Physics

Academic Year: 2014-2015 Marks: 70
Date & Time: 3rd October 2015, 4:00 pm
Duration: 3h

Section 1

Question 1 | Attempt any one of the following : [7]

Question 1.1: [7]

State an expression for the moment of intertia of a solid uniform disc, rotating about an axis passing through its centre, perpendicular to its plane. Hence derive an expression for the moment of inertia and radius of gyration:
i. about a tangent in the plane of the disc, and
ii. about a tangent perpendicular to the plane of the disc.

Solution: The M. I of a thin uniform disc about an axis passing through its centre and perpendicular to its plane is given by,

$I_C = (1/2)MR^2$

i. According to theorem of parallel axis,

$I_T = I_d + Mh^2 = I_d + MR^2$ [$\because h = R$]

But $I_d = MR^2/4$

$$\therefore I_T = \frac{MR^2}{4} + MR^2$$

$$\therefore I_T = \frac{5}{4}MR^2$$

Now , radius of gyration is given by,

$$K = \sqrt{\frac{I}{M}}$$

$$\therefore K = \sqrt{\frac{5MR^2}{4M}}$$

$$\therefore K = \frac{\sqrt{5}}{2}R$$

ii. Applying theorem of parallel axis,

$I_T = I_O + Mh^2 = I_O + MR^2$ [$\because$ h = R]

But $I_O = MR^2/2$

$$I_T = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

Now, radius of gyration is given by,

$$K = \sqrt{\frac{I}{M}}$$

$$\therefore K = \sqrt{\frac{3MR^2}{2M}}$$

$$\therefore K = \sqrt{\frac{3}{2}}R$$

Question 1.1:

In a set, 21 turning forks are arranged in a series of decreasing frequencies. Each tuning fork produces 4 beats per second with the preceding fork. If the first fork is an octave of the last fork, find the frequencies of the first and tenth fork.

Solution: Given: N = 21, x = 4, nF = 2 n_L

To find:

i. Frequency of first fork (n_F)

ii. Frequency of tenth fork (n_{10})

Formula: $n_L = n_F - (N - 1)x$

Calculation:-

i. When tuning forks are arranged in the decreasing order of frequencies, the frequency of the p^{th} tuning fork is,

$n_L = n_F - (N - 1)x = n_1 - (21 - 1)\ 4$

$\therefore n_L = n_F - 80$ ....(1)

As frequency of first fork is an octave of last,

$\therefore n_F = 2n_L$

$\therefore n_L = n_F/2$

From equation (1),

$n_F/2 = n_F - 80$

∴ $n_F - (n_F/2) = 80$

∴ $n_F/2 = 80$

∴ $n_F = 160$ Hz

The frequency of the first fork is 160 Hz.

ii. For 10th fork,

$n_{10} = n_1 - (10 - 1)x$

$= 160 - 9 * 4 = 160 - 36$

$n_{10} = 124$Hz

The frequency of the tenth fork is 124 Hz.

OR

Question 1.2: Discuss the composition of two S.H.M.s along the same path having same period. Find the resultant amplitude and initial phase. **[7]**

Solution: Analytical treatment:

i. Let the two linear S.H.M's be given by equations,
$x_1 = A_1 \sin(\omega t + \alpha_1)$ ...(1)

$x_2 = A_2 \sin(\omega t + \alpha_2)$ ...(2)

Where A_1, A_2 are amplitudes; α_1, α_2 are initial phase angles and x_1, x_2 are the displacement of two S.H.M's in time 't'. ω is same for both S.H.M's.

ii. The resultant displacement of the two S.H.M's is given by,

$x = x_1 + x_2$ ....(3)

iii. Using equations (1) and (2) , equation (3) can be written as,

$x = A_1 \sin(\omega t + \alpha_1) + A_2 \sin(\omega t + \alpha_2)$

$= A_1 [\sin \omega t \cos \alpha_1 + \cos \omega t \sin \alpha_1] + A_2 [\sin \omega t \cos \alpha_2 + \cos \omega t \sin \alpha_2]$

$= A_1 \sin \omega t \cos \alpha_1 + A_1 \cos \omega t \sin \alpha_1 + A_2 \sin \omega t \cos \alpha_2 + A_2 \cos \omega t \sin \alpha_2$

$= [A_1 \sin \omega t \cos \alpha_1 + A_2 \sin \omega t \cos \alpha_2] + [A_1 \cos \omega t \sin \alpha_1 + A_2 \cos \omega t \sin\alpha_2]$

∴ $x = \sin \omega t [A1 \cos \alpha_1 + A_2 \cos \alpha_2] + \cos \omega t [A_1 \sin \alpha_1 + A_2 \sin \alpha_2]$ ...(4)

iv. Let $A_1 \cos \alpha_1 + A_2 \cos \alpha_2 = R \cos \delta$ ...(5)

and $A_1 \sin \alpha_1 + A_2 \sin \alpha_2 = R \sin \delta$ ...(6)

v. Using equations (5) and (6), equation (4) can be written as,

$x = \sin \omega t. R \cos \delta + \cos \omega t.R \sin \delta$

$= R [\sin \omega t \cos \delta + \cos \omega t \sin \delta]$

$\therefore x = R \sin (\omega t + \delta)$ ....(7)

Equation (7) represents linear S.H.M. of amplitude R and initial phase angle δ with same period.

Resultant amplitude (R):

Squaring and adding equations (v) and (vi) we get,

$$(A_1 \cos \alpha_1 + A_2 \cos \alpha_2)^2 + (A_1 \sin \alpha_1 + A_2 \sin \alpha_2)^2 = R^2\cos^2\delta + R^2\sin^2\delta$$

$\therefore$ $A_1^2\cos^2 \alpha 1 + A_2^2 \cos^2 \alpha_2 + 2A_1 A_2 \cos\alpha_1 \cos\alpha_2 + A_1^2 \sin^2 \alpha_1 + A_2^2\sin^2 \alpha_2 + 2A_1A_2 \sin\alpha_1 \sin\alpha_2 = R^2 (\cos^2 \delta + \sin^2 \delta)$

$\therefore$ $A_1^2 (\cos^2 \alpha 1 + \sin^2 \alpha_1) + A_2^2 (\cos^2 \alpha_2 + \sin^2 \alpha_2) + 2A_1 A_2 (\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2) = R^2$

$\therefore$ $A_1^2 + A_2^2 + 2A_1 A_2 \cos (\alpha_1 - \alpha_2) = R^2$

$$\therefore \quad R = \pm\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2)} \qquad \text{..........(8)}$$

Equation (8) represents resultant amplitude of two S.H.M's.

Resultant (intial) phase (δ):

Dividing equation (6) by (5), we get

$$\frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2} = \frac{R\sin\delta}{R\cos\delta}$$

$$\therefore \frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2} = \tan\delta$$

$$\therefore \delta = \tan^{-1}\left[\frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2}\right] \qquad \text{.................(9)}$$

Equation (9) represents resultant or intial phase of two S.H.M's.

Question 1.2: A sonometer wire is in unison with a tuning fork of frequency 125 Hz when it is stretched by a weight. When the weight is completely immersed in water, 8 beats are heard per second. Find the specific gravity of the material of the weight. **[7]**

Solution: $n_1 = 125$ Hz, $n_2 = n_1 - 8 = 125 - 8 = 117$ Hz

As,

$$\frac{n_1}{n_2} = \sqrt{\frac{P}{P - 1}}$$

$$\frac{125}{117} = \sqrt{\frac{P}{P - 1}}$$

$$1.068 = \sqrt{\frac{P}{P - 1}}$$

$\Rightarrow P = 1.141P - 1.141$

$1.141 = 1.141P - P$

$$P = \frac{1.141}{0.141}$$

$\therefore P = 8.09$

Question 2: Select and write the most appropriate answer from the given alternatives for each sub-question : [7]

Question 2.1: Which of the following substances is ductile? [1]

Glass
High carbon steel
Steel
Copper

Solution: Copper

Question 2.2: Angle of contact for the pair of pure water with clean glass is _______. [1]

acute
obtuse
90°
0°

Solution: 0°

Question 2.3: A seconds pendulum is suspended in an elevator moving with constant speed in downward direction. The periodic time (T) of that pendulum is _______. [1]

less than two seconds
equal to two seconds
greater than two seconds
very much greater than two seconds

Solution: Equal to two seconds

Question 2.4: The equation of a progressive wave is y = 7 sin (4t - 0.02x), where x and y are in cms and time t in seconds. The maximum velocity of a particle is _______. [1]

28 cm/s
32 cm/s
49 cm/s
112 cm/s

Solution: 28 cm/s

Question 2.5: The dimensions of emissive power are [1]
[$M^1 L^{-2} T^{-3}$]
[$M^1 L^2 T^{-3}$]
[$M^1 L^0 T^{-3}$]
[$M^1 L^0 T^{-2}$]

Solution: (C) [$M^1 L^0 T^{-3}$]

Question 2.6: The pressure (P) of an ideal gas having volume (V) is 2E/3V, then the energy E is ______. [1]

translational kinetic
rotational kinetic
vibrational kinetic
inversely proportional to pressure

Solution: translational kinetic

Question 2.7: The fundamental frequency of transverse vibration of a stretched string of radius r is proportional to ______. [1]

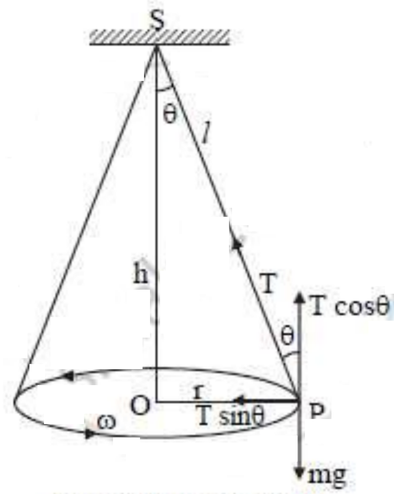
r^{-2}
r^{-1}
$r^{-1/2}$
r^2

Solution: r^{-1}

Question 3: Attempt any SIX: [12]

Question 3.1: Draw a neat labelled diagram of conical pendulum. State the expression for its periodic time in terms of length. [2]

Solution:



Conical pendulum

Where, S: rigid support

T : tension in the string

l : length of string

h : height of support from bob

v : velocity of bob

r : radius of horizontal circle

θ: semi-vertical angle

mg : weight of bob

i) Consider a bob of mass m tied to one end of a string of length 'l' and other the end is fixed to a rigid support.

ii) Let the bob be displaced from its mean position and whirled around a horizontal circle of radius 'r' with constant angular velocity ω, then the bob performs U.C.M.

iii) During the motion, a string is inclined to the vertical at an angle θ as shown in the above figure.

iv) In the displaced position P, there are two forces acting on the bob.

a. The weight mg acting vertically downwards.

b. The tension T acting upward along the string.

v) The tension (T) acting in the string can be resolved into two components:
a. T cos θ acting vertically upwards.
b. T sin θ acting horizontally towards centre of the circle.

vi) Vertical component T cos θ balances the weight and horizontal component T sin θ provides the necessary centripetal force.

∴ T cos θ = mg . ...(1)

$T \sin\theta = \frac{mv^2}{r} = mr\omega^2$ ....(2)

vii) Dividing equation (2) by (1),

$\tan\theta = \frac{v^2}{rg}$ ......(3)

Therefore, the angle made by the string with the vertical is $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Also, from equation (3),

$v^2 = rg\tan\theta$

$\therefore v = \sqrt{rg\tan\theta}$

The period, $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

It can be seen that r - = l sin θ

$T = 2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

The period of a conical pendulum is $2\pi\sqrt{\frac{l\cos\theta}{g}}$

Question 3.2: A raindrop of diameter 4 mm is about to fall on the ground. Calculate the pressure inside the raindrop. [Surface tension of water T = 0.072 N/m, atmospheric pressure = 1.013×10^5 N/m^2] **[2]**

Solution: Given:

T = 0.072 N/m,

d = 4mm ∴ r = 2 * 10^{-3} m,

P_o = 1.013 * 10^5 N/m^2

To find: Pressure inside the raindrop (P_i)

Formula: Pi = Po + 2T/r

Calculation: From formula,

$$P_i = 1.013 \times 10^5 + \frac{2 \times 0.072}{2 \times 10^{-3}}$$

$$P_i = 1.013 \times 10^5 + 0.072 \times 10^3$$

$$= 1.013 \times 10^5 + 0.00072 \times 10^5$$

$$\therefore P_i = 1.01372 \times 10^5 Pa$$

The pressure inside the raindrop is 1.01372 * 10^5 Pa.

Question 3.3: Explain why an astronaut in an orbiting satellite has a feeling of weightlessness. **[2]**

Solution: When an astronaut is in an orbiting satellite, the astronaut and satellite are attracted towards the centre of the Earth and both will fall towards the Earth with the same acceleration. This acceleration is the same as 'g' at the satellite. Thus, the astronaut is unable to exert weight on the floor of the satellite. Because of this, the satellite does not provide a normal reaction on the astronaut, and hence, the astronaut feels weightlessness.

Question 3.4: The periodic time of a linear harmonic oscillator is 2π second, with maximum displacement of 1 cm. If the particle starts from extreme position, find the displacement of the particle after π/3 seconds. **[2]**

Solution: Given:-

$$a = 1cm, t = \frac{\pi}{3}s, \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

To find: Displacement (x)

Formula: x = A cos ωt

Calculation: From formula,

$$x = 1\cos\left(1 \times \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore x = 0.5cm$$

The displacement of the particle after π/3 seconds is 0.5 cm.

Question 3.5: State and prove : Law of conservation of angular momentum. **[2]**

Solution 1: Statement:-

The angular momentum of a body remains constant, if resultant external torque acting on the body is zero.

Proof:-

a. Consider a particle of mass m, rotating about an axis with torque 'τ'.

Let $\vec{p}$ be the linear momentum of the particle and $\vec{r}$ be its position vector.

b. By definition, angular momentum is given by, $\vec{L} = \vec{r} \times \vec{p}$ ....(1)

c. Differentiating equation (1) with respect to time t, we get,

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right)$$

$$\therefore \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \vec{p} \times \frac{d\vec{r}}{dt} \text{ ....(2)}$$

d.

$$\text{But,} \frac{d\vec{r}}{dt} = \vec{v}, \frac{d\vec{p}}{dt} = \vec{F} \quad \text{and} \quad \vec{p} = mv$$

∴ Equation (2) becomes,

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + 0 \qquad [\because \vec{v} \times \vec{v} = v^2 \sin 0^\circ = 0]$$

e. Also, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\therefore \frac{d\vec{L}}{dt} = \vec{\tau}$$

f. If resultant external torque (τ) acting on the particle is zero, then $\frac{d\vec{L}}{dt} = 0.$

$\therefore \overrightarrow{L} = \text{constant}$

Hence, angular momentum remains conserved.

Solution 2: Principle (or law) of conservation of a body is conserved if the resultant external torque on the body is zero.
Proof: Consider a particle of mass m whose position vector with respect to the origin at any instant is $\overrightarrow{r}$

Then, at this instant, the linear velocity of this particle is $\overrightarrow{v} = \frac{\overrightarrow{dr}}{dt}$, its linear momentum is $\overrightarrow{p} = m\overrightarrow{v}$ and its angular momentum about an axis through the origin is $\overrightarrow{l} = \overrightarrow{r} \times \overrightarrow{p}$

Its angular momentum $\overrightarrow{l}$ may change with time due to a torque on the particle.

$$\frac{\overrightarrow{dl}}{dt} = \frac{d}{dt}\left(\overrightarrow{r} \times \overrightarrow{p}\right)$$

$$= \frac{d\overrightarrow{r}}{dt} \times \overrightarrow{p} + \overrightarrow{r} \times \frac{d\overrightarrow{p}}{dt}$$

$$= \overrightarrow{v} \times \overrightarrow{mv} + \overrightarrow{r} + \overrightarrow{F}$$

$$= \overrightarrow{r} \times \overrightarrow{F}$$

$$= \overrightarrow{r} \times \overrightarrow{F} \qquad (\because \overrightarrow{v} \times \overrightarrow{v} = 0)$$

$$= \overrightarrow{t}$$

Where = $\frac{d\overrightarrow{p}}{dt} = \overrightarrow{F}$, the force on the particle.

Hence if $\overrightarrow{t} = 0, \frac{\overrightarrow{dl}}{dt} = 0$

$\therefore \overrightarrow{l}$ = constant, i.e $\overrightarrow{l}$ is conserved. This proves the principle (or law) of conservation of angular momentum.

Question 3.6: A pinhole is made in a hollow sphere of radius 5 cm whose inner wall is at temperature 727°C. Find the power radiated per unit area. [Stefan's constant σ = 5.7 x 10^{-8} J/m^2 s K4, emissivity (e) = 0.2] **[2]**

Solution: Given:-

T = 727 + 273 = 1000 K,

$\sigma = 5.7 \times 10^{-8}$ J/m^2sK^4, e = 0.2

To find:- Power radiated (E)

Formula:-

$$(E)\frac{Q}{At} = \sigma e T^4$$

Calculation:- From formula,

$E = 0.2 \times 5.7 \times 10^{-8} \times (1000)^4$

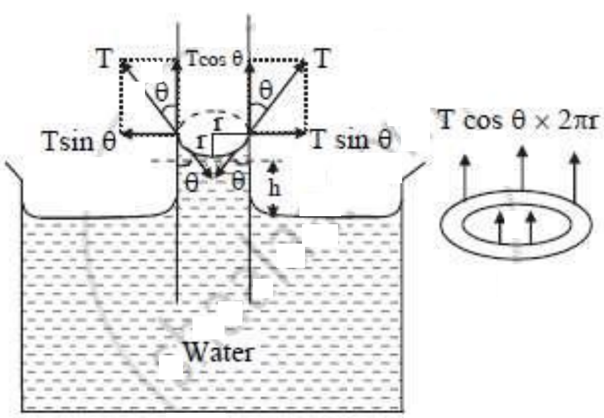
= 11400

∴ $E = 1.14 \times 10^4$ $watt/m^2$

The power radiated per unit area is 1.14×10^4 $watt/m^2$.

Question 3.7: Draw a neat labelled diagram showing forces acting on the meniscus of water in a capillary tube. **[2]**

Solution:



Rise of liquid in capillary tube

r = radius of capillary tube

h = height of liquid level in the tube

T = surface tension of liquid

ρ = density of liquid

g = acceleration due to gravity

Question 3.8: Compute the temperature at which the r.m.s. speed of nitrogen molecules is 832 m/s. [Universal gas constant, R = 8320 J/k mole K, molecular weight of nitrogen = 28.] **[2]**

Solution: Given:-

c_{rms} = 832 m/s,

R = 8320 J/k mole K,

M = 28

To find:

R.M.S velocity (c_{rms})

Formula:-

$$c_{rms} = \sqrt{\frac{3RT}{M}}$$

Calculation: From formula,

$$832 = \sqrt{\frac{3 \times 8320 \times T}{28}}$$

$$\therefore T = \frac{832 \times 832 \times 28}{3 \times 8320}$$

$$T = \frac{832 \times 28}{30}$$

$$T = 776.5K$$

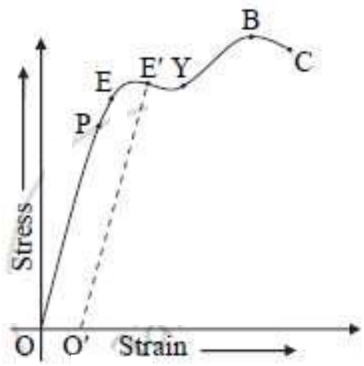
∴ T = 776.5 K or 503.5° C

The r.m.s. speed of nitrogen molecules is 832 m/s at 776.5 K or 503.5° C

Question 4 | Attempt any THREE : **[9]**

Question 4.1: Discuss the behaviour of wire under increasing load. **[3]**

Solution: Stress v/s strain graph:-



P : Proportional limit
E : Elastic limit
Y : Yield point
B : Breaking stress
C : Breaking point
OO' : Permanent set

The behaviour of wire under increasing load can be explained with the help of stress v/s strain graph.

a. Proportional limit: The initial portion OP of the graph is a straight line indicating stress is directly proportional to strain. Thus Hooke's law is completely obeyed in the region OP. Point P is called point of proportional limit.

b. Elastic limit: Beyond the point P, the stress-strain variation is not a straight line as indicated by the part PE of the graph.

If the wire is unloaded at point E, the graph between stress and strain follows the reverse path EPO, then the point E is called elastic limit. The portion between O and E is called elastic region.

c. Permanent set: If the load is increased so that stress becomes greater than that corresponding to the point E, the graph is no longer a straight line and the wire does not obey Hooke's law. If the wire is strained upto E′ beyond point E and then the load is removed, the wire does not regain its original length and there is a permanent increase in length. A small strain corresponding to OO' is set up permanently in the wire, called permanent set. However, the wire is still elastic and if loaded again, gives a linear relation shown by the dotted line O'E'.

d. Yield point: As the stress is increased beyond the elastic limit the graph is a curve and reaches a point Y where the tangent to the curve is parallel to the strain axis. This shows that for the stress corresponding to point Y the strain increases even without any increase in the stress. This is known
as plastic flow.

Point Y on the curve is called yield point.

The value of stress corresponding to yield point is called yield stress.

e. Breaking stress: When the wire begins to flow, its cross-section decreases uniformly and hence, the stress increases steadily. Later a neck or constriction begins to form at a weak point. The maximum stress corresponding to the point B is breaking stress.

f. Breaking point: Once the neck is formed, the wire goes on stretching even if the load is reduced, until the breaking point C is reached when the wire breaks.

Elastic limit:-

The maximum stress to which an elastic body can be subjected without causing permanent deformation is called as elastic limit.

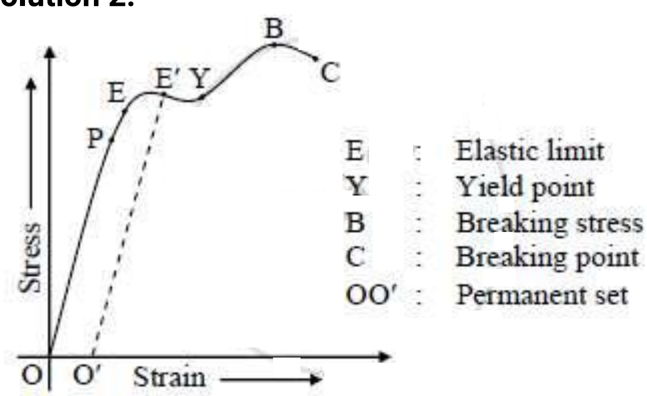
Yield point:-

The point on stress-strain curve at which the strain begins to increase without any increase in the stress is called yield point.

Breaking point:-

The point on stress-strain curve at which the wire breaks, is known as the breaking point.

Solution 2:



a) **Elastic limit**: The initial portion OE of the graph is a straight line, which indicates that upto the point E stress is directly proportional to strain. Hence, Hooke's law is obeyed upto point E. In this region, wire is perfectly elastic and it completely regains its original length when the load is removed. Point E represents limit of proportionality between stress and strain.

b) **Permanent set**: If the load is increased so that stress becomes greater than that corresponding to the point E, the graph is no longer a straight line and the wire does not obey Hooke's law.

If the wire is strained upto E' beyond point E and then the load is removed, the wire does not regain its original length and there is a permanent increase in length. A small strain corresponding to OO' is set up permanently in the wire, called permanent set. However, the wire is still elastic and if loaded again, gives a linear relation shown by the dotted line O'E'.

c) **Yield point**: As the stress is increased beyond the elastic limit the graph is a curve and reaches a point Y where the tangent to the curve is parallel to the strain axis. This shows that for the stress corresponding to point Y the strain increases even without any increase in the stress. This is known as plastic flow.
Point Y on the curve is called yield point.

The value of stress corresponding to yield point is called yield stress

e) **Breaking point**: Once the neck is formed, the wire goes on stretching even if the load is reduced, until the breaking point C is reached when the wire breaks.

Question 4.2: Determine the binding energy of satellite of mass 1000 kg revolving in a circular orbit around the Earth when it is close to the surface of Earth. Hence find kinetic energy and potential energy of the satellite. [Mass of Earth = 6 x 10^{24} kg, radius of Earth = 6400 km; gravitational constant G = 6.67 x 10-11 Nm^2 /kg2] **[3]**

Solution: Given:- m = 1000 kg, M = 6 x 10^{24} kg, R = 6400 km, G = 6.67 x 10^{-11} N m^2/kg^2

To find:-

i. Binding Energy (B.E.)

ii. Kinetic Energy (K.E.)

iii. Potential Energy (P.E.)

Formulae:- For satellite very close to earth,

i. B.E. =(1/2)*(GMm/R)

ii. K.E. = B.E.

iii. P.E. = -2K.E.

Calculation: From formula (i),

$$B.E = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000}{2 \times 6.4 \times 10^{6}}$$

$$B.E = \frac{6.67 \times 6}{12.8} \times 10^{10}$$

= antilog[log 6.67 + log 6 – log 12.8] x 10^{10}

= antilog[0.8241 + 0.7782 – 1.1072] x 10^{10}

= antilog[0.4951] * 10^{10}

= 3.127 x 1010

∴ B.E. = 3.1265 x 10^{10} J

The binding energy of the satellite is 3.1265 x 10^{10} J.

From formula (ii),

K.E. = 3.1265 x 10^{10}

∴ K.E. = 3.1265 x 10^{10} J

The kinetic energy of the satellite is 3.1265 x 10^{10} J.

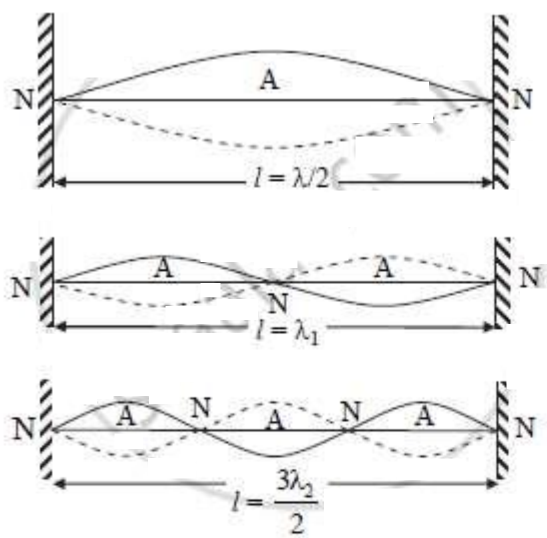
From formula (iii),

P.E. = -2(3.1265 x 1010)

∴ P.E. = -6.2530 x 1010 J

The potential energy of the satellite is -6.2530 x 10^{10} J.

Question 4.3: Show that all harmonics are present on a stretched string between two rigid supports. [3]



Solution:

Vibration of string for different mode

Modes of vibrations in stretched string:-

a. Consider a string stretched between two rigid supports and plucked. Due to plucking, string vibrates and loops are formed in the string. Vibrations of string are as shown in figure.

b. Let,

p = number of loops

l = length of string

∴ Length of one loop = l /p ......(1)

c. Two successive nodes form a loop. Distance between two successive nodes is λ/2.

∴ Length of one loop = λ/2 ....(2)

From equations (1) and (2),

λ/2 = l/p

∴ λ = 2l/p ....(3)

d. Velocity of transverse wave is given by,

$$v = \sqrt{\frac{T}{m}}$$

e. Frequency of string is given by,

n = v/λ

Substituting λ from equation (3),

$$n = \frac{\sqrt{\frac{T}{m}}}{\frac{2l}{p}}$$

$$\therefore n = \frac{p}{2l}\sqrt{\frac{T}{m}} \qquad ....(4)$$

f. Fundamental mode or first harmonic:-

In this case, p = 1

∴ From equation (4),

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This frequency is called fundamental frequency.

g. First overtone or second harmonic:-

In this case, p = 2

∴ From equation (4),

$$n_1 = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2 \times \frac{1}{2l}\sqrt{\frac{T}{m}} = 2n$$

$$\therefore n_1 = 2n$$

h. Second overtone or third harmonic:-

In this case, p = 3

Using equation (4),

$$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3 \times \frac{1}{2l}\sqrt{\frac{T}{m}} = 3n$$

$$\therefore n_2 = 3n$$

i. (p - 1)th overtone or pth harmonic:-

$$n_{(p-1)} = p \times \frac{1}{2l}\sqrt{\frac{T}{m}} = pn$$

For pth overtone,

$$n_p = \frac{p+1}{2l}\sqrt{\frac{T}{m}} = (p+1)n$$

j. Thus, in the vibration of stretched string, frequencies of vibrations are n, 2n, 3n,.....so on.

Hence, all harmonics (even as well as odd) are present in the vibrations of stretched string.

Question 4.4: A stone of mass 100 g attached to a string of length 50 cm is whirled in a vertical circle by giving velocity at lowest point as 7 m/s. Find the velocity at the highest point. [Acceleration due to gravity = 9.8 m/s2] **[3]**

Solution: Given:-
m = 100 g = 0.1 kg,
r = 50 cm = 0.5m,
g = 9.8 m/s^2,
v_2 = 7 m/s

To find:- Velocity at the highest point (v_H)

Formula:- $v_H = \sqrt{\frac{2(TE_H)}{m} - 4gr}$

Calculation:-

The total energy at the bottom,

E_{bot} (= KE + PE = $\frac{1}{2}$ 2.45 J)

T.E.(H)= K.E. at lowest point = $(1/2)mv_L^2$

$\therefore$ $T.E_{(H)} = \frac{1}{2} \times 0.1 \times 7^2$

=2.45 J

The total energy at the top,

E_{top} = KE + PE = $\frac{1}{2}mv_1^2 + mg(2r)$

$E_{top} = \frac{1}{2}(0.1)v_1^2 + (0.1)(9.8)(2 \times 0.5)$

$E_{top} = 0.05v_1^2 + 0.98$

By the Principle of conservation of energy,

$E_{top} = E_{bot}$

$0.05v_1^2 + 0.98 = 2.45$

$$v_1^2 = \frac{2.45 - 0.98}{0.05} = \frac{147}{5} = 29.4$$

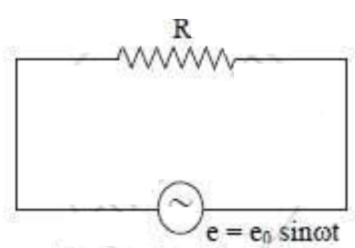
The required velocity $v_1 = \sqrt{29.4}$ = 5.422 m/s.

Section II

Question 5 | Attempt any one of the following : [7]

Question 5.1: Obtain an expression for average power dissipated in a purely resistive A.C. circuit. Define power factor of the circuit and state its value for purely resistive A.C. circuit. [7]

Solution: Expression for average power in purely resistive circuit:-



AC circuit with resistor

i. Let, $e = e_0 \sin \omega t$ be applied e.m.f across a resistor of resistance 'R' as shown in figure. At certain instant, current $I = I_0 \sin \omega t$ is flowing through the resistor. In this case both 'e' and 'I' are in phase.

ii. Instantaneous power in circuit is given by, P = eI = (e_0 sinωt) (I_0 sin ωt)

∴ P = e_0I_0 $\sin^2$ ωt ....(1)

iii. Average power for a complete cycle can be obtained by integrating equation (1).

$$\therefore P_{\text{av}} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T Pdt}{T}$$

$$= \frac{\int_0^T e_0 I_0 \sin^2 \omega tdt}{T}$$

$$\therefore P_{\text{av}} = \frac{e_0 I_0}{T}\left(\int_0^T \sin^2 \omega tdt\right) \qquad \text{...............(2)}$$

iv. But $\int_0^T \sin^2 \omega tdt = \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right)dt$

$$= \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega tdt\right]$$

$$= \frac{1}{2}\left[t - \frac{\sin e2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{2}\left[T - \frac{\sin 2\omega \times \frac{2\pi}{\omega}}{2\omega}\right]$$

$$= \frac{1}{2}\left[T - \frac{\sin 4\pi}{2\omega}\right]$$

$$= \frac{T}{2} \quad \text{..}[\because \sin 4\pi = 0]$$

From equation (2),

$$\therefore P_{\text{av}} = \frac{e_0 I_0}{T}\left(\frac{T}{2}\right)$$

$$= \frac{e_0 I_0}{2}$$

$$= \left(\frac{1}{2}\right)e_0 I_0$$

$$\therefore P_{\text{av}} = \frac{e_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}$$

$= e_{rms} \times I_{rms}$

$\therefore P_{av} = e_{rms} \times I_{rms}$

Power factor:-

Power factor of an a.c circuit is defined as the cosine of the phase difference between the applied voltage and the circuit current.

For purely resistive circuit, power factor = 1

Question 5.1: A rectangular coil of a moving coil galvanometer contains 50 turns each having area 12 cm2 . It is suspended in radial magnetic field 0.025 Wb/m^2 by a fibre of twist constant 15 $\times 10^{-10}$ Nm/degree. Calculate the sensitivity of the moving coil galvanometer.

Solution: Given:-

N = 50, A = 12 cm^2 = 12 x 10^{-4} m^2,

B = 0.025 Wb/m^2,

C = 15 x 10^{-10} Nm/degree

To find:- Sensitivity (S_i)

Formula: S_i = NAB/C

Calculation: From formula,

$$S_i = \frac{50 \times 12 \times 10^{-4} \times 0.025}{15 \times 10^{-10}}$$

$$\therefore S_i = 10^6 \text{div/A}$$

The sensitivity of a moving coil galvanometer is 10^6 div/A.

OR

Question 5.2: State Bohr's third postulate for hydrogen (H2) atom. Derive Bohr's formula for the wave number. Obtain expressions for longest and shortest wavelength of spectral lines in ultraviolet region for hydrogen atom **[7]**

Solution: Third postulate (Transition and frequency condition):-

As long as electron remains in one of the stationary orbits, it does not radiate energy. Whenever an electron jumps from higher stationary orbit to lower stationary orbit, it radiates energy equal to the difference in energies of the electron in the two orbits.

Bohr's formula for spectral lines in hydrogen spectrum:-

i. Let, E_n = Energy of electron in nth higher orbit

E_p = Energy of electron in pth lower orbit

ii. According to Bohr's third postulate,

$E_n - E_p = hv$

$\therefore v = (E_a - E_p)/h$ ....(1)

iii. But $E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$ ....(2)

$E_p = -\frac{me^4}{8\varepsilon_0^2 h^2 p^2}$ ....(3)

iv. From equations (1), (2) and (3),

$$v = \frac{\left(\frac{-me^4}{8\varepsilon_0^2 h^2 n^2}\right) - \left(-\frac{me^4}{8\varepsilon_0^2 h^2 p^2}\right)}{h}$$

$$\therefore v = \frac{me^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{n^2} + \frac{1}{p^2}\right]$$

$$\therefore \frac{c}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \quad [\because v = c/\lambda]$$

where, c = speed of electromagnetic radiation

$$\therefore \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{p^2} - \frac{1}{n^2}\right]$$

v. $\frac{1}{\lambda} = R\left[\frac{1}{p^2} - \frac{1}{n^2}\right]$ ....(4)

where, $\frac{me^4}{8\varepsilon_0^2 h^3 c} = R = \text{Rydberg's constant}$

Equation (4) represents Bohr's formula for hydrogen spectrum.

vi. $1/\lambda$ is called wave number ($\bar{v}$) of the line.

$$\therefore \bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

For longest wavelength in ultraviolet region (Lyman series),

p = 1; n = 2

$$\therefore \frac{1}{\lambda_{L_1}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\therefore \lambda_{L_1} = \frac{4}{3R}$$

For shortest wavelength in ultraviolet region

P = 1; n = ∞

$$\therefore \frac{1}{\lambda_{L_2}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$$

$$\therefore \lambda_{L_2} = \frac{1}{R}$$

Question 5.2: The photoelectric current in a photoelectric cell can be reduced to zero by a stopping potential of 1.8 volt. Monochromatic light of wavelength 2200Å is incident on the cathode. Find the maximum kinetic energy of the photoelectrons in joules. [Charge on electron = 1.6×10^{-19} C]

Solution: Given:- V_0 = 1.8 V, e = 1.6×10^{-19} C, λ = 2200 Å

To find:- Maximum kinetic energy $(K.E.)_{max}$

Formula:- $(K.E.)_{max} = eV_0$

Calculation: Using formula,

$(K.E.)_{max} = 1.6 \times 10^{-19} \times 1.8$

∴ $(K.E)max = 2.88 \times 10^{-19}$ J

Maximum kinetic energy of emitted photoelectron is 2.88×10^{-19} J.

Question 6 | Select and write the most appropriate answer from the given alternatives for each sub-question : **[7]**

Question 6.1: Which one of the following particles cannot be accelerated by a cyclotron? **[1]**

(A) Electrons
(B) Protons
(C) Deuterons
(D) α- particles

Solution: A cyclotron **cannot** accelerate the following particles

Electrons
Neutrons

Question 6.2: In biprism experiment two interfering waves are produced due to division of _______. **[1]**

amplitude

wavefront
amplitude and wavefront
neither wavefront nor amplitude

Solution: wavefront

Question 6.3: The output of NOR gate is high, when ______. [1]

all inputs are high
all inputs are low
only one of its inputs is high
only one of its inputs is low

Solution: all inputs are low

Question 6.4: Light of a certain wavelength has a wave number $\bar{v}$ in vacuum. Its wave number in a medium of refractive index n is ______. [1]

$\frac{n}{\bar{v}}$
$\frac{1}{n\bar{v}}$
$\frac{\bar{v}}{n}$
$n\bar{v}$

Solution: $n\bar{v}$

Question 6.5: If the radius of a sphere is doubled without changing the charge on it, then electric flux originating from the sphere is _____. [1]

double
half
same
zero

Solution: Same

Question 6.6: The momentum of a photon of de Broglie wavelength 5000Å is ______.
[1]

[Planck's constant = 6.63 $\times 10^{-34}$ J.s.]
1.326 $\times 10^{-28}$ kg-m/s
7.54 $\times 10^{-28}$ kg-m/s
1.326 $\times 10^{-27}$ kg-m/s
7.54 $\times 10^{-27}$ kg-m/s

Solution: 1.326 $\times 10^{-27}$ kg-m/s

Question 6.7: Ionosphere mainly consists of ______. [1]

positive ions and electrons
water vapour and smoke
ozone layer
dust particles

Solution: Positive ions and electrons

Question 7 | Attempt any SIX: [12]

Question 7.1: State any 'two' possible sources of errors in meter-bridge experiment. How can they be minimised? [2]

Solution: Two possible sources of errors in meter- bridge experiment are:-

a. Error due to sliding of jockey on the wire:- Due to sliding of the jockey, heat is produced due to friction. Thus the area of wire can be deformed and resistance of the wire may change.

To minimise this error, jockey is tapped on the wire.

b. Error due to heating effect of the cell current:- Heating effect of the wire leads to thermal expansion in the wire. Due to thermal expansion, resistance of wire increases.

To minimise this error, circuit is switched off after every reading.

Question 7.2: A potentiometer wire has resistance of per unit length of 0.1 Ω/m. A cell of e.m.f. 1.5V balances against 300 cm length of the wire. Find the current in the potentiometer wire. [2]

Solution: Given:- $\sigma = 0.1\ \Omega/m$, $E = 1.5\ V$,

$L = 300\ cm = 3\ m$.

To find:- Current through potentiometer (I).

Formulae: i. $\sigma = R/L$ ii. $I = E/R$

Calculation:- From formula (i),

$R = \sigma L$

$= 0.1 \times 3 = 0.3\ \Omega$

From formula (ii),

$I = 1.5/0.3 = 5\ A$

Current through the potentiometer wire is 5 A.

Question 7.3: Give any 'two' points of differences between diamagnetic and ferromagnetic substances. [2]

Solution:

	Diamagnetic Substance		Ferromagnetic Substance
1	Weakly repelled by a magnet	1	Strongly attracted by a magnet
2	When kept in a non-uniform magnetic field, it shows moderate tendency to move from stronger to the weaker part of the field.	2	When kept in a non-uniform magnetic field, it shows strong tendency to move from weaker to the stronger part of the field.

Question 7.4: An iron rod of area of cross-section 0.1m2 is subjected to a magnetising field of 1000 A/m. Calculate the magnetic permeability of the iron rod. [Magnetic susceptibility of iron = 59.9, magnetic permeability of vacuum = $4\pi \times 10^{-7}$ S. I. unit] **[2]**

Solution: Given:- H= 1000 A/m, $\chi = 59.9$, $\mu_0 = 4\pi \times 10^{-7}$ S.I. unit

To find:- Permeability (μ)

Formula:- $\mu = \mu_0 (1 + \chi)$

Calculation:- From formula,

$\mu = 4\pi \times 10^{-7} (1 + 59.9)$

$= 4 \times 3.142 \times 10^{-7} \times 60.9$

$= \text{antilog} [\log(4) + \log(3.142) + \log(60.9)] \times 10^{-7}$

$= \text{antilog} [0.6021 + 0.4972 + 1.7846] \times 10^{-7}$

$= \text{antilog} [2.8839] \times 10^{-7}$

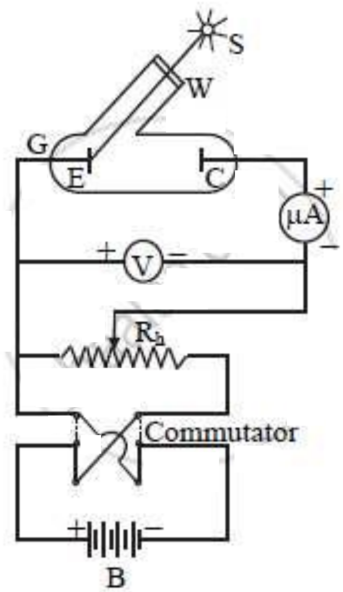
$= 765.4 \times 10^{-7}$

$\therefore \mu = 7.654 \times 10^{-5}$ Wb/A-m

The magnetic permeability of the iron rod is 7.654×10^{-5} Wb/A-m.

Question 7.5: Draw a neat labelled circuit diagram of experimental arrangement for study of photoelectric effect. **[2]**

Solution:



Experimental arrangement for study of photoelectric effect.

S : monochromatic source

E : emitter electrode

C : collector electrode

G : evacuated glass tube

W : quartz window

μA : micrometer

R_h : rheostat

V : voltmeter

B : Battery

Question 7.6: A coil of 100 turns, each of area $0.02 m^2$ is kept in a uniform field of induction 3.5×10^{-5} T. If the coil rotates with a speed of 6000 r.p.m. about an axis in the plane of the coil and perpendicular to the magnetic induction, calculate peak value of e.m.f. induced in the coil. **[2]**

Solution: Given: f = 6000 r.p.m = 100 Hz, N = 100, A = 0.02 m^2, B = 3.5×10^{-5} T

To find: peak e.m.f (e_0)

Formula: $e_0 = 2\pi f BNA$

Calculation: From formula,

$e_0 = 2 \times 3.14 \times 100 \times 100 \times 0.02 \times 3.5 \times 10^{-5}$

∴ $e_0 = 43.99 \times 10^{-3}$ V

The peak value of e.m.f. induced in the coil is 43.99 x 10^{-3} V.

Question 7.7: Define modulation and transducer **[2]**

Solution: Modulation:-

The process of superimposing a low frequency signal on a high frequency wave, which acts as a carrier wave for long distance transmission is known as modulation.

Transducer:-

A transducer is a device, which converts one form of energy into another.

Question 7.8: In a biprism experiment, when a convex lens was placed between the biprism and eyepiece at a distance of 30 cm from the slit, the virtual images of the slits are found to be separated by 7 mm. If the distance between the slit and biprism is 10 cm and between the biprism and eyepiece is 80cm, find the linear magnification of the image. **[2]**

Solution: Given : D = 10 + 80 = 90 cm, d = 30 cm, d_1 = 90 - 30 = 60 cm

To find : Linear magnification (m)

Formula : Linear magnification $m = \frac{d_1}{d} = \frac{v}{u}$

Calculation : From formula,

$$\frac{d_1}{d} = \frac{v}{u} = \frac{60}{30}$$

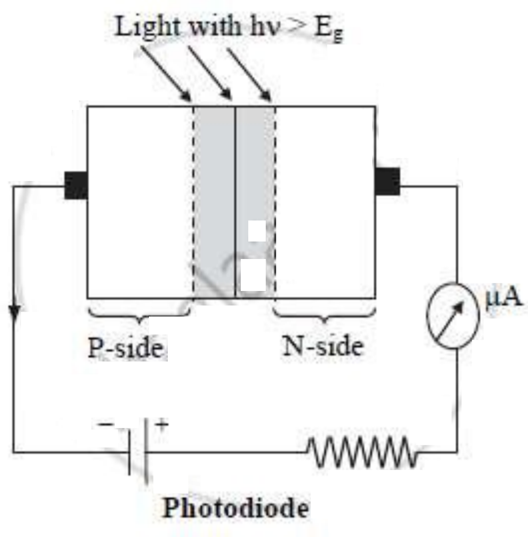
∴ m = 2

The linear magnification of the image is 2.

Question 8 | Attempt any THREE: **[9]**

Question 8.1: With the help of a neat circuit diagram, explain the working of a photodiode. **[3]**

Solution:



a. A photodiode is a special purpose P-N junction diode fabricated with a transparent window to allow light to fall on the diode.

b. When the photodiode is illuminated with light (photons) with energy h greater than the energy gap Eg of the semiconductor, then electron-hole pairs are generated due to the absorption of photons.

c. The diode is fabricated such that the generation of electron-hole pairs takes place in or near the depletion region of the diode.

d. Due to electric field of the junction, electrons and holes are separated before they recombine.

e. The direction of the electric field is such that electrons reach N-side and holes reach P side. Electrons are collected on N-side and holes are collected on P-side giving rise to an e.m.f.

f. When an external load is connected, current flows. The magnitude of the photocurrent depends on the intensity of incident light.

g. It is easier to observe the change in the current with change in the light intensity, if a reverse bias is applied. Thus, photodiode can be used as a photodetector to detect optical signals.

Question 8.1: State its any 'two' uses of photodiode.

Solution: Uses Of Photodiode:-

i. Photodiode is used in opt couplers which are used to couple isolated circuits.

ii. Photodiodes are widely used to detect optical signals.

Question 8.2: A parallel beam of monochromatic light is incident on a glass slab at an angle of incidence 60°. Find the ratio of width of the beam in the glass to that in the air if refractive index of glass is 3/2. **[3]**

Solution: Given: i = 60°, μ_g = 1.5,

Let d_g = width of beam in glass slab,

d_a = width of beam in air

To find: Ratio of widths $\left(\frac{d_g}{d_a}\right)$

Formulae:

i. $\mu_g = \frac{\sin i}{\sin r}$

ii. $\frac{d_g}{d_a} = \frac{\cos r}{\cos i}$

Calculation: From formula (i),

sin r = sin i/μ_g

∴ sin r = sin60°/1.5 = 0.8660/1.5 = 0.5773

∴ r = $\sin^{-1}$ (0.5773) = 35°16'

From formula (ii),

$$\frac{d_g}{d_a} = \frac{\cos r}{\cos i} = \frac{\cos 35^\circ 16'}{\cos 60^\circ}$$

$$\therefore \frac{d_g}{d_a} = \frac{0.8164}{0.5} = 1.633$$

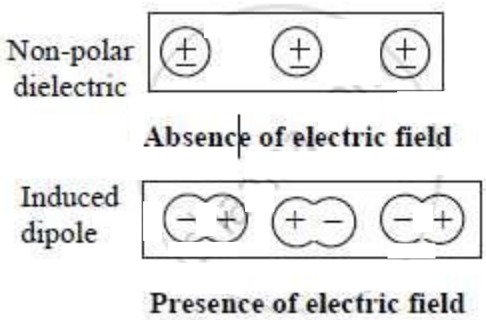
$$\therefore \frac{d_g}{d_a} = 1.633 : 1$$

∴ Ratio of the widths of beam = 1.633 : 1

The ratio of widths of the beam in glass to that in air is 1.633 : 1.

Question 8.3: With the help of neat diagram, explain how non-polar dielectric material is polarised in external electric field of increasing intensity. Define polarisation in dielectrics. **[3]**

Solution:



If a dielectric (non-polar) molecule is placed in an external electric field, a small induced dipole moment is created because the positive charge in each atom is pushed in the direction of the field and negative charge is pushed in the opposite direction as shown in the figure.

Polarization is the amount of induced surface charge per unit area or the surface density of polarization charges appearing at right angles to applied external electric field.

Question 8.4: In a single slit diffraction pattern, the distance between first minima on the right and first minima on the left of central maximum is 4 mm. The screen on which the pattern is displaced, is 2m from the slit and wavelength of light used is 6000Å. Calculate width of the slit and width of the central maximum. **[3]**

Solution: Given:- $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m,

D = 2 m,

$X_1 + X_2 = 4$ mm $= 4 \times 10^{-3}$ m

To find:- Width of slit (a)

Width of central maximum (W)

Formula:-

i. $X_1 + X_2 = \frac{2\lambda D}{a}$

ii. $W = X_1 + X_2$

Calculation: From formula (i),

$$4 \times 10^{-3} = \frac{2 \times 6 \times 10^{-7} \times 2}{a}$$

$$\therefore a = 6 \times 10^{4} m$$

The width of the slit is 6×10^{-4} m.

From formula (ii),

Width of central maximum = 4×10^{-3} m

The width of the central maximum is 4×10^{-3} m.