CBSE Class 12 - Mathematics

Sample Paper 05 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks

ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions

iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.

ii. Section I comprises of 16 very short answer type questions.

iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.

ii. Section III comprises of 10 questions of 2 marks each.

iii. Section IV comprises of 7 questions of 3 marks each.

iv. Section V comprises of 3 questions of 5 marks each.

v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Show that an onto function $f : \{1, 2, 3\} \longrightarrow \{1, 2, 3\}$ is always one-one.

OR

Let the function f : R → R be defined by f(x) = 4x - 1, ∀ x ∈ R. Then, show that f is one-one.

2. Let f : R → R be defined by

$$f(x) = \begin{cases} 2x+3, & \text{when } x < -2 \\ x^2-2, & \text{when } -2 \le x \le 3 \\ 3x-1, & \text{when } x > 3 \end{cases}$$

Find f(4)

OR

A Relation R : A→A is said to be Symmetric if ________ ∀ a, b,∈A

3. Show that the function given by f(x) = 3x + 17 is strictly increasing on R.
4. Give an example of matrices A and B such that A ≠ O, B ≠ O, AB = O and BA ≠ O.
5. Construct a 2 × 3 matrix A = [a_{ij}] whose elements a_{ij} are given by:

a_{ij} = i + j

OR

Construct a 2×3 matrix A = [a_{ij}] whose elements are given by a_{ij} = $\frac{i-j}{i+j}$.

6. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor of the element a_{23}.
7. Evaluate: $\int \frac{dx}{\sqrt{2-4x+x^2}}$

OR

Evaluate: $\int \sin^3 x \cos x \, dx$

8. Find the area above x-axis ,bounded by the line x = 4 and the curve y = f(x) ,where f(x) $= x^2, 0 \le x \le 1$ and f(x) =$\sqrt{x}, x \ge 1$
9. Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{1}{dy/dx} = 2.$

OR

Find the general solution for differential equation: $\frac{dy}{dx} = \frac{x-1}{y+2}$.

10. Find a vector of magnitude 9, which is perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$.

11. If $\overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{BO} + \overrightarrow{OC}$, prove that A, B, C are collinear points.

12. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 6$, Find $|\vec{a} + \vec{b}|$ and $|\vec{a} - \vec{b}|$

13. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

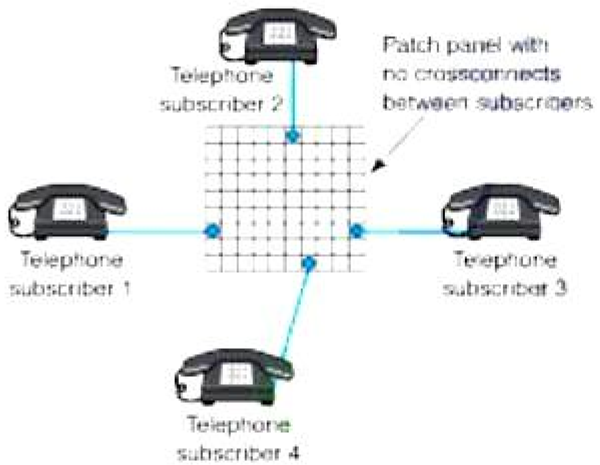
14. Show that the line $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$. Also, find the distance between the given line and the given.

15. If P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$ find P $(A \cap B)$ if A and B are independent events.

16. 7 coins are tossed simultaneously. What is the probability that a tail appears an odd number of times?

Section - II

17. A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increases the annual subscription and it is believed that for every increase of 1 one subscriber will discontinue the service.



i. If x be the annual subscription then the total revenue of the company after increment will be:
   a. $R(x) = -x^2 + 200x + 150000$
   b. $R(x) = x^2 - 200x - 140000$
   c. $R(x) = 200x^2 + x + 150000$
   d. $R(x) = -x^2 + 100x + 100000$

ii. To find maximum profit we put
   a. $R'(x) = 0$

b. $R'(x) > 0$

c. $R'(x) < 0$

d. $R''(x) = 0$

iii. How much fee the company should increase to have maximum profit?

a. Rs. 150

b. Rs. 100

c. Rs. 200

d. Rs. 250

iv. Find the maximum profit that the company can make if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

a. 25

b. 44

c. 45

d. 49

v. Find both the maximum and minimum value respectively of $3x^4 - 8x^3 + 48x + 1$ on the interval [1, 4].

a. -63, 257

b. 258, -63

c. 257, -63

d. -63, -257

18. Three bags contain a number of red and white balls as follow:

Bag 1: 3 red balls,

Bag 2: 2 red balls and 1 white ball

Bag 3: 3 white balls.

The probability that bag **i** will be chosen and a ball is selected from it is: $\frac{i}{6}$ where i = 1,2,3.

i. What is the probability that a red ball will be selected?

a. $\frac{5}{9}$
b. $\frac{7}{18}$
c. $\frac{5}{18}$
d. $\frac{11}{18}$

ii. What is the probability that a white ball is selected?

a. $\frac{18}{11}$
b. $\frac{11}{18}$
c. $\frac{13}{18}$
d. $\frac{7}{18}$

iii. If a white ball is selected, what is the probability that it came from Bag 2?

a. $\frac{2}{11}$
b. $\frac{11}{2}$
c. $\frac{12}{11}$
d. $\frac{7}{11}$

iv. If a white ball is selected, what is the probability that it came from Bag 3

a. $\frac{5}{11}$
b. $\frac{11}{9}$
c. $\frac{7}{11}$
d. $\frac{9}{11}$

v. If a white ball is selected, what is the probability that it came from Bag 1

a. 1
b. $\frac{11}{18}$
c. $\frac{2}{11}$
d. 0

Part - B Section - III

19. Find the principal value of $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$.

20. Show that $\begin{vmatrix} \sin 10^0 & -\cos 10^0 \\ \sin 80^0 & \cos 80^0 \end{vmatrix} = 1$

OR

Let $\begin{vmatrix} 4 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 4 & 1 \end{vmatrix}$,find all the possible values of x and y if x and y are natural numbers.

21. Prove that the function f defined by $f(x)=\begin{cases}\frac{x}{|x|+2x^2}, if\, x\neq 0\\ k, if\, x=0\end{cases}$ remains discontinuous at x = 0, regardless the choice of k.

22. The length x of a rectangle is decreasing at the rate of 3 cm/ min and the width y is increasing at the rate of 2cm/min. when x = 10cm and y = 6cm, find the ratio of change of (a) the perimeter (b) the area of the rectangle.

23. Evaluate $\int \frac{\sqrt{1+x^2}}{x^4}dx$

OR

Evaluate: $\int$ sin 3x sin 2x dx

24. Find the area of the region bounded by the curve y^2 = 2y - x and the Y-axis.

25. Solve the differential equation $x\cos ydy=(xe^x\log x+e^x)\,dx$

26. For any two vectors $\vec{a}$ and $\vec{b}$, prove that: $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$

27. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

28. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards.

What is the probability that first two cards are kings and the third card drawn is an ace?

OR

Prove that if A and B are independent events then, $P(A)=P(A\cap B)+P\left(A\cap\overline{B}\right)$

Section - IV

29. Let L be the set of all lines in plane and R be the relation in L define if R = {(L_1, L_2) : L_1 is $\perp$ to L_2}. Show that R is symmetric but neither reflexive nor transitive.

30. If sin y = x sin (a + y), prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

31. Prove that the function $f(x)=\begin{cases}\frac{\sin x}{x}, & x<0\\ x+1, & x\geq 0\end{cases}$ is everywhere continuous.

OR

If y = $\cos^{-1}(2x)$ + 2 $\cos^{-1}\sqrt{1-4x^2}$, $-\frac{1}{2}<x<0$, find $\frac{dy}{dx}$.

32. Find the equation of the tangent line to the curve $= \sqrt{5x-3} - 2$ is parallel to the line 4x - 2y + 3 = 0.

33. Evaluate $\int e^{-3x} \cos^3 x dx$

34. Find the area of the segment of the parabola $y = x^2 - 5x + 15$ cut off by the straight line y = 3x + 3.

OR

Find the area lying above the x-axis and under the parabola y = 4 x - x^2

35. Solve the initial value problem: $xe^{y/x}$ - y sin $\left(\frac{y}{x}\right)$ + $x\frac{dy}{dx}$ sin $\left(\frac{y}{x}\right)$ = 0, y(1) = 0

Section - V

36. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A + 4I = 0$ and hence find A^{-1}

OR

Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

37. Prove that if a plane has the intercepts a,b,c is at a distance of p units from the origin then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

OR

Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect Also, find the equation of the plane containing them.

38. Solve the Linear Programming Problem graphically:

Maximize Z = 3x + 2y subject to $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0$

OR

Minimise Z = 400x + 200y ,subject to,$5x + 2y \geqslant 30, 2x + y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0$

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Solution

Part - A Section - I

1. Suppose f is not one-one.

   Then there exists two elements, say 1 and 2 in the domain whose image in the co-domain is the same.

   Also, the image of 3 under f can be only one element.

   Therefore, the range set can have at the most two elements of the co-domain {1, 2, 3}, showing that f is not onto, a contradiction. Hence, f must be one-one.

OR

For any two elements $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$, we have

$4x_1 - 1 = 4x_2 - 1$

$\Rightarrow 4x_1 = 4x_2$, i.e., $x_1 = x_2$

Hence f is one-one.

2. The given function is $f(x) = \begin{cases} 2x+3, & \text{when } x < -2 \\ x^2-2, & \text{when } -2 \le x \le 3 \\ 3x-1, & \text{when } x > 3 \end{cases}$

   Since f(x) = 3x - 1, when x = 4

   $\therefore f(4) = (3 \times 4) - 1 = 12 - 1 = 11$

   $\therefore f(4) = 11$.

OR

(a, b) $\in$ R,

$\Rightarrow$ (b, a) $\in$ R

3. Given: $f(x) = 3x + 17$

   $\therefore f'(x) = 3(1) + 0 = 3 > 0$ i.e., positive for all $x \in R$

   Therefore, f(x) is strictly increasing on R.

4. We choose A, and B, such that

A = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ and B = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}$, then A $\neq$ O, B $\neq$ O.

But, AB = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ = O

and, BA = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} \neq$ O

5. $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2$, $a_{12} = 1 + 2 = 3$, $a_{13} = 1 + 3 = 4$

$a_{21} = 2 + 1 = 3$, $a_{22} = 2 + 2 = 4$, $a_{23} = 2 + 3 = 5$

therefore,

A = $\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\end{bmatrix}$

OR

We have, $a_{ij} = \frac{i-j}{i+j}$, $1 \leq j \leq 3$. Therefore,

$a_{11} = 0$, $a_{12} = -\frac{1}{3}$, $a_{13} = -\frac{1}{2}$,

$a_{21} = \frac{1}{3}$, $a_{22} = 0$ and $a_{23} = -\frac{1}{5}$

$\therefore$ A = $\begin{bmatrix}0 & -\frac{1}{3} & -\frac{1}{2}\\\frac{1}{3} & 0 & -\frac{1}{5}\end{bmatrix}$

6. Minor of the element $a_{23} = \begin{vmatrix}5 & 3\\1 & 2\end{vmatrix} = 10 - 3 = 7$

7. We have

$\int \frac{dx}{\sqrt{2-4x+x^2}} = \int \frac{dx}{\sqrt{x^2-4x+4-2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}}$

$= \log|(x-2)+\sqrt{(x-2)^2-2}+C$

$= \log|x-2+\sqrt{x^2-4x+2}+C$ ......ANSWER

OR

Let sin x = t

$\Rightarrow$ cosx dx = dt

$\Rightarrow \int \sin^3 x \cos x\, dx = \int t^3\, dt$

$\Rightarrow \frac{t^4}{4} + c$

Resubstituting the value of t = sin x we get

$\Rightarrow \frac{\sin^4 x}{4} + c$

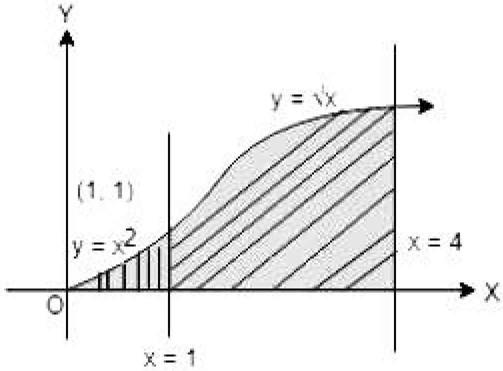
8. Required area = the area above x-axis ,bounded by the line x = 4 and the curve y = f(x) ,where f(x) $= x^2, 0 \leq x \leq 1$ and f(x)=$\sqrt{x}, x \geq 1$

$= \int_0^1 x^2 \, dx + \int_1^4 \sqrt{x} \, dx$

$= \left(\frac{x^3}{3}\right)_0^1 + \left(\frac{2x^{3/2}}{3}\right)_1^4$

$= \frac{1}{3} + \frac{2}{3}(8-1)$

= 5 sq units



9. $\left(\frac{dy}{dx}\right)^2 + \frac{1}{dy/dx} = 2$

Multiply $\frac{dy}{dx}$ both sides, we get,

$\left(\frac{dy}{dx}\right)^3 + 1 = 2\frac{dy}{dx}$

$\left(\frac{dy}{dx}\right)^3 - 2\frac{dy}{dx} + 1 = 0$

So, the degree of the given differential equation is 3.

OR

The given differential equation can be rewritten as,

(y + 2)dy = (x - 1)dx

Integrating on both sides,

$\int(y+2)dy = \int(x-1)dx$

$\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + c$

$y^2 + 4y - x^2 + 2x = c$

10. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. Then vector perpendicular to both vectors a & b is ;

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 3\\ -2 & 1 & -2\end{vmatrix}=(2-3)\hat{i}-(-8+6)\hat{j}+(4-2)\hat{k}=-\hat{i}+2\hat{j}+2\hat{k}$$

$\Rightarrow |\vec{a}\times\vec{b}|=\sqrt{(-1)^2+2^2+2^2}=3$

$\therefore$ Required vector $=9\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\frac{9}{3}(-\hat{i}+2\hat{j}+2\hat{k})=-3\hat{i}+6\hat{j}+6\hat{k}$

11. Given,

$\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$

$\Rightarrow \overrightarrow{AO}-\overrightarrow{BO}=\overrightarrow{OC}-\overrightarrow{OB}$

$\Rightarrow \overrightarrow{OB}-\overrightarrow{OA}=\overrightarrow{OC}-\overrightarrow{OB}$

$\Rightarrow \overrightarrow{AB}=\overrightarrow{BC}$

Hence A, B and C are collinear points.

12. We know that $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+2(\vec{a}\cdot\vec{b})+|\vec{b}|^2$ and $|\vec{a}-\vec{b}|^2=|\vec{a}|^2-2(\vec{a}\cdot\vec{b})+|\vec{b}|^2$

$\Rightarrow |\vec{a}+\vec{b}|^2=9+12+4$ and $|\vec{a}-\vec{b}|^2=9-12+4$

$\Rightarrow |\vec{a}+\vec{b}|^2=25$ and $|\vec{a}-\vec{b}|^2=1$

$\Rightarrow |\vec{a}+\vec{b}|=5$ and $|\vec{a}-\vec{b}|=1$

13. According to the question, the normal to the plane is equally inclined with coordinates axes, and the distance of the plane from origin is $5\sqrt{3}$ units

$\therefore$ the direction cosines are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$

The required equation of plane is

$\frac{1}{\sqrt{3}}\cdot x+\frac{1}{\sqrt{3}}\cdot y+\frac{1}{\sqrt{3}}\cdot z=5\sqrt{3}$

$\Rightarrow x+y+z=5\times 3$

$\Rightarrow \quad x+y+z=15$

[$\because$ If l, m and n are direction cosines of normal to the plane and P is a distance of a plane from origin, then the equation of plane is given by $lx+my+nz=p$]

14. The given line is in the form $\vec{r}=\vec{a}+\lambda\vec{b}$ and the given plane is in the form $\vec{r}\cdot\vec{n}=q$, where

$\vec{a}=(2\hat{i}-2\hat{j}+3\hat{k}),\vec{b}=(\hat{i}-\hat{j}+4\hat{k}),\vec{n}=(\hat{i}+5\hat{j}+\hat{k})$ and q = 5

We know that the line $\vec{r}=\vec{a}+\lambda\vec{b}$ is parallel to the plane $\vec{r}\cdot\vec{n}=q$ only when $\vec{b}\cdot\vec{n}=0$

Here, $\vec{b}\cdot\vec{n}=(\hat{i}-\hat{j}+4\hat{k})\cdot(\hat{i}+5\hat{j}+\hat{k})$

$=(1\times 1)+(-1)\times 5+4\times 1=0$

Therefore, the given line is parallel to the given plane.

Distance between the given line and the given plane

$$= \frac{|\vec{a}\cdot\vec{n}-q|}{|\vec{n}|} = \frac{|(2\hat{i}-2\hat{j}+3\hat{k})\cdot(\hat{i}+5\hat{j}+\hat{k})-5|}{|\hat{i}+5\hat{j}+\hat{k}|}$$

$$= \frac{|(2\times1)+(-2)\times5+(3\times1)-5|}{\sqrt{1^2+5^2+1^2}}$$

$$= \frac{|2-10+3-5|}{\sqrt{27}} = \frac{10}{3\sqrt{3}} \text{ units.}$$

Therefore,the required distance between the line and the plane is $\frac{10}{3\sqrt{3}}$ units.

15. As, A and B are independent events.

Therefore, $P(A\cap B)$ = P (A). P (B)= $\frac{3}{5}\times\frac{1}{5}=\frac{3}{25}$

16. As 7 coins are tossed simultaneously, therefore the total number of outcomes are $2^7 = 128$

The favourable number of outcomes that a tail appears an odd number of times will be,

${}^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7 = 64$

Therefore, required probability

$$= \frac{\text{The favourable outcomes}}{\text{The total number of outcomes}}$$

$$= \frac{64}{128}$$

$$= \frac{1}{2}$$

Section - II

17. i. (a) $-x^2 + 200x + 150000$

ii. (a) R'(x) = 0

iii. (b) Rs.100

iv. (d) 49

v. (c) 257, -63

18. Let E_2 , E_3 be the events when bag 2, 3 are selected and ball is chosen from them.

and Let F is the event of getting a White ball.

$P(E_2) = \frac{2}{6}$

$P(E_3) = \frac{3}{6}$

Using Baye's theorem, we have

i. (b)

Let E be the event that a red ball is selected. Then, Probability that red ball will be selected.

$$P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right)$$

$= \frac{1}{6} \cdot \frac{3}{3} + \frac{2}{6} \cdot \frac{2}{3} + \frac{3}{6} \cdot 0$
$= \frac{1}{6} + \frac{2}{9} + 0$
$= \frac{3+4}{18} = \frac{7}{18}$

ii. (b)

Let F be the event that a white ball is selected.

$\therefore P(F) = P(E_1) \cdot P\left(\frac{F}{E_1}\right) + P(E_2) \cdot P\left(\frac{F}{E_2}\right) + P(E_3) \cdot P\left(\frac{F}{E_3}\right)$
$= \frac{1}{6} \cdot 0 + \frac{2}{6} \cdot \frac{1}{3} + \frac{3}{6} \cdot 1 = \frac{1}{9} + \frac{3}{6} = \frac{11}{18}$

Note: P(F) = 1 - P(E) $= 1 - \frac{7}{18} = \frac{11}{18}$ [since, we know that P(E) + P(F) = 1]

iii. (a)

$P(E_2/F) = \frac{P(E_2) \cdot P(F/E_2)}{P(E_1) \cdot P(F/E_1) + P(E_2) \cdot P(F/E_2) + P(E_3) \cdot P(F/E_3)}$
$= \frac{\frac{2}{6} \cdot \frac{1}{3}}{\frac{1}{6} \cdot 0 + \frac{2}{6} \cdot \frac{1}{3} + \frac{3}{6} \cdot 1} = \frac{\frac{2}{18}}{\frac{2}{18} + \frac{3}{6}}$
$= \frac{2/18}{\frac{2+9}{18}} = \frac{2}{11}$

iv. (d)

$P(E_3/F) = \frac{P(E_3) \cdot P(F/E_3)}{P(E_1) \cdot P(F/E_1) + P(E_2) \cdot P(F/E_2) + P(E_3) \cdot P(F/E_3)}$
$= \frac{\frac{3}{6} \cdot 1}{\frac{1}{6} \cdot 0 + \frac{2}{6} \cdot \frac{1}{3} + \frac{3}{6} \cdot 1}$
$= \frac{\frac{3}{6}}{\frac{2}{18} + \frac{3}{6}} = \frac{3/6}{\frac{2}{18} + \frac{9}{18}} = \frac{9}{11}$

v. (d) 0

Part - B Section - III

19. $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$
$= \cos^{-1}\left(\tan\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right)$
$= \cos^{-1}(-1)$

For any $x \in [-1, 1]$, $\cos^{-1}x$ represents an angle in $[0, \pi]$ whose cosine is x.

$\cos^{-1}(-1) = \pi$

$\therefore$ Principle value of $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$ is π.

20. $= \sin 10^o \cos 80^o + \cos 10^o \sin 80^o$
$= \sin(10^o + 80^o)$ [as sin(A + B) = sinA cosB + cosA sinB]
$= sin 90^o = 1$

OR

4 – xy = 4 - 8

xy = 8

The possible values of x and y are

x = 8, y = 1 or x = 4, y = 2 or y = 8, x = 1 or y = 4, x = 2

21. $f(x)=\begin{cases}\dfrac{x}{|x|+2x^2}, if\, x\neq 0\\ k, if\, x=0\end{cases}$

At x = 0, $LHL=\lim\limits_{x\to0^-}\frac{x}{|x|+2x^2}=\lim\limits_{h\to0}\frac{(0-h)}{|0-h|+2(0-h)^2}$

$=\lim\limits_{h\to0}\frac{-h}{h+2h^2}=\lim\limits_{h\to0}\frac{-h}{h(1+2h)}=-1$

$RHL=\lim\limits_{h\to0^+}\frac{x}{|x|+2x^2}=\lim\limits_{h\to0}\frac{0+h}{|0+h|+2(0+h)^2}$

$=\lim\limits_{h\to0}\frac{h}{h+2h^2}=\lim\limits_{h\to0}\frac{h}{h(1+2h)}=1$

And f(0) = k

Since, LHL $\neq$ RHL for any value of k.

Hence, f(x) is discontinuous at x = 0 regardless the choice of k.

22. Given $\frac{dx}{dt}=-3cm/\text{min}$, $\frac{dy}{dt}=2cm/\text{min}$

a. Let P be the perimeter

   $P=2\,(x+y)$

   $\frac{dp}{dx}=2\left(\frac{dx}{dt}+\frac{dy}{dt}\right)$

   = 2(-3 + 2)

   = -2 cm/min (i.e perimeter is decreasing)

b. Now area of rectangle A = xy

   $\frac{dy}{dt}=x\frac{dy}{dx}+y.\frac{dx}{dt}$

   $=10\,(2)+6\,(-3)$

   $=20-18$

   $=2cm^2/\text{min}$

23. Let $I=\int\frac{\sqrt{1+x^2}}{x^4}dx=\int\frac{\sqrt{1+x^2}}{x}.\frac{1}{x^3}dx$

$=\int\sqrt{\frac{1+x^2}{x^2}}.\frac{1}{x^3}dx=\int\sqrt{\frac{1}{x^2}+1}.\frac{1}{x^3}dx$

Put $1+\frac{1}{x^2}=t^2\Rightarrow\frac{-2}{x^3}dx=2tdt$

$\Rightarrow \frac{1}{x^3} = -tdt \therefore I = -\int t^2 dt = -\frac{t^3}{3} + C = -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + C$

OR

Let I = $\int$sin 3x sin 2x dx. Then,

$I = \frac{1}{2}\int 2$ sin 3x sin 2x dx

= $\frac{1}{2}\int$ (cos x - cos 5x) dx

= $\frac{1}{2}\left\{\sin x - \frac{\sin 5x}{5}\right\} + c$

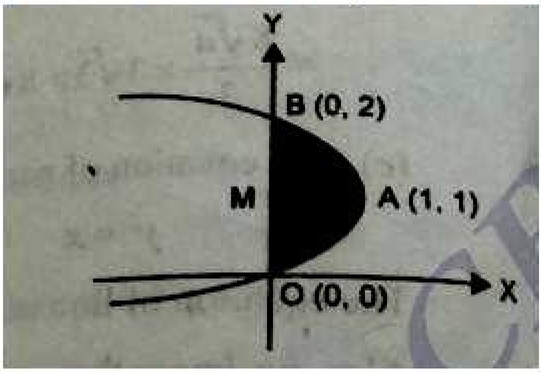
24. The given curve is $y^2 = 2y - x$ ....(1)

or $y^2 - 2y = -x$

or $y^2 - 2y + 1 = -x + 1$

or $(y-1)^2 = -(x-1)$,

which is a left handed parabola with vertex at (1,1).



putting x=0 in (1),we get,

$y^2 - 2y = 0$

y(y - 2) = 0

y = 0 ,2

Therefore, curve meets Y-axis in O(0,0) , B(0,2)

Required area=2(area OAM)

= $2\int_0^1 (2y - y^2)dy$

= $2\left[y^2 - \frac{y^3}{3}\right]_0^1$

= $2\left[(1-\frac{1}{3}) - (0-0)\right]$

= $\frac{4}{3}$ sq.units.

25. Here, it is given

xcos y dy =$(xe^x \log x + e^x)\,dx$

$\Rightarrow \cos y dy = e^x\left(\log x + \frac{1}{x}\right)dx$ [on separating the variables]

$\Rightarrow \int \cos y\, dy = \int e^x \left(\log x + \frac{1}{x}\right) dx$ [integrating both sides]

$\Rightarrow \sin y = e^x \log x + c\ [\because \int e^x f(x) + f'(x)]\, dx = e^x f(x)]$

Therefore, sin y = $e^x (\log x)$ + C is the required solution.

26. We have, $|\vec{a} + \vec{b}|^2 = \overrightarrow{(a + b)} \cdot (\overrightarrow{a} + \vec{b})$

$\Rightarrow \quad |\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot \vec{a} + (\vec{a} + \vec{b}) \cdot \vec{b} \left[|\vec{x}|^2 = \vec{x} \cdot \vec{x}\right]$

$\Rightarrow \quad |\vec{a} + \vec{b}|^2 = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$ [By distributivity of dot product over vector addition]

$\Rightarrow \quad |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$ [By distributivity of dot product over vector addition][$\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$]

27. x_1 = -3, y_1 = 1, z_1 = 5

a_1 = -3, b_1 = 1, c_1 = 5

x_2 = -1, y_2 = 2, z_2 = 5

a_2 = -1, b_2 = 2, c_2 = 5

$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$ = 2(5 - 10) + 1(-5 + 15) = 0

Therefore, lines are coplanar.

28. Let's define the events

K: card drawn is king and A: card drawn is an ace. Clearly, we have to find P (KKA)

Now $P(K) = \frac{\text{Number of kings}}{\text{Total number of cards}} \Rightarrow \frac{4}{52} = \frac{1}{13}$

Also, P (K|K) is the probability of second king with the condition that one king has already been drawn.

Now there are three kings in (52 - 1) = 51 cards.

Therefore P(K|K) = $\frac{3}{51} = \frac{1}{17}$

Lastly, P(A|KK) is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn. Now there are four aces in left 50 cards.

Therefore P(A|KK) = $\frac{4}{50} = \frac{2}{25}$

By multiplication law of probability, we have

P(KKA) = P(K) P(K|K) P(A|KK)

= $= \frac{1}{13} \times \frac{1}{17} \times \frac{2}{25} = \frac{2}{5525}$

OR

We have to show, $P(A) = P(A \cap B) + P\left(A \cap \overline{B}\right)$

Here, $RHS = P(A \cap B) + P\left(A \cap \overline{B}\right)$

$= P(A) \cdot P(B) + P(A) \cdot P(\overline{B})$

$= P(A)\left[P(B) + P(\overline{B})\right]$

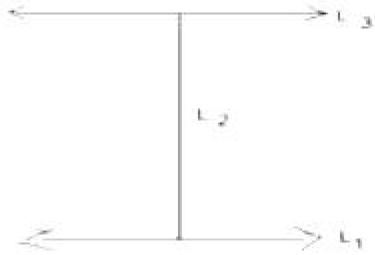
$= P(A)\left[P(B) + 1 - P(B)\right]\left[\because P(\overline{B}) = 1 - P(B)\right]$

= P(A) = LHS

Hence proved.

Section - IV

29. R is not reflexive, as a line L_1 cannot be $\perp$ to itself i.e $(L_1, L_1) \notin R$



Let $(L_1, L_2) \in R$

$\Rightarrow L_1 \perp L_2$

$\Rightarrow L_2 \perp L_1$

$\Rightarrow (L_2, L_1) \in R$

$\Rightarrow$ R is symmetric

Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$,then

$L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be $\perp$ to L_3 in fact $L_1 \,||\, L_3$

i.e $(L_1, L_2) \in R$, $(L_2, L_3) \in R$.

But $(L_1, L_3) \notin R$

R is not transitive.

30. $\sin y = x \cdot \sin(a+y)$

$\Rightarrow x = \frac{\sin y}{\sin(a+y)}$ ...(i)

Differentiating equation (i) w.r.t. y, we get

$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$

$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$

$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

31. Given $f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ x+1, & x \geq 0 \end{cases}$

When x < 0, we have

$f(x) = \frac{\sin x}{x}$

We know that sin x, as well as the identity function x, are everywhere continuous

So is the quotient function $\frac{\sin x}{x}$

When x > 0, we have

f(x) = x + 1

$\therefore$ f(x) is continuous at each x > 0

We have

(LHL at x = 0) = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} f(-h)$

$= \lim_{h\to 0}\left(\frac{\sin(-h)}{-h}\right) = \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right) = 1$

(RHL at x = 0) = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(h) = \lim_{h\to 0}(h+1) = 1$

Also,

f(0) = 0 + 1 = 1

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Thus, f(x) is continuous at x = 0

Hence, f(x) is everywhere continuous.

OR

Here, y = $\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put 2x = $\cos\theta$

$\therefore y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow y = \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$

$\Rightarrow y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\theta\right)\right]$ ...(i)

Now, $-\frac{1}{2} < x < 0$

$\Rightarrow$ -1 < 2x < 0

$\Rightarrow$ -1 < $\cos\theta$ < 0

$\Rightarrow \frac{\pi}{2} < \theta < \pi$ and $\Rightarrow -\frac{\pi}{2} > -\theta > -\pi$

$\Rightarrow \left(\frac{\pi}{2}-\frac{\pi}{2}\right) > \left(\frac{\pi}{2}-\theta\right) > \left(\frac{\pi}{2}-\pi\right)$

$\Rightarrow 0 > \left(\frac{\pi}{2}-\theta\right) > -\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < \left(\frac{\pi}{2}-\theta\right) < 0$

So, from equation (i)

$y=\theta+2\left[-\left(\frac{\pi}{2}-\theta\right)\right]$

$\left[\text{since}, \cos^{-1}\cos(\theta)=\theta, \text{ if } \theta \in [0,\pi], \cos^{-1}\cos(\theta)=-\theta, \text{ if } \theta \in [-\pi,0]\right]$

$y=\theta-2\times\frac{\pi}{2}+2\theta$

$y=-\pi+3\theta$

$y=-\pi+3\cos^{-1}(2x)$ ... [Since, 2x = cos θ]

Differentiate it with respect to x using chain rule,

$\frac{dy}{dx}=0+3\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$

$\Rightarrow \frac{dy}{dx}=\frac{-3}{\sqrt{1-4x^2}}\times 2$

$\therefore \frac{dy}{dx}=-\frac{6}{\sqrt{1-4x^2}}$

The differentiation of the given function y is as above.

32. Let the point of contact of the tangent line parallel to the given line be $P(x_1, y_1)$.

The equation of the curve is $y=\sqrt{5x-3}-2$.

Differentiating both sides with respect to x, we get

$\frac{dy}{dx}=\frac{5}{2\sqrt{5x-3}} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\frac{5}{2\sqrt{5x_1-3}}$

Since the tangent at (x_1, y_1) is parallel to the line 4x - 2y + 3 = 0. Therefore,

$\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$ = (Slope of the line 4x - 2y + 3 = 0)

$\Rightarrow \quad \frac{5}{2\sqrt{5x_1-3}}=\frac{-4}{-2}$

$\Rightarrow 4\sqrt{5x_1-3}=5$

$\Rightarrow 16(5x_1-3)=25$

$\Rightarrow x_1=\frac{73}{80}$

Since (x_1, y_1) lies on $y=\sqrt{5x-3}-2$. Therefore,

$y_1=\sqrt{5x_1-3}-2$

$\Rightarrow \quad y_1=\sqrt{5\times\frac{73}{80}-3}-2=-\frac{3}{4}\left[\because x_1=\frac{73}{80}\right]$

So, the coordinates of the point of contact are $\left(\frac{73}{80},-\frac{3}{4}\right)$

Hence, the required equation of the tangent is

$y-\left(-\frac{3}{4}\right)=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1)$

$\Rightarrow \quad y-\left(-\frac{3}{4}\right)=2\left(x-\frac{73}{80}\right)\left[\because\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=2\right]$

$\Rightarrow \frac{(4y+3)}{4}=2[\frac{(80x-73)}{80}]$

$\Rightarrow$ 40y +30 = 80x - 73

$\Rightarrow$ 80x - 40y - 103 = 0

Hence the equation

33. Given integral is: $\int e^{-3x}\cos^3 x dx$

Using trigonometric identity cos 3x = 4 $\cos^3$ x - 3 cos x

$\Rightarrow \int e^{-3x}\cos^3 x dx = \frac{1}{4}\int e^{-3x}(\cos 3x+3\cos x)dx$

$\Rightarrow \frac{1}{4}\int e^{-3x}(\cos 3x+3\cos x)dx = \frac{1}{4}\int e^{-3x}\cos 3x dx+\frac{3}{4}\int e^{-3x}\cos x dx$ ...(i)

Using a general formula i.e.

$\Rightarrow \int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}$ (a cos bx + b sin bx)

$\Rightarrow \int e^{-3x}\cos 3x dx = \frac{e^{-3x}}{(-3)^2+3^2}$ ((-3) cos 3x + 3 sin 3x)

$\Rightarrow \frac{e^{-3x}}{(-3)^2+3^2}$ ((-3) cos 3x + 3 sin 3x) = $\frac{e^{-3x}}{6}$ (sin 3x - cos 3x) .....(ii)

$\Rightarrow \int e^{-3x}\cos x dx = \frac{e^{-3x}}{(-3)^2+1^2}$ ((-3) cos 3x + 3 sin 3x) = $\frac{e^{-3x}}{10}$ (sin x - 3 cos x)

$= \frac{e^{-3x}}{10}$ (sin x - 3 cos x) .....(iii)

On putting (ii) and (iii) in (i)

$\Rightarrow \frac{1}{4}\int e^{-3x}\cos 3x dx + \frac{3}{4}\int e^{-3x}\cos x dx = \frac{e^{-3x}}{4\times 6}(\sin 3x-\cos 3x) + \frac{3e^{-3x}}{4\times 10}$ (sin x - cos x)

$\Rightarrow \int e^{-3x}\cos^3 x dx = \mathrm{e}^{-3x}\left\{\frac{(\sin 3x-\cos 3x)}{24}+\frac{3(\sin x-3\cos x)}{40}\right\}+C$

34. The equation of parabola is

y = x^2 - 5x + 15 ....(1)

The equation of line is

y = 3x + 3 ....(2)

From (1) and (2), we get

3x + 3 = x^2 - 5x + 15

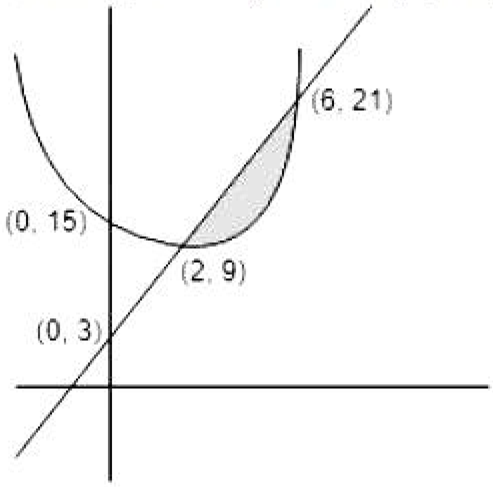
x^2 - 8x + 12 = 0

(x - 2)(x - 6) = 0

x = 2, 6

Therefore, from (1), y = 9, 21

$\therefore$ line (2) meets parabola (1) in (2, 9) and (6, 21).



Required area = the area of the segment of the parabola y = x^2 - 5x + 15 cut off by the straight line y = 3x + 3

= $\int_2^6 [(3x+3)-(x^2-5x+15)]dx$ $[\because\ Area = \int_a^b (y_2 - y_1)dx]$

= $\int_2^6 (-x^2+8x-12)\,dx$

= $\left[-\frac{x^3}{3}+4x^2-12x\right]_2^6$

= $(-72+144-72)-\left(-\frac{8}{3}+16-24\right)$

= $0+\frac{32}{3}$

= $\frac{32}{3}$ sq.units

OR

The given equation y = 4x - x^2 represents a parabola opening downwards and cutting x-axis at O(0, 0) and B(4, 0)

Slicing the region above x-axis in vertical strips of length = |y| and width = dx, area of corresponding rectangle is = |y| dx

Since the corresponding rectangle can move from z = 0 to z = 4,

Therefore Required area of shaded region OABO is

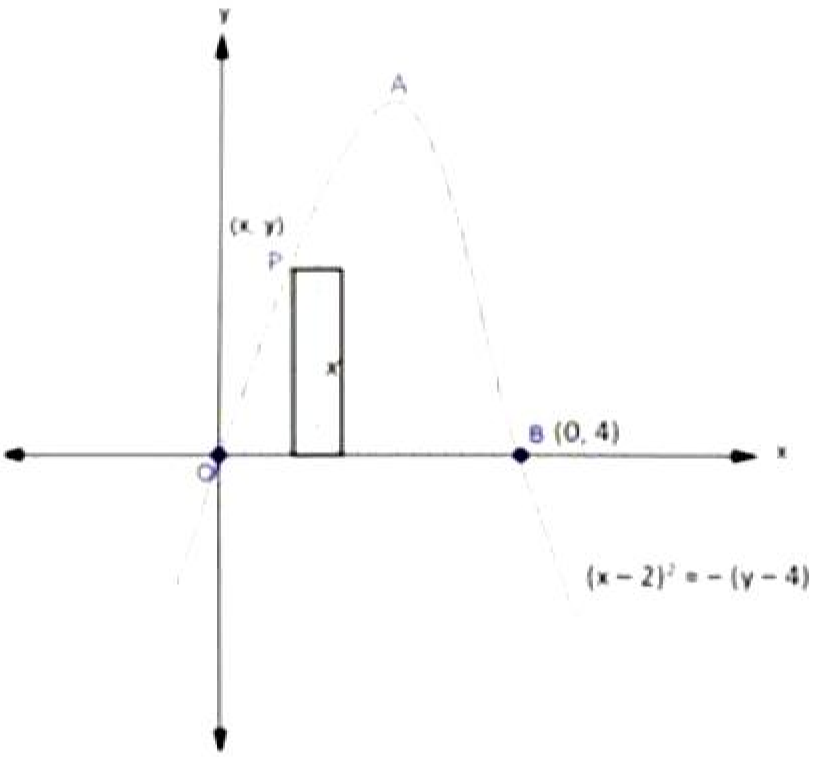
$A = \int_0^4 |y|dx = \int_0^4$ ydx ... [As, y > 0 for $0 \le x \le 4 \Rightarrow |y|$ = y]

$\Rightarrow A = \int_0^4 (4x - x^2)\,dx$

$\Rightarrow A = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$

$A = 32 - \frac{64}{3}$

$\Rightarrow A = \frac{32}{3}$ square units



35. The given differential equation is,

$$xe^{y/x} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - xe^{y/x}}{x\sin\left(\frac{y}{x}\right)}$$

This is a homogeneous differential equation.

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, then,we have,

$$v + x\frac{dv}{dx} = \frac{v\sin v - e^v}{\sin v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v\sin v - e^v}{\sin v} - v$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{e^v}{\sin v}$$

$\Rightarrow e^{-v} \sin v\ dv = -\frac{dx}{x}$

$\Rightarrow \int e^{-v} \sin v\ dv = -\int \frac{1}{x}\ dx$

$\Rightarrow \frac{e^{-v}}{2}$ (-sin v - cos v) = -log |x| + log C [$\because \int e^{ax} \sin bx\ dx = \frac{e^{ax}}{a^2+b^2}$ (a sin bx - b cos bx)]

$\Rightarrow -\frac{1}{2}e^{-y/x}\left\{\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\}$ = -log |x| + log C

$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\}$ = 2 log |x| - 2 log C

$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\}$ = log $|x|^2$ - 2 log C ...(ii)

It is given that y(1) = 0 i.e., y = 0 when x = 1. Putting these values in (ii), we get

1 = 0 - 2 log C $\Rightarrow$ log C = $-\frac{1}{2}$

Putting log C = $-\frac{1}{2}$ in (ii), we get

$e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = log $|x|^2$ + 1 as the required solution.

Section - V

36. Given: $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$

$\therefore A^2=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}4+1+1&-2-2-1&2+1+2\\-2-2-1&1+4+1&-1-2-2\\2+1+2&-1-2-2&1+1+4\end{bmatrix}=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}$

Now $A^3=A^2.A=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$

$=\begin{bmatrix}12+5+5&-6-10-5&6+5+10\\-10-6-5&5+12+5&-5-6-10\\10+5+6&-5-10-6&5+5+12\end{bmatrix}$

$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}$

L.H.S. $=A^3-6A^2+9A-4I$

$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}-6\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}$

$+9\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}-4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}-\begin{bmatrix}36&-30&30\\-30&36&-30\\30&-30&36\end{bmatrix}$

$+\begin{bmatrix}18&-9&9\\-9&18&-9\\9&-9&18\end{bmatrix}-\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} + \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1}, multiplying $A^3 - 6A^2 + 9A - 4I^{-1} = 0.A^{-1}$ by A^{-1}

$\Rightarrow A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4I . A^{-1} = 0A^{-1}$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} -6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} +9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

OR

We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A' = A^{-1}$

$\Rightarrow AA' = AA^{-1}$ [$\because AA^{-1} = I$]

$\Rightarrow AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2-z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$

$\Rightarrow 4y^2 + z^2 = 1$

$\Rightarrow 2.z^2 + z^2 = 1$

$z = \pm\frac{1}{\sqrt{3}}$

$\therefore y^2 = \frac{z^2}{2} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

Also, $x^2 + y^2 + z^2 = 1$

$\Rightarrow x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$

$= 1 - \frac{3}{6} = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$\therefore x = \pm\frac{1}{\sqrt{2}},\ y = \pm\frac{1}{\sqrt{6}}$

and $z = \pm\frac{1}{\sqrt{3}}$

37. The equation of the plane in the intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

Distance of this plane from the origin is given to be p.

$$\therefore p = \frac{\left|\frac{1}{a}\times 0 + \frac{1}{b}\times 0 + \frac{1}{c}\times 0 - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow p = \frac{1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \frac{1}{p} = \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}$$

$$\Rightarrow \frac{1}{p^2} = \left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

OR

Given lines are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$

and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

On comparing both equations of lines with

$\vec{r} = \vec{a} + \lambda\vec{b}$ respectively, we get ,

$\overrightarrow{a_1} = \hat{i} + \hat{j} - \hat{k}, \overrightarrow{b_1} = 3\hat{i} - \hat{j}$

and $\overrightarrow{a_2} = 4\hat{i} - \hat{k}, \overrightarrow{b_2} = 2\hat{i} + 3\hat{k}$

Now $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$= \hat{i}(-3 - 0 - \hat{j}(9 - 0) + \hat{k}(0 + 2)$

$= -3\hat{i} - 9\hat{j} + 2\hat{k}$

and $\overrightarrow{a_2} - \overrightarrow{a_1} = \left(4\hat{i} - \hat{k}\right) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) = (3\hat{i} - \hat{j}).(-3\hat{i} - 9\hat{j} + 2\hat{k})$

= - 9 + 9 = 0

Hence, given lines are coplanar.

Now, cartesian equations of given lines are

$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3}$

Then, equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y - 1 & z + 1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

(x - 1) (-3- 0) - (y - 1) (9- 0)+(z+1)(0+2)=0

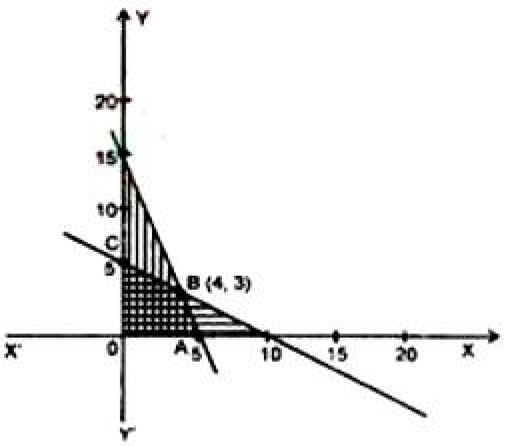
- 3x + 3 - 9y + 9 + 2z + 2 = 0

3x + 9y - 2z = 14

38. Consider $x + 2y \leqslant 10$

Let x + 2y = 10

$\Rightarrow \frac{x}{10} + \frac{y}{5} = 1$



Since, (0, 0) satisfies the inequation, therefore the half plane containing (0, 0) is the required plane.

Again $3x + 2y \leqslant 15$

Let 3x + y = 15

$\Rightarrow \frac{x}{5} + \frac{y}{15} = 1$

It also satisfies by (0, 0) and its required half plane contains (0, 0).

Now double shaded region in the first quadrant contains the solution.

Now OABC represents the feasible region.

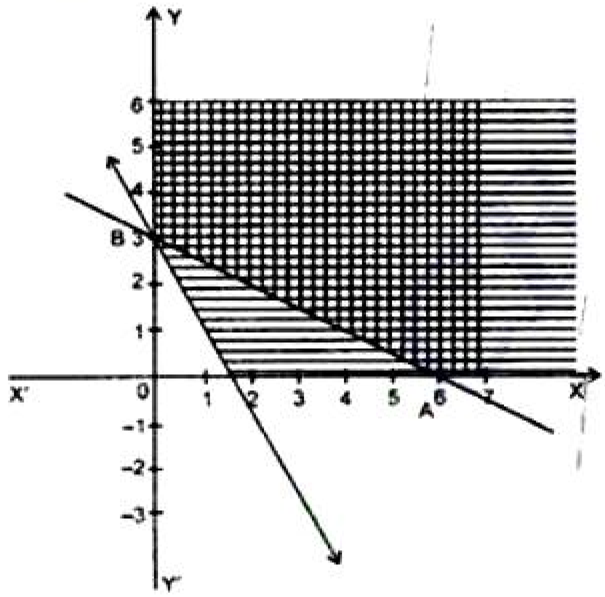
Z = 3x + 2y

At O (0, 0) Z = 3 × 0 + 2 × 0 = 0

At A (5, 0) Z = 3 × 5 + 2 × 0 = 15

At B (4, 3) Z = 3 × 4 + 2 × 3 = 18

At C (0, 5) Z = 3 × 0 + 2 × 5 = 10



Hence, Z is maximum i.e., 18 at x = 4, y = 3.

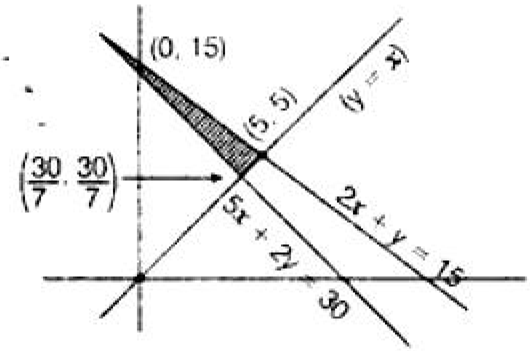
OR

we have to minimise Z = 400x + 200y, subject to $5x + 2y \geqslant 30$

$2x + y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0.$

On solving x - y = 0 and 5x + 2y = 30, we get

$y = \frac{30}{7}, x = \frac{30}{7}$



On solving x - y = 0 and 2x + y = 15, we get x = 5, y = 5

So, from the shaded feasible region it is clear that coordinates of corner points are (0, 15), (5, 5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	**Corresponding Value of X = 400x + 200y**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7}$ = 2571.43 (minimum)

Hence, the minimum value is Rs 2571.43.