ELECTRO MAGNETIC THEORY

(1.) Basics
(2.) static electric field.
(3.) Static magnetic field.
(4.) Time varying Fields & maxwell eqn
(5.) Uniform plane waves
(6.) X^n lines.

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(1.) Basics →

(1.) Scalar → If the physical quantity has magnitude & sign (±) then it is called scalar quantity.

eg. →
T → time (sec)
I → Current (amp)
V → Voltage (volts)
Q → charge (coulombs)
ϕ → magnetic flux (webers)

(2.) Vector → If the physical quantity has magnidue & dirn then it is called vector quantity.

eg. →
$\vec{E}$ → Electric field intensity $\left(\frac{\text{Volt}}{\text{meter}}\right)$
$\vec{H}$ → Magnetic Field intensity $\left(\frac{\text{Amp}}{\text{meter}}\right)$
$\vec{D}$ → Electric flux density $\left(\frac{\text{Coulomb}}{m^2}\right)$
$\vec{B}$ → Magnetic flux density $\left(\frac{\text{weber}}{m^2}\right)$
$\vec{F}$ → Force (Newton)

* Different configuration →

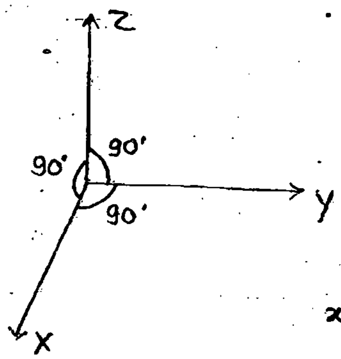
$x, y, z \to$ Spacial variables

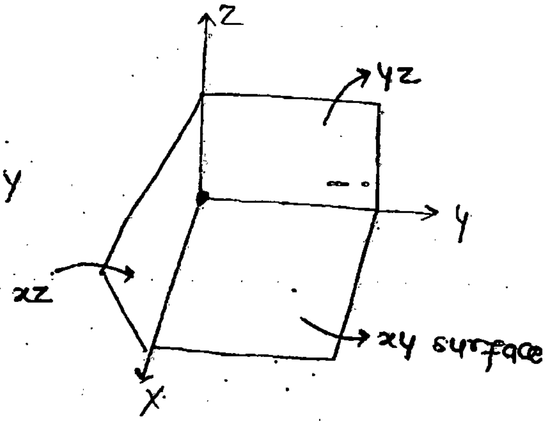
(1) Point →

$$\left.\begin{array}{l} x \to 0 \\ y \to 0 \\ z \to 0 \end{array}\right\}$$

No variables (or) 3 constants (x, y, z)

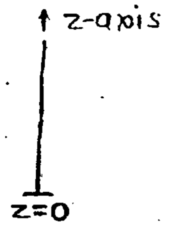
* Point is obtained by joining 3 surfaces.





* Point is a sphere of radius $r \to 0$

(2) Line →

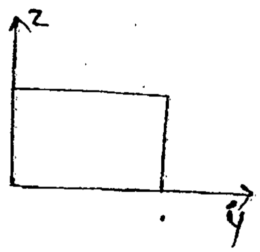


$x = 0$
$y = 0$
$z =$ variable

1 variable or 2 Constants

* Line is obtained by joining 2 surfaces.
* Line is a cylinder of radius $\rho = 0$.
* Line is obtained by joining many points.
* Line length 1 meter is defined as vector.

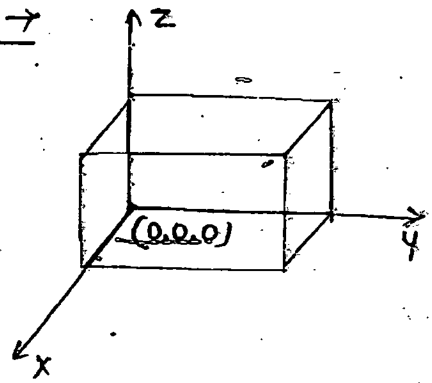
3) Surface →



2 Variables or 1 Constants

* ~~lz~~ surface is obtained by joining many lines.
* Surface is a plane whose thickness $\rightarrow 0$
* surface area (m^2) is defined as a vector.

(4.) Volume →



$\boxed{\text{3 variables (or) No constants}}$

* Placing many surfaces one up on one we get a volume.
* Volume (m^3) is defined as a scalar.

Note:-

(1.) To multiply with meter take line integral ($\int d\vec{l}$)
(2.) To multiply with meter² take surface integral ($\iint d\vec{s}$)
(3.) To multiply with meter³ take volume integral ($\iiint dv$)

Formula →

(1.) scalar (scalar) = scalar
Eg:- $2 \cdot 4 = 8$

(2.) scalar (vector) = Vector
Eg:- $2(2a_x + 3a_y + 4a_z) = 4a_x + 6a_y + 8a_z$

(3.) (vector) . (vector) = Scalar
(dot product)

(4.) (vector) X (vector) = vector
(cross product)

(5.) $\vec{A}\cdot(\vec{B}\times\vec{C}) = [\vec{A}\,\vec{B}\,\vec{C}] = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

(6.) $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = (\nabla\times\vec{A})\cdot B - (\nabla\times\vec{B})\cdot\vec{A}$

(7.) $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\cdot\vec{C}$

(8.) $\bar{\nabla}\times(\bar{\nabla}\times\bar{A}) = (\bar{\nabla}\cdot\bar{A})\bar{\nabla} - (\bar{\nabla}\cdot\bar{\nabla})\bar{A} = (\nabla\cdot\bar{A})\bar{\nabla} - \nabla^2\bar{A}$

(9.) $\bar{\nabla}\cdot\bar{\nabla}V = \nabla^2 V$

(10.) $\bar{\nabla}\cdot(\bar{\nabla}\times\bar{A}) = 0$

divergence of curl of a vector is always 0.

(11.) $\bar{\nabla}\times\bar{\nabla}V = 0$

Curl of a gradient is always 0

** Vector mathematics has 2 branches :-

(1.) Vector algebra
* vector addition
* vector substraction
* vector multiplication

(New physical quantity is not produced)

(2.) Vector Analysis
* vector differential operator $\left[\bar{\nabla} = \frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right]$
* Gradient $(\bar{\nabla}V)$
* divergance $(\bar{\nabla}\cdot\bar{A})$
* curl $(\bar{\nabla}\times\bar{A})$

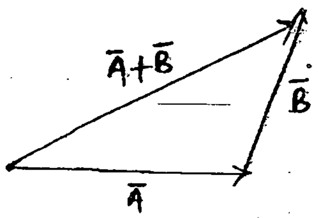
differentiation

Integration
* vector integrals [line integral, surface integral, volume integral]
* vector integrals theorems [stokes theorem, Divergance theorem]

(New physical quantity is produced)

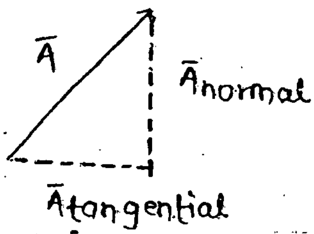
Vector Algebra →

(1.) Vector addition →



$\bar{A} = A_x a_x + A_y a_y + A_z a_z$

$\bar{B} = B_x a_x + B_y a_y + B_z a_z$



* Any vector ($\vec{A}$) can be written in terms of its tengential ($\bar{A}_{tan}$), Normal vector ($\bar{A}_{normal}$)

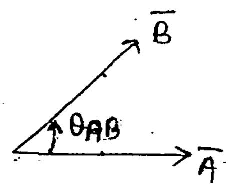
* $\vec{A} = \vec{A}_{tan} + \vec{A}_{Normal}$

(2) Vector multiplication →

(i.) Scalar product (or) dot product →

By defination

$$\boxed{\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}| \cos\theta_{(AB)}}$$



θ_{AB} = Angle between $\bar{A}, \bar{B}$ vectors

$$\boxed{a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1}$$

$$\boxed{a_x \cdot a_y = a_x a_z = a_y \cdot a_z = 0}$$

(ii) $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$ [Dot product is commutative]

(iii) Vector (or) Cross product →

By defination.

$$\boxed{\bar{A}\times\bar{B} = |A||B| \sin(\theta_{AB})\, \hat{a}_N}$$

$\hat{a}_N$ = Normal unit vector to the surface on which both $\bar{A}, \bar{B}$ vectors are present (outward dirⁿ)

$\bar{A} = A_x a_x + A_y a_y + A_z a_z$

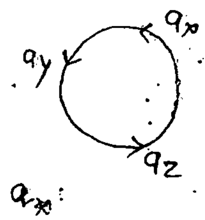
$\bar{B} = B_x a_x + B_y a_y + B_z a_z$

$$\bar{A} \times \bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

* $\hat{a}_x \times a_x = |a_x||a_x| \sin(0)\, \hat{a}_N = 0$

$$\boxed{a_x \times a_x = a_y \times a_y = a_z \times a_z = 0}$$

* $a_x \times a_y = |a_x||a_y| \sin(90^\circ)\, a_z = a_z$

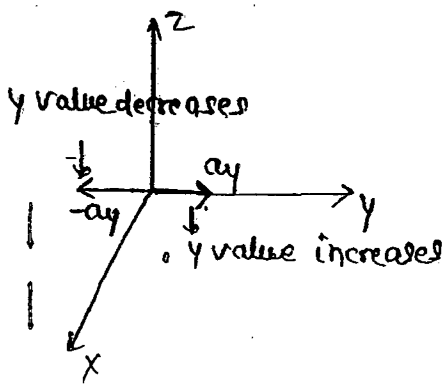


$a_x \times a_y = a_z$ $a_y \times a_z = a_x$ $a_z \times a_x = a_y$	$a_y \times a_x = -a_z$

* $\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}$ [Cross product is not commutative]

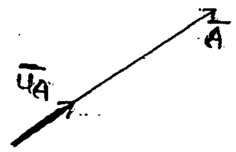
<u>Unit vector</u> → It has unit magnitude & dirⁿ is in its incremental componant dirⁿ.



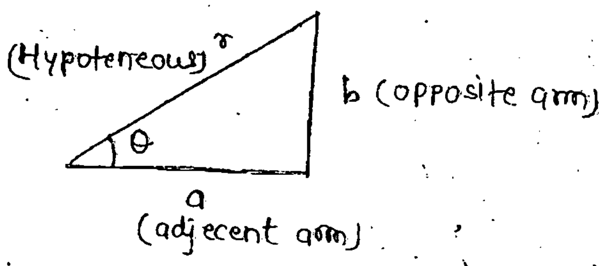
a_x $[u_x, \hat{i}, \hat{i}_x]$ → Unit vector in x dirⁿ
a_y $[u_y, \hat{j}, \hat{i}_y]$ → Unit vector in y dirⁿ
a_z $[u_z, \hat{k}, \hat{i}_z]$ → Unit vector in z dirⁿ

* If $\bar{A} = A_x a_x + A_y a_y + A_z a_z$ then unit vector in the dirⁿ of $\bar{A}$ is

$$\bar{u}_A = \frac{\bar{A}}{|\bar{A}|} = \frac{A_x a_x + A_y a_y + A_z a_z}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$



Co-ordinate system → The purpose of the co-ordinate sys. is to locate the object in space wrt reference point [origin]



By defination;

$$\cos\theta = \frac{\text{adjecent}}{\text{hypoteneous}}$$

$$\cos\theta = \frac{a}{r}$$

$$\boxed{\begin{array}{c} a = r\cos\theta \\ \text{ady} = \text{hyp}\cos\theta \end{array}} \text{ —— (i)}$$

By defination;

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypote.}}$$

$$\sin\theta = \frac{b}{r}$$

$$\boxed{\begin{array}{c} b = r\sin\theta \\ \text{opp} = \text{hyp}\sin\theta \end{array}} \text{ —— (ii)}$$

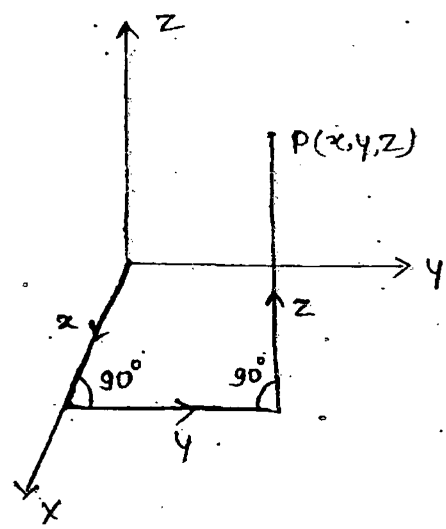
$$\sqrt{(1)^2+(2)^2} \Rightarrow \sqrt{a^2+b^2} = \sqrt{r^2(\sin^2\theta+\cos^2\theta)}$$

$$\boxed{r=\sqrt{a^2+b^2}}$$

*** Relation b/n co-ordinates →**

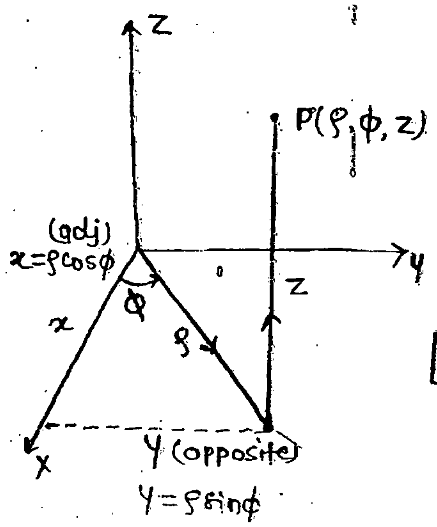
$$\begin{bmatrix} (x,y,z) \text{ to } (\rho,\phi,z) \\ (x,y,z) \text{ to } (r\,\theta\,\phi) \\ (\rho,\phi,z) \text{ to } (r,\theta,\phi) \end{bmatrix}$$

(1) Rectangular (cartision) Co-ordinate →



$$\boxed{a_x \perp a_y \perp a_z}$$

(2) Cylindrical co-ordinate →



$$\boxed{\begin{aligned} x &= \rho\cos\phi \quad \text{---(i)} \\ y &= \rho\sin\phi \quad \text{---(ii)} \\ z &= z \quad \text{---(iii)} \end{aligned}}$$

$$\boxed{a_\rho \perp a_\phi \perp a_z}$$

$\sqrt{(1)^2+(2)^2} \Rightarrow \sqrt{x^2+y^2} = \sqrt{\rho^2(\cos^2\phi+\sin^2\phi)}$

$$\boxed{\sqrt{x^2+y^2} = \rho} \quad \text{---(4)}$$

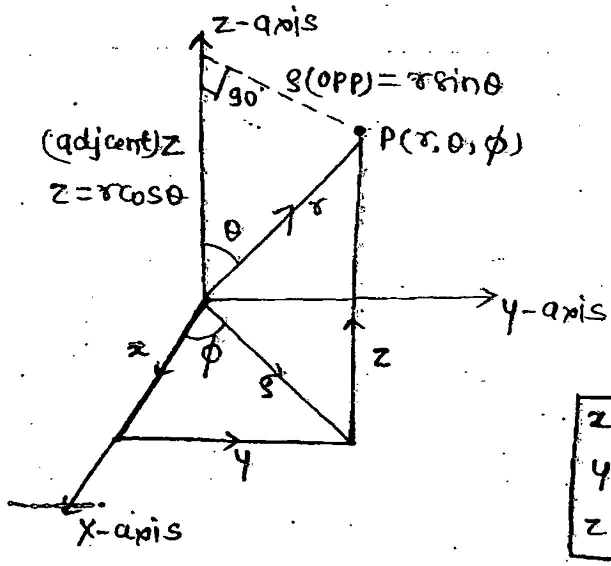
$\frac{(2)}{(1)} \Rightarrow \frac{y}{x} = \tan\phi \; ; \; \boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)} \quad \text{---(5)}$

$(3) \Rightarrow \boxed{z = z} \quad \text{---(6)}$

ρ is $\perp$ to z

ϕ is the angle b/w x-axis & radial line "ρ"

(3) Spherical co-ordinate →



$\boxed{a_r \perp a_\theta \perp a_\phi}$

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \quad \text{---(i)} \\ y &= r\sin\theta\sin\phi \quad \text{---(ii)} \\ z &= r\cos\theta \quad \text{---(iii)} \end{aligned}}$$

$x = \rho\cos\phi = r\sin\theta\cos\phi$

$y = \rho\sin\phi = r\sin\theta\sin\phi$

From eq^n (i) & (ii) & (iii) $\sqrt{x^2+y^2+z^2} = r$ ---(iv)

From eq^n (2) ÷ (i) $\frac{y}{x} = \tan\phi, \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$ ---(v)

From eq^n (3) $\theta = \cos^{-1}\left(\frac{z}{r}\right) \Rightarrow \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$ ---(vi)

r = Radius of sphere

θ = Angle of radius (r) with z-axis

ϕ = Angle between x-axis & radius when it is on xy plane.

* Relation between cylindrical & spherical co-ordinate sys →

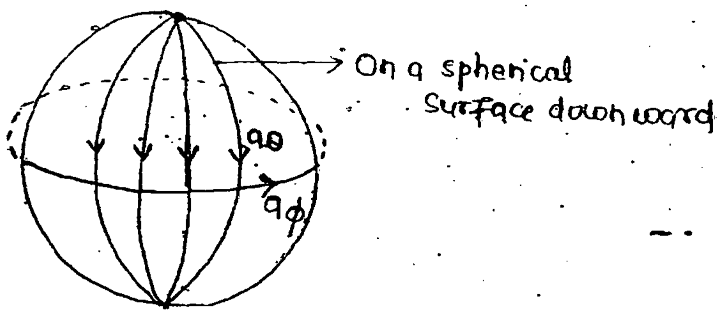
$$\rho = r\sin\theta \quad \text{——— (i)}$$
$$\phi = \phi \quad \text{——— (ii)}$$
$$z = r\cos\theta \quad \text{——— (iii)}$$

From eq^n (i) & (iii) $\sqrt{(i)^2+(iii)^2} \Rightarrow r = \sqrt{\rho^2+z^2}$ ——— (iv)

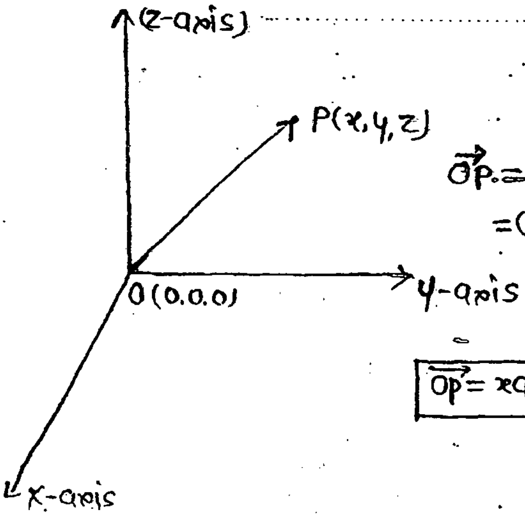
From eq^n (iii) $\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{z}{\sqrt{\rho^2+z^2}}\right)$ ——— (v)

from eq^n (ii) $\phi = \phi$ ——— (vi)

Rectangular	x	y	z
cylindrical	ρ	ϕ	z
spherical	r	θ	ϕ



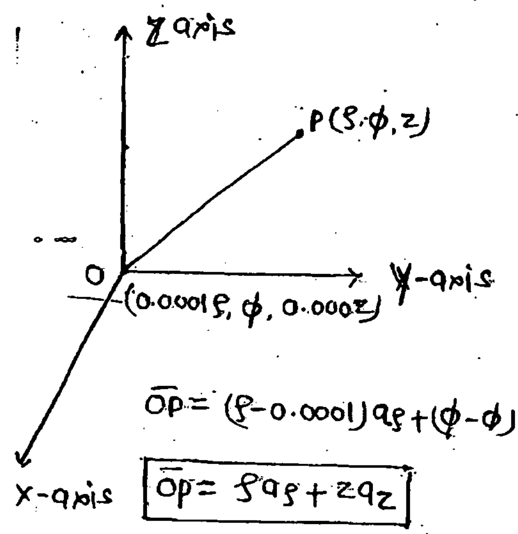
* Position vector →



$\vec{OP} = \cancel{P-O}$

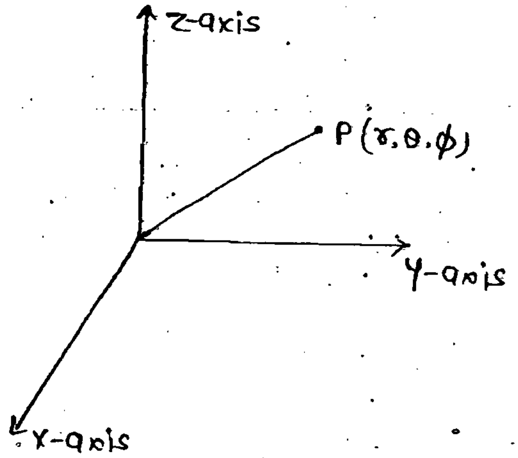
$= (x-0)a_x + (y-0)a_y + (z-0)a_z$

$$\boxed{\vec{OP} = x a_x + y a_y + z a_z}$$



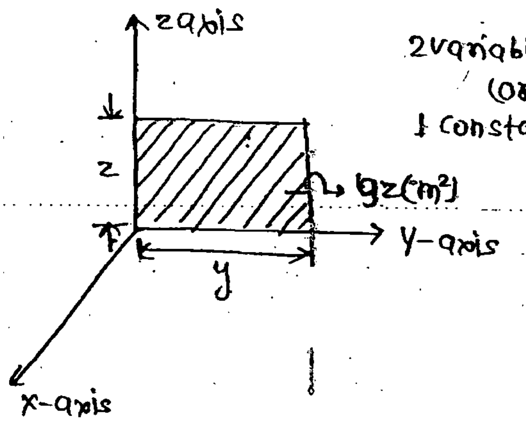
$$\overline{OP} = (\rho - 0.0001)a_\rho + (\phi - \phi)a_\phi + (z - 0.0001)a_z$$

$$\boxed{\overline{OP} = \rho a_\rho + z a_z}$$



$$\boxed{\overline{OP} = r a_r}$$

* Representation of surfaces in vector form →



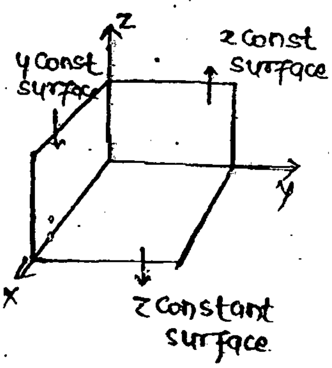
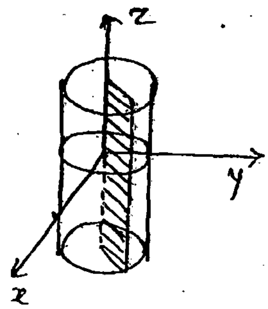
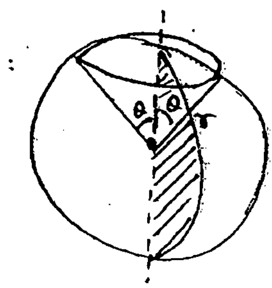
2 variables (y, z)
(or)
1 constant (x)

surface vector is defined as

$$\boxed{\overline{S} = (area)\,\hat{a}_N}$$

where $\hat{a}_N \rightarrow$ Normal unit vector (in outward dirn) to the surfa

$\vec{S} = yz\, a_x$

	Rectangular	cylindrical	spherical
1.) Name of the general point	$P(x, y, z)$	$P(\rho, \phi, z)$	$P(r, \theta, \phi)$
2.) Unit vectors	$a_x \perp a_y \perp a_z$	$a_\rho \perp a_\phi \perp a_z$	$a_r \perp a_\theta \perp a_\phi$
3.) Range of the Variables	$\left.\begin{matrix} -\infty < x < \infty \\ -\infty < y < \infty \\ -\infty < z < \infty \end{matrix}\right\}$ Biggest cube	$0 \le \rho < \infty$ $0 \le \phi < 2\pi$ $-\infty < z < \infty$ (biggest cylinder)	$0 \le r < \infty$ $0 \le \theta \le \pi$ $0 \le \phi < 2\pi$ (biggest sphere)
4.) Shapes of the surfaces			
	$\left.\begin{matrix} x\text{-const} \\ y\text{-const} \\ z\text{-const} \end{matrix}\right\}$ Rectangular surface	ρ = constant cylindrical surface ϕ = constant Rectangular surface z = constant circular plane	r = constant Spherical surface θ = constant conical surface ϕ = constant Semi-circular plane
5.) Differential lengths	$\boxed{dx, dy, dz}$ $\downarrow$ x, $\downarrow$ y, $\downarrow$ z	$\boxed{d\rho, \rho d\phi, dz}$ Curve length = angle × radius	$dr, r d\theta, \rho d\phi$ $\boxed{dr, r d\theta, r\sin\theta d\phi}$ $\rho = r\sin\theta$
6.) Differential length vector ($\vec{dl}$)	$dx\,a_x + dy\,a_y + dz\,a_z$ (meter)	$d\rho\, a_\rho + \rho\, d\phi\, a_\phi + dz\, a_z$	$dr\, a_r + r\, d\theta\, a_\theta + r\sin\theta\, d\phi\, a_\phi$
7.) Differential ($\vec{ds}$) surface	$dy\,dz\,a_x + dx\,dz\,a_y + dx\,dy\,a_z$	$\rho\, d\phi\, dz\, a_\rho + d\rho\, dz\, a_\phi + \rho\, d\rho\, d\phi\, a_z$	$r^2 d\theta \sin\theta\, d\phi\, a_r + r\sin\theta\, dr\, d\phi\, a_\theta + r\, dr\, d\theta\, a_\phi$

(8.) Differential volume (dV) —— $dx\,dy\,dz$. $\rho\, d\rho\, d\phi\, dz$ $r^2\sin\theta\, dr\, d\theta\, d\phi$

(9.) Problem solⁿ Parallel plate capacitor Cylindrical capacitor Spherical capacitor

Vector Analysis →

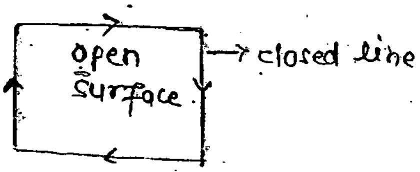
Integral →

(1.) Line integral → * It is the integral calculated on a line (or) curve.

Eg. → $\int \bar{A}\cdot d\bar{l}$ ——→—— (open line)

* If the integral is calculated ~~on~~ a closed line then it is called line integral

Eg. → $\oint \bar{A}\cdot d\bar{l}$



* If the closed line is zero then it is called conservative (or) irrotational. $\oint \bar{A}\cdot d\bar{l} = 0$

(2.) Surface integral → * It is calculated on its surface.

Eg:- $\iint \bar{A}\cdot d\bar{s}$ (or) $\int_s \bar{A}\cdot d\bar{s}$

* If the surface integral is calculated on a closed surface then it is called surface integral.

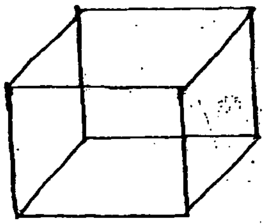
Eg:- $\oiint \bar{A}\cdot d\bar{s}$ (or) $\oint\oint \bar{A}\cdot d\bar{s}$ (or) $\oint_s \bar{A}\cdot d\bar{s}$

* If the closed surface integral is 0 then it is called solenoidal (or) divergence surface.

(3.) Volume integral → It is calculated in a volume.

Eg:- $\iiint \rho_v\, dv$ (or) $\int_v \rho_v\, dV$

Observation →

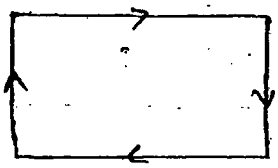


Closed surface

$$\boxed{\oiint \bar{A}\cdot \bar{ds} = \iiint (\bar{\nabla}\cdot\bar{A})\, dv}$$

(Divergance Theorem) → Volume

[Inside the closed surface volume is present]



Closed line

$$\boxed{\oint \bar{A}\cdot \bar{dl} = \iint (\bar{\nabla}\times\bar{A})\cdot \bar{ds}}$$

(Stokes Theorem) → Open surface

[Inside closed line open surface is present]