2 - 3)a^2 + (3 + 1)b^3 + (-7 - 5)c + (2 + 1)$

$= -a^2 + 4b^3 - 12c + 3$

Column Method

$$\begin{array}{r} 2a^2 + 3b^3 - 7c + 2 \\ 3a^2 - b^3 + 5c - 1 \\ - \quad + \quad - \quad + \\ \hline -a^2 + 4b^3 - 12c + 3 \\ \hline \end{array}$$

Example-7 : From the sum of $2x^2 + 7x - 2$ and $3x^2 - 8x + 7$ Subtract $2x^2 + x - 1$

Sol. We first add $2x^2 + 7x - 2$ and $3x^2 - 8x + 7$

$$\begin{array}{r} 2x^2 + 7x - 2 \\ 3x^2 - 8x + 7 \\ \hline 5x^2 - x + 5 \qquad (1) \\ \hline \end{array}$$

Now we subtract $2x^2 + x - 1$ from sum (1)

$$\begin{array}{r} 5x^2 - x + 5 \\ 2x^2 + x - 1 \\ - \quad - \quad + \\ \hline 3x^2 - 2x + 6 \\ \hline \end{array}$$

EXERCISE - 12.2

1. Fill in the blanks :-

(*i*) $5y + 7y =$ ..... (*ii*) $3xy + 2xy =$ .....

(*iii*) $12a^2 - 7a^2 =$ ..... (*iv*) $8mn^2 - 3mn^2 =$ .....

2. Add the following algebraic expressions

(*a*) $3x^2y^2, 7xy^2$ (*b*) $7x, -3x, 2x$

(*c*) $12p^2q, 3p^2q, -5p^2q$ (*d*) $3x^2, -8x^2, -5x^2, 13x^2$

3. Add the following algebraic expressions.

(*a*) $x + y$ and $2x - 3y$ (*b*) $5a + 7b$ and $3a - 2b$

(*c*) $3m + 2n, 7m - 8n, 2m - n$ (*d*) $3x^2 + 2x - 7$ and $5x^2 - 7x + 8$

(*e*) $m^2 + 2n^2 - p^2, -3m^2 + n^2 + 2p^2$ and $4m^2 - 3n^2 + 5p^2$

(*f*) $3xy + 7x^2 - 2y^2, 2xy + y^2$ and $2x^2 + y^2$

4. Simplify the following algebraic expressions by combining like terms.

(*a*) $-5ax + 3xy + 2xy - 8ax$

(*b*) $3m - 2n + 5m - 3m + 8n$

(*c*) $3pq - 15r^2 - 3\,l^2m^2 + 2r^2 + 2l^2m^2 - 5pq.$

(*d*) $4x^3 + 7x^2 - 3x + 2 - 2x^3 - 2x^2 + 7x - 3$

5. Subtract the algebraic expressions.

(*a*) $-3x^2$ from $7x^2$ (*b*) $-3ab$ from $10ab$

(*c*) $a + b$ from $a - b$ (*d*) $15m + 10n$ from $2m - 16n$

(*e*) $2x + 8y - 3z$ from $-3x + 2y + z$

(*f*) $18m^2 + 3n^2 - 2mn - 7$ from $3m^2 - 2n^2 + 8mn - 8m + 4$

6. What should be subtracted from $l - 2m + 5n$ to get $2l - 3m + 4n$

7. What should be added to $3x^2 + 2xy - y^2$ to obtain $x^2 - 7xy + 3y^2$.

8. Subtract $3a^2 + 2b^2 - 8ab + 8$ from the sum of $a^2 - b^2 + 7ab + 3$ and $2a^2 + 4b^2 - 18ab + 7$

9. How much $x^2 + 3xy + y^2$ is less than $2x^2 + 5xy - y^2$.

10. Multiple Choice Questions :

(*i*) The algebraic expression for "Number 5 added to three times the product of numbers m and n" is.

(*a*) $5 + 3mn$ (*b*) $3 + 5mn$

(*c*) $(5 + 3)mn$

(*ii*) The sum of algebraic expressions $3x + 11$ and $2x - 7$ is

(*a*) $5x + 4$ (*b*) $x + 4$

(*c*) $5x - 18$

(*iii*) Subtraction of $a + b$ from $2a + 3b$

(*a*) $a + 2b$ (*b*) $-a - 2b$

(*c*) $3a + 4b$ (*d*) $a + b$

VALUE OF AN ALGEBRAIC EXPRESSION

The value of an Algebriac expression varies with the change in the value of variable forming the expression. There are number of situation in which we need to find the value of an algebraic expression such as when we want to check whether a particular value of variable satisfies a given equation or not. The process of finding the value of an algebraic expression by replacing the variable by their particular value is called substitution.

Example-1 : Find the value of the following expressions for $x = 1$

(a) $x + 5$ (b) $3x - 7$ (c) $7x^2 - 2x$

Sol. (a) $x + 5$

Putting $x = 1$ in $x + 5$ we get

$= 1 + 5$

$= 6$

(b) $3x - 7$

Putting $x = 1$ in $3x - 7$ we get

$= 3\,(1) - 7$

$= 3 - 7$

$= -4$

(c) $7x^2 - 2x$

Putting $x = 1$ in $7x^2 - 2x$ we get

$= 7\,(1)^2 - 2\,(1)$

$= 7(1) - 2 = 7 - 2$

$= 5$

Example-2 : Find the value of the following expressions when $p = -3$

(a) $p^2 - 7$ (b) $3p^2 + p - 2$ (c) $10p^3 - 100p^2$

Sol. (a) $p^2 - 7$

Putting $p = -3$ in $p^2 - 7$ we get

$= (-3)^2 - 7 = 9 - 7 = 2$

(b) $3p^2 + p - 2$

Putting $p = -3$ in $3p^2 + p - 2$ we get

$= 3(-3)^2 + (-3) - 2 = 3(9) - 3 - 2$

$= 27 - 3 - 2$

$= 27 - 5 = 22$

(c) $10p^3 - 100p^2$

Putting $p = -3$ in $10p^3 - 100p^2$ we get

$= 10(-3)^3 - (100)\,(-3)^2$

$= 100\,(-27) - 100\,(9)$

$= -2700 - 900$

$= -3600$

Example-3 : Find the value of the following expressions when $a = -2$, $b = 3$

(i) $a + b$ (ii) $a^2 + b^2$ (iii) $10a - 8b$ (iv) $a^2 + 2ab + b^2$

Sol. (i) $a + b$

Putting $a = -2$, $b = 3$ in $a + b$ we get

$= (-2) + 3 = +1$

(ii) $a^2 + b^2$

Putting $a = -2$, $b = 3$ in $a^2 + b^2$ we get

$= (-2)^2 + (3)^2 = 4 + 9 = 13$

(*iii*) $10a - 8b$

Putting $a = -2$, $b = 3$ in $10a - 8b$ we get

$= 10(-2) - 8(3) = -20 - 24 = -44$

(*iv*) $a^2 + 2ab + b^2$

Putting $a = -2$, $b = 3$ in $a^2 + 2ab + b^2$ we get

$= (-2)^2 + 2(-2)(3) + (3)^2$

$= 4 - 12 + 9$

$= 13 - 12$

$= 1$

USING ALGEBRAIC EXPRESSIONS – FORMULA AND RULE

We have already learnt some rules and formulas in mathematics. Here we shall see that these rules and formula can be written in a concise and general form using algebraic expressions.

Perimeter formula :

(*i*) To find the perimeter of an equilateral triangle if we denote the length of the side of the equilateral triangle by l. Then the perimeter of the equilateral triangle = $3l$.

(*ii*) To find the perimeter of rectangle we use the algebraic expression $2(l + b)$ where l and b are length and breadth of a rectangle.

(*iii*) To find the perimeter of a square we use the algebraic expression 4s. Where 's' is the side of square.

Area Formula :

(*i*) If length of side of a square is 's'. Then area of square = s^2.

(*ii*) If l is the length and b is the breadth of a rectangle. Then area of rectangle = $l \times b$

Once a formula i.e the algebraic expression for a given quantity is known. Then we can compute the value of the quantity according to requirement.

For example : If the length of a rectangle is 4 unit and breadth is 6 unit. Then perimeter of rectangle is.

$= 2(4 + 6) = 2(10) = 20$ unit

Area of rectangle $= 4 \times 6 = 24$ sq. unit

Rules for numbers :

1. If n is any natural number then its successor is $n + 1$. We can check this for any natural number. For example. If $n = 15$, Then $n + 1 = 15 + 1 = 16$ which is succesor of 15.
2. If n is any natural number, then $2n$ is always an even number and $2n-1$ is an odd number for example if $n = 3$.

$2n = 2(3) = 6$ is an even number

$2n-1 = 2(3) - 1 = 6 - 1 = 5$ is an odd number

3. If n is an odd number n^3 is also an odd number and if n is an even number n^3 is also an even number for example.

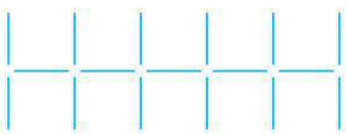
If $n = 5$

$n^3 = 5^3 = 5 \times 5 \times 5 = 125$ is an odd number

If $n = 4$

$n^3 = 4^3 = 4 \times 4 \times 4 = 64$ is an even number

Take some matchsticks, Tooth picks or pieces of straw cut into smaller pieces of equal length. Join them in patterns as show in given figure.



It consist of the repetition of the letter H made from 5 line segments. As we note that the number of letters (H) formed and the number of the line segment required to form these letters. We get the following table.

Number of letters formed	Number of line segments required
1	5
2	8
3	11
4	14
....	....

i.e. If n is the number of letter, then the algebraic expression to represent the number of required line segments is $3n + 2$.

You may verify this by taking different values for n. such that $n = 1, 2, 3, ...$ so on.

For example : If the number of letter formed is 5. then the number of line segment required is $3n + 2 = 3(5) + 2 = 15 + 2 = 17$ as seen from figure.

Pattern of shapes of Rectangles :

In consist of the repetition of the shapes  made from 6 line segment. We note that number of shapes of rectangle formed and number of line segment required to form the shapes we get the following table.

Number of shapes formed	Number of line segments required
1	6
2	11
3	16
4	21
...	...

i.e. If n is the number of shapes, then the algebraic expression to represent the number of required line segment $5n + 1$.

For example, if the number of shapes formed is 3 than the number of line segment required is $5(3) + 1 = 15 + 1 = 16$ as seen from figure

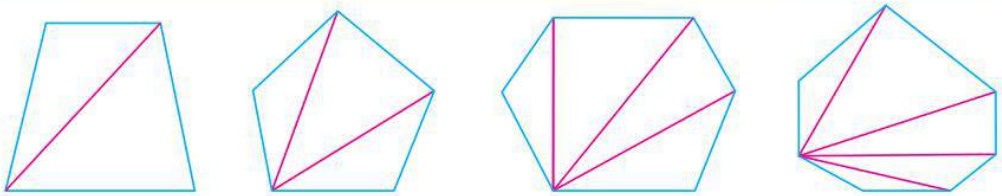
Pattern in Geometry : Here we are going to check the number of diagonals from one vertex in a simple closed polygon. Take four polygons, a quadrilateral, a pentagen, a hexagon and a heptagon

Number of diagonals from one vertex of a quadrelateral = 1

Number of diagonals from one vertex of a pentagon = 2

Number of diagonals from one vertex of a hexagon = 3

Number of diagonals from one vertex of a heptagon = 4



We observe that number of diagonals, we can draw from one vertex of a polygon of 'n' sides is $(n-3)$ check it for octagon. By drawing figure check what is the number of diagonals for a triangle.

For example, If a polygon has 12 sides i.e $n = 12$ then the number of diagonals, we can draw from one vertex $= n - 3 = 12 - 3 = 9$

EXERCISE - 12.3

1. Fill in the Table by substituting the values in the given expressions.

Expression	Value of the expression for			
	$x = 1$	$x = -2$	$x = 3$	$x = 10$
(*i*) $3x + 7$				
(*ii*) $x^2 - 2x + 3$				
(*iii*) $8x^3 - 3x^2$				
(*iv*) $-10x^2 + 20x$				

2. If a = 1, $b = -2$ find the value of given expressions

(*i*) $a^2 - b^2$

(*ii*) $a + 2\,ab - b^2$

(*iii*) $a^2b + 2ab^2 + 5$

3. Simplify the following expressions and find their values for m = 1, $n = 2$, $p = -1$.

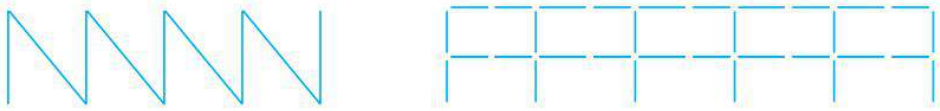
(*i*) $2m + 3n - p + 7m - 2n$ (*ii*) $3p + n - m + 2n$

(*iii*) $m + p - 2p + 3m$ (*iv*) $3n + 2m - 5p - 3m - 2n + p$

4. What should be the value of a if the value of $2a^2 + b^2 = 10$ when $b = 2$?

5. Find the vlaue of x if $-3x + 7y^2 = 1$ when $y = 1$?

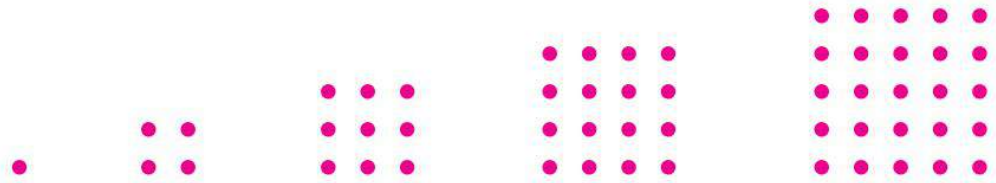
6. Observe the pattern of shapes of letters formed from line segment of equal lengths.



(*i*) (*iii*)

If n shapes of letters are formed, then write the algebraic expression for the number of line segment requered for making these n shapes in each case.

7. Observe the following pattern of squares made using dots.

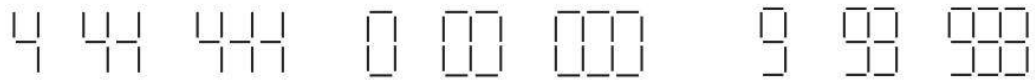


If n is taken as the number of dots in each row then find the algebraic expression for number of dots in nth figure. Also find number of dots if.

(*i*) $n = 3$ (*ii*) $n = 7$

(*iii*) $n = 10$

8. Observe the pattern of shapes of digits formed from line segment of equal lengths.



If n shapes of digits are formed then write the algebraic expression for the numbers of line segment required to make n shapes.

9. Multiple Choice Questions :

(*i*) If l is the length of the side of the regular pentagon, perimeter of a regular Pentagon is.

(*a*) $3\,l$ (*b*) $4\,l$ (*c*) $5\,l$ (*d*) $8\,l$

(*ii*) The value of the expression $5n–2$ when $n = 2$ is.

(*a*) 12 (*b*) –12 (*c*) 8 (*d*) 3

(*iii*) The value of $3x^2 – 5x + 6$ when $x = 1$

(*a*) 3 (*b*) 4 (*c*) –8 (*d*) 14

WHAT HAVE WE DISCUSSED ?

1. A symbol having a fixed numerical value is called a constant.

2. A symbol which takes on various numerical value is called a variable.

3. An algebraic expression is formed by using mathematical operations (addition, subtraction, multiplication and division) on variables and constant.

4. Expressions are formed by addition of terms. Terms can be negative as well as positive.

5. The numerical factor of the term is called the numerical coefficient of the term.

6. The terms having same algebraic factors are called like terms and the terms having different algebraic factors are called unlike terms.

7. Monomial, Binomial and trinomials have one, two and three (unlike) terms respectively.

8. Two or more algebraic expressions can be added by arranging their terms and combining the like terms.

9. To find the value of an algebraic expression, we replace the variable of the expression with their respective values.

10. Algebraic expressions are used in finding perimeters and areas of various geometrical shapes and also in forming patterns etc.

LEARNING OUTCOMES

After completions of the chapter the students are now able to :

1. Differentiate between constants and variables.
2. Generate algebraic expressions.
3. Add and subtract algebraic expressions.
4. Find the value of an expression for a particular value of a variable.
5. Find an algebraic expression for a given pattern.
6. Apply their knowledge about algebraic expressions in their daily life.

EXERCISE 12.1

1. (*i*) $a + b$ (*ii*) z^2 (*iii*) $xy + mn$

(*iv*) $\frac{p}{5}q$ (*v*) $2t + \frac{z}{2}$ (*vi*) $x^2 + z^2$

(*vii*) $xy - (x + y)$

2. Constant Terms 7, $\frac{72}{3}$

Variable Terms xy, $\frac{3x^2}{2}$, $\frac{72}{3}z$, $\frac{-8z}{3x^2}$

3.

	Expreission	Terms	Factors
(*a*)	$2x^2 + 3xy$	$2x^2$	2, *x*, *x*
		$3xy$	3, *x*, *y*
(*b*)	$15x^2y + 3xy^2$	$15x^2y$	15, *x*, *x y*
		$3xy^2$	3, *x*, *y*, *y*
(*c*)	$-7x\,y\,z^2$	$-7xy\,z^2$	–7, *x*, *y*, *z*, *z*
(*d*)	$100pq + 10p^2q^2$	$100pq$	100, *p*, *q*
		$10p^2q^2$	10, *p*, *p*, *q*, *q*
(*e*)	$xy + 3x^2y^2$	xy	*x*,*y*
		$3x^2y^2$	3, *x*, *x*, *y*, *y*
(*f*)	$-7x^2yz + 3xy^2z$	$-7x^2\,yz$	–7, *x*, *x*, *y*, *z*
	$+2xy\,z^2$	$3xy^2z$	3, *x*, *y*, *y*, *z*
		$2xyz^2$	2, *x*, *y*, *z*, *z*

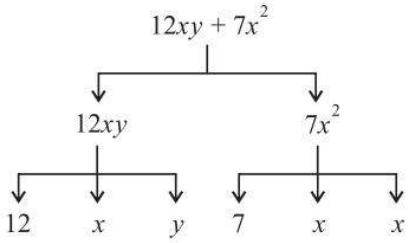
4. (*a*) Binomial (*b*) Binomial (*c*) Trinomial
(*d*) Monomial (*e*) Binomial (*f*) Monomial
(*g*) Binomial

5. (*a*) 2 (*b*) $\frac{-3}{2}$ (*c*) $\frac{7}{2}$
(*d*) -1 (*e*) -5

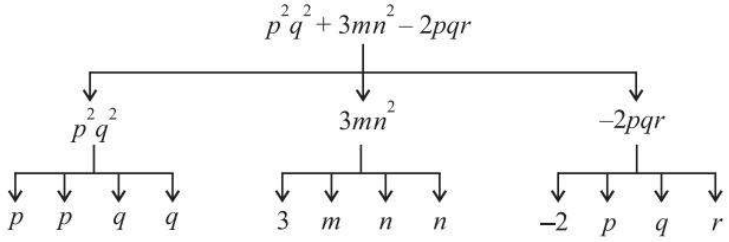
6. (a) Like (*b*) Unlike (*c*) unlike
(*d*) unlike (*e*) unlike (*f*) like

7. (*a*) xy (*b*) $15x$ (*c*) pr^2
(*d*) 1 (*e*) xy

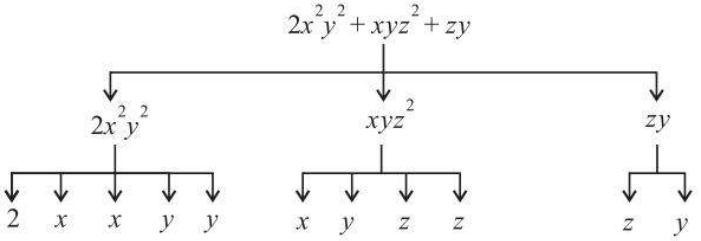
8. (*a*)



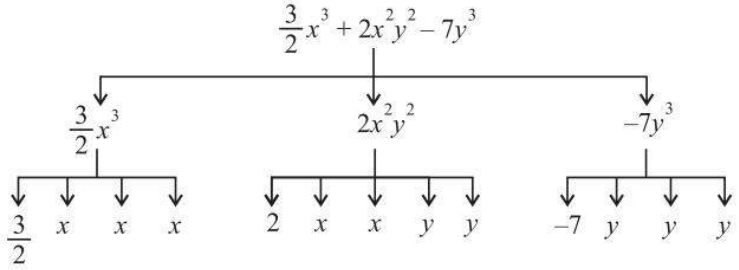
(*b*)



(*c*)



(*d*)



9. (*i*) *a* (*ii*) *a*
(*iii*) *b* (*iv*) *a*

EXERCISE 12.2

1. (*i*) 12y (*ii*) $5xy$
(*iii*) $5a^2$ (*iv*) $5mn^2$

2. (*a*) $10xy^2$ (*b*) $6x$
(*c*) $10p^2q$ (*d*) $3x^2$

3. (*a*) $3x - 2y$ (*b*) $8a + 5b$
(*c*) $12m - 7n$ (*d*) $8x^2 - 5x + 1$
(*e*) $2m^2 + 0n^2 + 6p^2$ (*f*) $5xy + 9x^2$

4. (*a*) $-13ax + 5xy$ (*b*) $5m + 6n$
(*c*) $-2pq - 13r^2 - 2\,l^2m^2$ (*d*) $2x^3 + 5x^2 + 4x - 1$

5. (*a*) $4x^2$ (*b*) $13ab$
(*c*) $-2b$ (*d*) $-13m - 26n$
(*e*) $-5x - 6y + 4z$ (*f*) $-15m^2 - 5n^2 + 10m\,n - 8m + 11$

6. $-l + m + n$ **7.** $-2x^2 - 9xy + 4y^2$

8. $2b^2 - 3ab + 2$ **9.** $x^2 + 2xy - 2y^2$

10. (*i*) *a* (*ii*) *a*
(*iii*) *a*

EXERCISE 12.3

1. (*i*) 10, 1, 16, 37 (*ii*) 2, 11, 6, 83
(*iii*) 5, –76, 189, 7700 (*iv*) 10, –80, –30, –800

2. (*i*) –3 (*ii*) –7
(*iii*) 9

3. (*i*) 12 (*ii*) 2
(*iii*) 5 (*iv*) 5

4. $a = 3$ **5.** $x = 2$

6. (*i*) $2n + 1$ (*ii*) $4n + 2$

7. (*i*) n^2 (*ii*) 9
(*iii*) 49 (*iv*) 100

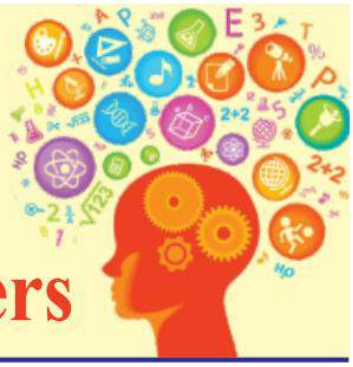
8. (*i*) $3n + 1$ (*ii*) $4n + 2$
(*iii*) $5n + 1$

9. (*i*) *c* (*ii*) *c*
(*iii*) *b*

CHAPTER

13

T3Z6U9



Exponents And Powers

Learning Objectives :-

In this chapter, you will learn :-

1. To identify the base and exponent.
2. About the exponential notation.
3. To write the number in exponential form.
4. To apply the laws of exponents.
5. About the standard form of numbers.

OUR NATION'S PRIDE

Apastamba has been known as India's most complicated Mathematician, who is known to have lived around 600 BC. According to the Hindu tradition, he was the student of Baudhayana. Sulabha Sutras given by Apastamba, are known to be one of the oldest known Mathematics text in existence. His major contribution in the field of mathematics include the numeric solution of Pythagoras Theorem. Apastamba's rules for altar construction led to the discovery of irrational numbers though he has never been given the due credit for the same.

INTRODUCTION

In many situations, we come across numbers that are very large or very small. For example, the age of the universe in years, the mass of the earth in tons, the distance of the sun from the earth (in km), the size of bacteria etc. are numbers that are either very large or very small. Such numbers are generally approximate (due to complexity of their measurement). Therefore these are represented by some numbers followed by certain number of zeros. For example, the age of the universe is approximated to the 8,000,000,000 years and the mass of the Earth is approximately 5980000000000000000 metric tons. Such numbers are usually written by using exponents. For example, the number 8000000000 can be written as 8×10^9 or 80×10^8 or 800×10^7. Similarly, $598000000000000000000 = 598\times10^{19}$. This is known as exponential notation. The exponential notation helps us to write very large and very small numbers easily.

EXPONENTIAL FORM

Consider the number 125 we write $125 = 5 \times 5 \times 5$. Therefore $125 = 5^3$, 5^3 is the exponential form of 125. Here '5' is base and '3' is exponent. The number 5^3 is read as 5 raised to power of 3 or simply 5 cubed.

Consider another number $\frac{16}{81}$.

$$\frac{16}{81} = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \left(\frac{2}{3}\right)^4$$

Therefore, the exponential form of $\frac{16}{81}$ is $\left(\frac{2}{3}\right)^4$.

Here $\frac{2}{3}$ is base and 4 is exponent.

This leads to, if a is any rational number and n is a natural number then $a^n = a \times a \times a$ .... multiplied n times, where a is called the base and n is called the exponent or index and a^n is the exponential form, a^n is read as a raised to the power n. In particular, $a^1 = a$.

For example : $10^4 = 10 \times 10 \times 10 \times 10$ *i.e.* $10^4 = 10000$ here base = 10, exponent (or index) = 4 and 10^4 is the exponential form of the number 10000.

Example-1 : Expand the following :

(*i*) $(-3)^4$ (*ii*) 2^6 (*iii*) $(-1)^5$ (*iv*) $\left(-\frac{1}{2}\right)^2$

Sol. (*i*) $(-3)^4$ means that -3 is multiplied to itself 4 times.

Therefore, $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$

$= (+9) \times (+9) = 81$

(*ii*) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(*iii*) $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

(*iv*) $\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = +\frac{1}{4}$

Example-2 : Express the following in the exponential form :

(*i*) 343 (*ii*) 3125

Sol. (*i*) 343

$343 = 7 \times 7 \times 7 = 7^3$

7	343
7	49
7	7
	1

(*ii*) 3125

$3125 = 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$

5	3125
5	625
5	125
5	25
5	5
	1

Example-3 : Which is greater 5^3 or 3^5

Sol.
$$5^3 = 5 \times 5 \times 5 = 125$$
$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$
$$243 > 125$$
$$\therefore \quad 3^5 > 5^3$$

Example-4 : Express 540 as product of powers of their prime factors.

Sol. 540
$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$
$$= 2^2 \times 3^3 \times 5$$

2	540
2	270
3	135
3	45
3	15
5	5
	1

Example-5 : Simplify (*i*) $5^2 \times 3^3$ (*ii*) 0×10^2

Sol. (*i*) $5^2 \times 3^3 = 5 \times 5 \times 3 \times 3 \times 3$
$$= 25 \times 27 = 675$$
(*ii*) $0 \times 10^2 = 0 \times 10 \times 10$
$$= 0 \times 100 = 0$$

Example-6 : Find the value of x if $3^x = 729$

Sol. $3^x = 729$
$3^x = 3^6$
$\therefore \quad x = 6$

3	729
3	243
3	81
3	27
3	9
3	3
	1

Example-7 : Check : $(1)^5$, $(-1)^3$, $(-1)^4$, $(-10)^3$, $(-5)^4$

Sol. (*i*) We have $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1$

In fact, you will realise that 1 raised to any power is 1

(*ii*) $(-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$ $\quad [(-1)^{\text{odd number}} = -1]$

(*iii*) $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 = 1$ $[(-1)^{\text{even number}} = +1]$

You may check that (– 1) raised to any odd power is (– 1) and (– 1) raised to any even power is (+ 1).

(*iv*) $(-10)^3 = (-10) \times (-10) \times (-10) = 100 \times (-10) = -1000$

(*v*) $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 25 \times 25 = 625$

$$(-1)^{\text{odd number}} = -1$$
$$(-1)^{\text{even number}} = 1$$

EXERCISE - 13.1

1. Fill in the blanks :

(*i*) In the expression 3^7, base = _____ and exponent = _____.

(*ii*) In the expression $\left(\frac{2}{5}\right)^{11}$, base = _____ and exponent = _____.

2. Find the value of the following :

(*i*) 2^6 (*ii*) 9^3 (*iii*) 5^5

(*iv*) $(-6)^4$ (*v*) $\left(-\frac{2}{3}\right)^5$

3. Express the following in the exponential form :

(*i*) $6 \times 6 \times 6 \times 6$ (*ii*) $b \times b \times b \times b$ (*iii*) $5 \times 5 \times 7 \times 7 \times 7$

4. Simplify the following :

(*i*) 2×10^3 (*ii*) $5^2 \times 3^2$ (*iii*) $3^2 \times 10^4$

5. Simplify :

(*i*) $(-3) \times (-2)^3$ (*ii*) $(-4)^3 \times 5^2$ (*iii*) $(-1)^{99}$

(*iv*) $(-3)^2 \times (-5)^2$ (*v*) $(-1)^{132}$

6. Identify the greater number in each of the following :

(*i*) 4^3 or 3^4 (*ii*) 5^3 or 3^2 (*iii*) 2^3 or 8^2

(*iv*) 4^5 or 5^4 (*v*) 2^{10} or 10^2

7. Write the following numbers as power of 2 :

(*i*) 8 (*ii*) 128 (*iii*) 1024

8. Write the following numbers as power of 3 :

(*i*) 27 (*ii*) 2187

9. Find the value of x in each of the following :

(*i*) $7^x = 343$ (*ii*) $9^x = 729$ (*iii*) $(-8)^x = -512$

10. To what power (– 2) should be raised to get 16 ?

11. Write the prime factorization of the following numbers in the exponential form :

(*i*) 72 (*ii*) 360 (*iii*) 405

(*iv*) 648 (*v*) 3600

LAWS OF EXPONENTS

We can multiply and divide **rational numbers** expressed in exponential form.

Multiplication of Identical bases with different powers

Let us find $2^4 \times 2^3$

$= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

Let us find $(-3)^2 \times (-3)^3$

$[(-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$

$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) = (-3)^5$

Note that $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$

In fact, this is true in general we have :

Law 1 : If a is any **rational number** and m, n are integers, then $a^m \times a^n = a^{m+n}$

Division of Identical bases with different Powers

$$\text{Let us find } 5^7 \div 5^4 = \frac{5^7}{5^4} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$$

$$= 5 \times 5 \times 5 = 5^3$$

$$\text{Note that } 5^7 \div 5^4 = \frac{5^7}{5^4} = 5^{7-4} = 5^3$$

In fact, this is true in general we have :

Law 2 : If a is any (non zero) **rational number** and m, n are integers such that $\boldsymbol{m > n}$, then

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}.$$

Zero Exponent

$$\text{Let us find } \frac{3^3}{3^3} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3} = \frac{27}{27} = 1$$

$$\text{Also } \frac{3^3}{3^3} = 3^{3-3} = 3^0$$

Though we have calculated $\frac{3^3}{3^3}$ in two different ways, the answer remains the same. It follows, that $3^0 = 1$.

In fact, this is true in general. So, we have :

Law 3 : If a is any (non-zero) rational number, then $a^0 = 1$

Taking power of a power

$$(2^3)^2 = 2^3 \times 2^3 = 2^{3+3}$$

$$= 2^6 = 2^{3 \times 2}$$

Thus $(2^3)^2 = 2^{3 \times 2}$

In fact, this is true in general we have.

Law 4 : If a is any **rational number** and m, n are integers then $(a^m)^n = a^{m\times n}$

Multiply different bases with same exponent

Let us find $3^4 \times 5^4$

$$= (3\times3\times3\times3)\times(5\times5\times5\times5)$$
$$= (3\times5)\times(3\times5)\times(3\times5)\times(3\times5)$$
$$= (3\times5)^4$$

In fact, this is true in general. We have :

Law 5 : If a, b are any rational numbers and n is an integer then $a^n \times b^n = (ab)^n$

Division of different bases with same exponent

$$\text{Let us find } \frac{2^4}{7^4} = \frac{2\times2\times2\times2}{7\times7\times7\times7} = \left(\frac{2}{7}\right)^4$$

In fact, this is true in general. We have :

Law 6 : If a, $(b \neq 0)$ are any numbers and n is an integer, then $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ or

$$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

NEGATIVE EXPONENT

For any (non – zero) **rational number** a and natural number n, we have

$$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}, \text{ thus } a^{-n} = \frac{1}{a^n}$$

Law 7 : If a is any (non – zero) **rational number** and n is an integer, then $a^{-n} = \frac{1}{a^n}$ In partcular, $a^{-1} = \frac{1}{a}$.

Example-1 : Simplify and write in exponential form :-

(a) $2^3 \times 2^2$ **(b)** $4^2 \times 4^3$ **(c)** $3^2 \times 3^3 \times 3^4$ **(d)** $(-4)^3 \times (-4)^2$

Sol. (a) $2^3 \times 2^2 = 2^{3+2} = 2^5$

(b) $4^2 \times 4^3 = 4^{2+3} = 4^5$

(c) $3^2 \times 3^3 \times 3^4 = 3^{2+3+4} = 3^9$

(d) $(-4)^3 \times (-4)^2 = (-4)^{3+2} = (-4)^5$

Example-2 : Simplify and write in exponential form

(a) $13^6 \div 13^4$ **(b)** $10^4 \div 10$ **(c)** $18^{16} \div 18^{10}$ **(d)** $(-5)^6 \div (-5)^2$

Sol. (a) $13^6 \div 13^4 = 13^{6-4} = 13^2$

(b) $10^4 \div 10 = 10^{4-1} = 10^3$

(c) $18^{16} \div 18^{10} = 18^{16-10} = 18^6$

(d) $(-5)^6 \div (-5)^2 = (-5)^{6-2} = (-5)^4$

Example-3 : Simplify and express in exponential form

(a) $(3^2)^3$ **(b)** $(4^3)^2$ **(c)** $[(10)^2]^3$ **(d)** $(2^{100})^2$

Sol. (a) $(3^2)^3 = 3^{2\times3} = 3^6$

(b) $(4^3)^2 = 4^{3\times2} = 4^6$

(c) $[(10)^2]^3 = (10)^{2\times3} = (10)^6$

(d) $(2^{100})^2 = 2^{100\times2} = 2^{200}$

Example-4 : Simplify (a) $\left(\frac{2}{5}\right)^4$ **(b)** $\left(\frac{-1}{3}\right)^3$ **(c)** $\left(\frac{-6}{7}\right)^2$

Sol. (a) $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{2\times2\times2\times2}{5\times5\times5\times5} = \frac{16}{625}$

(b) $\left(\frac{-1}{3}\right)^3 = \frac{(-1)^3}{3^3} = \frac{-1\times-1\times-1}{3\times3\times3} = -\frac{1}{27}$

(c) $\left(\frac{-6}{7}\right)^2 = \frac{(-6)^2}{7^2} = \frac{-6\times-6}{7\times7} = \frac{36}{49}$

Example-5 : Simplify and express each of the following in the exponential form.

(a) $[(5^2)^3 \times 5^4] \div 5^7$ **(b)** $125^4 \div 5^3$ **(c)** $[(2^2)^3 \times 3^6] \times 5^6$

Sol. (a) $[(5^2)^3 \times 5^4] \div 5^7$

$= (5^{2\times3} \times 5^4) \div 5^7$

$= (5^6 \times 5^4) \div 5^7$

$= 5^{6+4} \div 5^7$

$= 5^{10} \div 5^7$

$= 5^{10-7} = 5^3$

(b) $125^4 \div 5^3$

$125^4 = (5 \times 5 \times 5)^4 = (5^3)^4 = 5^{12}$

$125^4 \div 5^3 = 5^{12} \div 5^3 = 5^{12-3} = 5^9$

(c) $[(2^2)^3 \times 3^6] \times 5^6 = (2^{2\times3} \times 3^6) \times 5^6$

$= (2 \times 3)^6 \times 5^6$

$= 6^6 \times 5^6$

$= (6 \times 5)^6$

$= 30^6$

Example-6 : Simplify and express each of the following in exponential form :

(i) $\frac{2^3 \times 3^4 \times 4}{3 \times 32}$ **(ii)** $(3^0 + 2^0) \times 5^0$ **(iii)** $\frac{25 \times 5^2 \times a^8}{10^3 \times a^4}$

Sol. (i) $4 = 2 \times 2 = 2^2$ and $32 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore \quad \frac{2^3 \times 3^4 \times 4}{3 \times 32} = \frac{2^3 \times 3^4 \times 2^2}{3 \times 2^5} = \frac{2^5 \times 3^4}{3^1 \times 2^5}$

$= 2^{5-5} \times 3^{4-1}$

$= 2^0 \times 3^3 = 1 \times 3^3 = 3^3$

(*ii*) $(3^0 + 2^0) \times 5^0 = (1 + 1) \times 1 = 2 \times 1 = 2 = 2^1$

(*iii*) $$\frac{25 \times 5^2 \times a^8}{10^3 \times a^4} = \frac{5^2 \times 5^2 \times a^8}{(2 \times 5)^3 \times a^4} = \frac{5^2 \times 5^2 \times a^8}{2^3 \times 5^3 \times a^4}$$

$$= \frac{5^{2+2-3} \times a^{8-4}}{2^3} = \frac{5a^4}{2^3}$$

Example-7 : Express each of the following rational numbers in exponential form :-

(*i*) $\frac{64}{343}$ (*ii*) $\frac{-27}{125}$ (*iii*) $\frac{-1}{243}$

Sol. (*i*) $$\frac{64}{343} = \frac{4^3}{7^3} = \left(\frac{4}{7}\right)^3$$

(*ii*) $$\frac{-27}{125} = \frac{(-3)^3}{5^3} = \left(\frac{-3}{5}\right)^3$$

(*iii*) $$\frac{-1}{243} = \frac{-1}{243} = \frac{(-1)^5}{3^5} = \left(\frac{-1}{3}\right)^5$$

Example-8: Simplify $(3^0 + 2^0 - 6^0) \div (100)^0$ and write the answer as a power of 5.

Sol. $(3^0 + 2^0 - 6^0) \div (100)^0$

$= (1 + 1 - 1) \div 1$ $(\because x^0 = 1)$

$= 1 \div 1 = 1 = 5^0$ $(\because 5^0 = 1)$

EXERCISE - 13.2

1. Using laws of exponents, simplify and write the following in the exponential form.

(*i*) $2^7 \times 2^4$ (*ii*) $p^5 \times p^3$

(*iii*) $(-7)^5 \times (-7)^{11}$ (*iv*) $20^{15} \div 20^{13}$

(*v*) $(-6)^7 \div (-6)^3$ (*v*) $7^x \times 7^3$

2. Simplify and write the following in exponential form.

(*i*) $5^3 \times 5^7 \times 5^{12}$ (*ii*) $a^5 \times a^3 \times a^7$

3. Simplify and write the following in the exponential form :-

(*i*) $(2^2)^{100}$ (ii) $(5^3)^7$

4. Simplify and write in the exponential form :

(*i*) $(2^3)^4 \div 2^5$ (*ii*) $2^3 \times 2^2 \times 5^5$

(*iii*) $[(2^2)^3 \times 3^6] \times 5^6$

5. Simplify and write in the exponential form :

(*i*) $5^4 \times 8^4$ (*ii*) $(-3)^6 \times (-5)^6$

6. Simplify and express each of the following in the exponential form :

(*i*) $\dfrac{\left(3^2\right)^3 \times (-2)^5}{(-2)^3}$ (*ii*) $\dfrac{3^7}{3^4 \times 3^3}$

(*iii*) $\dfrac{2^8 \times a^5}{4^3 \times a^3}$ (*iv*) $3^0 \times 4^0 \times 5^0$

7. Express each of the following rational number in the exponontial form :-

(*i*) $\dfrac{25}{64}$ (*ii*) $\dfrac{-64}{125}$

(*iii*) $\dfrac{-125}{216}$ (*iv*) $\dfrac{-343}{729}$

8. Simplify :-

(*i*) $\dfrac{(2^5)^2 \times 7^3}{8^3 \times 7}$ (*ii*) $\dfrac{2 \times 3^4 \times 2^5}{9 \times 4^2}$

9. Express each of the following as a product of prime factors in the exponential form

(*i*) 384×147 (*ii*) 729×64

(*iii*) 108×92

10. Simplify and write the following in the exponential form :

(*i*) $3^3 \times 2^2 + 2^2 \times 5^0$ (*ii*) $\left(\dfrac{3^7}{3^2}\right) \times 3^5$

(*iii*) $8^2 \div 2^3$

Multiple Choice Questions :-

11. $\left(\dfrac{-5}{8}\right)^0$ is equal to

(*i*) 0 (*ii*) 1

(*iii*) $\dfrac{-5}{8}$ (*iv*) $\dfrac{-8}{5}$

12. $(5^2)^3$ is equal to

(*i*) 5^6 (*ii*) 5^5

(*iii*) 5^9 (*iv*) 10^3

13. $a \times a \times a \times b \times b \times b$ is equal to

(*i*) $a^3 b^2$ (*ii*) $a^2 b^3$

(*iii*) $(ab)^3$ (*iv*) $a^6 b^6$

14. $(-5)^2 \times (-1)^1$ is equal to

(*i*) 25 (*ii*) –25

(*iii*) 10 (*iv*) –10

DECIMAL NUMBER SYSTEM

Consider the expansion of the number 753015, we know that

$753015 = 7 \times 100000 + 5 \times 10000 + 3 \times 1000 + 0 \times 100 + 1 \times 10 + 5 \times 1$

Using power of 10 in the exponent, we can write it as

$753015 = 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 5 \times 10^0$

$= 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 1 \times 10^1 + 5 \times 10^0$

In fact, the expansion of every number can be written using power of 10 in the exponent.

STANDARD FORM OF NUMBERS

The standard form of a number in of the form $k \times 10^n$ where k is a number between 1 and 10 and n is an integer.

Look at the following

$76 = 7.6 \times 10 = 7.6 \times 10^1$

$763 = 7.63 \times 100 = 7.63 \times 10^2$

$7630 = 7.63 \times 1000 = 7.63 \times 10^3$

$76300 = 7.63 \times 10000 = 7.63 \times 10^4$ and so on.

SCIENTIFIC NOTATION

Scientific notation is a way of writing numbers that accommodates value too large to be conveniently written in decimal notation.

In scientific notation all numbers are written as $k \times 10^n$ where k is decimal number such that $1 < k < 10$ and n is a whole number. The decimal k is called significand. Scientific notation is also known as standard form.

Ordinary decimal notation	Scientific notation
500	5×10^2
47,000	4.7×10^4
9,830,000,000	9.83×10^9

Example-1 : Write the following numbers in the standard form.

(*i*) 763.4 (*ii*) 83,500 (*iii*) 573,000

Sol. (*i*) $763.4 = 7.634 \times 10^2$

(*ii*) $83, 500 = 8.3500 \times 10^4 = 8.35 \times 10^4$

(*iii*) $573, 000 = 5.73000 \times 10^5 = 5.73 \times 10^5$

Example-2 : Write the following as usual decimal notation.

(*i*) 5.37×10^4 (*ii*) 7.501×10^7 (*iii*) 2.3049×10^{11}

Sol. (*i*) $5.37 \times 10^4 = 53700$

(*ii*) $7.501 \times 10^7 = 75010000$

(*iii*) $2.3049 \times 10^{11} = 230490000000.$

Example-3 : Express the numbers appearing in the following statement in scientific notation (or standard form)

(*i*) The radius of the earth is 6366000 metres

(*ii*) The distance between the sun and the earth is 149, 600, 000, 000 *m*

(*iii*) The speed of light in vacum = 299, 800, 000 *m*/sec

(*iv*) The mass of the earth is 5, 976, 000, 000, 000, 000, 000, 000, 000 kg

Sol. (*i*) The radius of the earth is = 6366000 = 6.366×10^6 m

(*ii*) The distance between the sun and the earth = 149, 600, 000, 000 *m* = 1.496×10^{11} m

(*iii*) The speed of light in vacuum = 299, 800, 000 *m*/sec

$= 2.998 \times 10^8$ *m*/sec

(*iv*) The mass of earth = 5, 976, 000,000,000,000,000,000, 000 kg

$= 5.976 \times 10^{24}$ *kg*

Example-4 : Compare the following numbers

(*i*) 2.7×10^{12} ; 1.5×10^8 **(*ii*) 3.547×10^9; 6.02×10^9**

Sol. (*i*) The given numbers are 2.7×10^{12} and 1.5×10^8,

Note that both the numbers are in standard form. Since the power of 10 in 2.7×10^{12} is greater than the power of 10 in 1.5×10^8,

$\therefore$ $2.7 \times 10^{12} > 1.5 \times 10^8$

(*ii*) The given numbers are 3.54×10^9 and 6.02×10^9 and 6.02×10^9. Note that both the numbers are in standard form. Also we note that both the numbers have equal power of 10. There fore, we compare their significands.

The significand in 3.547×10^9 is 3.547 and the significand in 6.02×10^9 is 6.02

As $6.02 > 3.547$, so $6.02 \times 10^9 > 3.547 \times 10^9$

EXERCISE - 13.3

1. Write the following numbers in the expanded exponential form :

(*i*) 104278 (*ii*) 20068

(*iii*) 120719 (*iv*) 3006194

(*v*) 28061906

2. Find the number from each of the following expanded form :

(*i*) $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(*ii*) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(*iii*) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(*iv*) $8 \times 10^7 + 3 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 8 \times 10^1$

3. Express the following numbers in standard form :

(*i*) 3, 43,000 (*ii*) 70,00,000

(*iii*) 3, 18,65,00,000 (*iv*) 530.7

(*vi*) 5985.3 (*v*) 3908.78

4. Express the number appearing in the following statements in standard form :

(*i*) The distance between the earth and the moon is 384, 000,000*m*

(*ii*) The diameter of the earth is 1,27, 56,000*m*.

(*iii*) The diameter of the sun is 1,400,000,000*m*.

(*vi*) The universe is estimated to be about 12,000,000,000 years old.

(*v*) Mass of uranis is 86,800,000, 000,000,000,000,000,000kg

5. Compare the following numbers :

(*i*) 4.3×10^{14}; 3.01×10^{17}.

(*ii*) 1.439×10^{12}; 1.4335×10^{12}

WHAT HAVE WE DISCUSSED ?

- **If a is any rational number and *n* is an integer then.**

  $a^n = a \times a \times a$ ..... multiplied n times.

  Where a is called the base and n is called the exponent or index and a^n is the exponential form a^n is read as 'a' raised to the power n or a to the power.'n'

  In particular $a^1 = a$

  $(-1)^{\text{odd natural number}} = -1$

  and $(-1)^{\text{even natural number}} = 1$

- **Laws of exponents**

  Law 1. : If a is any rational number and m, n are integers then $a^m \times a^n = a^{m+n}$

  Law 2. : If a is any (non – zero) rational number and m, n are integers such that $m > n$, then $a^m \div a^n = a^{m-n}$.

  Law 3. : If a is any (non – zero) rational number, then $a^0 = 1$

  Law 4. : If a is any rational number and m, n are integers, then $(a^m)^n = a^{m \times n}$

  Law 5. : If a, b are any rational numbers and n is an integer then $a^n \times b^n = (ab)^n$

  Law 6. : If a, b $(b \neq 0)$ are any rational numbers and n is an integer, than $a^n \div b^n = \left(\frac{a}{b}\right)^n$ or $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$.

  Law 7. : If a is any (non – zero) rational number and n is an integer then $a^{-n} = \frac{1}{a^n}$

- **Standard form or scientific notation**

  A number is said to be in the standard form if it is expressed as $k \times 10^n$, where k is a decimal number such that $1 \leq k < 10$ and n is a whole number.

  The standard form of a number is also known as scientific notation

  The decimal number k is called significand.

- **Convert from decimal notation to standard form**

  (*i*) Move the decimal point to the left till you get just one digit to the left of decimal place.

  (*ii*) Write the given number as the product of the number obtained in step (*i*) and 10^n, where n is the number of places the decimal point has been moved to the left.

- **Conversion from standard form to usual form.**

  Take the significand and move the decimal point to the right by the number of places indicated by the exponent of 10^n adding trailing zeros as necessary.

- **Comparing numbers in standard form.**

  (*i*) The number with greater power of 10 is greater.

  (*ii*) If the power of 10 are equal in both numbers then compare their significands. The number with greater significand is greater.

LEARNING OUTCOMES

After completion of the chapter, students are now able to

1. Understand the base and the exponent.
2. Apply the laws of exponents including multiplication and division of power with same base.
3. Evaluate the zero exponents.
4. Understand that exponents with different base can't be multiplied or divided.
5. Write standard form of numbers.
6. Use exponential form of numbers to simplify problems involving multiplication and division of large numbers.

ANSWERS

EXERCISE 13.1

1. (*i*) 3, 7 (*ii*) $\frac{2}{5}, 11$

2. (*i*) 64 (*ii*) 729 (*iii*) 3125
(*iv*) 1296 (*v*) $\frac{-32}{243}$

3. (*i*) 6^4 (*ii*) b^4 (*iii*) $5^2 \times 7^3$

4. (*i*) 2000 (*ii*) 225 (*iii*) 90000

5. (*i*) 24 (*ii*) –1600 (*iii*) –1
(*iv*) 225 (*v*) 1

6. (*i*) 3^4 (*ii*) 5^3 (*iii*) 8^2
(*iv*) 4^5 (*v*) 2^{10}

7. (*i*) 2^3 (*ii*) 2^7 (*iii*) 2^{10}

8. (*i*) 3^3 (*ii*) 2^7 (*iii*) 2^{10}

9. (i) 3 (*ii*) 3 (*iii*) 3

10. 4

11. (*i*) $2^3 \times 3^2$ (*ii*) $2^3 \times 3^2 \times 5^1$ (*iii*) $5^1 \times 3^4$
(*iv*) $2^3 \times 3^4$ (*v*) $2^4 \times 3^2 \times 5^2$

EXERCISE 13.2

1. (*i*) 2^{11} (*ii*) p^8 (*iii*) $(-7)^{16}$
(*iv*) 20^2 (*v*) $(-6)^4$ (*vi*) 7^{x+3}

2. (*i*) 5^{22} (*ii*) a^{15}

3. (*i*) 2^{200} (*ii*) 5^{21}

4. (*i*) 2^7 (*ii*) 10^5 (*iii*) 30^6

5. (*i*) 40^4 (*ii*) 15^6

6. (*i*) $3^6 \times 2^2$ (*ii*) 1^1 (*iii*) $(2a)^2$
(*iv*) 1^1

7. (*i*) $\left(\frac{5}{8}\right)^2$ (*ii*) $\left(\frac{-4}{5}\right)^3$ (*iii*) $\left(\frac{-5}{6}\right)^3$ (*iv*) $\left(\frac{-7}{9}\right)^3$

8. (*i*) 98 (*ii*) 36

9. (*i*) $2^7 \times 3^2 \times 7^2$ (*ii*) $3^6 \times 2^6$ (*iii*) 28×34

10. (*i*) $2^4 \times 7^1$ (*ii*) 3^{10} (*iii*) 2^3

11. (*i*) **12.** (*i*)

13. (*iii*) **14.** (*ii*)

EXERCISE 13.3

1. (*i*) $104278 = 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0$
(*ii*) $20068 = 2 \times 10^4 + 6 \times 10^1 + 8 \times 10^0$
(*iii*) $120719 = 1 \times 10^5 + 2 \times 10^4 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$
(*iv*) $3006194 = 3 \times 10^6 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$
(*v*) $28061906 = 2 \times 10^7 + 8 \times 10^6 + 6 \times 10^4 + 1 \times 10^3 + 9 \times 10^2 + 6 \times 10^0$

2. (*i*) 47561 (*ii*) 30705 (*iii*) 405302
(*iv*) 80037580

3. (*i*) 3.43×10^5 (*ii*) 7.0×10^6 (*iii*) 3.1865×10^9
(*iv*) 5.307×10^2
(*iv*) 5.9853×10^3 (*v*) 3.90878×10^3

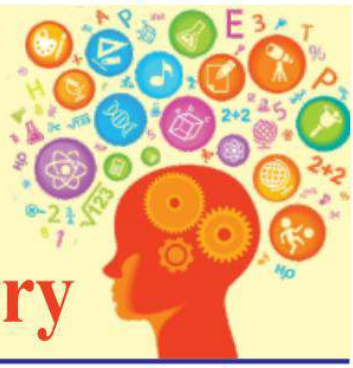
4. (*i*) 3.84×10^8 (*ii*) $1.2756 \times 10^7 m$ (*iii*) $1.40 \times 10^9 m$.
(*iv*) 1.2×10^{10} years (*v*) 8.68×10^{28} kg.

5. (*i*) $3.01 \times 10^{17} > 4.3 \times 10^{14}$ (*ii*) $1.439 \times 10^{12} > 1.4335 \times 10^{12}$.

CHAPTER 14

C1S6F7

Symmetry



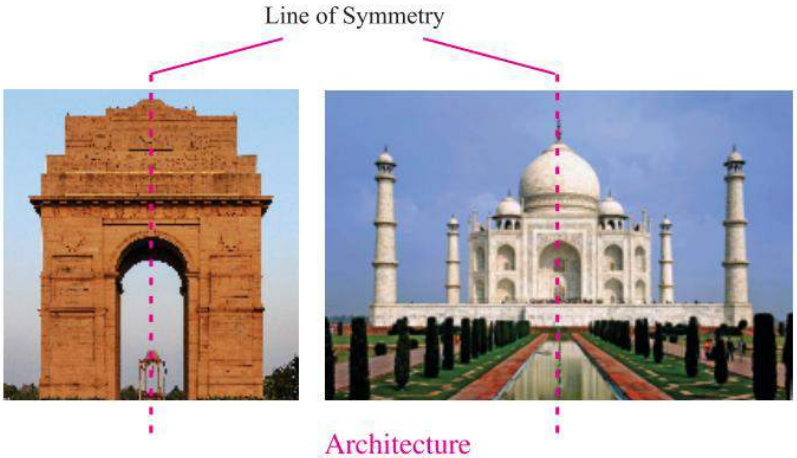
Learning Objectives :-

In this chapter, you will learn :-

1. To differentiate symmetrical and asymmetrical figures.
2. To draw lines of symmetry.
3. The concepts of rotational symmetry, centre of rotation, angle of rotation and order of rotational symmetry.
4. About the shapes that have both lines of symmetry and rotational symmetry.
5. To make use of symmetry in completing the missing half of symmetrical figures.
6. To relate the concept of symmetry with your daily life situations and develop aesthetic sense in you.

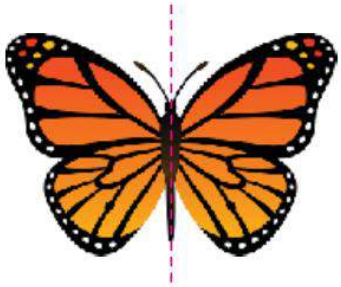
INTRODUCTION

Symmetry is an important geometrical concept that is commonly used in almost every activity of our daily life. Various professionals like architects, car manufacturers, engineers, and designers use the concept of symmetry. In class VIth we have learnt about the line of symmetry, which refers to the line that divides the shape in two identical parts. We have seen the presence of lines of symmetry in many man made things as well as in nature. Flowers, Leaves, Fish, birds, animals, human, architecture and in religious symbols everywhere we find symmetry.

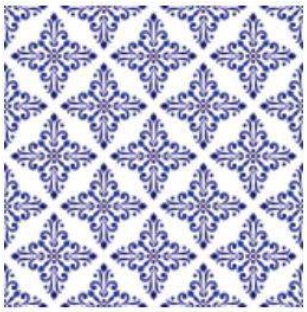


Architecture

The above architecture designs of India Gate and Taj Mahal look beautiful because of their symmetry.



Symmetry in nature



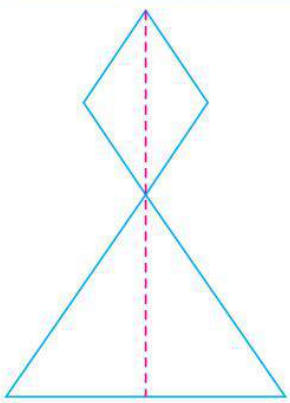
Symmetry in Cloth Designing

Symmetry in Enginnering

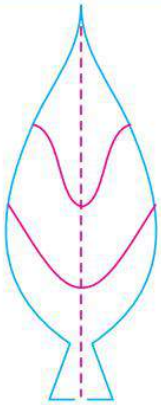
Asymmetrical figures : The objects or figures that do not have any line of symmetry are called asymmetrical figures.

Line of symmetry : Look at the following plane figures and pictures of objects.

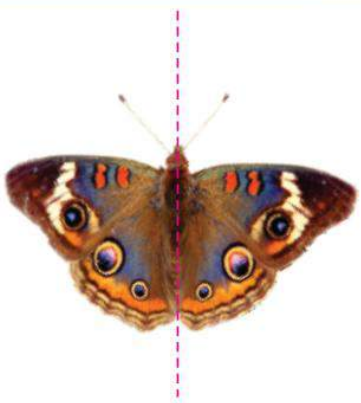
We observe that if these figures or pictures are folded along a dotted line shown in each figure on picture the left hand of dotted line fits exactly on the right side of dotted line i.e; each figure is divided into two coincident parts about the dotted line.



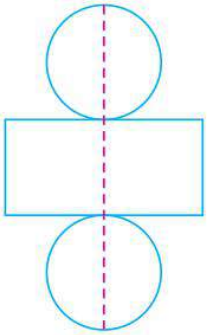
(*i*)



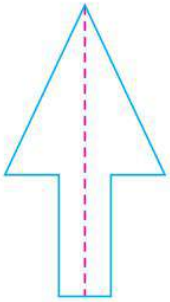
(*ii*)



(*iii*)



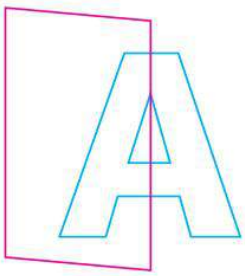
(*iv*)



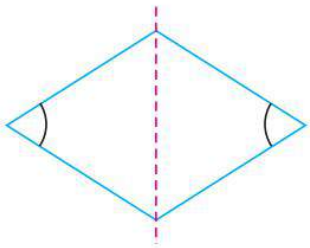
(*v*)

Thus if a figure can be divided into two coincident parts by a line then the figure is called symmetrical about the line and the line is called **line of symmetry or axis of symmetry.**

Mirror reflection : The concept of symmetry is closely related to mirror reflection. A shape has line symmetry when one half of it is the mirror image of the other half. as in (fig) (*i*) A mirror line thus helps to visualise a line of symmetry (fig *ii*)

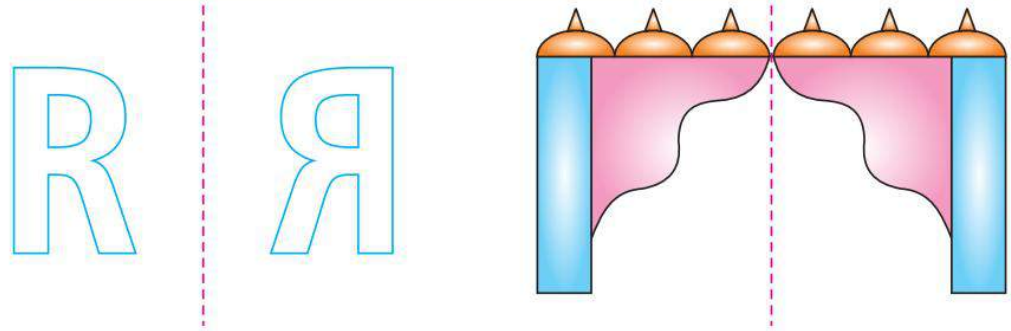


(*i*)



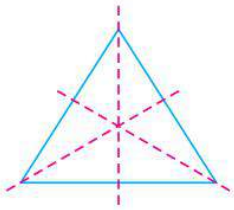
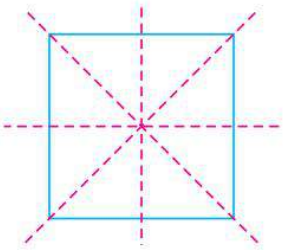
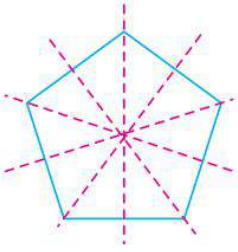
(*ii*)

While dealing with mirror reflection, care is needed to note down the left right changes (directional changes) every thing of the image (Shape size are same but reversed). Some examples of mirror image are as shown below.

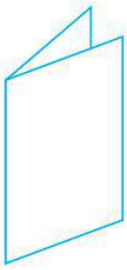


Lines of symmetry for regular polygons : A simple closed figure made up of several line segments is called a polygon. A polygon has minimum three line segments. If all the sides of a polygon are of equal length and its angles are also equal then it is said to be a regular polygon The regular polygons are symmetrical figures and have more than one line of symmetry. **In fact each regular polygon has as many line of symmetry as the number of sides.**

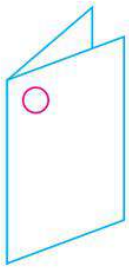
Some regular polygon with lines of symmetry

Sr. No.	Name of regular polygons and their features	Figure with lines of symmetry	No. of lines of symmetry
1.	**Equilateral triangle :** An equilateral triangle is regular because all of its sides have same length and measures of each interior angle is 60°.		3
2.	**Square :** All its four sides are of equal length and each of its interior angle is 90°. Its diagonals are prependicular bisector of each other.		4
3.	**Regular pentagon :** All its five sides are of equal length and measure of each interior angles is 108°.		5

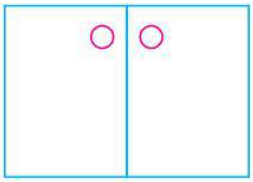
Punching of Paper : In punching of a paper for symmetrical design, the line of fold is the line of symmetry as shown below.



(Fold a sheet into two halves)



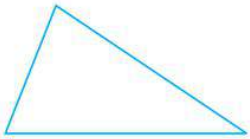
(Punch a hole)



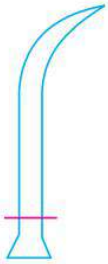
(Two holes about the symmetric fold)

Example-1 : Which of the following figures are asymmetrical ?

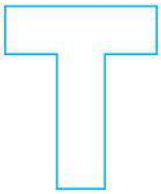
(a)



(b)



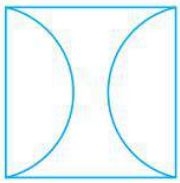
(c)



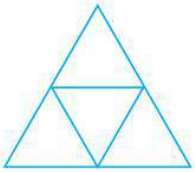
(d)

Sol. (*b*) and (*c*) are asymmetrical figures (*a*) and (*d*) are symmetrical figures

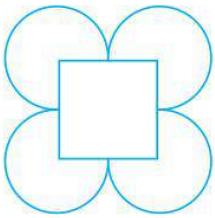
Example-2 : Draw lines of symmetry. If any in each of the following figures.



(a)

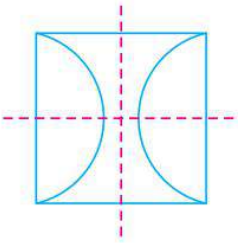


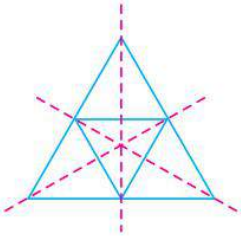
(b)

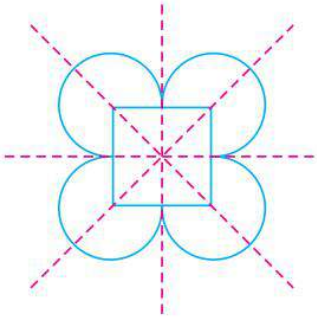


(c)

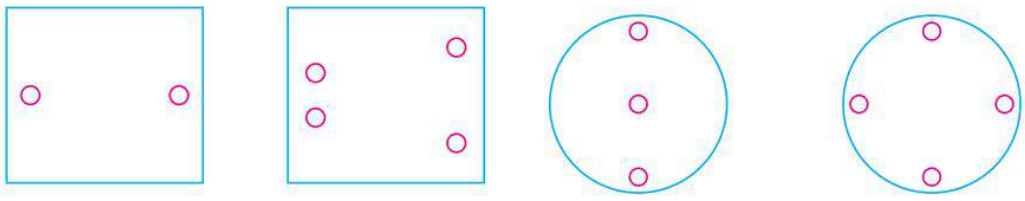
Sol.





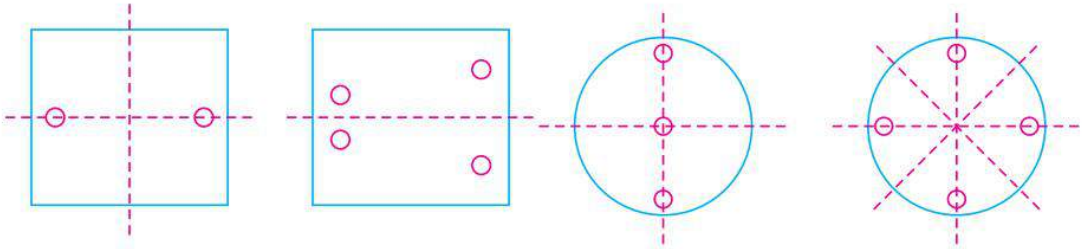


Example-3.: Copy the figures with punched holes and find the axes of symmetry for the following.

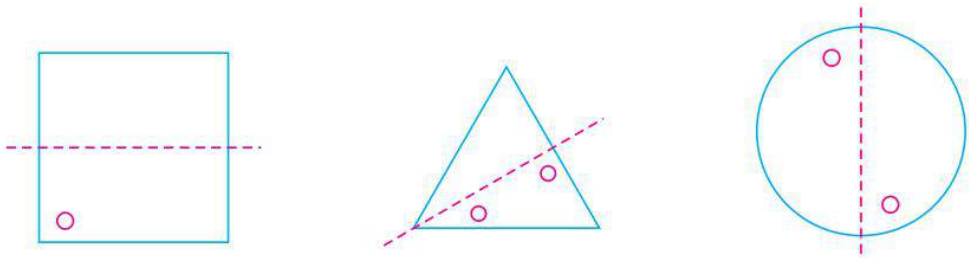


(a) (b) (c) (d)

Sol.

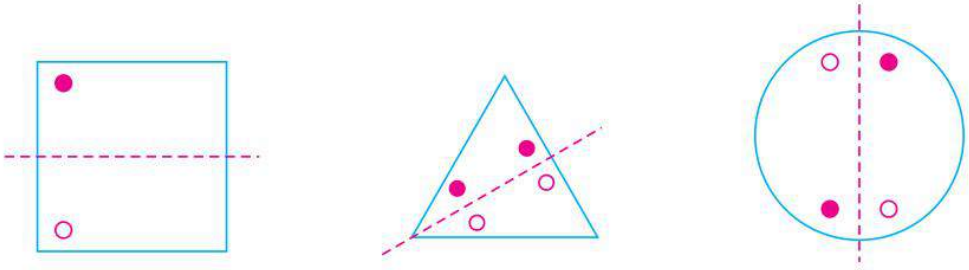


Example-4 : In the following figures mark the missing holes in order to make them symmetrical about the dotted line.

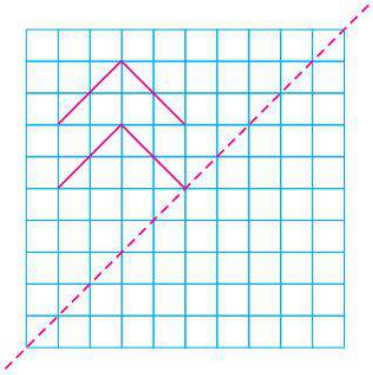


(a) (b) (c)

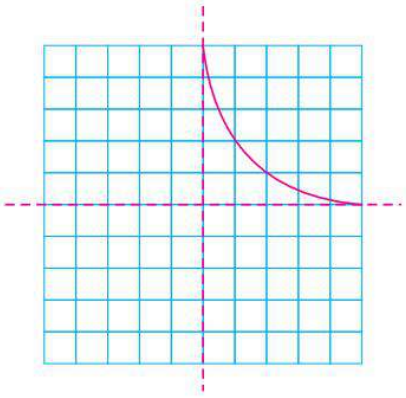
Sol. The missing holes are marked by dark punches (small circles) in each of the following figures.



Example-5 : Copy each diagram on a squared paper and complete each shape to be symmetrical about the mirror lines shown dotted.

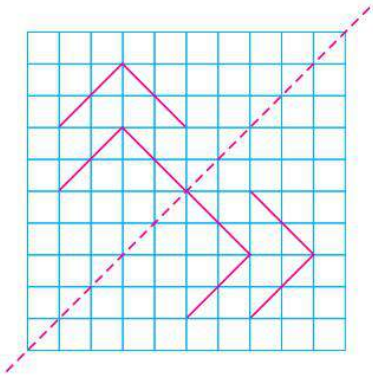


(a)

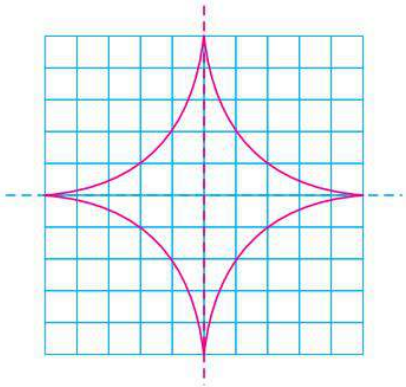


(b)

Sol. The complete shapes are given below



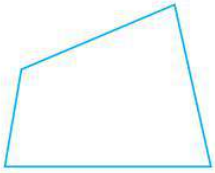
(a)



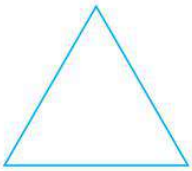
(b)

EXERCISE - 14.1

1. Which of the following figures are asymmetrical ?



(a)

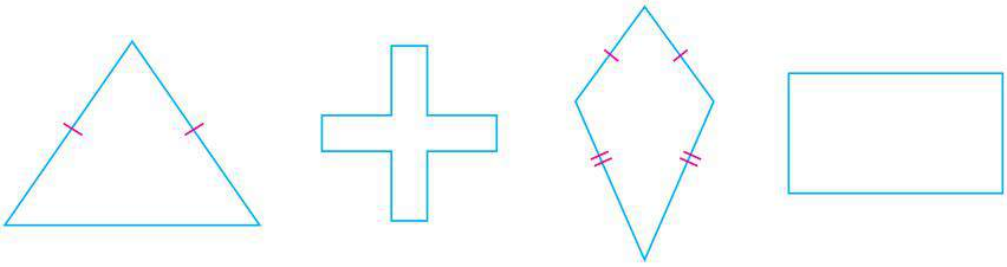


(b)

(c)

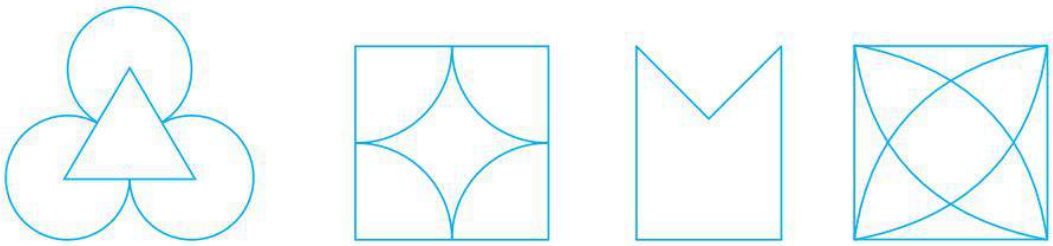
(d)

2. Draw the lines of symmetry in the following figures



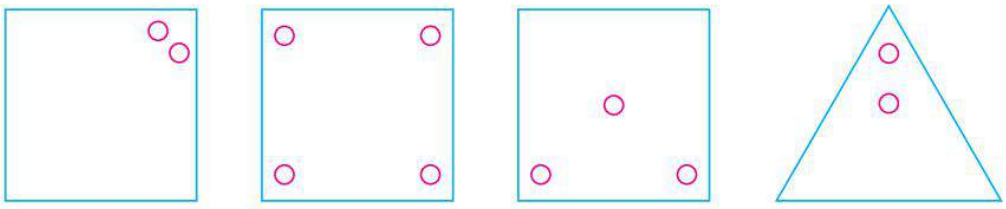
(a) (b) (c) (d)

3. Draw all lines of symmetry if any in each of the following figures.

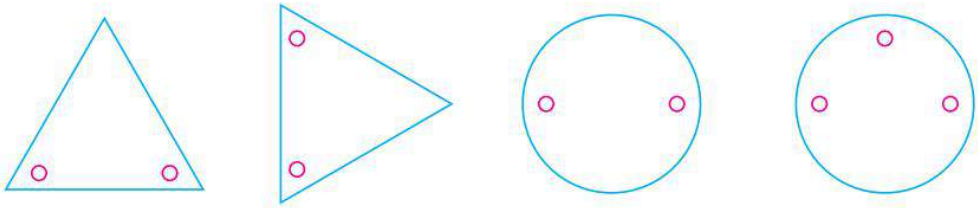


(a) (b) (c) (d)

4. Copy the figures with punched holes and find the axes of symmetry for the following.

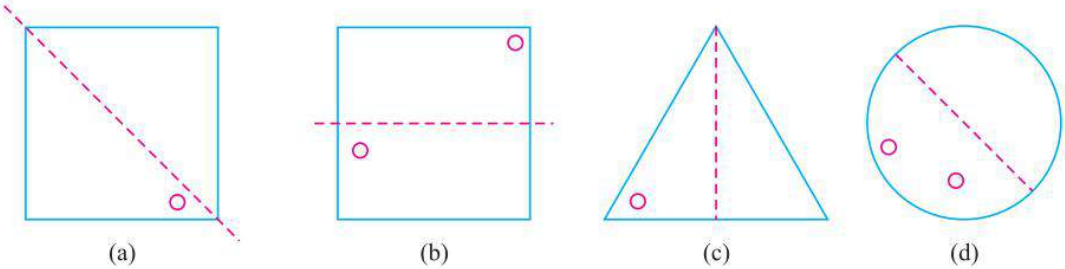


(a) (b) (c) (d)



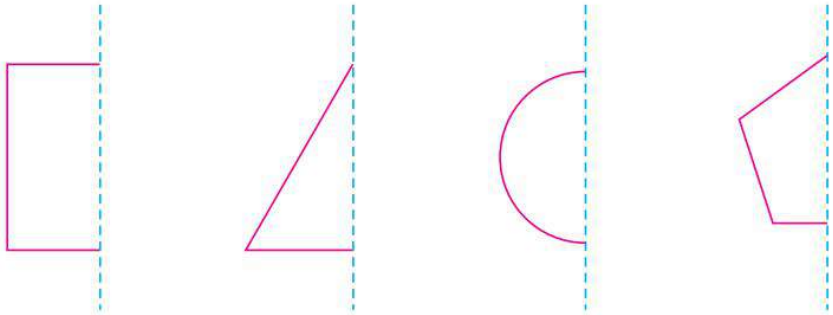
(e) (f) (g) (h)

5. In the following figures mark the missing holes in order to make them symmetrical about the dotted line.



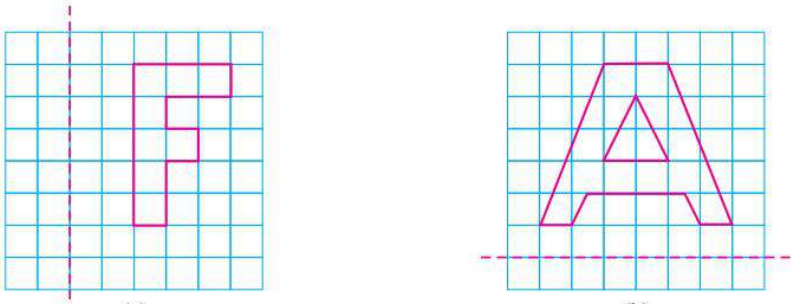
(a) (b) (c) (d)

6. In each of the following figures, the mirror line (i.e; the line of symmetry) is given as dotted line complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of figure you complete.



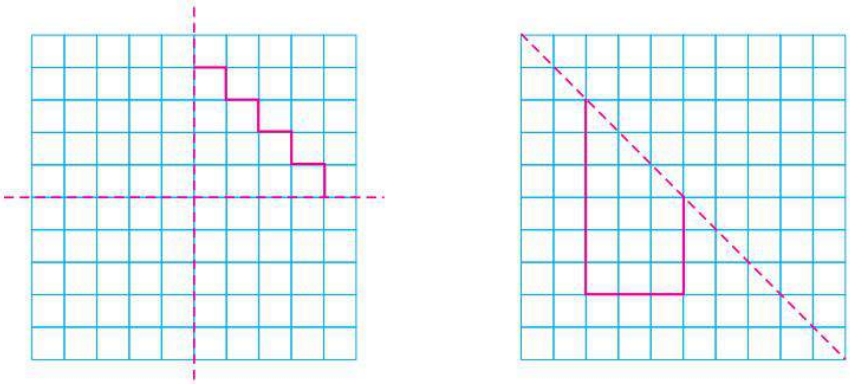
(a) (b) (c) (d)

7. Draw the reflection of the following letter in the given mirror line



(a) (b)

8. Copy each diagram on a squared paper and complete each shape to be symmetrical about the mirror lines shown dotted



(a) (b)

9. State the number of lines of symmetry for the following figures

(*a*) A scalene triangle (*b*) A rectangle

(*c*) A rhombus (*d*) A parallelogram

(*e*) A regular hexagon (*f*) A circle

10. Multiple Choice Questions.

(*i*) Which of the following triangles have no line of symmetry ?

(*a*) An equilateral triangle (*b*) An Isosceles triangle

(*c*) A scalene triangle (*d*) All of above

(*ii*) What is the other name for a line of symmetry of a circle ?

(*a*) An arc (*b*) A sector

(*c*) A diameter (*d*) A radius

(*iii*) How many lines of symmetry does a regular polygon have ?

(*a*) Infinitely many (*b*) As many as its sides

(*c*) One (*d*) Zero

(*iv*) In the given figure, the dotted line is the line of symmetry which figure is formed if the given figure is reflected in the dotted line 

(*a*) Square (*b*) Rhombus

(*c*) Triangle (*d*) Pentagon

(*v*) What is other name for a line of symmetry of an Isosceles triangle ?

(*a*) Side (*b*) Median

(*c*) Radius (*d*) Angle

(*vi*) Which of the following alphabets has a vertical line of symmetry ?

(*a*) M (*b*) Q

(*c*) E (*d*) B

(*vii*) Which of the following alphabets has a horizontal line of symmetry ?

(*a*) C (*b*) D

(*c*) K (*d*) All the above

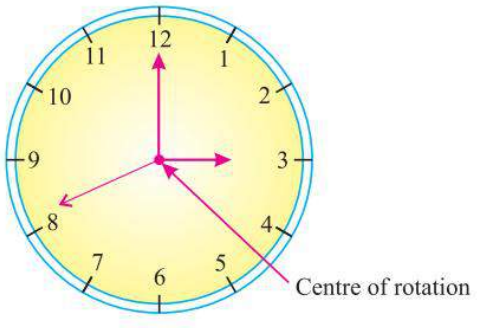
(*viii*) Which of the following alphabets has no line of symmetry ?

(*a*) A (*b*) B

(*c*) P (*d*) O

Rotational symmetry : In our daily life we see objects that rotate. Rotation is the circular movement of an object about a point. Rotation can be clockwise or anticlockwise for example, when we open the cap of a bottle the rotation is anticlockwise and when we close the cap of a bottle, the rotation is clockwise.

Centre of rotation : The fixed point about which an object rotated is called centre of rotation. For example, the centre of rotation for the hands of clock is the point where all the three hands are joined as shown in figure.



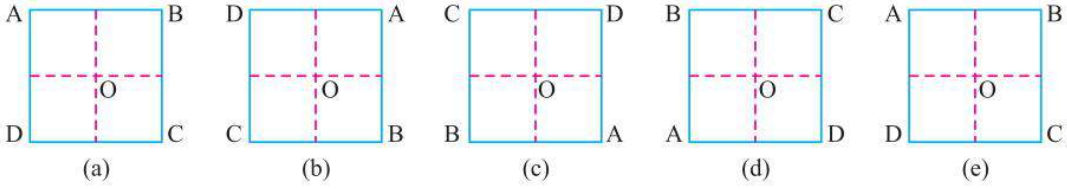
Angle of rotation : The smallest angle through which an object (or a figure) rotates about a fixed point (Centre of rotation) so that it looks the same is called angle of rotation. An object is said to take a full turn if it rotates by 360°. A half turn means a rotation by 180° and a quarter-turn means a rotation by 90°.

Order of rotational symmetry : If A° is the smallest angle through which a figure can be rotated and still looks the same, then it has a rotational symmetry of order = $\frac{360}{A°}$

For a figure having a rotational symmetry, A° must be less than or equal to 180°.

EXAMPLE OF ROTATION SYMMETRY

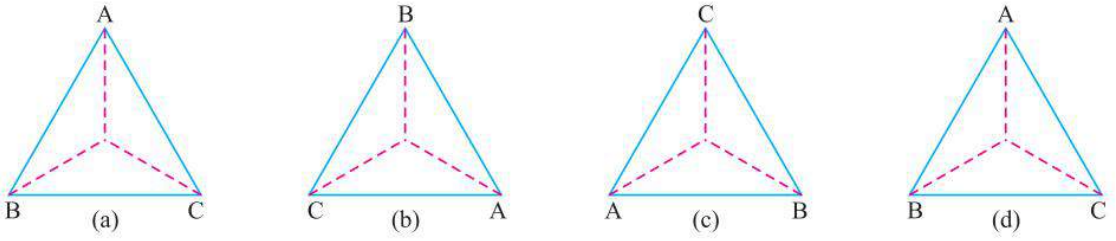
(*i*) **Rotational symmetry of a square :** Let us rotate a square ABCD in fig (*a*) to a full turn, i.e; through four positions *i.e*, 90° 180°, 270° and 360° to attain the positions shown in fig (*b*), fig (*c*), fig (*d*) and fig (*e*) respectively.



(a) Original form; (b) Rotated through 90°; (c) Rotated through 180°; (d) Rotated through 270°; (e) Rotated through 360°

Clearly, after four rotations square regains its original position so It has a rotational symmetry of order 4

(*ii*) **Rotational symmetry of Equilateral triangle :** Let us rotate an equilateral triangle ABC through an angle of 120° we observe that in a full turn, there are precisely three positions (on rotation through 120°, 240° and 360°)



(a) Original form; (b) Rotated through 120°; (c) Rotated through 240°; (d) Rotated through 360°

Thus an equilateral triangle has a rotational symmetry of order 3.

Note that in this case

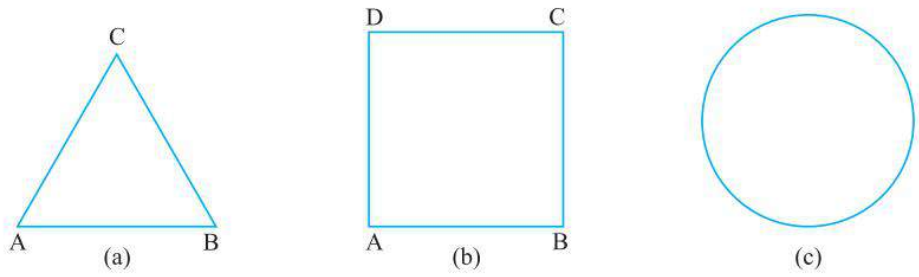
(*i*) The centre of rotation is the point of concurrence of the bisectors of the angles of triangle.

(*ii*) Angle of rotation is 120°.

(*iii*) The direction of rotation is clockwise.

(*iv*) The order of rotational symmetry is 3.

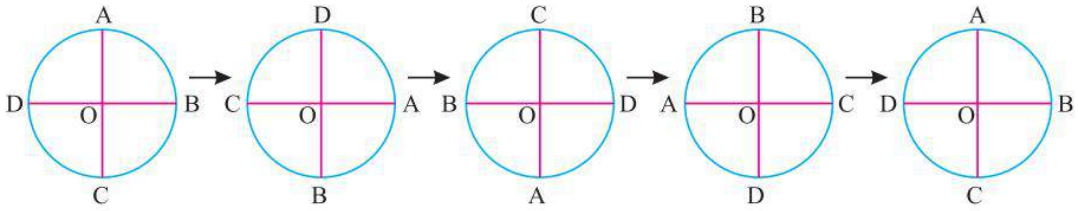
Example-1 : Write the order of rotation for the following figures



Sol.

Sr. No.	Name of figure	Order of rotation
1.	Equilateral triangle	3
2.	Square	4
3.	Circle	Infinite

Example-2 : Specify the centre of rotation, direction of rotation, angle of rotation and order of rotation for the following.



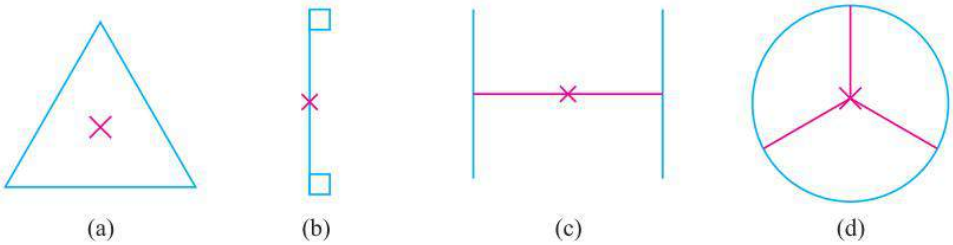
Sol. (*i*) The centre of rotation is O.

(*ii*) The direction of rotation is clockwise

(*iii*) The angle of rotation is 90°.

(*iv*) The order of rotation is 4.

Example-3 : Which of the following figures have rotational symmetry about the marked point, specify angle of rotation and order of rotation of the figures.



Sol. Figure (*a*) has rotational symmetry of order 3 about the marked point through an angle of 120°.

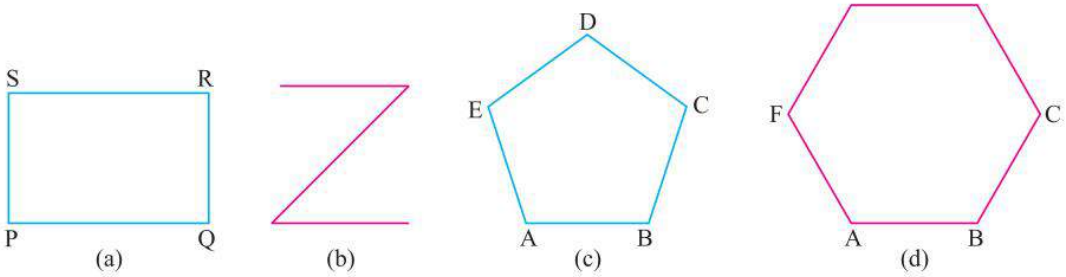
Figure (*b*) has no rotational symmetry about the marked point

Figure (*c*) has a rotational symmetry of order 2 about the marked point through an angle of 180°.

Figure (*d*) has a rotational symmetry of order 3 about the marked point through an angle of 120°.

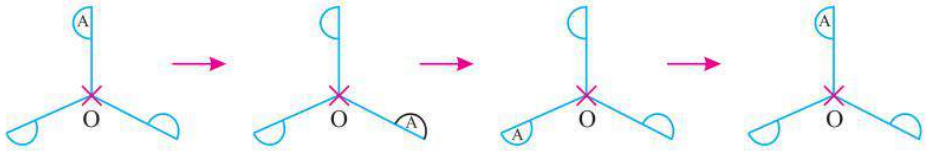
EXERCISE - 14.2

1. Write the order of rotation for the following figures.

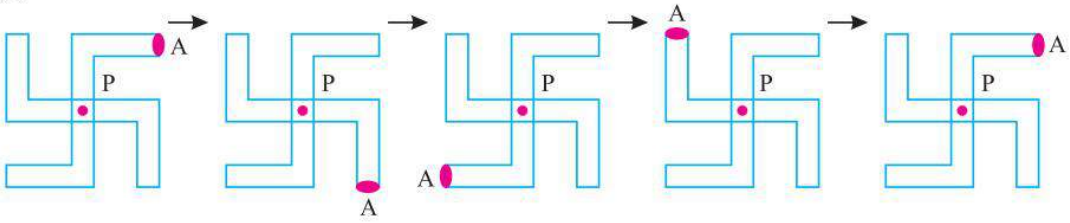


2. Specify the centre of rotation, direction of rotation, angle of rotation and order of rotation for the following.

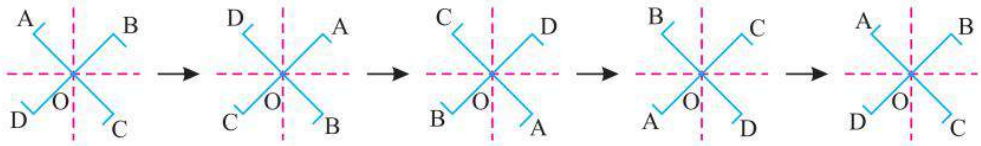
(*i*)



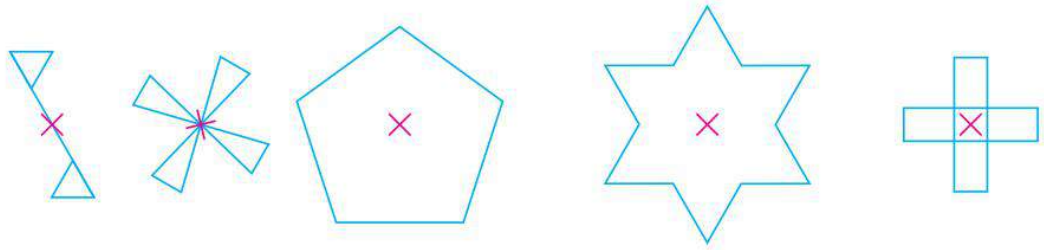
(*ii*)



(*iii*)



3. Which of the following figures have rotational symmetry about the marked point (×) give the angle of rotation and order of the rotation of the figures.



(a) (b) (c) (d) (e)

4. Multiple choice questions :-

(*i*) The angle of rotation in an equilateral triangle is

(*a*) 60° (*b*) 70°

(*c*) 90° (*d*) 120°

(*ii*) A square has a rotational symmetry of order 4 about its centre what is the angle of rotation ?

(*a*) 45° (*b*) 90°

(*c*) 180° (*d*) 270°

(*iii*) What is the order of rotational symmetry of the english alphabet Z ?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

(*iv*) Which of these letters has only rotational symmetry ?

(*a*) S (*b*) E

(*c*) B (*d*) P

(*v*) If the smallest angle of rotation is 90° then order of symmetry is ?

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) 2

Line symmetry and rotational symmetry : We have learnt about various figure and their symmetries. Some figures have only line symmetry, some have rotational symmetry and some have both line symmetry and rotational symmetry for example :

(*i*) An isosceles triangle has a line symmetry but not rotational symmetry

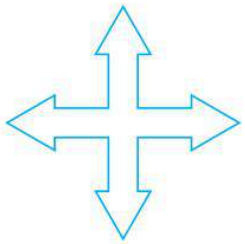
(*ii*) A parallelogram has rotational symmetry but no line symmetry.

(*iii*) A square has both line symmetry as well as rolational symmetry. In fact a square has four lines of symmetry and has rotational symmetry of order 4.

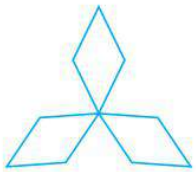
(iv) A circle is the most perfect symmetrical figure because its has an infinite number of lines of symmetry and it can be rotated about its centre at any angle and still works the same.

Note : If a figure has two or more lines of symmetry then it also has rotational symmetry.

Example-1 : In the following figures, find the number of symmetry and angle of rotation for rotational symmetry.



(a)



(b)

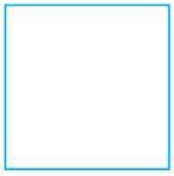
Sol. (*a*) Number of lines of symmetry = 2

Angle of rotation = 90°

(*b*) Number of line of symmetry = 3

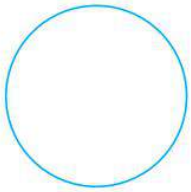
Angle of rotation = 120°

Example-2 : Following shapes have both line symmetry and rotational symmetry. Write number of lines of symmetry, also specify centre of rotation and write order of rotational symmetry.



(a)

(b)



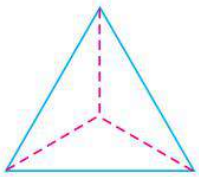
(c)

Sol.

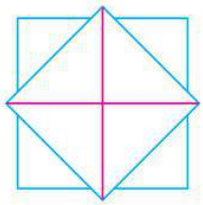
Sr. No	Name of figure	Number of lines of symmetry	Centre of rotation	Order of rotational symmetry
1.	Square	4	Intersection of diagonals	4
2.	Rectangle	2	Intersection of diagonals	2
3.	Circle	Infinite	Centre	Infinite

EXERCISE - 14.3

1. In the following figures, find the number of lines of symmetry and angle of rotation for rotational symmetry.



(a)

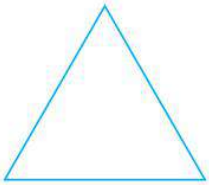


(b)

2. Name any two figures that have both line of symmetry and rotational symmetry.

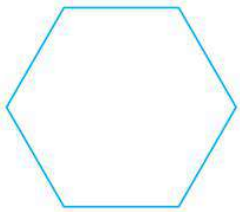
3. If a figure has two or more lines of symmetry should it have a rotational symmetry of order more than 1 ?

4. Following shapes have both, line symmetry and rotational symmetry. Find the number of lines of symmetry, centre of rotation and order of rotational symmetry.



(a)

(b)



(c)

5. Some of the english alphabets have fascinating symmetrical structures. Which capital letters have just one line of symmetry (Like E) ? Which capital letters have a rotational symmetry of order 2 (Like I) ? By attempting to think on such lines, you will be able to fill in the following table.

English Alphabet	Line Symmetry	Number of lines Symmetry	Rotational Symmetry	Order of Rotational symmetry
Z	No		Yes	
S		0		2
H		2		
O	Yes			4
E	Yes	1		
N			Yes	
C	Yes			1

6. Multiple choice questions :-

(*i*) If 60° is the smallest angle of rotation for a given figure what will be the angle of rotation for same figure.

(*a*) 150° (*b*) 180°

(*c*) 90° (*d*) 330°

(*ii*) Which of these can not be a measure of an angle of rotation for any figure.

(*a*) 120° (*b*) 180°

(*c*) 17° (*d*) 90°

(*iii*) Which of the following have both line symmetry and rotational symmetry ?

(a) An isosceles triangle (b) A scalene triangle

(c) A square (d) A parallelogram

(*iv*) Which of the alphabet has both multiple line and rotational symmetries ?

(a) S (b) O

(c) H (d) L

(*v*) In the word 'MATHS' which of the following pairs of letters shows rotational symmetry?

(a) M and T (b) H and S

(c) A and S (d) T and S

WHAT HAVE WE DISCUSSED ?

1. A figure has line symmetry, if there is a line about which the figure may be folded so that two parts of figure will coincide.
2. Each regular polygon has as many lines of symmetry as the number of Its sides.
3. Mirror reflection leads to symmetry, under which the left right orientation have to be taken care of.
4. Rotation turns an object about a fixed point
   - This fixed point is the centre of rotation
   - The angle by which the object rotates is the angle of rotation.
   - A full turn means rotation by 360°, A half turn means rotation by 90° rotation may be clockwise or anticlockwise.
5. A plane figure has a rotational symmetry if on rotation through some angle ($\leq 180°$) about a point, it looks the same as it did in starting position.
6. If A° ($\leq 180°$) is the smallest angle through which a figure can be rotated and still looks the same, then it has a rotationals symmetry of order = $\frac{360°}{A°}$
7. If the order of rotational symmetry is 1, Then the figure is said to have no rotational symmetry
8. Some figure (or shapes) have only line symmetry, some have only rotational symmetry and some have both line symmetry as well as rotational symmetry.

LEARNING OUTCOMES

After completion of this chapter the students are now able to :

1. Differentiate between symmetrical and asymmetrical figures.
2. Draw lines of symmetry.

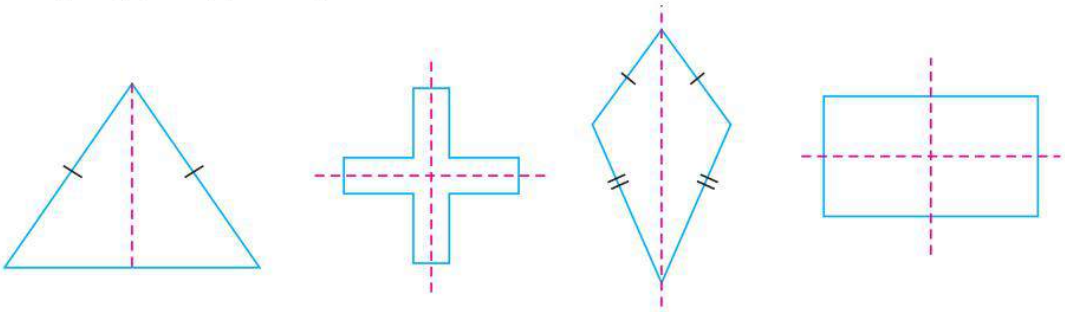
3. Differentiate between line symmetry and rotational symmetry.
4. Find the centre of rotation, the angle of rotation and the order of rotation.
5. Derive equivalent positions for mirrors and certain rotations.

ANSWERS

EXERCISE 14.1

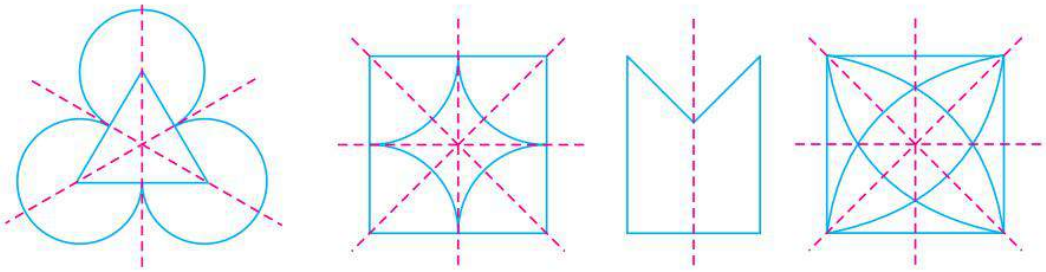
1. Figure (*a*) and (*c*) are asymmetrical

2.



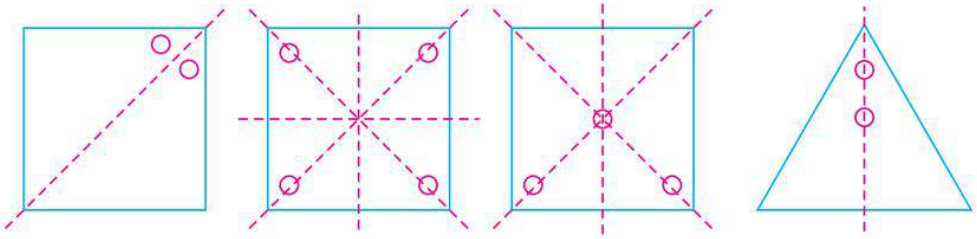
(a) (b) (c) (d)

3.

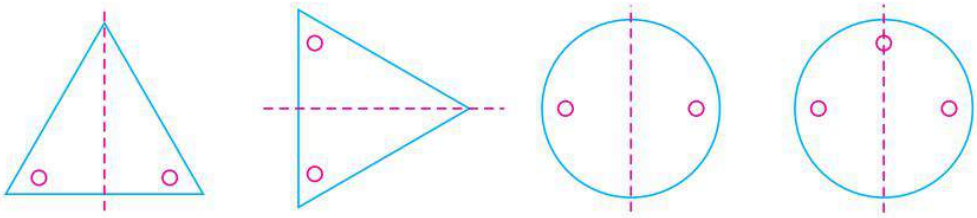


(a) (b) (c) (d)

4.

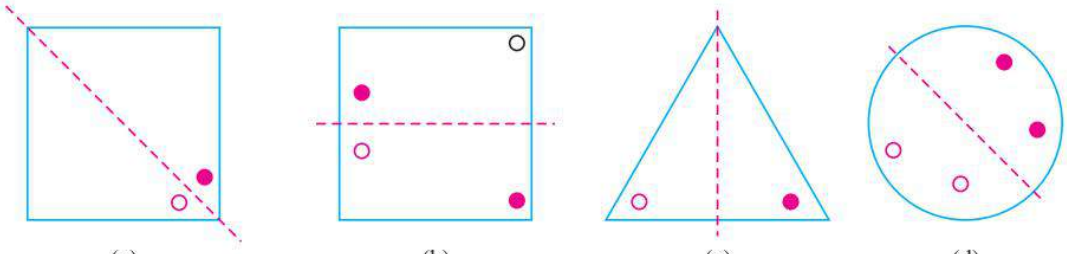


(a) (b) (c) (d)



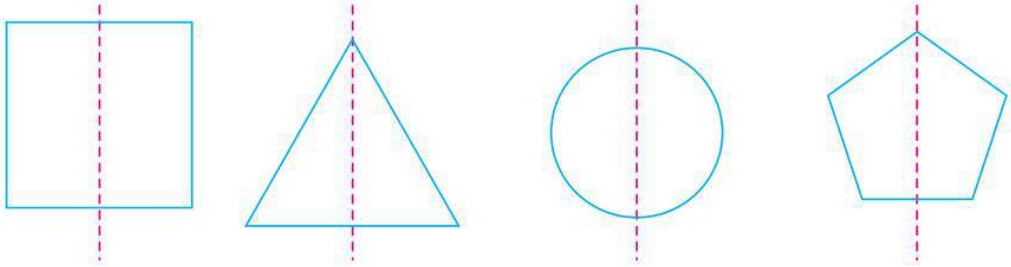
(e) (f) (g) (h)

5. The missing holes are marked by dark punches (small circles) in each of following figures



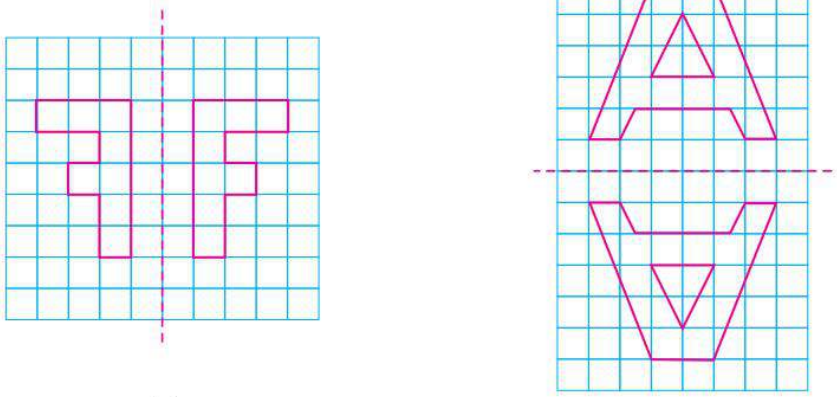
(a) (b) (c) (d)

6.



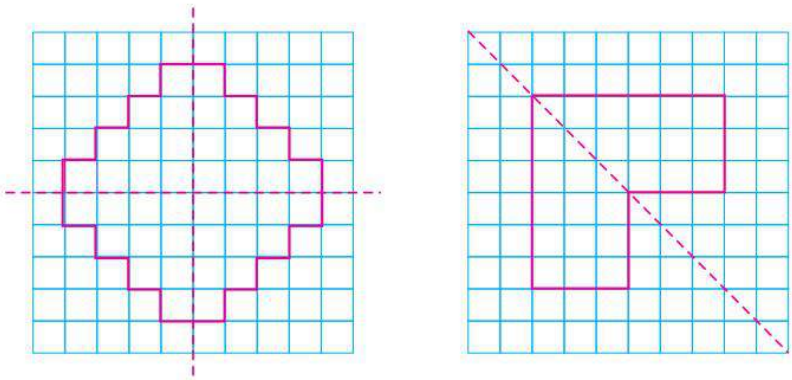
(a) Square (b) Triangle (c) Circle (d) Pentagon

7.



(a) (b)

8.



(a) (b)

9. (*a*) 0 (*b*) 2 (*c*) 2
(*d*) 0 (*e*) 6 (*f*) Infinite

10. (*i*) *a* (*ii*) *c* (*iii*) *b*
(*iv*) *b* (*v*) *b* (*vi*) *a*
(*vii*) *d* (*viii*) *c*

EXERCISE 14.2

1. (*a*) 2 (*b*) 2
(*c*) 5 (*d*) 6

2. (*i*) Centre of rotation is O, direction of rotation is clockwise, Angle of rotation is 120° and order of rotation is 3.
(*ii*) Centre of rotation is P, direction of rotation is clockwise, Angle of rotation is 90° and order of rotation is 4.
(*iii*) Centre of rotation is O, direction of rotation is clockwise, Angle of rotation is 90° and order of rotation is 4.

3. (*a*) It has rotational symmetry, angle of rotation 180° and order of rotation 2
(*b*) It has rotational symmetry angle of rotation is 90° and order of rotation 4
(*c*) It has rotational symmetry angle of rotation is 72° and order of rotation 5
(*d*) It has rotational symmetry angle of rotation is 60° and order of rotation 6
(*e*) It has rotational symmetry angle of rotation is 90° and order of rotation 4

4. (*i*) *d* (*ii*) *b* (*iii*) c
(*iv*) *a* (*v*) *c*

EXERCISE 14.3

1. (*a*) Line of symmetry 3, angle of rotation 120°.
(*b*) Line of symmetry 4, angle of rotation 90°.

2. Equilateral triangle and circle

3. Yes, Square has four lines of symmetry and rotational symmetry of order 4.

4. (*a*) 3, centroid , 3 (*b*) 2, Intersection of diagonals, 2
(c) 6, centre of hexagon, 6

5.

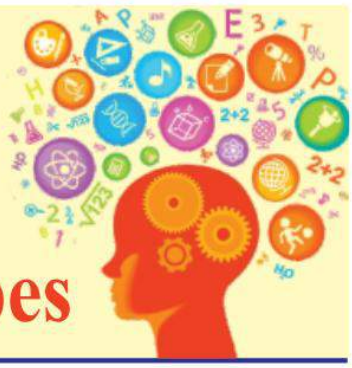
Alphabet Letters	Line Symmetry	No of lines Symmetry	Rotational Symmetry	Order of Rotational symmetry
Z	No	0	Yes	2
S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	2	Yes	4
E	Yes	1	Yes	1
N	No	0	Yes	2
C	Yes	1	Yes	1

6. (*i*) *b* (*ii*) *c* (*iii*) *c*
(*iv*) *b* (*v*) *b*

CHAPTER

15

Q6E9Y6



Visualising Solid Shapes

Learning Objectives :-

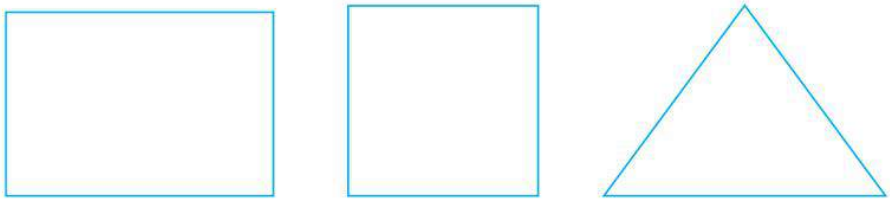
In this chapter, you will learn :-

1. To associate 2–D shapes with 3-D shapes.
2. To understand and identify faces, edges and vertices of solid figures.
3. To identify nets of different 3–D shapes and use them to form those 3–D shapes.
4. About oblique sketches and isometric sketches and also their differences.
5. To visualize solid shapes in different ways and also to see the hidden parts of a solid.
6. To apply your knowledge of solids in your day to day life.

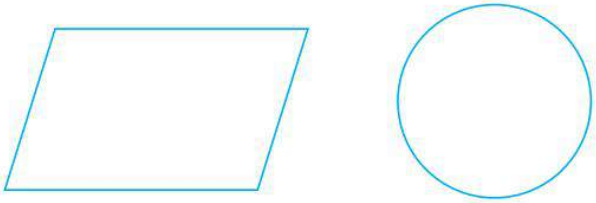
INTRODUCTION

In this chapter we will discuss about plane figures and solid shapes.

Plane figures : In previous classes, we have learnt to draw some figures like square, rectangle, triangle, quadrilateral, circle etc. These figure have two dimensions namely length and breadth and can be drawn on a paper, these figure are called two dimensional (2 – D) figure or plane figures. Some types of plane figure are as follows.



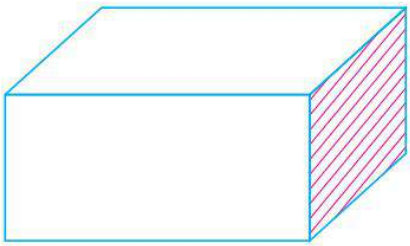
(i) Rectangle (ii) Square (iii) Triangle



(iv) Quadrilateral (v) Circle

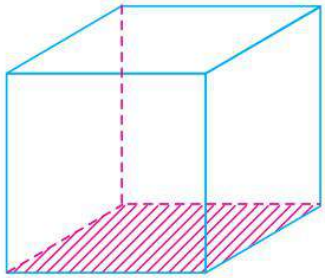
Solid Shapes : In Our daily life we come across various objects like books, packing boxes, road rollers, balls and ice cream cones etc. These type of objects having length, breadth and height are known as three dimensional (3 – D) figures or solids, because they have a definite shape and occupy space. Some types of solid shapes are as follows.

(i)



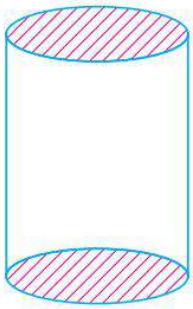
Cuboid

(ii)



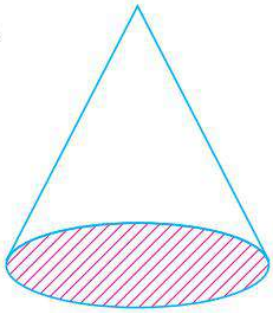
Cube

(iii)



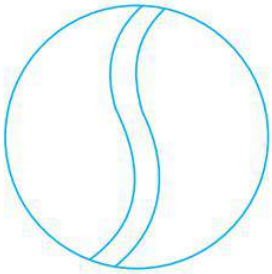
Cylinder

(iv)



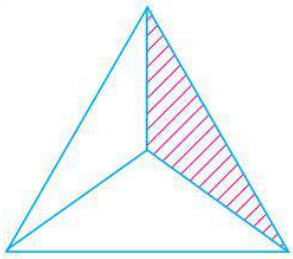
Cone

(v)



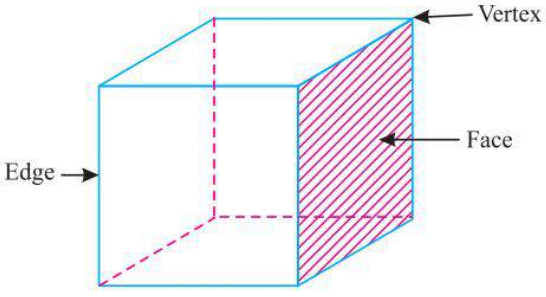
Sphere

(vi)



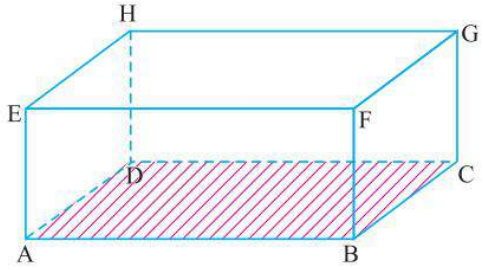
Triangular pyramid

Faces, edges and vertices : You have studied faces, edges and vertics of solid shapes. Let us revise the terms related to solid shapes.



DIFFERENT SOLID SHAPES AND THEIR FEATURES

Cuboid : A solid bounded by six rectangular faces at right angle to each other is called a cuboid. The figure shows a cuboid ABCDEFGH having :

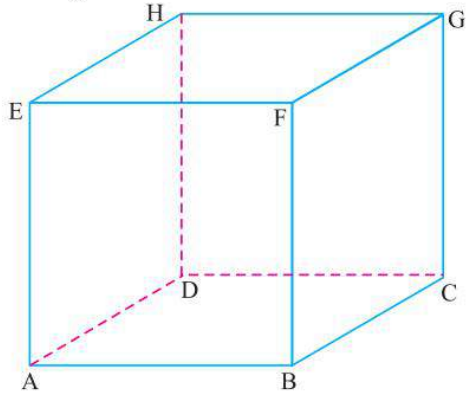


(*i*) **Faces :** It has 6 rectangular faces ABCD, EFGH, ADHE, BCGH, ABEF and DCGH. Out of these six faces ABFE, DCGH, BCGE and ADHE are called lateral faces of cuboid

(*ii*) **Edges :** It has 12 edges AB, BC, CD, DA, EF, FG, GH, HE, BF, CG, AE and DH

(*iii*) **Vertices :** It has 8 vertices A, B, C, D, E, F, G and H

Cube : A cuboid whose length, breadth and height are equal is called a cube the figure shows a cube ABCDEFGH having :



(*i*) **Faces :** It has 6 square faces ABCD, EFGH, ADHE, BCGH, ABEF and DCGH and lateral faces are ABFE, DCGH, BCGF and ADHE.

(*ii*) **Edges :** It has 12 edges AB, BC, CD, DA, EF, FG, GH, HE, BF, CG, AE and DH

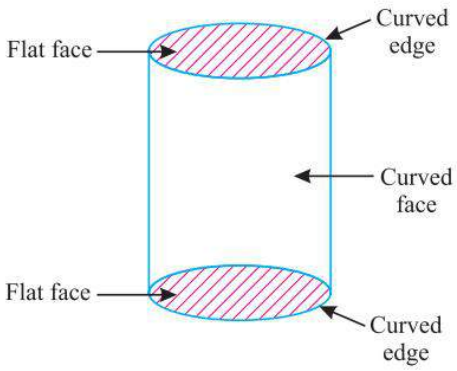
(*iii*) **Vertices :** It has 8 vertices A, B, C, D, E, F and H.

Cylinder : A cylinder is a three dimensional solid that contains two parallel bases connected by a curved surface. The bases are usally circular in shape for example :- pipes, cold drink cans, road roller. The figure shows a cylinder having

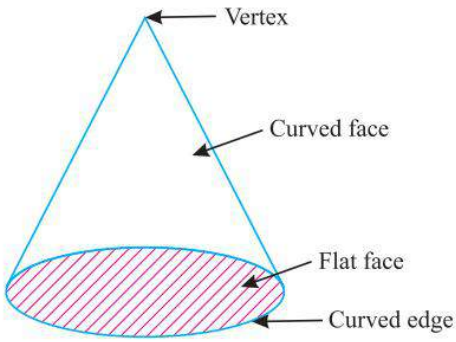
(*i*) **Faces :** It has two flat faces and one curved face.

(*ii*) **Edges :** It has two curved edges.

(*iii*) **Vertices :** It has no vertex.



Cone : A cone is a three dimensional shape that tapers smoothly from a flat base to a point called vertex for example : Ice cream cone, funnel, a conical tent. The figure shows a cone having

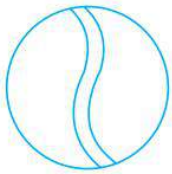


(*i*) **Faces :** It has one flat and one curved face.

(*ii*) **Edge :** It has one cured edge

(*iii*) **Vertex :** It has one vertex

Sphere : A three dimensional figure which is absolutely round like a ball is called a sphere.



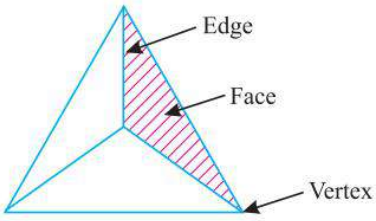
The figure shows a sphere

(*i*) It has a curved surface

(*ii*) It has no edge

(*iii*) It has no vertex

Triangular Pyramid : A triangular pyramid is a Pyramid which has a triangular base. The figure shows a triangular pyramid having :-



Faces : It has 4 faces

Edges : It has 6 edges

Vertices : It has 4 vertices

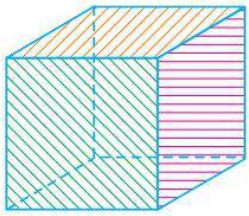
SUMMARY

Sr No.	Name of the solid	Number of faces	Number of edges	Number of vertices
1.	Cuboid	6	12	8
2.	Cube	6	12	8
3.	Cylinder	3	12	NIL
4.	Cone	2	1	1
5.	Sphere	1	NIL	NIL
6.	triangular pyramid	4	6	4

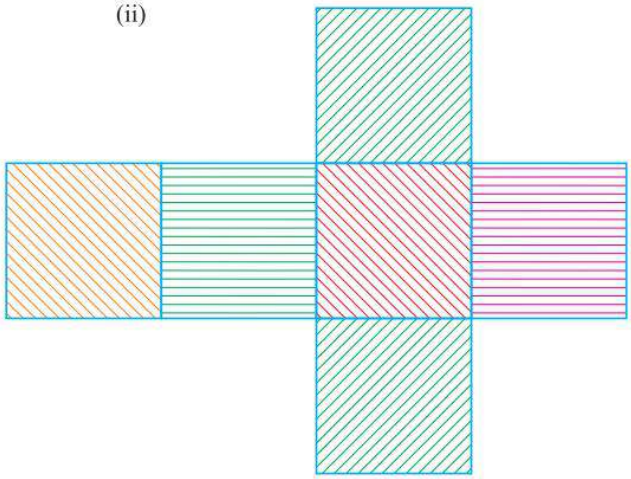
Nets for Building 3 – D shapes : Net is a two dimensional shape that can be folded to form a three dimensional shape or a solid. A solid may have different nets.

The figure (*i*) Shows a cube and figure (*ii*) is net of cube.

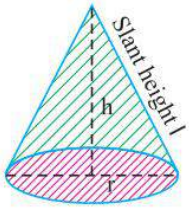
(i)

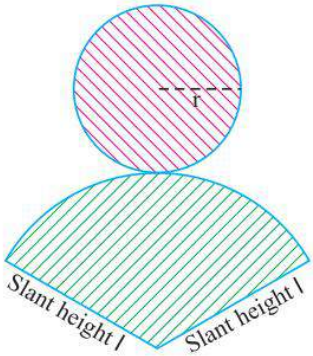


(ii)

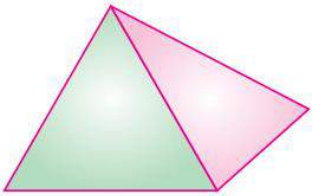


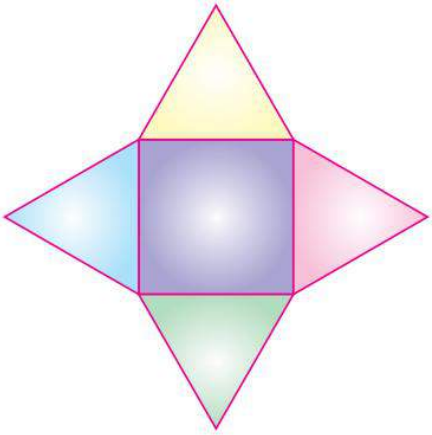
Similarly, you can get a net for a cone by cutting a slit along its slant surface.





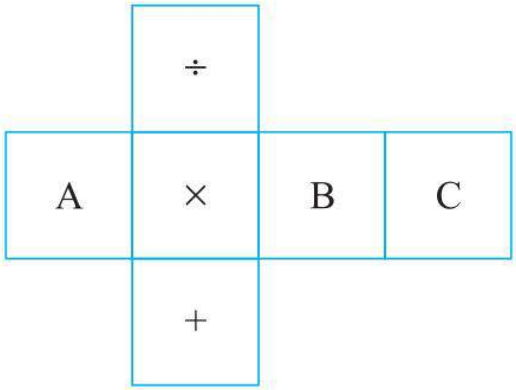
Net of great pyramid of Egypt which has a square base and triangles on the four side is as follows.



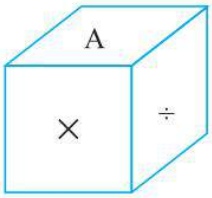


We can also make 3 – D shapes from different nets.

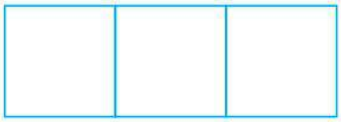
Example-1 : Draw the correct solid shape with the help of the given net.



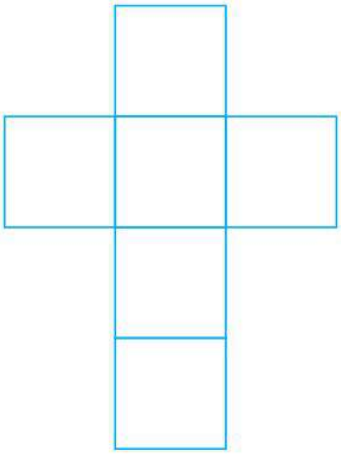
Sol.

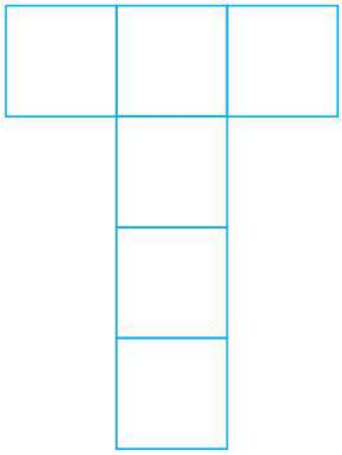


Example-2 : The following net is an in complete net for making a cube complete it in atleast two ways (seprate diagrams) Remember that a cube has six faces.

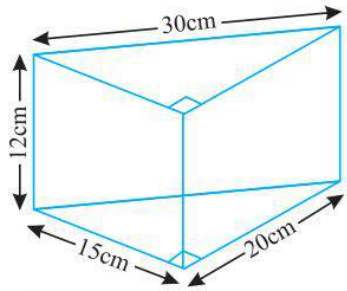


Sol. A cube has six faces so net for making a cube in atleast two different ways are as follows.

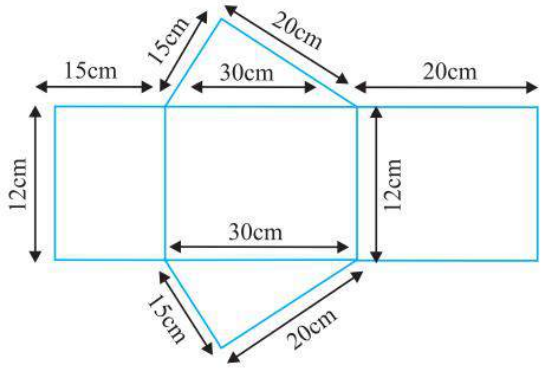




Example-3 : Draw the net of the solid given in the figure

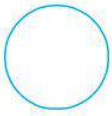
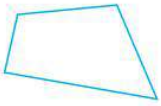


Sol. The net of given solid figure is



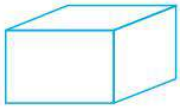
EXERCISE - 15.1

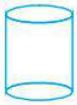
1. Match the two dimensional figure with the names

(*i*)		(*a*)	Square
(*ii*)		(*b*)	Circle
(*iii*)		(*c*)	Quadrilateral
(*iv*)		(*d*)	Triangle
(*v*)		(*e*)	Rectangle

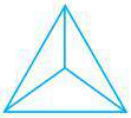
2. Match the three dimension shapes with the names.

(*i*) 

(*ii*) 

(*iii*) 

(*iv*) 

(*v*) 

(*a*) Cylinder

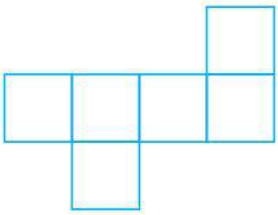
(*b*) Triangular pyramid

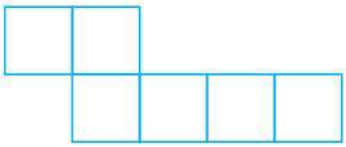
(*c*) Sphere

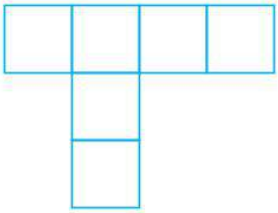
(*d*) Cone

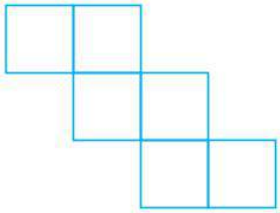
(*e*) Cuboid

3. Identify the nets which can be used to make cubes (cut out copies of the nets and try it)

(i) 

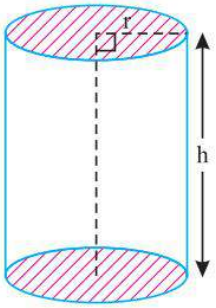
(ii) 

(iii) 

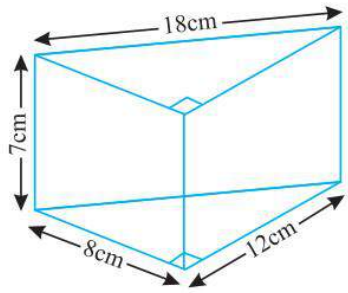
(iv) 

4. Draw the net for a square pyramid with base as square of sides 5*cm* and slant edges 7*cm*.

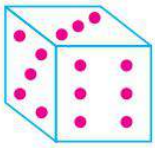
5. Draw a net for the following cylinder.



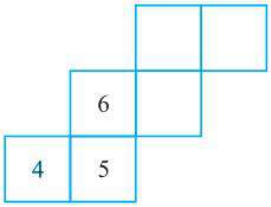
6. Draw the net of the solid given in figure.



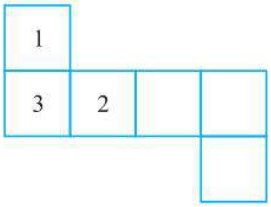
7. Dice are cubes with dots on each face oppsite faces of a die always have a total of seven dots on them following are two nets to make dice (cuber) the number inserted in each square indicate the number of dots in that box insert suitable number in the blank squares.



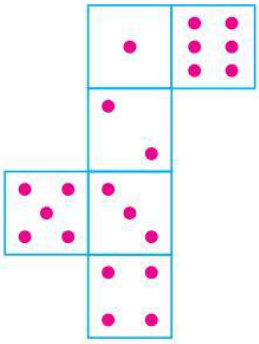
(i)



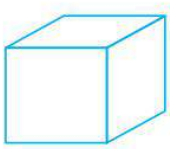
(ii)



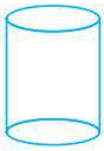
8. Which solid will be obtained by folding the following net.



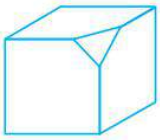
9. Complete the following table



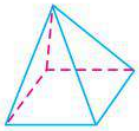
(i)



(ii)



(iii)



(iv)

Face		3		
Edges	12			8
Vertices	8		10	

10. Multiple choice questions

(*i*) Out of following which is 3–D figure ?

(*a*) Square (*b*) Triangle

(*c*) Sphere (*d*) Circle

(*ii*) Total number of faces a cylinder has

(*a*) 0 (*b*) 2

(*c*) 1 (*d*) 3

(*iii*) How many edges are there in a square pyamid ?

(*a*) 5 (*b*) 8

(*c*) 7 (*d*) 4

(*iv*) Sum of number on the opposite faces of a die is

(*a*) 8 (*b*) 7

(*c*) 9 (*d*) 6

(*v*) Which is not a solid figure ?

(*a*) Cuboid (*b*) Sphere

(*c*) Quadrilateral (*d*) Pyramid

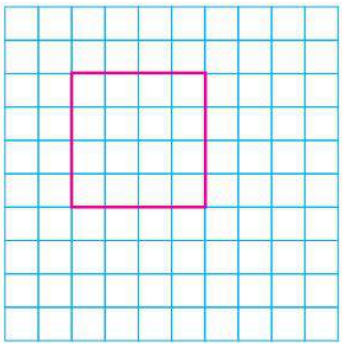
DRAWING SOLIDS ON A FLAT SURFACE

When we draw a solid shape, the images are somewhat distorted because our drawing surface is paper, which is flat. To make them appear three dimensional, there are two ways to draw 3–D figures.

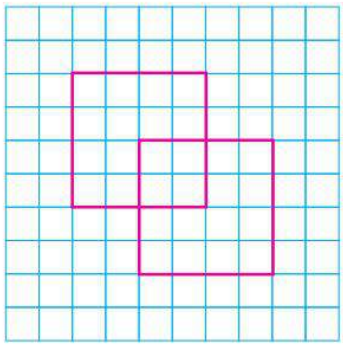
1. Oblique sketches

2. Isometric sketches

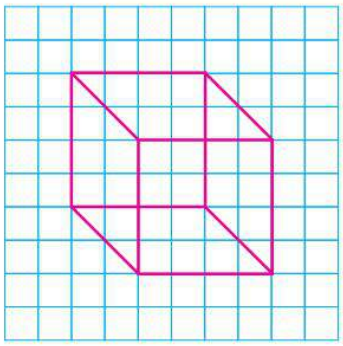
Oblique sketches : The oblique sketching is a pictorial representation of an object, in which the diagram is intended to depict the perspective of object in three dimensions. Following are the steps to draw oblique sketch of a cube.



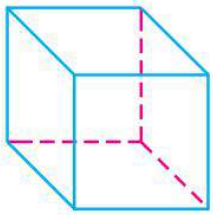
Step 1
Draw a square



Step 2
Draw the second square where the midpoints of two sides of both square coincide



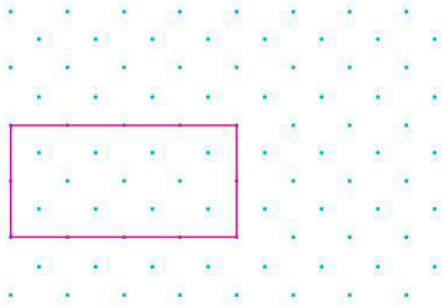
Step 3
Join the corresponding vertices
of both the squares



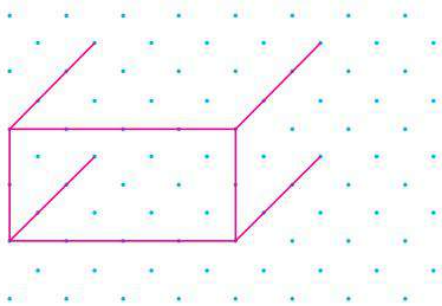
Step 4
Redraw using dotted lines
for hidden edges

Similarly we could make an oblique sketch of cuboid (Remember faces of cuboid are rectangles)

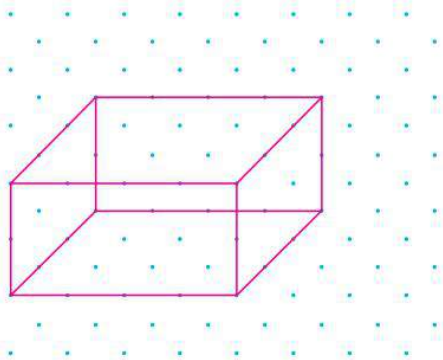
***(ii)* Isometric sketches :** An Isometric sheet is a special sheet on which dots are formed a pattern of equilateral triangles. In an isometric sketch measurement are kept proportional. Following are steps to draw isometric sketch of a cuboid having dimensions $4 \times 3 \times 2$



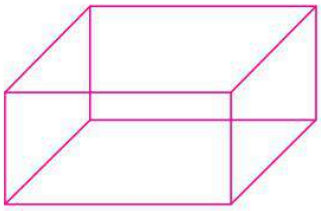
Step 1
Draw a rectangle to depict the front face



Step 2
Draw four parallel line segments
of length starting from each of
the four corners rectangle



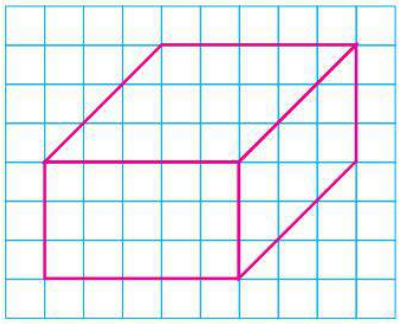
Step 3
Join the matching corners with an
appropriate line segments



Step 4
This is an isometric sketch
of a cuboid

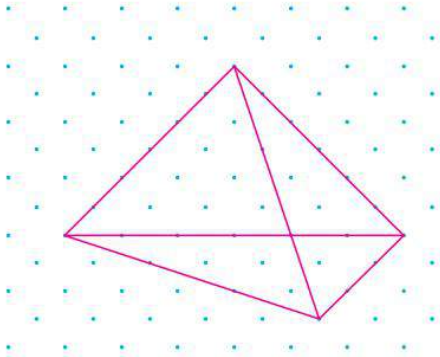
Example-1: Length of a cuboid is 5*cm*, Breadth is 4*cm* and height is 3*cm* draw oblique sketch of this cuboid.

Sol. Oblique sketch of a cuboid with length 5*cm*, breadth 4*cm* and height 3*cm* is as follows.



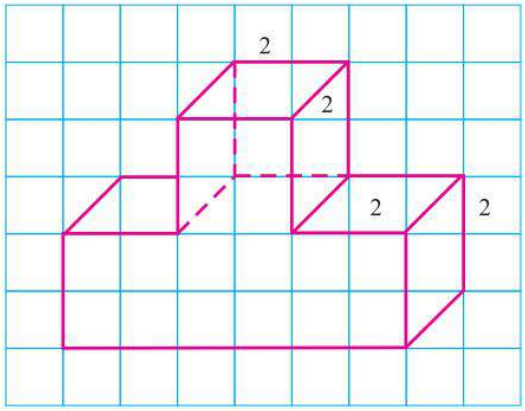
Example-2 : Draw an Isometric sketch of a trianglar pyramid

Sol. Isometric sketch of triangular pyramid is as follows.

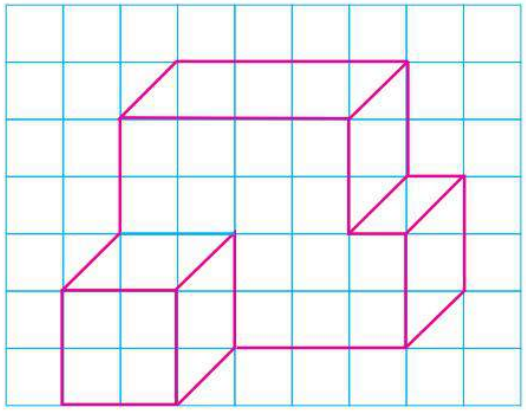


EXERCISE - 15.2

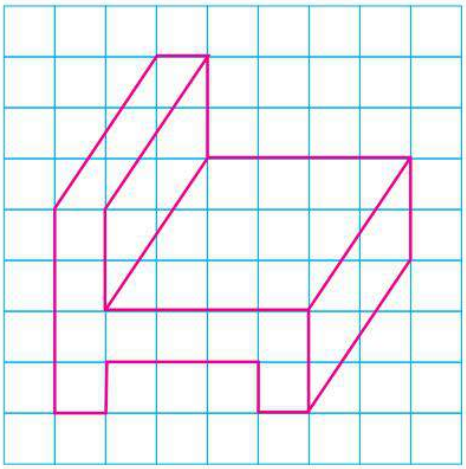
1. Use Isometric dot paper to make an Isometric sketch of the following figures



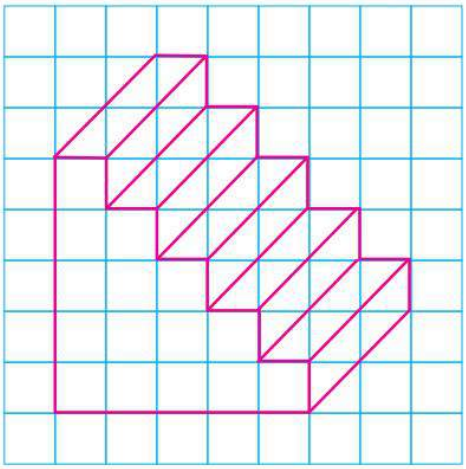
(i)



(ii)



(iii)



(iv)

2. Draw (*i*) an oblique sketch (*ii*) Isometric sketch for

(*a*) A cube with a edge of 4*cm* long

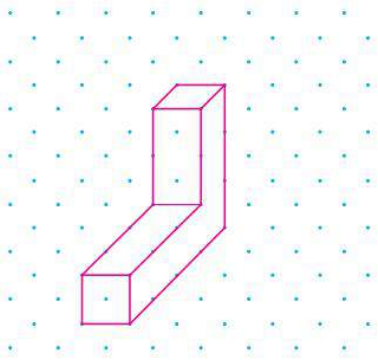
(*b*) A cuboid of length 6*cm*, breadth 4*cm* and height 3*cm*

3. Two cubes each with edge 3*cm* are placed side by side to form a cuboid, sketch oblique and isometric sketch of this cuboid.

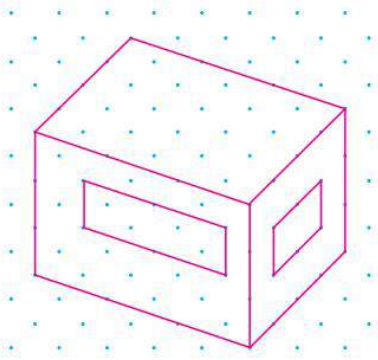
4. Draw an Isometric sketch of triangular pyramid with base as equilateral triangle of 6*cm* and height 4*cm*.

5. Draw an Isometric sketch of square pyramid

6. Make an oblique sketch for each of the given Isometric shapes

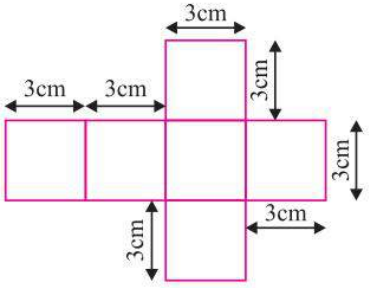


(i)



(ii)

7. Using an isometric dot paper draw the solid shape formed by the given net.



8. Multiple choice questions :-

(*i*) An oblique sheet is made up of

(*a*) Rectangles (*b*) Squares

(*c*) Right angled triangles (*d*) Equilateral triangles

(*ii*) An isometric sheet is made up of dots forming

(*a*) Squares (*b*) Rectangles

(*c*) Equilateral triangles (*d*) Right angled triangle

(*iii*) An oblique sketch has

(*a*) Proportional lengths (*b*) Parallel lengths

(*c*) Non proportional lengths (*d*) Prependicular lengths

(*iv*) An Isometric sketch has.

(*a*) Non propertional lengths (*b*) Parallel lengths

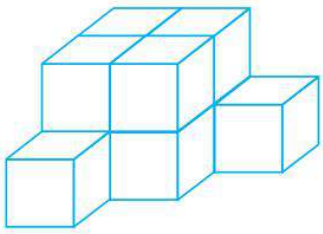
(*c*) Prependicular lengths (*d*) Proporional lengths

(*v*) Isometric sketches shows objects of

(*a*) Two dimensions (*b*) Shadows

(*c*) Three dimensions (*d*) One dimension

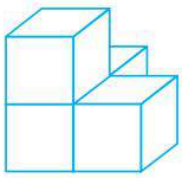
Visualising solid objects : When we look at a solid object, It is not necessary that the whole of it can be seen from one place. The view of the solid, also depend upon the direction from where it is seen. when some combined shapes are viewed, some of the shapes remains hidden from the viewer. There fore visualising solid shapes is a very useful skill through which we can see the hidden parts of a solid shape.



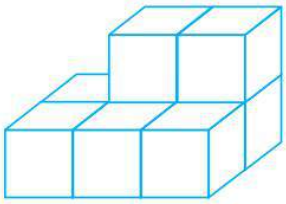
Look at above combined shape and how many cubes do you think have been used to make this structure. A little thinking will help you to find the right answer. This structure contains 10 cubes.

Example-1 : Count the number of cubes in the following structures

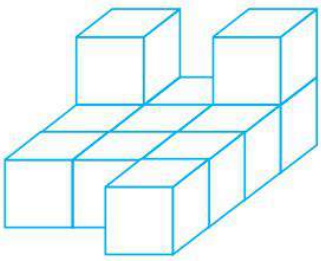
(i)



(ii)



(iii)

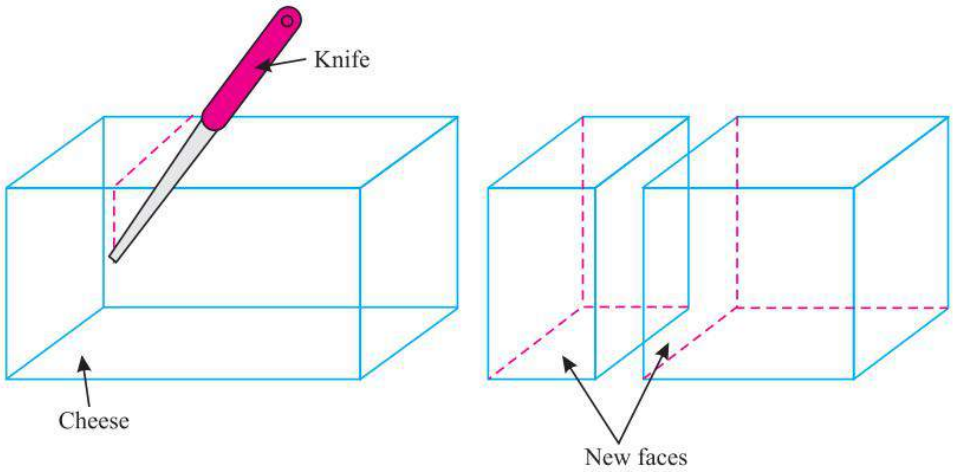


Sol. (*i*) 4 (*ii*) 8 (*iii*) 12

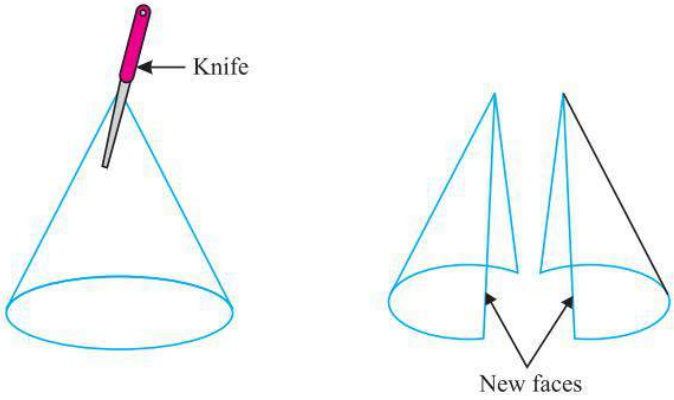
VIEWING DIFFERENT SECTIONS OF A SOLID

Different ways of viewing *a* 3-D object are

1. By cutting or slicing : A solid or *a* 3-D object can be cut into a number of parts. When we cut a 3-D object into two parts by using a knife, we get two new faces of solid. These new faces are called cross-sections of a solid. For example, If we cut a piece of cheese vertically we get two new faces as shown in figure.



Similarly, If we cut a solid cone vertically we get two new faces as shown in figure.



From above it is clear that when we give a cut we get a plane face. This plane face is called a 'Cross Section' and its boundary is a plane curve.

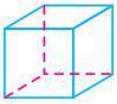
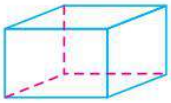
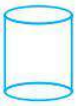
Example-2 : What cross section is made by (*i*) Vertical cut (*ii*) Horizontal cut in the following solid :

(*a*) Cube (*b*) Cuboid

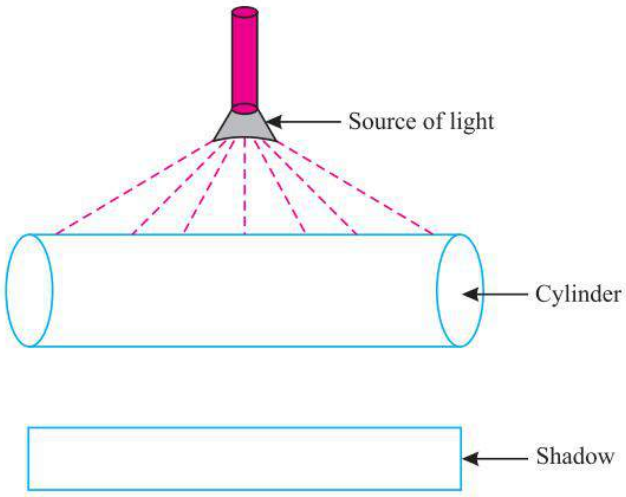
(*c*) Cylinder (*d*) Sphere

(*e*) Cone (*f*) Triangular prism

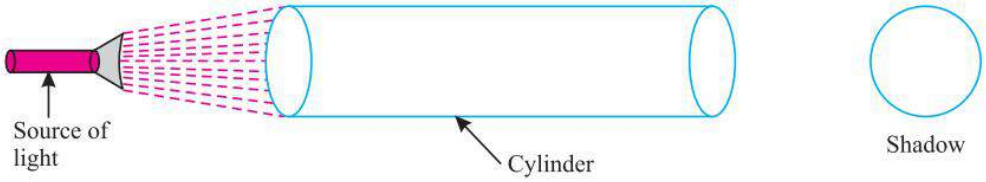
also draw rough sketch of the solids

	Name of solid	Sketch	Vetical Cut	Horizontal Cut
a	Cube		Square	Square
b	Cuboid		Rectangle	Rectangle
c	Cylinder		Rectangle	Circle
d	Sphere		Circle	Circle
e	Cone		Triangle	Circle
f	Triangular Prism		Rectangle	Triangle

2. By shadow of a 3-D object : The shadow of a 3-D object is a 2-D Image. The shadow of an object is not a fixed image with a change in the position of the source of light, the shadow of the object changes. If we throw light on the cylinder from the top we get a shadow in the shape of a rectangle.



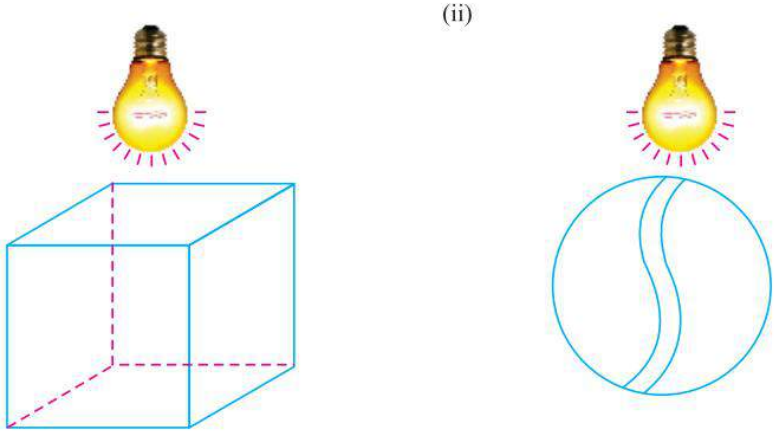
But, if we throw the light on a cylinder from left, we get the shadow in the shape of a circle.



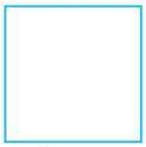
We observe that the shadow of an object depends not only on different position of the solid but also on the position of the source of light.

Example-3 : If we throw light on the following solids from the top. Name the shape of shadow obtained in each case and also give a rough sketh of the shadow.

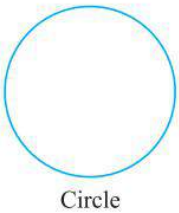
(i) (ii)



Sol. (*i*) Shadow of cube looks like a square.

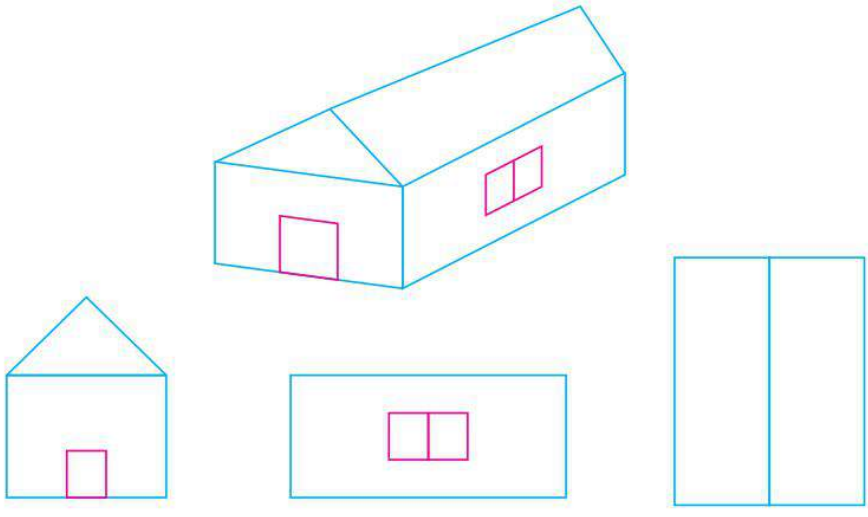


(*ii*) The shape of shadow of a ball looks like a circle.

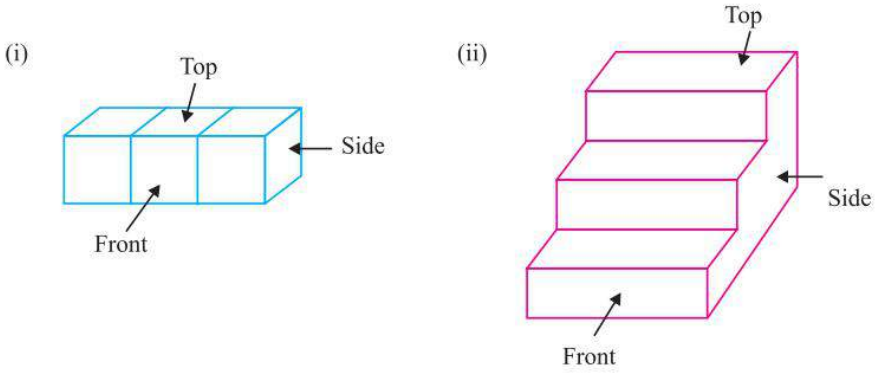


3. By looking at a solid from certain angles : Another way is to look at the shape from different angles i.e; the front view, the side view and the top view, which can provide a lot of information about the shape observed.

We can see three views i.e; front, side and top views of a house as follows.

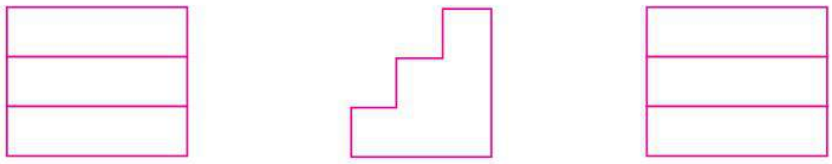


Example-4 : For the given solids sketch the front side and top view.



Sol. (i)

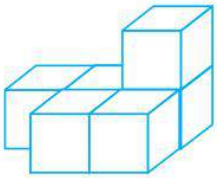
(ii)



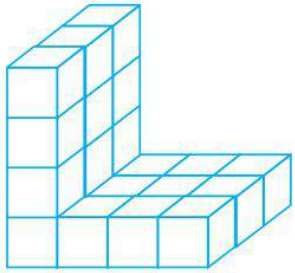
EXERCISE - 15.3

1. Count the number of cubes in each of the following figures.

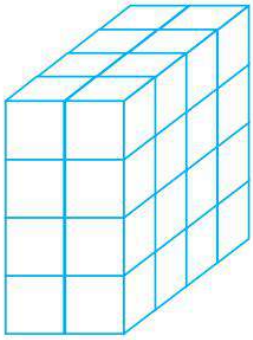
(i)



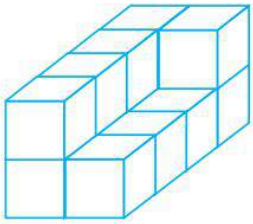
(ii)



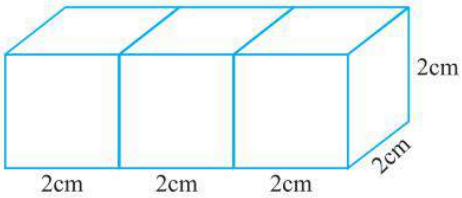
(iii)



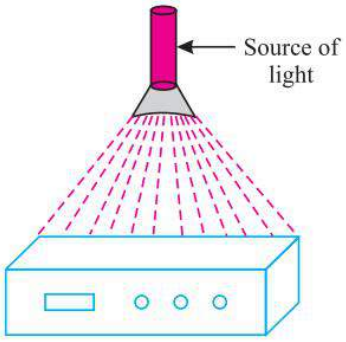
(iv)



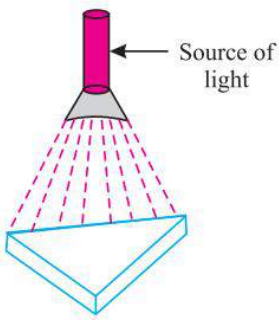
2. If three cubes of dimensions $2cm \times 2cm \times 2cm$ are placed side by side, what would be the dimensions of resulting cuboid ?



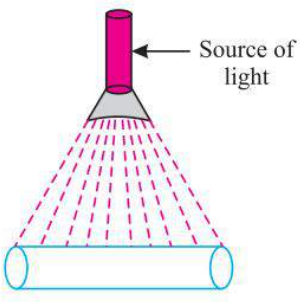
3. If we throw light on the following solids from the top name the shape of shadow obtained in each case and also give a rough sketch of the shadow.



(i) DVD player



(ii) Sandwich



(iii) Straw

3. What cross-sections do you get when you given a.

(*i*) Vertical cut (*ii*) Horizontal cut

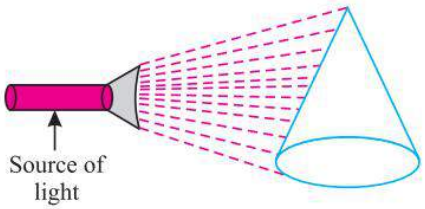
to the following solids ?

(*a*) A die (*b*) A square pyramid

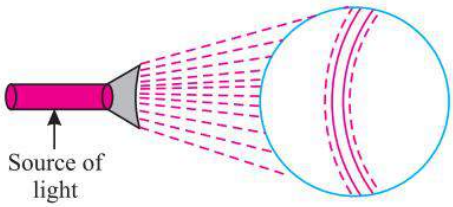
(*c*) A round melon (*d*) A circular pipe

(*e*) A brick (*f*) An ice cream cone

5. If we throw light on following solids, from left name the shape of shadow in each and also give a rough sketch of the shadow.

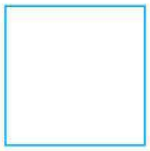


A cone



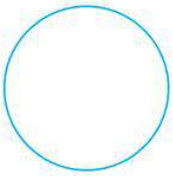
A ball

6. Here are the shadows of some 3 – D objects, when seen under the lamp of an overhead projector. Identify the solids that match each shadow (There may be multiple answers for these)

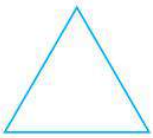


A Square

A Rectangle



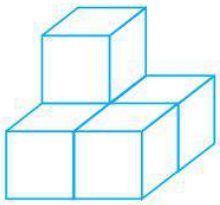
A Circle



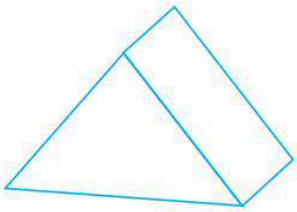
A Triangle

7. Sketch the front, side and top view of the following figures.

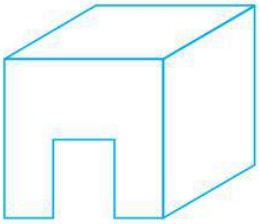
(i)



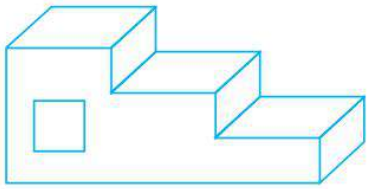
(ii)



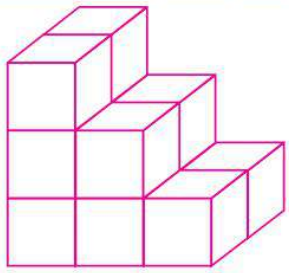
(iii)



(iv)



8. Multiple choice questions :-



(*i*) The number of cubes in the given structure is ?

(*a*) 12

(*b*) 10

(*c*) 9

(*d*) 8

(*ii*) The number of unit cubes to be added in above students to make a cuboid of dimensions 4 unit × 2 unit × 3 unit is ?

(*a*) 11 (*b*) 12

(*c*) 13 (*d*) 14

(*iii*) What cross-section is made by vertical cut in a cuboid

(*a*) Square (*b*) Rectangle

(*c*) Circle (*d*) Triangle

(*iv*) What cross section is made by horizontal cut in a cone

(*a*) Triangle (*b*) Circle

(*c*) Square (*d*) Rectangle

(*v*) Which solid cost a shadow of triangle under the effect of light

(*a*) Sphere (*b*) Cylinder

(*c*) Cone (*d*) Cube

WHAT HAVE WE DISCUSSED ?

1. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of plane figures; the cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of solid shapes.

2. Plane figures are two dimensional (2–D) and solid shapes are three dimensional (3–D)

3. A face is a flat surface, An edge is where two faces meet and a vertex is a corner where edges meet and the plural of vertex is vetices.

4. Net is a two dimensional shape that can be folded to form a three dimensional shape or a solid. A solid can have several type of nets

5. Solid shapes can be drawn on a flat surface i.e, on paper, as 2–D representation of a 3–D shape. The two methods to draw sketches of 3–D shapes are :-

(*i*) **Oblique sketch :** An oblique sketch does not have proportional lengths, still it conveys all important aspects of the appearance of a solid.

(*ii*) An isometric sheet is a special sheet on which dots are formed on a pattern of equilateral triangles. In an isometric sketch measurement are kept proportional.

6. Visualising solid shapes is a very usefull skill through which we can seen the hidden parts of a solid shape. Different ways of viewing a 3–D object are :-

(*i*) **By cutting or slicing :** A solid or a 3–D object can be cut into a number of parts. When we cut it into two parts, we get two new faces called crossections of a solid.

(*ii*) **By shadow of a 3–D object :** In this way we observe a 2–D shadow of a 3–D shape.

(*iii*) **By looking at a solid from certain angles :** In this way we look at the shape from different angles, i.e. the front view, the side view and the top view. Which can provide a lot of information about the shape observed.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Relate 2–D shapes with 3–D shapes.
2. Identify and draw nets of various 3–D shapes and are able to form solids using these nets.
3. Count the faces, edges and vertices of solids (3–D Shapes)
4. Draw 3–D shapes on a flat surface by oblique and isometric sketch method.
5. See the hidden parts of a solid by cutting or slicing and will also be able to visualize solids from different angles.
6. Apply their knowledge about shapes in their day to day life.

ANSWERS

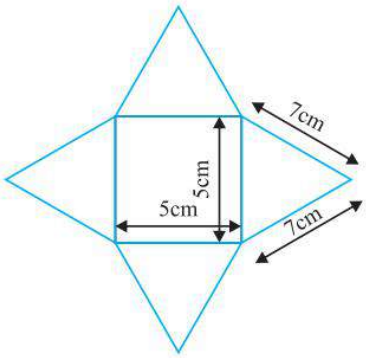
EXERCISE 15.1

1. (*i*) *e* (*ii*) *d*
(*iii*) *a* (*iv*) *b*
(*v*) *c*

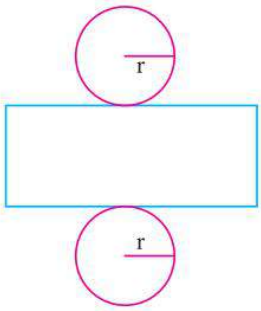
2. (*i*) *d* (*ii*) *e*
(*iii*) *a* (*iv*) *c*
(*v*) *b*

3. (*i*), (*iv*)

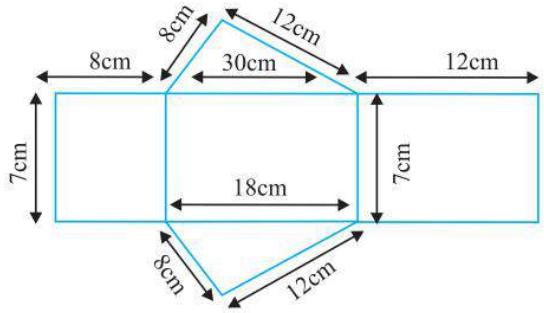
4.



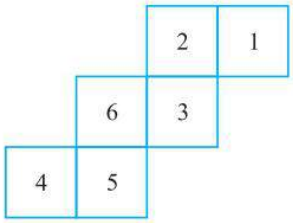
5.



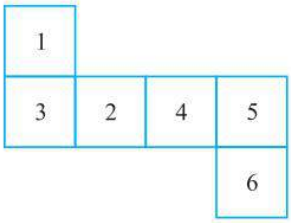
6.



7. (i)



(ii)



8. Dice 

9. (*i*) Faces : 6 (*ii*) Edges : 2, vertices : NIL (*iii*) Faces : 7, Edges : 15 (*iv*) faces : 5, vertices : 5

10. (*i*) *c* (*ii*) *d* (*iii*) *b* (*iv*) *b* (*v*) *c*

EXERCISE 15.2

8. (*i*) *b* (*ii*) *c* (*iii*) *c*
(*iv*) *d* (*v*) *c*

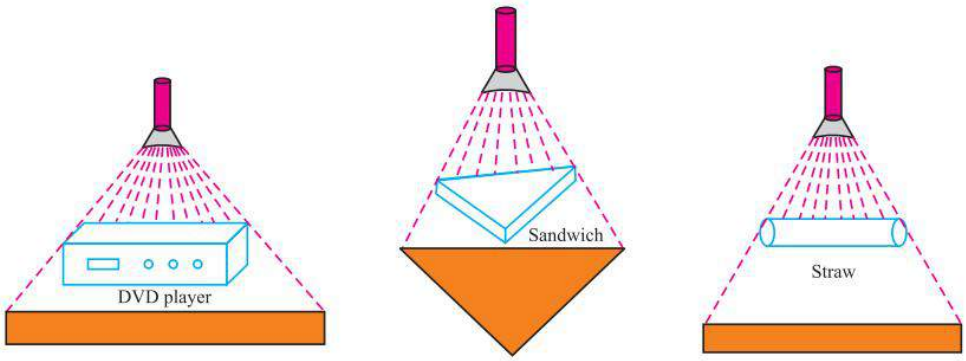
EXERCISE 15.3

1. (*i*) 6 (*ii*) 21
(*iii*) 32 (*iv*) 13

2. Length 6*cm*, breadth 2*cm* and height 2*cm* (3) (*a*) Svuare, Svuare (*b*)Triangle, Svuare.

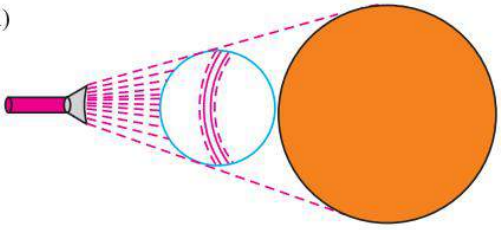
3. Circle, circle (*d*) Circle, Rectangle (*e*) Rectangle, Rectangle (*f*) Triangle, Circle

4.

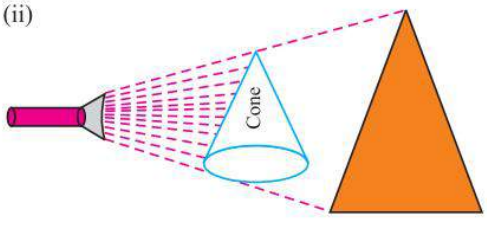


(i) Rectangle (ii) Triangle (iii) Rectangle

5. (i)

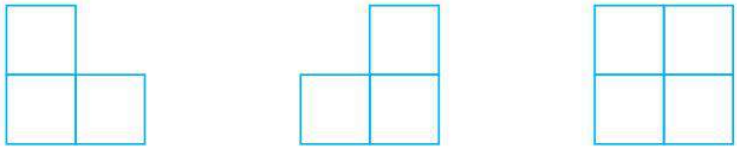


(ii)



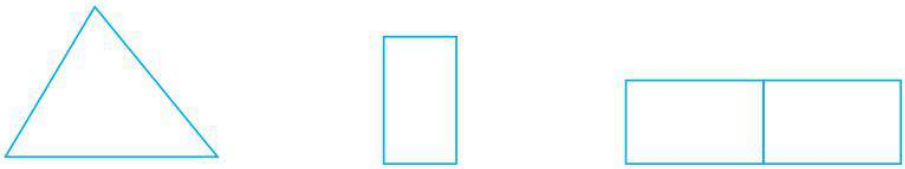
6. (*i*) Dice, chalk box etc (*ii*) Book, Mobile, DVD player etc
(*iii*) Cricket ball, Disc etc (*iv*) Birthday cap, Icecream cone etc.

7. (i)



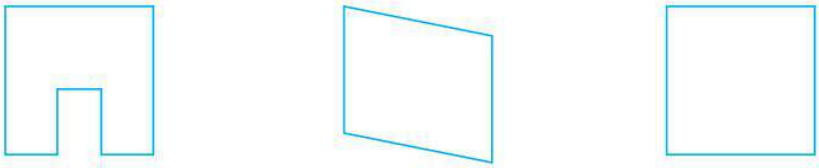
Front view Side view Top view

(ii)



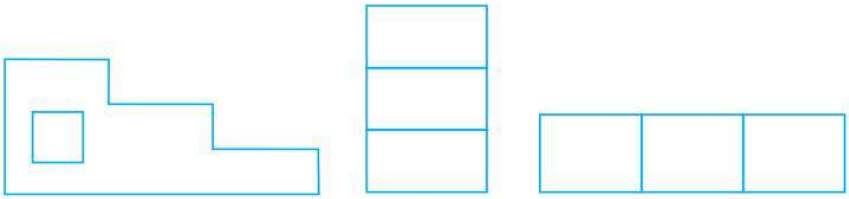
Front view Side view Top view

(iii)



Front view Side view Top view

(iv)



Front view Side view Top view

8. (*i*) *a* (*ii*) *b* (*iii*) *b*
(*iv*) *b* (*v*) *c*

ISOMETRIC SHEET

ISOMETRIC SHEET